Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

MASTER THESIS

Determinants of Economic Growth: A Bayesian Model Averaging

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Academic Year: **2012/2013**

Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Prague, May 17, 2013

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Acknowledgments

I am grateful to my supervisor doc. Roman Horvath Ph.D., for his consultations and valuable comments. I would like to thank Tomas Havranek for his suggestions and help in writing the thesis.

I appreciate the use of the BMS package in R developed by Stefan Zeugner and Martin Feldkircher.

Abstract

The paper estimates the economic growth determinants across 72 countries using a Bayesian Model Averaging. Unlike the other studies we include debt to GDP ratio as an explanatory variable among 29 growth determinants. For given values of the other variables debt to GDP ratio up to the threshold level is positively related with the growth rate. The coefficient on the ratio has nearly 0.8 posterior inclusion probability suggesting that debt to GDP ratio is an important long term growth determinant. We find that the initial level of GDP, life expectancy and equipment investments have a strong effect on the GDP per capita growth rate together with the debt to GDP ratio.

JEL Classification	C11, C15, E01, E22, F43, H63, O47
Keywords	Economic Growth, Bayesian Model Averaging, Posterior Inclusion Probability

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Acronyms

BMA Bayesian Model Averaging

BACE Bayesian Averaging of Classical Estimates

PIP Posterior Inclusion Probability

PMP Posterior Model Probability

GDP Gross Domestic Product

TFP Total Factor Productivity

Master Thesis Proposal

Author	Bc. Nikoloz Kudashvili
Supervisor	doc. Roman Horvath Ph.D.
Proposed topic	Determinants of Economic Growth: A Bayesian Model Averaging

Topic characteristics

Economic growth is one of the crucial factors to identify countries' different development levels across the world.Thus, number of works are devoted to explore the main determinants of economic growth. Classical growth theories take economic growth as a dependent variable and regress it on explanatory variables. However, the set of explanatory variables is not certain. This is known as a model uncertainty in the literature. Instead of choosing a single set of explanatory variables i.e. choosing a single model, BMA allows to include 2^k combination of explanatory variables (where k is number of regressors) and assinging a posterior inclusion probability to each variable. Extreme bound tests introduced in Leamer (1983,1985), however rejected the significance of some specific variables, which often is a source of omitted variable bias. Hence, BMA gives an advantage to avoid omitted variable bias.

Exploring economic growth determinants in somewhat new way of modelling will allow us to draw consistent conclusions, improve the explanatory power of each potential variable (in our case there are 42 explanatory variables). Additionally, the estimates of the growth determinants change over different set of explanatory variables. Analysing all models with their corresponding probabilities gives us opportunity to deal with this problem. We will explore how the public education share is correlated to the growth. Furthermore we will include investments, rule of law and other broadly accepted growth determinants in the BMA analysis.

We will use data by Fernandez C. et al (2001) which is a modification of the data used by Xavier Sala-i-Martin exploring in cross country growth patterns (1997). We will use sub-sampling to observe behavior in developing and developed countries, countries with different levels of rule of law (dividing data in two sub-samples: Africa and the western countries). Comparing results to the baseline regressions will explain the different behaviour of different regressors in various countries. We will do forecasting via BMA analyisis.

"Model Uncertainty In Cross-Country Growth Regressions" by C. Fernandez et al. (2001), "Determinants of Long-Term Growth: A bayesian Averaging of Classical Estimates (BACE) Approach" by Xavier Sala-i-Martin et al. (2004), "Research and Development and Long-term Economic Growth? (2011); Does Trust Promote Growth?" (2012) by Roman Horvath will be main sources for our work.

Hypotheses

Hypothesis #1: We will explore which subset of the explanatory variables will have significant effect on economic growth and test whether the results are different from the one obtained by extreme bounds analysis.

Hypothesis #2: How the behaviour of estimates on growth determinants changes in different sub-samples. Is explanatory power of explanatory variables are different in different models?

Hypothesis #3: We will test whether the estimates are the same across different sub-samples. We will test whether estimates from sub-samples are equal to the ones from the baseline regression.

Other hypotheses: We will test whether the findings of modern endogenous growth theories are confirmed using a BMA analysis. Additionally we check whether fluctuations in exchange rates have a negative effect on economic growth. Furthermore we will test whether the degree of capitalism and number of years of open economy are significant.

Methodology

We will use a Bayesian Model Averaging mainly based on seminal works by Fernandez, Ley and Steel (2001), Sala-i-Martin (1997) and Hoeting et al. (1999).Economic growth is a dependent variable Y which is $(NX1)$ matrix and k regressors denoted by X. Due to model uncertainty, we will take $l = 2^k$ subsets of X's. Correspondingly, we will estimate $M_1, M_2, .., M_l$ models. By assigning Prior Inclusion Probabilities, we measure whether the specific regressor is in the set of the explanatory variables of the true model. Since, beliefs on the parameter priors are less consistent we implement uninformative priors.

Outline

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6. Conclusion

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Chapter 1

Introduction

We address the central question of macroeconomics - why countries differ in development levels and economic growth rates. The neoclassical growth theories based on the papers by Ramsey (1928), Solow (1956), Swan (1956), Cass (1965) and Koopmans (1965) consider technological progress as the main source of the economic growth, while the endogenous theories include the human capital and spillover effects, which generate different results in large extent. Hence, the number of the explanatory variables started to increase once the endogenous growth theories proved themselves dominant during the last decades of the twentieth century. The reason of the dominance of the latter group of growth models is the non-plausibility of the main premise in the neoclassical theories - the main source of the economic growth comes out of the model. Neoclassical growth models do not name a specific source of the economic growth rate. Instead the answer is very general - as technology is the only source of the growth (in effectie terms) in the neoclassical macroeconomics.

The endogenous growth theories include real sources of the growth in the model, which is one of the main advantages of them compared to the exogenous theories. More specifically, the human capital accumulation and investing in the R&D sector are the main sources of the economic growth together with the physical capital already discussed in the neoclassical theories. The neoclassical theories hinge to the premise of the diminishing returns to the production factors, which basically generated the broadly accepted feature of those theories known as the conditional convergence. The conditional convergence is not predicted by the endogenous growth theories due to the inclusion the different set of the sources into the model which do not lead to the diminishing marginal product of capital (or any other production factor).

Although the endogenous growth theories became dominant and seemed to explain the most of the variation across countries, the empirical works started in the late 1980s confirmed the existence of the conditional convergence (see Mankiw *et al.* (1992)). This is in contrast with the predictions of the endogenous growth theories. Empirical evidence suggests that the neoclassical models might be a good theoretical background to construct accurate explanatory variables which essentially are robust growth determinants. This clearly requires examination within the empirical framework.

The conditional convergence stresses how the initial level of the real GDP per capita is important in evolution of the growth rate. Holding all else equal country with smaller starting level of the real GDP per capita has a higher predicted growth rate. In order to examine the conditional convergence one may include the initial level of GDP in the model. If the estimate on the initial level of GDP is negative, then the conditional convergence is presented. Hence, the contribution of the exogenous growth theories gives one of the most frequently used growth determinant - initial GDP level. Influential empirical work by Mankiw *et al.* (1992) shed light to the neoclassical theories to be fit data better. Bloom *et al.* (2002) find that conditional convergence could be explained through technological diffusion.

This motivated researchers to generate more extensions of the neoclassical theories to explain the cross-country growth variations further. More specifically, the extended versions include government policiy measures affecting the level of consumption; property rights regulations and financial market indicators. Government policies and regulations of the financial institutions clearly have a significant effect on the long-run growth rates. Therefore number of the new explanatory variables are constructed to control for the efficiency of the government policy. Hence, more developments of the growth models brought more potential explanatory variables and short models were replaced by models with large number of the explanatory variables.

The discussion on the paramet uncertainty became subject of the prominent empirical papers. Parameter uncertainty is usually referred to the fact when the probability of including or excluding a potential explanatory variable in the model is unknown. Furthermore, there is a model uncertainty meaning that the number of the explanatory variables should be included in the regression is not well-defined. Additionaly, there is an issue of the different regressor combinations of the model. Therefore, model uncertainty is related to the qualitative and quantitative characteristics of the potential explanatory variables for the regression.

Model and parameter uncertainty, together with the potential omitted variable bias due to the incomplete set of the explanatory variables made Bayesian econometrics more tractable. The BMA analysis is the most optimal tool to estimate cross-country variations and figure out the robust growth determinants. To account for a model uncertainty Hoeting *et al.* (1999) show how BMA results are superior to the estimates obtained using standard econometric practice. We heavily follow the influential paper by Sala-i-Martin *et al.* (2004) estimates the model with 67 explanatory variables. However, we depart from their methodology in the sense we do not include any fixed variables that is there are no variables which are included in all regressions. Also we do not choose a fixed model prior, instead we take uniform.

In the sensitivity analysis we show that uniform model prior is more analytically tractable compared to any other prior including hyper model prior. In contrast with Sala-i-Martin *et al.* (2004) and similar to Fernandez *et al.* (2001a) we consider all the possible combinations of explanatory variables, rather than selecting a subset of the growth determinants. By estimating a large number of models and taking a posterior mean of the coefficient (a weighted sum over the model space) results are getting more precise and consistent with the theoretical predictions.

Unlike Levine & Renelt (1992) our results suggest that there are some robust growth determinants explaining the cross-country growth variation. Specifically, there is a conditional convergence of GDP per capita without any additional assumption or the restriction on the set of explanatory variables. Additionally, number of country specific variables appear to be important to determine the growth rate. These result is in line with the findings of Barro (1996) and Fernandez *et al.* (2001a). The latter work is based on the BMA analysis, while Barro (1996) uses OLS to determine growth determinants. For the BMA analysis we heavily follow book by Koop (2003) which examines the influence of different priors on the posterior results. For more technical derivations we use book by Koop *et al.* (2007) which provides with more rigorous description of the model.

We use relatively small number of explanatory variables compared to Sala-i-Martin *et al.* (2004). The dataset is identical to the data used by Fernandez *et al.* (2001b). We estimate the baseline regression and the results are similar to citetLey & Steel (2009). Additionally, we run the BMA for the second sample which contains 21 OECD countries due to the shortage of the data for the rest of the countries. Since OECD countries are similar in terms of the development level, it is interesting to test whether estimates differ for developed and developing countries. In contrast with the abovementioned BMA estimations, we include an additional explanatory variable public debt to GDP ratio. This variable is especially crucial in the exploration growth rates of the developed countries. During the last decades the developed countries including the US experience large increase in the debt to GDP ratio. We want to capture the effect of the debt burden on the growth rates. More specifically we test whether debt to GDP ratio is a robust growth determinant for the developed countries.

