Referee report on doctoral thesis

Structural graph theory

by **José Zamora**.

The thesis is a compilation of four research articles:

- Martín Matamala, José Zamora: Nowhere-zero 5-flows and $(1, 2)$-factors. Electronic Notes in Discrete Mathematics 30: 279–284 (2008),
- Natacha Astromujoff, Mathieu Chapelle, Martín Matamala, Ioan Todinca, José Zamora: Injective coloring with arithmetic constraints (unpublished),
- José Aliste-Prieto, José Zamora: Proper caterpillars are distinguished by their symmetric chromatic function (arXiv:1208.2267),
- Maya Stein, José Zamora: The relative degree and large complete minors in infinite graphs. Electronic Notes in Discrete Mathematics 37: 129–134 (2011).

These sources should be acknowledged in the thesis.

As the first part has already appeared as a part of José Zamora's doctoral thesis "Etiquetamiento de grafos: flujos y coloreamientos inducidos de aristas", Universidad de Chile, 2008. I consider in my report only the remaining three parts.

In the section "Injective coloring with arithmetic constraints" a new concept of vertex coloring is introduced, in particular *additive coloring* is an assignment of integer labels to vertices, s.t. differences of labels of adjacent vertices yield a proper edge coloring. This chapter relates this concept to analogous labelings (injective coloring $L(p, q)$-labeling), provides lower and upper bounds, as well as several computational results: a polynomial algorithm for trees, and NP-hardness reductions for more general cases. This part is rich in terms of variety of results, though several of them are inspired by related works. I particularly liked the adaptation of the algorithm of Chang and Kuo to the new concept. I would appreciate if some of the constructions were depicted (as in the other chapters).

The two remaining chapters are focused on rather specific problems, solved by more technically involved approach: In the chapter "Proper caterpillars are distinguished by their symmetric chromatic function" a result of M. Morin has been extended to all proper caterpillars, i.e. caterpillars without vertices of degree two. Though the results are expressed by using weighted graph polynomial U_G, I suggest to include also the definition of the chromatic symmetric function X_G, mentioned in the title. (Another oddity related to the titles: two are called the same "Proof of the main result".)

In the final chapter it is shown the existence of complete graphs as a topological minors, or of finite subgraphs of given average degree in infinite graphs. This part was difficult to follow — among others the central notion $\delta^{V,\Omega}$ has not been properly defined. Also more care should been taken during the compilation of the thesis to avoid phrases like "The first author showed:" (just before Theorem 5.1.2, page 35).

The thesis should have a concluding section that summarizes the main results and proposes directions of further research.

In despite to the issues mentioned in my report, I recommend to accept the submitted doctoral thesis for the specialization Discrete models and algorithms.

Jiří Fiala
Eugene, May 21, 2013