ABSTRACT:

In this thesis we consider four different problems in structural graph theory:

We start studying the structure of graphs having a nowhere-zero 5-flow. We give a characterization of the graphs that have a nowhere-zero 5-flow in terms of the existence of a \((1, 2)\)-factor.

For the second problem we introduce a new type of labeling of graphs that we call additive coloring. This coloring is a variation of the injective coloring. Indeed, it is an injective coloring with arithmetic restrictions determined by the graph. We study the properties and the structure of graphs admitting this type of labeling. Moreover, we study the computational complexity of the problem of computing this labeling for a graph with a fixed number of colors.

In the third problem we study how the structure of caterpillars is encoded by the chromatic symmetric function or, equivalently, the \(U\)-polynomial. Stanley conjectured that the symmetric chromatic polynomial distinguishes non-isomorphic trees. In this thesis we prove that the conjecture is true for proper caterpillars (caterpillars without vertices of degree 2).

Finally, we study the structure of infinite graphs having a complete graph as a minor or as a topological minor. It is known that bounds on the degree of the vertices is not enough to ensure the existence of a complete graph minor in an infinite graph. So, we define a new notion of degree for the ends of an infinite graph. Then, we prove that a condition of minimum degree for the vertices and the ends of the graph ensure the existence of a complete graph as a minor and as a topological minor.

Keywords: nowhere-zero flows, graph labeling, graph polynomial, infinite graphs.