Reinhart *et al.* (2012) find that the ratio is negatively related with the GDP growth once the ratio exceeds 90%. It is worthy to note that the negative correlation between high debt to GDP ratio and economic growth is empirically confirmed. However, this does not mean causality. Specifically, a negative estimate on the debt to GDP ratio - exceeding some threshold level does not necessarily mean that high debt causes low growth. Instead, high debt might be due to the low economic growth that is countries take more debt because they have low economic growth and they need to finance projects stimulating economy. Recent research paper by Reinhart & Rogoff (2010a) emphasized the negative effect of high public debt to GDP ratio on output growth, which was questioned by Herndon *et al.* (2013). The latter critique is based on the the error found in the work by Reinhart & Rogoff (2010b) and Reinhart & Rogoff (2010a). The error found in the data and hence in the estimation results changed the outcomes of the paper substantially.

The authors acknowledged the error but asserted that the error does not change the results much. In contrast number of the prominent researchers including Nobel prize laureate Paul Krugman defended the position of Herndon *et al.* (2013) stressing that data does not show the causal relationship from high debt to low growth. As suggested this issue requires further research to test whether there is a causal relation. However, based on the empirical evidence discussed in the related literature high government debt does not necessarily cause low economic growth. Chapter four contains the descriptive statistics of the public debt to GDP ratio which we construct. Since the ratio gets value below 90%[1] the estimate on the ratio is expected to be positive. Low variation in the data on debt and only limited values which are lower than 90% do not allow us to address causality problem and this requires further research.

[1]the ratio is higher than 90% only for one country. We do not consider this exception to be important due to the estimation results given in chapter five

The main contribution of the paper is to explore the complete set of the explanatory variables, and draw the corresponding conclusions. Together with the 28 explanatory variables we test jointly how the debt to GDP ratio affects the growth rates in the advanced economies. We find that debt to GDP ratio is positively related to the GDP growth for given observations. Except this ratio we find that initial level of GDP per capita, life expectancy, equipment investment, geographical location, ethnical and religion fractionalization are robust growth determinants. More importantly initial level of GDP growth is negatively related with the GDP growth and has the highest PIP for both samples. In contrast, countries with high life expectancy and equipment investment which not located in Sahara have relatively high economic growth. In addition fraction confucian is also an important variable which enhances economic growth. Debt to GDP ratio was found as an important driver of economic growth, but has not been examined under the BMA framework. Due to the advantages of the BMA analysis and obtaining a weighted sum of the coefficients across all the model space we believe that our estimate on the ratio is more precise. The sign and magnitude of the coefficient on the ratio is in line with the growth literature mentioned above.

The paper is organized as follows, the second chapter reviews the existing theoretical and empirical literature and briefly describes the main findings of growth theories. The third chapter briefly explains the basics of the Bayesian econometrics which are essential for understanding the methodology part. Chapter four describes data and methodology, also provides with detailed description of the explanatory variables and gives intuition why specific variable should be included in cross country regressions. The fifth chapter descrubes the posterior results under different model priors, hence links variables with high PIP to the averaged growth rate of GDP per capita. Robustness checks are given in chapter six. Additionally the sixth chapter includes the non-technical summary of the fifth chapter and gives analytical explanations of the results given in the fifth chapter. The seventh and final chapter concludes the results and analytically explains the results, briefly summarizes the main findings and discusses the contribution of the paper.

Chapter 2

Literature Review

2.1 Growth Theories

This chapter assesses the development of the endogenous and exogenous growth theories and thus figures out the main determinants of the long-term economic growth in the literature. The classic growth theories start with the famous Solow-Swan model (1956). Although, it is based on simple assumptions such as constant savings rate and exclusion of the household utility maximization problem, it still generates plausible outcomes, which are the sources in forming the set of explanatory in growth regressions. Furthermore, other growth models even though having fundamentally different structure including assumptions and building blocks, Solow-Swan model remains as the benchmark model when it comes comparative analysis of the cross-country growth variations.

The model predicts that the technological progress is the main source of GDP (in effective terms) growth and illustrating that the physical capital accumulation cannot generate increase in the real output, hence unable to explain the large differences across countries. The CRS production function satisfying the essentiality assumption and Inada conditions for the capital and labor imply the most featured property of the neoclassical growth theories, known as conditional convergence.

Barro (1996) shows that the further developments of the growth theories were not fruitful from the empirical point of view, since the majority of empirical works confirm the existense of the conditional convergence. By further developments we mainly mean the endogenous growth theories which predict that initial level of capital does not affect the long term growth rates in contrast with exogenous growth theories.This is why we consider crucial to review more of the neoclassical model. The detailed description of the explanatory variables are represented in chapter four. For the theoretical predictions we follow Barro & Sala-i-Martin (2004).

The second influential model known as Ramsey-Kass-Koopmans (1965) takes into account that the savings rate might not be constant. Furthermore includes the households' lifetime utility maximization problem accounting for the different types of the households with different preferences over the consumption today and tomorrow. It can be shown that Solow model is the special case of Ramsey model. Hence, both models predict the decisive role of the government policies to change the growth rates of the consumption and capital and thus, the output per capita. The late empirical works take into account the contribution of the neoclassical growth theories and include explanatory variables such as initial income, government consumption share in GDP, investment price, population growth and others.

GDP growth rate per capita in the Solow model depends merely on the growth rate of the technology in the Solow model. While in the Ramsey model, the growth rate is affected by the preference parameter usually denoted by ρ, risk aversion parameter θ, interest rate (or marginal product of the capital) and the technology level itself. Neoclassical models fit to the data well compared to the endogenous growth models, but on the other hand the main source of the real output growth - technology growth is not a real source in the sense that it is exogenous, or does not come from the model. The respond to the "weakness" of the exogenous growth theories listed above was the evolution of the endogenous growth theories.

The endogenous growth theories include investments R&D sector, which captures the technological progress. In other words, the higher the investments in the R&D sector the higher the growth of the technology and thus, the higher the increase in the output. Romer (1986) (given in Romer (1994)) and Lucas (1988)[1] fail to effectively include the R&D goods market in their models, since the models generate the increasing returns scale. The late revisions by Romer (1987, 1990) imposed imperfect competition in contrast with the neoclassical growth models. This worked out well as these models generated plausible results.

Specifically, Romer (1990), Grossman & Helpman (1991), Aghion & Howitt (1992) created a new class of the growth theories, which are based on the markets characterized with imperfect competition taking into account separated

[1] Models without a source are taken from the textbook by Barro & Sala-i-Martin (2004)

markets for consumption and technology goods. In addition innovation plays a key role in determination of growth rate. This shed light to the further importance of the government policies such as financing/promoting production of the R&D goods, protection of the intellectual rights and other actions, which could possibly stimulate the R&D sector.

Although, the conclusions of the endogenous growth theories are consistent and they are strengthened by the mathematical tractability of the models, the endogenous models were attacked by the empirically dominant conditional convergence property. Since the endogenous growth models did not predict the conditional convergence, the extended versions of endogenous models started to tackle this issue. Specifically, Barro (1996) shows that diffusion endogenous models generate the conditional convergence, which reconciles the previous inconsistencies.

All the abovementioned models are the core of the growth theories, which are in the sake of explanation the cross-country growth variation. The discussion is rather complicated whether those models capture the aggregate growth across the world. There is little consensus regarding the statement that these growth models explain the growth rates. Durlauf *et al.* (2008) asserts that this is matter of the model specification. In other words, depending on the model specification the set of the significant explanatory variables are different. Therfore, the theoretical approach to explain the growth rates started to be accompanied by the rigorous empirical models. Due to the model uncertainty the usage of the BMA analysis became advantegous and hence dominant.

To sum up, the existing economic growth models have two features: either poorly perform as a theoretical model but having significant empirical evidence or vice versa. In other words, while the neoclassical models does not contain the real source of economic growth it still gets the empirical support which is exhibited in the conditional convergence property. The endogenous growth theories do include the source and the models are endogenously determined but the majority of them have hard times to fit data. The main contribution of the abovementioned theories is that they give the underlying theoretical background to form the set of explanatory variables such as initial income, government spending share and government policy indicators as the major determinants of the economic growth. In the empirical part of the second chapter we review how those variables are estimated, which proxies are used to measure those variables and how significant are they.

2.2 Empirical Literature

This chapter assesses the main findings of the models estimating the determinants of economic growth. Furthermore, we suggest that the debt to GDP ratio is significant variable to explain the cross-country variations and include the review of existing empirical works on the new variable.

One of the classic empirical works exploring the growth determinants is extended neoclassical model by Mankiw *et al.* (1992). The paper advocates the predictions drawn by the influential Solow-Swan model and gets slightly different results. Specifically, the extended Solow model with human capital confirms the existence of the conditional convergence, predicts relatively high return to the human capital compared to physical capital and shows that the savings rate is positively related to the economic growth. The role of the human capital accumulation is also considered as the source in the ednogenous growth theories discussed in the previous section.

The cornerstone of the endogenous growth theories is that changes in the main sources of the growth such as investments in R&D sector affect growth persistently. In contrast Jones (1995) finds that permanent changes in the sources do not generate persistent variations in GDP growth. Using the time series tests the study concludes that AK models are not consistent in explaining growth. The same is true for the R&D models since increase in the investments in R&D sector at a constant rate does not lead to a persistent GDP growth. Based on the abovementioned models it is hard to say which class of growth theories fit data empirically. Thus, we take into account both types of the theories in forming the set of the explanatory variables and also drawing conclusions on the estimated results.

Clearly, the results largely depend on the estimation methods. Therefore number of the studies find either fundamentally different long-term growth determinants or no robust long-term growth forces. The extremes bounds analysis is one of the conservative approaches which usually generates pessimistic results in the sense that very little number of the explanatory variables are significant or none of them.The extreme bounds test was used in several empirical works to identify the determinants of economic growth. It is worthy to note that this was the first try to address the model uncertainty in the early 1990s.

