June 27, 2013

Report on 'Structural Graph Theory' by José Zamora

The thesis consists in four research articles, each being a different chapter. (These four articles are co-authored and, while the co-authors are named and thanked in the introduction, it would be a important to give the precise references of the four articles, showing who was involved in that or that chapter of the thesis.)

The first chapter contains an interesting result about nowhere-zero 5-flows. (This chapter was already submitted as a part of José Zamora's doctoral thesis in Chile, five years ago.)

In the second chapter, a new notion of graph colouring (or, rather, graph labelling) is introduced : one wants to assign to each vertex v a colour $c(v)$ such that the mapping f defined on every edge $\{u, v\}$ of the graph by $|c(u) - c(v)|$ is a proper edge-colouring. The mapping c is then called an *additive colouring*. Additive colourings bear similarities with previously defined labellings, such as injective colourings and $L(p, q)$-colourings. José Zamora managed to obtain inspiration from works in this area and to adapt the techniques to additive colourings. The chapter contains upper and lower bounds, and the computational aspect is also addressed : a polynomial-time algorithm to compute the additive chromatic number of trees is designed and, as is expected for more general cases, the corresponding decision problem is shown to be NP-complete. The polynomial time algorithm for trees is a neat adaptation of an algorithm of Chang and Kuo for $L(2, 1)$-labellings. It would be interesting to investigate to which extent the other techniques developed for $L(2, 1)$-labellings and related notions (e.g., backbone colourings) do apply to additive colourings. In particular, the probabilistic method and the Combinatorial Nullstellensatz may be relevant. The term "additive colouring" is slightly unfortunate, since it is already used for a similar notion : a mapping assigning to every vertex a positive integer (called a *weight*) such that a proper vertex-colouring is obtained by assigning to each vertex the sum of the weights of its neighbours. In addition, it is unfortunate that no mention at all is made to a number of notions very close to that introduced, such as edge-weighting vertex-distinguishing labellings and harmonious colourings (the latter being the very same notion, except that the edge-colouring is not obtained by setting $c(\{u, v\}) := |c(u) - c(v)|$ but rather $c(\{u, v\}) := \{c(u), c(v)\}$, the condition being that no two adjacent edges are assigned the same set by c).

The third chapter deals with a celebrated conjecture by Stanley about the symmetric chromatic polynomial of a graph. As observed by Thatte, the Stanley conjecture is equivalent to a conjecture of Noble and Welsh that the U-polynomial distinguishes all trees. The purpose of this chapter is to establish that the U-polynomial distinguishes all *proper caterpillars*, that is, all trees T with no vertices of degree 2 and such that removing all leaves from T yields a path. Stanley's conjecture is notoriously difficult and the result obtained in this chapter is interesting. The proof is nice, building on earlier works about partition coarsenings by Billera, Thomas and van Willigenburg, who characterized integer compositions having the same multiset of partition coarsenings. More background would have been appreciated (for instance, the chromatic symmetric function appearing in Stanley's original statement is not defined).

The fourth and last chapter deals with sufficient conditions to force topological minors in infinite graphs. I found this part to be difficult to follow ; the write-up should have been better, in my opinion. More importantly, there seems to be some degree of imprecision and the text is not as formal as it should be. In particular, the central notion of minimum/relative degree is defined quite informally, which should not be the case.

In total, and although the thesis is rather short, it looks like it was written in a hurry : clearly, four research papers have been glued one after the author, with a one-page introduction and minimal changes in the texts of the papers. (For instance, one notes sentences as "The first author showed".) A number of typographic errors and typesetting problems are present.

I would have appreciated more efforts in either trying to tie the topics together by providing more hindsight on a global picture (for instance talking about flows/colouring duality), or providing better backgrounds and state-of-the-arts for the topics considered. In the present state, it looks as if the author just considered specific problems into several domains, but the text does not provide a good view on these domains and no or few insight on why the questions are important and how they relate to a bigger picture. More information on techniques and ideas that were developed in these domains would also have been a good thing (especially given the overall length of the thesis).

In the same vein, few open problems and directions for future research are given, leaving the reader to wonder about the extent to which the each domain was investigated.

To conclude, based on the scientific new results contained in the thesis, my opinion is that the thesis could be accepted for the specialization "Discrete Models and Algorithms".

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