Dipartimento di Matematica Applicata
« Ulisse Dini »

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To the committee of the Ph.D. thesis of Martina Hofmanová, Mathematics

Subject: Report of Franco Flandoli on the Ph.D. thesis of Martina Hofmanová entitled: *Degenerate Parabolic Stochastic Partial Differential Equations*

The Ph.D. thesis of Martina Hofmanová provides a significant contribution to the theory of stochastic conservation laws, with additional very interesting side results for the theory of SDEs and SPDEs. It is based on five papers, three of which accepted for publication. As just mentioned, it has a main central topic and two variations.

The central theme is the approach to stochastic conservation laws by means of the kinetic formulation. This subject, initiated by Debussche and Vovelle [16] (reference numbers here and below are those of the thesis), is developed here in two directions:
i) the case when a degenerate parabolic term is included;
ii) the convergence of the approximation called BGK.

Two additional topics are:
iii) a regularity result for strong solutions to semilinear parabolic equations;
iv) a new proof of weak existence for stochastic equations.

Topic (i) elaborates ideas and results of [16] and Chen-Perthame [13] and gives a complete result of existence and uniqueness for stochastic conservation laws with a degenerate parabolic term, following the approach of the kinetic formulation. It is technically a formidable work, as well as (ii) described below. Uniqueness comes from a non trivial comparison principle between kinetic solutions. Existence is very elaborate and innovative. First, a sequence of non-degenerate parabolic approximating problems is introduced, for which the results of topic (iii) below are used. Then suitable estimates are proved which will allow to take the limit in a strong sense, opposite to the (non-parabolic) case [16] where weak limit is sufficient. Finally, the limit is performed using part of the strategy described in issue (iv) below. We see

here that the two side questions of topics (iii) and (iv), although of interest in themselves, are technically related to part (i).

The subject treated in (ii) requires to master several delicate tools. The BKG approximation replaces the kinetic measure by a smoother and more explicit term, which allows one to study the new approximating equation by a perturbative approach (a fixed point arguments), based on a good theory for the linear part. The linear part here is stochastic and is investigated by the theory of stochastic flows of diffeomorphisms. Here there are remarkable differences with respect to the deterministic case because, due to the arbitrarily large (although rare) fluctuations of the noise, uniform controls are lost. At the end the necessary estimates on the BKG approximations are obtained in spite of these difficulties and the limit is controlled.

The results of topic (iii) are of interest in themselves and have been published as an original result. They deal with non degenerate parabolic stochastic equations with quite general semilinear terms. The semilinear part is of Nemytskij type but involving derivatives (of lower order than those of the leading parabolic operator); hence they cover applications to conservation laws with non-degenerate parabolic term, so that later they can be used in the proof of existence of solutions to the kinetic formulation in the case of degenerate parabolic term, point (i). Under suitable assumptions on the semilinear terms, a fixed point argument is applied in a Sobolev scale, using analytic semigroups theory. The final result is a regularity result for these equations, under regular initial conditions.

Topic (iv) is a side one with respect to stochastic conservation laws but it is very interesting for stochastic analysis and it has been published in two papers. It deals with a new proof of existence of weak (martingale) solutions to SDEs. Half of the idea comes from recent works on stochastic wave map equations, where a new method is developed to identify the limit of the approximating scheme (the classical identification was done by representation theorems for martingales; the new proof is much more elementary). The other half is new of these works and replaces the usual application of Skorokhod theorem (the one stating that a sequence of weakly converging laws can be realized by a sequence of a.s. converging random variables, on a suitable probability space). It is a very ingenious proof.

To conclude, the thesis deals with highly non-trivial topics on stochastic conservation laws and presents remarkable new results, ideas and techniques. The care of delicate and massive technical issues is really remarkable and the candidate was able to overcome several new difficulties. The candidate has

proved to be able to master very advanced topics and be able to perform research in Mathematics.

As a result, I believe that the Ph.D. thesis of Martina Hofmanová ought to be defended.

Yours sincerely,

Franco Flandoli
Professor in Probability and Statistics
University of Pisa, Italy
flandoli@dma.unipi.it