Non-surprisingly Levine & Renelt (1992) found nearly no significant determinant following the methodology discussed in Leamer (1983) and Leamer (1985). The extreme bounds test proved itself too restrictive, therefore it could be a reason of the loss of significance in explaining the variation. Sala-i-Martin (1997) also finds Leamer's extreme bounds test (1985) too strong. Alternatively, Sala-i-Martin establishes new analysis by distinguishing the normal and non-normal distributions of the estimates. Once the distributions are drawn, the estimates are calculated as the integrated likelihoods. The results obtained based on this model specification are consistent with the exogenous theories - suggesting the existence of the conditional convergence.

Fernandez *et al.* (2001a) uses the BMA analysis to pin down the determinants of the long-term economic growth. The paper estimates the data from the influential paper by Sala-i-Martin (1997) with some modifications. Specifically Fernandez takes 41 explanatory variables and establishes the benchmark prior distributions for the estimates and show that these prior distributions have little influence on the posterior results (on the inclusion probabilities). The number of the potential models is decreased in a standard way - using the Markov Chain Monte Carlo simulation Model Decomposition (MC^3). Although, the results are slightly different from the preceding influential paper by Sala-i-Martin (1997) the implications of the model do not change largely. The information on the estimates is much more detailed under the BMA framework and includes PIPs and PMPs for all explanatory variables and estimated models rather than including estimates based on single model.

Sala-i-Martin *et al.* (2004) address the model uncertainty problem by implementing the BACE analysis, which simply constructs the coefficients on the explanatory variables by averaging the corresponding OLS coefficients for all models. The paper does not follow to the fully Bayesian framework in order to limit the effects of the prior distribution of all the estimates conditioned on all the possible combinations of the models on the regression results. The paper estimates the average growth rate of GDP per capita for the period 1960-1996 for 139 countries. Out of the 67 explanatory variables, 18 are found significant. The fully Bayesian approach given in the study by Fernandez *et al.* (2001a) is superior compared to BACE approach, since the former work constructs benchmark priors and do not restrict all models by including some fixed number of explanatory variables (hence imposing a fixed model prior).

The explanatory variables in Sala-i-Martin *et al.* (2004) are chosen according to the underlying economic growth theories discussed in the first part of chapter two, the appendix also contains the source of the origin for each variable. Specifically, the following groups of the explanatory variables are included : religion, ethnicity, location, population, level of the healthiness, colonial experience, structure of the GDP, share of the government spending in the GDP, exchange rate fluctuations, education level, fertility level, institutions, initial level population and income, inflation, openness of the economy and investment features. The model predicts that under BACE the price of the investment, initial level of income, dummy for the East Asian countries, life expectancy, primary schooling and other 13 variables are significant. Those findings are in the line of the predictions implied by the basic neoclassical growth theories.

Sala-i-Martin *et al.* (2004) and Fernandez *et al.* (2001a) are considered as the core papers estimating cross-country growth variations using the Bayesian Model Averaging. The number of the explanatory variables in the recent empirical works varies from 40 and exceeds 140 Durlauf (2005). Although the number of the regressors differ even over the BMA framework due to the researchers different perceptions for the empirical proxies, eight groups of the explanatory variables are mostly common. By forming groups we depart from the forming principles provided by Durlauf *et al.* (2008).

The first class of the regressors is mainly formed in the early 1990s by Mankiw *et al.* (1992) and is known as the neoclassical growth determinants. It includes the initial level of GDP, human capital measured as primary school enrollment rate. The similar reasoning is drawn by Barro & Lee (1993), where the proxy for the human capital is education attainment. Although, there is a big divergence in the results on the robustness among empirical works in Bernanke & Gurkaynak (2001) also emphasize the role of education.

In contrast with Mankiw(1992) based on the extended Solow model Bernanke & Gurkaynak (2001) suggest that more developments of endogenous growth theories are needed and they are superior. Furthermore, the first group of the regressors contains population growth and investments. Empirical evidence (see Fernandez *et al.* (2001a) and Sala-i-Martin *et al.* (2004) suggests that the initial level of GDP is significant and negatively related with the growth rate of the economy, which confirms the existence of the conditional convergence.

Other findings are also in line with the predictions of the extended neoclassical models described in the first part of chapter two. Hence, we include those variables in our model to test the conditional convergence and also detect other important features of the neoclassical growth variables. In addition both empirical works confirm the importance of the investments in economic growth, which is a standard result for macro papers. Note that both papers distinguish different types of investments which may be helpful to decompose their effects on GDP growth rate.

Barro & McCleary (2003) and McCleary & Barro (2006) pioneer measuring the effect of the religion on the economic growth. The first formal try to include the time spent on the religion activities in the utility function was proposed by Azzi & Ehrenberg (1975). Besides the various correlations between the church-related activities and education, race or sex, the paper addresses the issue how the wage rate (income) and religious participation rate can be correlated. Based on the former two papers of this class, we conclude that religion affects the economic growth.

This might be due to the time spent on producing consumption good, or other indirect links which affect the personal development of the representative agent. Thus, the second group contains the fraction of the mainstream religions such as Orthodox, Catholic, Protestant, Jewish, Muslim and others. In major cases, the estimates of the fraction of specific religion are highly significant, meaning that the religion affects the growth rates. Therefore, we include this group of the regressors in our model and find that fraction Confucian has one of the highest PIPs among explanatory variables.

The third group, which contains variables such as ethnic and linguistic fractionalization, also plays an important role in the countries development according to Barro (1999). The estimates like fraction of the population speaking foreign language and English, together with ethno linguistic fractionalization are included in this group. The higher the share of the people speaking foreign language, the higher the economic performance of that economy is. The fractionalization issues are also discussed by Alesina *et al.* (2003). Outcomes of the latter two papers differ from each other, but implications are the same, suggesting that fractiolization characteristics play an important role in explaining cross-country growth variations.

The location of the country and other geographical features form the fourth group of the regressors which are also found to have an important influence on the economic growth. Besides the abovementioned core papers the issue is addressed by Gallup *et al.* (1998) and Sachs (2003) This is captured by including the continental and regional dummy variables. Non-surprisingly the estimates of the developing and poor countries are negative. Furthermore PIPs of the regressors belonging the fourth group are high enough[2].

Additional parameters such as urbanization and land area are not as robust as Sub-Saharan dummy but still affect the growth rate. Interesting implication is accompanied by including absolute latitude as the measure of the location

[2] see chapter five

provided by Barro (1999). Thus, we extend our model by including these variables which is in line with Henderson *et al.* (2009).

The fifth group of the regressors is related to the demographic features of the population. More specifically life expectancy, health conditions, ratios work force to the total population and other variables measuring the age structure of the population control for the individual differences which might be a source of cross-country growth variation. Bloom *et al.* (2007) finds that some of these features are important in explaining GDP growth.

The sixth group contains variables measuring the performance of the institutions, thus the large number of the works tried to show the linkage between development level of the institutions and economic growth. The methodology and proxies are different for the core papers, but most of them confirm the strong effect of the variables measuring the institutions development on the economic growth. In other words, the higher the development of the institutions and civil society, the higher is the GDP per capita growth. This is empirically confirmed by Acemoglu *et al.* (2001). The same relationship is found for rule of law.

Macroeconomic policy variables are grouped as the seventh group of growth determinants. Most of these macro indicators are described by Barro (1996). The set of variables include inflation rate, government consumption (the fraction of GDP), export share and other traditional macroeconomic policy instruments affecting the economic growth such as the level of the openness of economy. The proxies for the effects of the government policies are estimated in a somewhat different ways.

The eighth group of the determinants captures the fact whether the country has been colony of the Western European countries in the middle centuries. This is done by including the dummy variables named by colonial dummy of one of the Western European countries. This is little subtle to estimate since the number of papers use those colonial dummies to estimate the role of institutions on the GDP growth Acemoglu *et al.* (2001) or to instrument inflation Barro (1999). The results regarding the colonial dummies are similar in the classical empirical papers by Fernandez *et al.* (2001a) and Sala-i-Martin *et al.* (2004).

Except these growth determinants we add one more explanatory variable to detect the influence of the debt to GDP ratio on the GDP growth. According to Reinhart & Rogoff (2009) the ratio is negatively related to the economic growth after 90%, but the relationship has not been exploered under Bayesian framework. Since BMA proved itself superior compared to the standard econometric tools, it is interesting to figure out the effect of the ratio on the growth using BMA.

Clearly, the attention to that variable to explain the economic performance was always high. The developing countries have been experiencing the debt crisis in the second half of the twentieth century and this is well explored using the classical econometric techniques. Thus, we include the debt to GDP ratio in our basket of the explanatory variables.

Debt to GDP ratio belongs to the seventh group taking into consideration the character of the variable. Panizza & Presbitero (2012) discuss the issue and show that public debt has a significant effect on the economic growth for the OECD countries. Non-surprisingly the effect is negative, but once including the instrument variable accounting for the exchange rate volatility due to the change in the government debt, the effect of the public debt to GDP ratio to the economic growth gets insignificant.

Reinhart *et al.* (2012) also explore the developments of the public debt in the advanced economies and draws implications on the changes in the real interest rate. The paper seeks for the threshold level for the debt to GDP ratio and finds that the most plausible number is 90%. In other words, the paper considers the ratio below the 90% as a non-extreme level. Correspondingly, the ratio exceeding 90% is a signal that the economy is having hard times. However, number of recent critiques regarding the error in the calculations doubt the causal relationship between the ratio and GDP growth. The correlation is negative but the causal effect is not found empirically.

Reinhart & Rogoff (2010a) includes the analysis to figure out whether there is a causal relationship between debt level and GDP growth. The methodology and the threshold level is the same as described above. The empirical work finds that the causal relationship is weak for the countries with debt to GDP ratio smaller than 90%, while the relationship is significant and strong for the countries having the ratio above 90%. The results significantly differ for the developing and advanced economies. Specifically, if the government policies achieving fall in the debt to GDP ratio with no significant changes in the inflation (and rise in the GDP growth rate) in the developed economies, the story is different for the emerging economies - since the inflation rate sharply rises with the increase in the ratio.

Checherita & Rother (2010) estimate the impact of the high debt on the economic growth for the Euro area from 1970. The estimation does not follow the linear model and also uses different sets of the threshold level including 90%. The main contribution of the paper is that the authors identify the channels for the debt to affect the economic growth. The paper asserts that an increase in the debt level generates the smaller real interest rate, which automatically is reflected in the smaller private savings and public investments. The rest two channels, stress the effect of the high debt on the total factor productivity and the sovereignty of the nominal and real interest rates. The latter holds only for the twelve European countries and is in contrast with the findings by Reinhart & Rogoff (2010b). In other aspects the conclusions are similar since the first two traditional channels are common for both empirical works.

We stress the empirical works related to the latter explanatory variable, since this is one of the main distinction of our version of the BMA analysis from the influential papers by Sala-i-Martin and Fernandez discussed in the previous blocks. We consider that the existing literature and empirical works support our hypothesis that under the BMA framework the debt to GDP ratio will be a significant determinant of the long-term growth. We suspect that the estimate on the ratio to be positive in the line with the empirical evidence and the theoretical background, since the ratio is smaller than 90% nearly for all countries we consider.

Chapter 3

Bayesian Model Averaging

This chapter reviews the basics of the Bayesian Econometrics theory. However, this chapter does not aim to provide with the complete description of the underlying theory, since this is beyond this work. More detailed derivations can be seen in Koop (2003) and Koop *et al.* (2007).

Standard econometrics usually referred as a frequency inference treats the set of the parameters to be estimated as unknown. The estimated set of parameters are considered to be close to the true unobserved values. Various tests based on the classic linear regression assumptions are constructed to check whether the estimated coefficients differ from the true values. In contrast Bayesian econometrics treats coefficients to be estimated as the random variables. Due to the computational difficulties the latter framework was not popular during the last decades. The considerable progress in modelling regressions in computers in the late 1990s made Bayesian estimation tractable. We get an advantage to use Bayesian framework and exclude potential errors from the estimated coefficients.

3.1 Bayesian Theory

3.1.1 Bayes Theorem

Bayesian econometrics is based on the famous Bayes theorem. Suppose A and B are random variables, using the basic probability rules one can get

$$p(A, B) = p(A|B)p(B) \tag{3.1}$$

Alternatively (3.1) can be rewritten as

$$p(A, B) = p(B|A)p(A) \tag{3.2}$$

Using (3.1) and (3.2) Bayes's rule can be obtained:

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)} \tag{3.3}$$

Bayesian Model Averaging developed below is based on the simple rule given by equation (3.3). To relate Bayes rule to the model to be estimated we use standard notation. Let the dependent variable economic growth in 1960-1992 be y and the set of the explanatory variables X with coefficients θ. Clearly the subject of interest is the parameter θ. Estimating θ is crucial to identify the driving force of the economic growth in 72 countries. Replacing A by θ and B by y in equation (3.3) we obtain

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \tag{3.4}$$

In contrast with the standard econometric theory the Bayesian econometrics is concerned with $p(\theta|y)$. In other words $p(\theta|y)$ shows the probability of the estimator given the data. $p(\theta|y)$ is usually referred as a posterior density function, $p(y|\theta)$ as a likelihood function and $p(\theta)$ - the prior density of θ. As stated above the subject of interest is θ, since θ does not enter into the term $p(y)$, we can drop $p(y)$. In other words, the posterior density of θ is proportional to the product of the likelihood and prior. This can be written

$$p(\theta|y) \propto p(y|\theta)p(\theta) \tag{3.5}$$

Hence identification of the distribution of θ requires prior distribution of θ and the likelihood function to be known.

3.1.2 The Likelihood Function

The cross-country economic growth regression can be formally written as

$$Y = X\beta + \epsilon \tag{3.6}$$

Where Y is [72X1] matrix. Similarly X is $[72X42]$, β - $[42X1]$ and ϵ - $[72X1]$ matrix. Standardly, ϵ denotes the error term. Y and X are defined in section (3.1). θ denotes the set of the parameters - β and σ^2. Furthermore,$p(X|\gamma)$ is a probability density function of X depending on a parameter γ.

The general form of the likelihood function can be written

$$p(Y,X|\theta,\gamma) = p(Y|X,\theta)p(X|\gamma) \tag{3.7}$$

Where the equality follows from the standard rules of the conditional probability. Since we are not interested in the joint probability distribution of $Y\&X$ we work on the likelihood function given as the first term of (3.7) RHS. The likelihood function $p(Y|\theta)$[1] shows the probability of obtaining observed outcomes of Y given θ. The likelihood function can be explicitly given depending on the assumptions on the error term.

3.1.3 The Prior

Priors are based on the information about the estimates which is known for a researcher before analysing the data. Therefore $p(\theta)$ is not restricted to have a specific form and it may vary depending on the researcher's beliefs on the parameter θ. However, an arbitrary form of priors is not tractable due to the estimation issues. Therefore we classify different types of priors.

The most natural way to distinct priors is division between informative and non-informative priors. If the researcher does not have any specific information on the distribution of θ, then usually non-informative prior is used. In contrast informative prior gives additional information about the distribution of the parameter θ to be estimated, which usually affects the results of the Bayesian regression results more. A well-known example of non-informative class of priors is Jeffrey's prior.

$$p(\theta) \propto \left| E\left[-\frac{\partial^2 p(y|\theta)}{\partial\theta\partial\theta'} \right] \right| \tag{3.8}$$

Where the expectation of the second derivative of the likelihood function with respect to θ is usually known as the information matrix $I(\theta)$. Not-surprisingly non-informative priors do not set any restriction on the set of parameters to

[1]For the notational simplicity we drop conditioning Y on X and condition it only on θ.

be estimated, but simply cover all the possible outcomes which might occur observing the distribution of θ.

Furthermore, priors are classified into conjugated and non-conjugated priors. A conjugated prior distribution $p(\theta)$ is tractable because it together with the likelihood function $p(y|\theta)$ generates the posterior distribution which belongs the same family of distribution as the prior θ. Conjugated prior distributions facilitate the calculations and also are easy to interpret. A natural conjugate distribution is a special case of the conjugated prior. Specifically, natural conjugate prior distribution name coincides with the distribution of the likelihood function. This makes interpretations “natural”, as it is based on the data.

Additionally, priors can be either proper or improper. Proper prior distributions density function integrates to one, while this is not true for improper priors. Therefore, informative priors are proper and non-informative ones - improper. We are interested in the analytical results, hence we use informative priors.

3.2 The Model

In this section we shortly review a general model and show how the BMA inference works. Additionally, we list the classical assumptions which are typically used in the Bayesian analysis. As in the previous section denote a dependent variable by y. In line with the adopted notation we denote the explanatory variables by x and the disturbance by ε. Then the model can be rewritten as

$y = X\beta + \varepsilon$

The dimensions of the dependent and explanatory variables differ from the previous section, since we analyse the general model. The dependent variable y is NX1. Similarly ε is NX1

$$y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{bmatrix} \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_N \end{bmatrix}$$

The vector of coefficients β is KX1 and explanatory variable x is NXK

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ \beta_k \end{bmatrix} \quad X = \begin{bmatrix} 1 & x_{12} & ... & x_{1k} \\ 1 & x_{22} & ... & x_{2k} \\ . & . & ... & . \\ . & . & ... & . \\ 1 & x_{N2} & ... & x_{Nk} \end{bmatrix}$$

The error term is normally distributed with zero mean and variance-covariance matrix $\sigma^2 I_N$. Formally, this can be written as

$$\varepsilon \sim N(0_N, \sigma^2 I_N)$$

where 0_N is a vector of zeros of dimension NX1, similarly I_N is an identity matrix NXN. We assume homoskedasticity, more about the model under the assumption of heteroskedasticity can be found in Koop (2003). Additionally, we assume that the explanatory variables are independent of the error term and have a full rank. Having those assumptions we illustrate how θ can be calculated. Note that $\theta = [\beta, \sigma^2]$

3.2.1 The Likelihood Function

Since the error term is normally distributed one can derive the likelihood function $p(y|\theta)$, which takes the form

$$p(y|\beta, \sigma^2) = \frac{1}{[[2\pi]^{1/2}\sigma]^N} e^{\{[-\frac{1}{2\sigma^2}][y-X\beta]'[y-X\beta]\}} \tag{3.9}$$

The most natural way to estimate θ is minimizing sum of squares of residuals, which yields estimates usually referred as OLS. Correspondingly,

$$\widehat{\beta_{OLS}} = (X'X)^{-1}X'y \tag{3.10}$$

Similarly, σ^2 can be estimated by the standard deviation adjusted for the degrees of freedom k. Hence, the estimate of σ^2 takes form

$$\widehat{\sigma}^2 \equiv s^2 = \frac{[y - X\widehat{\beta_{OLS}}]'[y - X\widehat{\beta_{OLS}}]}{N - k} \tag{3.11}$$

Plugging (3.10) & (3.11) in (3.9) and doing some algebraic transformations one can derive the value likelihood function:

$$p(y|\beta,\sigma^2) = \frac{1}{[[2\pi]^{N/2}}\{\frac{1}{\sigma^2}e^{[-\frac{1}{2\sigma^2}(\beta-\widehat{\beta_{OLS}})'X'X(\beta-\widehat{\beta_{OLS}})]}\}\{\frac{1}{[\sigma]^{(N-k)}}e^{[-\frac{s^2(N-k)}{2\sigma^2}]}\} \tag{3.12}$$

(3.12) is tractable as the RHS of the equation can be decomposed between the probability density function (pdf) of normal distribution of β and pdf of Gamma distribution of σ^2 - second and third terms correspondingly.

3.2.2 The Prior

In the general description of the priors we described the natural conjugate class of priors. Given the functional form of the likelihood function (3.12) the natural conjugate prior has the distribution of Normal-Gamma. Then the conditional distribution of β becomes

$$\beta|\sigma^2 \sim N(\underline{\beta}, \sigma^2\underline{V}) \tag{3.13}$$

Where, $\underline{\Theta}$ denotes the value representing the prior information about Θ. Correspondingly V denotes the variance-covariance matrix of β. The lower case bar points that this is a prior information. As stated in the previous section the prior distribution is merely based on the researcher's beliefs on the parameters to be estimated.

Similarly, an unconditional prior distribution of σ^2 is

$$\sigma^{-2} \sim G(\underline{s}^{-2}, \underline{v}) \tag{3.14}$$

Where $\underline{v}$ denotes the prior information on the variance. Combining (3.13) & (3.14) and taking into account the functional form of the Normal-Gamma distribution we derive the prior distribution of θ

$$\beta, \sigma^{-2} \sim NG(\underline{\beta}, \underline{V}, \underline{s}^{-2}, \underline{v}) \tag{3.15}$$

Hence, depending on the prior information given to the researcher the prior density function gets a specific form containing all four parameters of Normal-Gamma distribution. Since we have conjugate normal priors the posterior results will have the same distribution which simplifies the analysis and hence the computational part.

3.2.3 The Posterior

Following the notations and the rules defined in the previous section and the property of the natural conjugate priors we get a joint posterior distribution

$$\beta, \sigma^{-2}|y \sim NG(\bar{\beta}, \bar{V}, (\bar{s})^{-2}, \bar{v}) \tag{3.16}$$

Where, $\bar{\Theta}$ denotes the value representing the posterior information about Θ. Following Koop (2003) the posterior distribution can be calculated using the following equations:

$$\bar{V} = [\underline{V}^{-1} + X'X]^{-1} \tag{3.17}$$

$$\bar{\beta} = \bar{V}[\underline{V}^{-1}\underline{\beta} + X'X\widehat{\beta_{OLS}}] \tag{3.18}$$

$$\bar{v} = \underline{v} + N \tag{3.19}$$

Posterior standard deviation can be found from

$$\bar{v}\bar{s}^2 = \underline{vs}^2 + vs^2 + (\widehat{\beta_{OLS}} - \underline{\beta})'[\underline{V} + (X'X)^{-1}]^{-1}[(\widehat{\beta_{OLS}} - \underline{\beta})] \tag{3.20}$$

Typically, under the BMA framework the subject of interest is the marginal distribution of β. This can be found using the simple rules of the probability theory. Specifically,

$$p(\beta|y) = \int p(\beta, \sigma^2|y)d(\sigma^2) \tag{3.21}$$

Hence, the resulting conditional marginal distribution is of a multivariate t distribution. Therefore, the conditional expectation of βequals its posterior mean. Formally, this can be rewritten as

$$E(\beta|y) = \bar{\beta} \tag{3.22}$$

Similarly, using the properties of the multivariate t distribution one can derive the conditional variance as follows

$$Var(\beta|y) = \bar{V}\frac{\bar{v}\bar{s}^2}{\bar{v} - 2} \tag{3.23}$$

Using the fact that posterior results have the form of Normal-Gamma distribution, the distribution of $\sigma^{-2}|y$ coincided with the unconditional Gamma distribution given by (3.14). Therefore, the first and second moments of the conditional variance can be represented

$$E(\sigma^{-2}|y) = \bar{s}^{-2} \tag{3.24}$$

and

$$E(\sigma^{-2}|y) = 2\frac{\bar{s}^{-2}}{\bar{v}} \tag{3.25}$$

Equation (3.18) illustrates how the posterior mean is calculated. The posterior mean of β is a weighted average of the prior $\underline{\beta}$ and $\hat{\beta}_{OLS}$. Furthermore, $\hat{\beta}$ depends on the posterior variance, which is defined by (3.17). Clearly, the latter is highly dependent on the variance in the explanatory variable and prior variance denoted as $\underline{V}$. Hence the distributions of the prior parameters are usually essential. If the researcher does not have specific beliefs regarding the prior distribution of θ, then the posterior mean and variance coincide to the corresponding OLS estimates. This is usually done by assigning small values to the prior parameters. Observing the latter two equations it is easy to guess that posterior estimates coincide with the OLS ones in case $\underline{V} = 0$ for N large enough. In case of a small N, this requires the second restriction, specifically $\underline{v} = 0$, Meaning that there is no variance for the estimate $\bar{s}^{-2}$. Hence, the Bayesian framework allows to account for the preliminary knowledge on estimates.

Chapter 4

Data and Methodology

4.1 Data

We consider two datasets: the dependent variable is GDP per capita growth rate in both cases. More specifically, GDP per capita is averaged for period 1960-1992. The first dataset consists of 72 countries and 41 explanatory variables. The dataset is taken from the famous paper by Sala-i-Martin (1997), which is also used by Fernandez *et al.* (2001a). Note that Fernandez uses relatively shortened data. This dataset is also available at homepage of Zeugner [1].

In the second data set we include an additional explanatory variable - debt to GDP ratio. In line with the measurement methodology used for GDP growth rate we average debt for the same period. Since data is not available for all the countries for this period we consider only 21 OECD countries and 29 explanatory variables. The number of explanatory variables is smaller compared to the dataset one due to the assumption on X (inverse of $(X'X)$ exists) to have a full rank. Since some of the variables such as geographical or nationality/religion dummies are not relevant to the second dataset we have fewer number of the economic growth determinants.

Table 4.1 shows the descriptive statistics of the dependent variable, which is averaged GDP per capita growth rate for the period 1960-1992. Most of countries have a positive economic growth, generating a positive mean of value 0.02, meaning that an average yearly GDP growth rate is around 2%.

The set of explanatory variables are based on Sala-i-Martin *et al.* (2004) and Fernandez *et al.* (2001a). X contains three dummy variables getting values one

[1] http://bms.zeugner.eu/tutorials/fls/

and zero correspondingly if a country was a Spanish, French or British colony. Those three dummy variables control for the influence of colonialism on the growth rate of a country. Furthermore, in line with the theory and number of papers exploring the economic growth determinants, colonial dummies seem to be important in explaining the growth.

Table 4.1
Summary Statistics, using the observations 1–72
for averaged growth rate of GDP (72 valid observations)

Mean	Median	Minimum	Maximum
0.0207285	0.0203055	−0.0207690	0.0661790
Std. Dev.	C.V.	Skewness	Ex. kurtosis
0.0182539	0.880620	0.324644	0.243273

If the country was a colony then it is expected to have a lower economic growth rate due to the negative effects of the colonialism. Among the negative effects low level of trust and underdevlopment of institutions are important (see Acemoglu *et al.* (2001). Another dummy variable controls for the war, similar to the colonial case war dummy gets value one if a country was involved in any war in 1960-1992 and zero otherwise. Intuition on the inlcusion of this variable is straightforward since countries unaffected by the war might have higher growth rates. In other words, infrastructure, industry development and other important factors of a country's development were not affected and hence, development level was not impeded.

Another class of the dummy variables control for the geographical location of the country. Following Sala-i-Martin *et al.* (2004) we include two regional dummy variables: Latin America and Sub-Sahara dummies. In line with the growth theories location affects the development, therefore those two dummy variables control for this effect. Moreover it is expected that countries located in the Sub-Sahara to have lower economic performance compared to the countries not located in this part of the world. The same logic applies for the Latin American countries.

Economic policy is an important determinant of growth, as a government can potentially stimulate implement a set of the policies which would bust the economic growth. This is why dummy variable - "Outward Orientation" is included as an explanatory variable. In line with the classical definition of the dummy variable it gets value one in case a country is oriented outward and zero otherwise. Outward orientation controls for the fact how the government policy affects the export-import structure of the country. Srinivasan & Bhagwati (1999) define outward orientation using Krueger's definition that outward orientation " is an entire set of policies oriented toward encouraging the production of goods and services efficiently". In other words, outwardly oriented country promotes free trade and does not take into account whether goods are of a domestic or foreign production. In line with the existing literature it is expected that countries with outward orientation to have higher efficiency in production and hence higher growth rates.

To control for the size of the economy we include area as an explanatory variable. In contrast with the previous specification of the explanatory variables, area is measured in absolute terms rather than defined as a dummy variable. Area is an closely related to the size of the economy and hence to the scales. From microeconomic studies scale effects are found to determine the costs of production. From macroeconomic point of view scale effects are important since, one needs to specify the form of the production function which usually can be either increasing, decreasing or constant returns to scale (CRS). In the neoclassical growth theories such as Solow-Swan model and Ramsey model, it is usually assumed to have CRS production function, due to its tractable properties. By including area as a growth determinant, we test whether size of the country and hence size of the economy is an important in explaining growth.

In line with the endogenous growth theories education has a positive effect on a country's development. Number of different measures have used to proxy the education. Following Barro (1996) we take three explanatory variables to measure a relation between education and economic growth. Primary School Enrolment, Higher Education Enrolment and Public Education Share. The first two variables are measured as a fraction enrolled students to total population of the same age, while the third variable is a share of a government's spending on the education out of the total state budget. Another explanatory variable is a life expectancy which is usually referred as a human capital other than educational one. Life expectancy is found as an important determinant in number of studies, this is why we check how robust are the previous studies by including it among other 40 explanatory variables.

The main feature of the neoclassical growth theories is the notion of the conditional convergence. GDP60 is the initial level of the GDP per capita in 1960. Intuitively, the coefficient on GDP60 is expected to be negative, since countries with smaller initial output level grow faster. This variable is important since we can test whether conditional convergence is presented. To decompose GDP Jones (1995) includes fraction of GDP in mining as an explanatory variable. For the period 1960-1992, it was more common to have different order of the organization of economy, compared to the latter decade. Therefore, we include EcoOrg to measure degree of capitalism. In line with Jones (1995) the degree of capitalism is positively correlated to the growth. The same argument applies for the variable measuring years of the open economy, which is also found significant determinant in a number of studies. To control for the average age of the population Barro (1996) propose to include average age as an explanatory variable, which is expected to be negatively correlated with the growth.

Barro (1996) finds that religion of a country affects the growth rate at a large extent. Therefore, fraction of Buddhist, Catholic, Protestant, Muslim, Jewish, Hindu and Confucian is included as explanatory variables. To control for the linguistic features Levine & Renelt (1992) define a variable ethnolinguistic fractionalization, which measures a probability that randomly chosen two people within the country speak the same language. Unlike the variable outward orientation, primary exports measures level of exports in 1970 and is found to be positively correlated with the growth.

Empirical evidence suggests that rule of law is an important variable and largely affects GDP per capita growth rate. To account for the population growth we include it as a growth determinant. In line with the exogenous growth theories population growth is negatively correlated with the growth. To distinguish between working force and population we include the ratio workers to population. Unsurprisingly, empirical evidence suggests that the ratio is positively related with GDP per capita, since higher participation generates higher output and hence higher GDP per capita.

To control for the stability Barro (1996) constructs a variable measuring the number of revolutions and military coups for the period 1960-1992. Clearly this variable is negatively correlated with the growth since big number of revolutions and military coups points instability of the country and hence the economy. In many cases low economic growth could be indirectly transmitted through low foreign direct investments in a country with a big number of revolutions. Although, there might be number of other channels which could strengthen and impede economic growth the country.

Barro (1996) finds that political rights are also an important determinant of the economic growth. If the political rights are violated then the probability that economic agents interests are not evenly represented in the legislation or other directives made by the government. This in turn affects income redistribution and as a result a small part of the people gets a big share of the income. This could demotivate agents to work, or limit the individuals development. The same reasoning applies to explain the relationship between civil rights and growth.

Jones (1995) finds that knowledge of English language is an advantage to internalize all the benefits of the globalization. Therefore they propose to include the fraction of English speaking population to measure its effect on economic growth. Intuitively, the higher fraction - English speaking to total population generates better economic performance of the country. Alternatively another explanatory variable can be constructed to control for the foreign language effect on the dependent variable. Specifically, more general form of the above mentioned fraction is taken - share of the people speaking foreign language. In line with the empirical evidence countries with bigger number of foreign language speaking people have a higher economic growth rate.

Currency depreciation or appreciation may affect aggregate demand by changing the price of domestic goods relative to foreign production. As a result exchange rate fluctuations seems to be a natural candidate to influence macroeconomic indicators of the economy and hence economic growth. Classic international trade theories suggest that currency depreciation makes country's exports relatively cheap. Additionally, imported goods are more expensive and therefore aggregate demand for domestic products increases in both domestic and foreign markets.

In line with the Keynesian models and their extensions increase in the aggregate demand generates higher output. Hence, domestic currency depreciation may be a stimulus for economic growth. Clearly, there are other factors which may strengthen or weaken the magnitude of the effects such as investments and capital movements. In contrast, domestic currency appreciation has a negative effect on net exports. Taking into all these factors Barro (1996) proposes to include exchange rate distortions as an explanatory variable to capture the above mentioned effects on economic growth.

Majority of the growth theories consider investments as a main determinant of economic growth. DeLong and Summers (1991) divide investments between equipment and non-equipment investments. Equipment investments are usually found to increase efficiency through improvements in the technology. Therefore equipment investments are more important in explaining cross-country growth variations.

To measure the influence of the black market Barro (1996) proposes black market premium and its standard deviation as an explanatory variable. The standard deviation of the black market premium is a good proxy for the uncertainty. Specifically, high standard deviation usually reflects larger degree of uncertainty, which creates disincentives for capital lenders to invest in such an economy.

Table 4.2 illustrates descriptive statistics of the fraction debt to GDP. Only one country has debt to GDP ratio which exceeds the threshold level, therefore it does not seem reasonable to include a dummy variable and control for the threshold level. In line with the existing literature it is expected that the coefficient on the debt to GDP ratio to be positive since all the values are smaller than the threshold level stated above.

Table 4.2
Summary Statistics, using the observations 1–21
for the variable Debt to GDP ratio (21 valid observations)

Mean	Median	Minimum	Maximum
0.404312	0.363840	0.0869192	0.944584
Std. Dev.	C.V.	Skewness	Ex. kurtosis
0.247921	0.613193	0.876190	−0.00324390

All the explanatory variables listed above are broadly used and estimated in the BMA framework. Debt to GDP ratio is examined using classic econometric tools (see Reinhart *et al.* (2012)) and found to be positively correlated with economic growth up to some threshold level. The threshold level is usually 90%. We include averaged public debt to GDP ratio for 21 OECD countries for the period 1980-1992[2]. For the US and other few countries full dataset (1960-1992) is available, hence by comparing averages those averages to the ones obtained for (1980-1992) we find negligible differences. Therefore, usage of the average ratio of the latter period are likely to avoid any measurement error or other related problems.

[2]data for the period 1960-1980 is not available, therefore we average public debt to GDP ratio for last twelve years.

4.2 Methodology

In the third chapter we reviewed basic features of BMA, but we did not address the problem how tractable is to estimate 2^{41} models. It is impossible to run the enormous number of models, therefore some techniques is needed to make BMA feasible. This computational issue is usually solved using Markov Chain Monte Carlo Model Composition (MC^3), which is described below.

4.2.1 BMA

The model has the following form

$$y = \alpha_r + X_r\beta_r + \varepsilon \tag{4.1}$$

where ε is identically independently distributed error term with variance σ^2. Formally $\varepsilon \sim N(0, \sigma^2 I)$. This model differs from the one given in the third chapter only in notation - the constant is separately written in the model and not in the standard form as in (3.6). Following Koop (2003) there are $R = 2^{41}$ models, denoting each model by M_r, where $r = 1, 2, ..., R$. Taking into account the notions defined in the third chapter we can write:

$$p(M_r|y, X) = \frac{p(y|M_r, y, X)p(M_r)}{p(y|X)} \tag{4.2}$$

Since the integrated likelihood $p(y|X)$ is constant for all models (4.2) can be rewritten as follows:

$$p(M_r|y, X) \propto p(y|M_r, y, X)p(M_r) \tag{4.3}$$

In other words probability of each model is the product of the marginal likelihood of the model and prior model probability. Note that similar to chapter three marginal likelihood $p(y|M_r, y, X)$ shows probability of the data given the specific model.

We are interested in the posterior results which is a weighted average of a coefficient all over the models.

$$p(\theta|y, X) = \sum_{r=1}^{2^k} p(\theta|y, M_r)p(M_r|y) \tag{4.4}$$

Combining (4.3) and (4.4) we get:

$$p(\theta|y) = \sum_{r=1}^{2^k} p(\theta|y, M_r) \frac{p(M_r|y)p(M_r)}{\sum\limits_{s=1}^{2^k} p(M_l|y)p(M_l)} \tag{4.5}$$

Note that we also use conditional probability rule and for notational simplicity we do not include conditioning on X, which does not affect the results. The functional form of the prior may affect the posterior results at a large extent. Therefore it is important to detect the priors that generate reasonable posterior results. Fernandez et al. (2001) introduce benchmark priors and using multiple simulations show that benchmark priors have a little effect on the posterior results and additionally they are close enough to the corresponding theoretical priors.

Choosing the best model requires comparing posterior odds ratios (see Koop (2003)). Therefore noninformative priors can be used for the subset of θ which are common for all models. However, parameters which might differ for at least one model are required to have informative priors in order to obtain reasonable posterior distribution of θ. Since constant α_r and σ^2 are common for all models we can use noninformative priors for these two parameters.

$$p(\alpha_r) \propto 1 \tag{4.6}$$

$$p(\sigma) \propto \sigma^{-1} \tag{4.7}$$

Following Fernandez et al. (2001b) we standardize all explanatory variables by demeaning. Clearly it will not affect coefficients β but will allow to interpret intercept parameter α_r in the similar fashion over all models. Using the OLS properties one can easily find that slope parameter measures the mean of the dependent variable y. The third set of parameters to be estimated is coefficients on the explanatory variables. Those coefficients will be different for various models since we take multiple combinations of growth determinants due to uncertainty described in the introduction. In other words, the number of coefficients included in model M_l and $M_{l'}$ are not equal, where $l \neq l'$. More importantly even the number of parameters are equal for two different models, qualitatively vector of coefficients will be different. Therefore noninformative priors cannot be used for β.

4.2.2 Zellner's g-prior

Rewriting (3.13) and taking into account that some of the variables might not be important in explaining growth we set the mean of $\underline{\beta_r}$ zero. Hence,

$$\underline{\beta_r|}\sigma^2 \propto N(0, \sigma^2 \underline{V_r}) \tag{4.8}$$

The second component of the prior of β is calculated using a g-prior. Zellner (1986) proposed g-prior as a common benchmark prior. The g-prior depends on the data, thus does not violate the conditional probability rule (note that we simply dropped likelihood function's conditioning on X to keep notations simple). The fact that g-prior is data dependent has an advantage since we include data characteristics to obtain the posterior results.

Formally,

$$\underline{V_r} = [g(X_r{}'X_r)]^{-1} \tag{4.9}$$

Replacing $\underline{V_r}$ in terms of g-prior in (4.8) we get

$$\underline{\beta_r|}\sigma^2 \propto N(0_r, \sigma^2[g(X_r{}'X_r)]^{-1}) \tag{4.10}$$

Depending on the researcher's beliefs parameter g gets small or large values. Relatively small g means that the variance of the prior coefficient is very little and therefore reflects information that researcher has a "complete" information on the coefficients (in this case complete information means that the researcher knows that all coefficients are zero). In contrast, high g is equivalent to the fact that there is a large variance of the prior coefficient, equivalently the researcher does not have enough information that indicate that all the coefficients are zero. Therefore, assigning large value to g the researcher allows for the high variation of the prior coefficient and hence the likelihood that coefficients are zero is not zero.

Using (3.22) we know that

$$E(\beta_r|y, M_r) = \bar{\beta_r} = \bar{V_r} X_r{}'y \tag{4.11}$$

Where $\bar{V_r} = [(1+g)(X_r{}'X_r)]^{-1}$. Using the notions from chapter three, one can derive an explicit form of $\bar{\beta_r}$

$$\bar{\beta_r} = \frac{g}{1+g}\hat{\beta}_{OLS} \tag{4.12}$$

Clearly, when $g \to\infty$ $\bar{\beta}_r$ approaches to $\hat{\beta}_{OLS}$. The variance-covariance matrix can be calculated using OLS variance formula adjusted for the coefficient β prior

$$Cov(\beta_r|y, X, M_r) = \frac{g}{1+g}\frac{(y-\bar{y})'(y-\bar{y})}{(N-2)}[1-\frac{g}{1+g}{R_r}^2]({X_r}'X_r)^{-1} \tag{4.13}$$

where N is sample size of a model, ${R_r}^2$ is the coefficient of determination for the corresponding model r. Similarly, the marginal likelihood can be derived as follows

$$p(y|M_r, X) \propto [\frac{g}{1+g}]^{r/2}[\frac{g}{1+g}(y-\bar{y})'(y-\bar{y}) + \frac{y'P_{X_r}y}{1+g}]^{-\frac{N-1}{2}} \tag{4.14}$$

where $P_{X_r} = I_N - X_r({X_r}'X_r)^{-1}{X_r}'$

4.2.3 Posterior Analysis and MCMC

Since we do not have any specific information about models we assume that models are distributed uniformly, that is each model has equal probability. In other words

$$p(M_r) = \frac{1}{R} \tag{4.15}$$

Equations (4.12) and (4.13) imply that if $g = 1$ then the posterior β is simply equally weighted sum of the prior and corresponding OLS coefficient. Fernandez et al. (2001) show that the most efficient g-prior is benchmark prior, which is

$$g = [max\{N, K^2\}]^{-1} \tag{4.16}$$

In our case $N = 72$ and $K^2 = 41^2$, hence $g = 1681^{-1}$ for the full sample. Since R is a large number, due to the computational difficulties to estimate each model we use Markov Chain Monte Carlo Model Composition. This approach was firstly proposed by Madigan and York (1995). $MCMC$ achieves to decrease in the number of the models to be estimated using selecting draws from the parameter space. Specifically, $MCMC$ takes into account a big number of models which have high PMP, and very few models with low PMP. Similar to the parameter selection process, choosing models is done randomly. $MCMC$ samplers are tractable since they allow the researcher to estimate models with high PMP rather than estimating each model which is an impossible task to do even for modern computers. In the empirical results we examine how close $MCMC$ approximation is to the exact posterior results.

Markov Chain Monte Carlo Model Composition is usually implemented using Metropolis-Hasting algorithm. This algorithm is similar to the importance sampling. Following our standard notation let θ be the parameter we are interested, then $p(\theta|y)$ can be found using random sampling rather than estimating each model and then finding posterior result. Denote $\theta^{(s)}$ a random draw having pdf $f(\theta)$, where s denotes sampled model and $s = 1, ..., S$. The density is known as the importance function too. In order to detect the importance of the Metropolis-Hasting algorithm suppose that the subject of interest is $g(\theta)$. Then the exact posterior result would be written as $E[g(\theta|y)]$. On the other hand the estimated sample value of $g(\theta)$ would be

$$\widehat{g(\theta)}_S = \frac{1}{S}\sum_{s=1}^{S} g(\theta^{(s)}) \tag{4.17}$$

Clearly, when S is large enough, or when $S \to \infty$, $\widehat{g(\theta)}_S$ does not necessarily converge to $E[g(\theta|y)]$. For this reason we use MCMC sampler. In the posterior results we will see that the correlation coefficient between analytical and MCMC PMPs is usually 0.99 which emphasizes the role of the sampler.

Chapter 5

Posterior Results

5.1 Posterior Results for the Full Sample

We run BMA for the full data sample. Following Fernandez *et al.* (2001b) we set number of burn-ins=1 000 000 and draws=2 000 000. We use BRIC prior which is described in the fourth chapter. Additionally, the $MCMC$ sampler is based on the birth-death sampler.

Figure 5.1 shows the descriptive statistics of the BMA analysis. The model size is 10.5 and the correlation coefficient between analytical posterior model inclusion probabilities and approximated $MCMC$ PMPs is 0.9939 for the best 2000 models.

The second section of figure 5.1 shows three best models[1]. The best model with 8.63% PMP includes Sub-Sahara dummy, life expectancy, GDP60, degree of capitalism (EcoOrg), fraction Confucian, Muslim and Protestants, rule of law and both types of investments. In other words with 8.63 % those ten determinants explain the growth. In the similar fashion can be interpreted the second and third best models, with PMPs 7.53% and 5.02% correspondingly.

Figure 5.2 shows the results based on $MCMC$ sampler, results based on the likelihoods -analytical PMPs which are usually referred as exact and are listed in the appendix[2]. Based on both $MCMC$ and analytical PMPs GDP60- initial level of GDP per capita in 1960 is important variable and its PIP is 99.9 %. The coefficient on GDP60 is negative in line with the theoretical prediction. In other words, countries with smaller level of initial GDP in 1960 have higher economic growth. The interpretation is standard and since the dependent variable is in

[1] for the sake of brevity not all the variables are listed, best models with all variables can be found in the appendix

[2] See figure A.4

Figure 5.1: Descriptive Statistics and Top 3 Models

Mean no. regressors	Draws	Burnins	Time
"10.4501"	"2e+06"	"1e+06"	"8.235971 mins"
No. models visited	Modelspace 2^K	% visited	% Topmodels
"521752"	"2.2e+12"	"2.4e-05"	"46"
Corr PMP	No. Obs.	Model Prior	g-Prior
"0.9940"	"72"	"uniform / 20.5"	"BRIC"
Shrinkage-Stats			
"Av=0.9994"			

Model Name	0046845800c	0046844800c	474440008
SubSahara	1	1	1
LifeExp	1	1	1
GDP60	1	1	1
Mining	0	0	1
EcoOrg	1	1	0
YrsOpen	0	0	1
Confucian	1	1	1
EthnoL	0	0	0
Hindu	0	0	0
Jewish	0	0	0
Muslim	1	1	1
Protestants	1	0	0
RuleofLaw	1	1	0
EquipInv	1	1	1
NequipInv	1	1	0
PMP (Exact)	0.0086255	0.0075332	0.0050197
PMP (MCMC)	0.0081650	0.0075820	0.0049750

Figure 5.2: BMA Coefficients and their PIPs

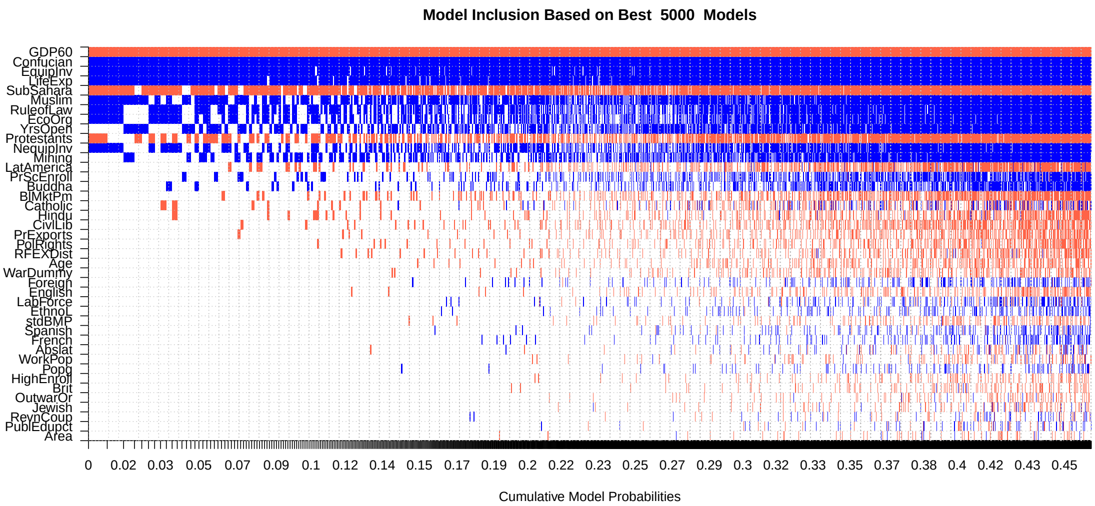
	PIP	Post Mean	Post SD	d.Pos.Sign	Idx
GDP60	0.99865	-0.01608	0.00316	0.00000	12
Confucian	0.98720	0.05631	0.01478	1.00000	19
LifeExp	0.93157	0.00084	0.00034	1.00000	11
EquipInv	0.92284	0.15964	0.06860	1.00000	38
SubSahara	0.73622	-0.01160	0.00851	0.00000	7
Muslim	0.63338	0.00866	0.00777	0.99903	23
YrsOpen	0.50635	0.00721	0.00796	0.99994	15
RuleofLaw	0.49399	0.00735	0.00835	0.99997	26
EcoOrg	0.46451	0.00121	0.00144	0.99995	14
Mining	0.45801	0.01900	0.02344	0.99999	13
Protestants	0.45747	-0.00577	0.00713	0.00000	25
NequipInv	0.43590	0.02501	0.03189	1.00000	39
LatAmerica	0.21615	-0.00182	0.00419	0.05986	6
PrScEnroll	0.20220	0.00415	0.00949	0.98880	10
Buddha	0.19720	0.00259	0.00596	0.99966	17
BlMktPm	0.17939	-0.00137	0.00334	0.00021	41
Catholic	0.13491	-0.00033	0.00319	0.36429	18
Hindu	0.12938	-0.00352	0.01192	0.05115	21
CivlLib	0.12620	-0.00028	0.00088	0.00736	34
PrExports	0.09859	-0.00096	0.00353	0.00499	24
PolRights	0.09628	-0.00015	0.00057	0.01651	33
Age	0.08645	0.00000	0.00002	0.00077	16
RFEXDist	0.08360	0.00000	0.00002	0.03519	37
WarDummy	0.07936	-0.00031	0.00129	0.00281	5
LabForce	0.07501	0.00000	0.00000	0.84912	29
English	0.06874	-0.00044	0.00200	0.00009	35
Foreign	0.06558	0.00028	0.00139	0.92909	36
EthnoL	0.05609	0.00032	0.00185	0.93394	20
Spanish	0.05558	0.00022	0.00156	0.84074	2
stdBMP	0.05044	0.00000	0.00000	0.03224	40
French	0.05036	0.00019	0.00118	0.97271	3
HighEnroll	0.04539	-0.00159	0.01100	0.03135	30
WorkPop	0.04429	-0.00030	0.00233	0.14165	28
Abslat	0.04202	0.00000	0.00003	0.52415	1
OutwarOr	0.03894	-0.00007	0.00060	0.09634	8
Popg	0.03834	0.00545	0.04832	0.89149	27
Brit	0.03707	-0.00006	0.00065	0.13489	4
Jewish	0.03644	-0.00023	0.00285	0.22865	22
PublEdupct	0.03232	0.00105	0.02605	0.60175	31
Area	0.02980	0.00000	0.00000	0.29555	9
RevnCoup	0.02792	0.00000	0.00094	0.50193	32

percentage terms, an increase in the initial level of GDP per capita by one unit would generate fall in the economic growth rate by 1.608 %.

Similarly, in line with the results obtained in Fernandez et al. (2001) fraction Confucian, life expectancy and equipment investment have PIPs more than 90%. Furthermore, the coefficients are positively related to the economic growth as expected. In contrast Sub-Sahara dummy has a negative posterior mean and relatively smaller PIP 0.73622.

Figure 5.3 shows the magnitude and signs of the determinants. On the horizontal axis top 5000 models are considered ordered according to their PMPs.

Figure 5.3: Model Inclusion Probabilities Based on Top 5000 Models

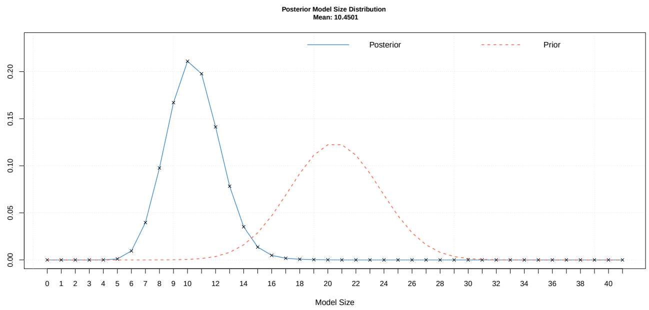


5.1.1 Model Size

Figure 5.1 shows that the posterior model size is 10.45. The posterior model distribution compared to the prior distribution is illustrated on Figure 5.4. As expected the prior distribution is symmetric with mean around 20.5 which is equal to one half of the total number of the determinants.

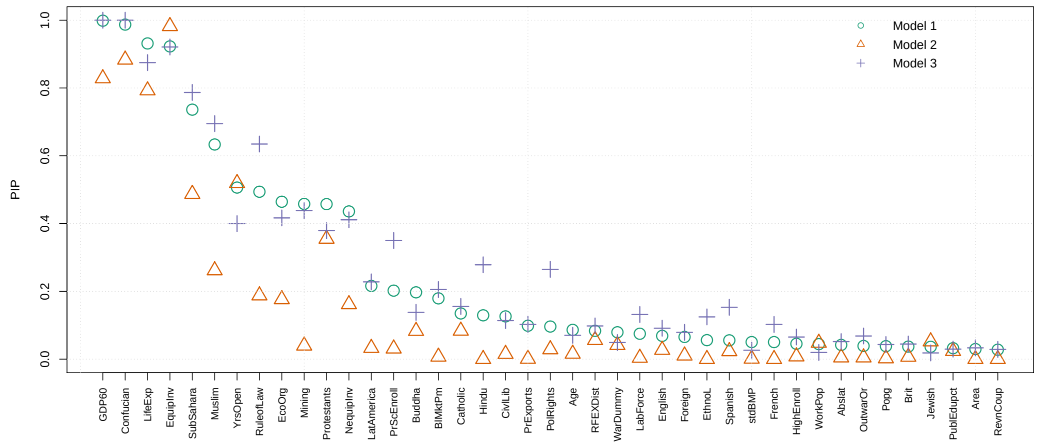
However, the posterior model size mean is much smaller, this can be also seen by observing the number of the included explanatory variables for the top 3 models. To examine the effects of the model prior distribution on the posterior results we run BMA with fixed and random model priors. Figure 5.5 shows that random model priors are similar to the uniform model prior. In contrast fixed model priors can be misleading, since PIPs under fixed prior are

Figure 5.4: Prior and Posterior Model Sizes



relatively low which are observed to be high. Note that model 1 corresponds to the posterior model size distribution under uniform model prior, while model 2 and model 3 represent the same feature under fixed and random model priors.

Figure 5.5: PIPs for Uniform, Fixed and Random Model Priors



5.1.2 Sensitivity Analysis of MCMC Sampler

Descriptive statistics of the BMA model presented in the previous section shows that correlation coefficient between analytical and MCMC PMPs is 0.994. The MCMC algorithm is based on the birth-death sampler which is most standard form of the sampler used under BMA framework. To check whether other samplers are not better in terms of the correlation coefficient, we run the same model based on the reversible-jump sampler. The correlation coefficient is slightly smaller for the reversible-jump sampler.

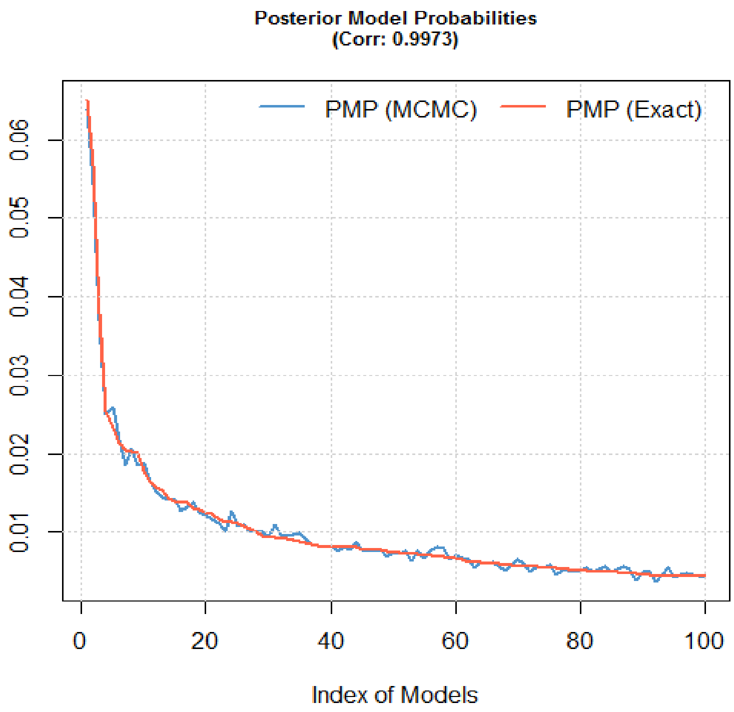
Figure 5.6: BMA with Different MCMC Samplers

BMA under Reversible-jump Sampler

Mean no. regressors	Draws	Burnins	Time	No. models visited
"10.4365"	"2e+06"	"1e+06"	"9.713742 mins"	"453936"
Corr PMP	No. Obs.	Model Prior	g-Prior	Shrinkage-Stats
"0.9923"	"72"	"uniform / 20.5"	"BRIC"	"Av=0.9994"
% Topmodels	Modelspace 2^K	% visited		
"47"	"2.2e+12"	"2.1e-05"		

BMA under Combined Sampler

Mean no. regressors	Draws	Burnins	Time	No. models visited
"10.4433"	"4e+06"	"2e+06"	"17.94971 mins"	"975688"
Corr PMP	No. Obs.	Model Prior	g-Prior	Shrinkage-Stats
"0.9965"	"72"	"uniform / 20.5"	"BRIC"	"Av=0.9994"
% Topmodels	Modelspace 2^K	% visited		
"46"	"2.2e+12"	"4.4e-05"		



Following the common practise we combine both samplers and get a better results. Figure 5.6 shows a comparative statics of the reversible-jump and birth-death samplers. MCMC sampler based on the combined sampler of the birth-death and reversible-jump samplers have a higher correlation coefficient compared to each sampler separately. This stresses the advantage of the combined sampler, hence the convergence of the MCMC results to the analytical likelihoods is more exact. Note that PMP correlation coefficient in the second section of figure 5.6 is slightly bigger than the reported in the table. This is due to the fact that the graph is based on top 100 models. Hence the correlation for these 100 models is higher compared to the other 4900 models which are chosen initially[3].

5.1.3 Densities of the Posterior Coefficients

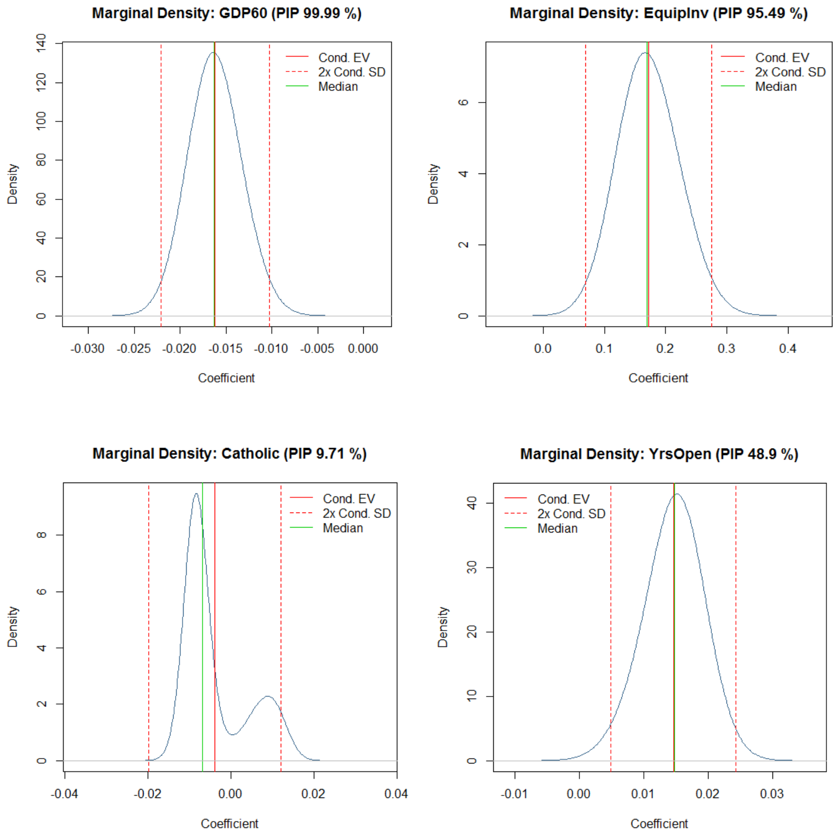
The descriptive statistics only gives the posterior mean of the coefficients. More detailed information about the coefficients can be observed on the density plots of the coefficients.

Figure 5.7 illustrates the densities of the selected posterior coefficients. The coefficient on GDP60 is negative and the integral of the density is 0.99865 which coincides with the analytical PMP of GDP60. The posterior mean of the GDP60 is negative for most of the models.

In contrast the coefficient on equipment investment is positive similar to the coefficient on the years of open economy. In line with Fernandez et al. (2001) the coefficient on Catholic is negative with mean around -0.003.

[3]see the command log and its description for more information about the number of models

Figure 5.7: Densities of Selected Coefficients



Chapter 6

Posterior Results for 21 OECD Countries

In the chapter four we describe the reasoning regarding the smaller number of explanatory variables for 21 OECD countries. Notably, 13 determinants (most of them are dummies) are dropped and one additional growth determinant - debt to GDP ratio is added, which has not been examined under the BMA framework. Hence there are 29 explanatory variables.

6.1 Top Models Including Debt to GDP Ratio

Similar to the baseline model in the previous section we take uniform model prior, birth-death MCMC sampler and Zellner's g-prior, which is k^{-2}. Figure 6.1 lists the descriptive statistics and top 5 models. In contrast the baseline class of models in the first section of the chapter five, the size of the model is bigger. The mean of the regressors is 13.63, hence the explanatory power of the top models is relatively higher compared to the baseline regression. This might also be due to the more similarities between countries and therefore similar economic growth determinants. The correlation coefficient between the analytical and MCMC PMPs is 0.9991 and hence higher than in the baseline regression. This is an indicator that the fit of the birth-death sampler to the analytical PMPs is improved.

The second section of the figure 6.1 lists top 5 models. The best model has 23.9 % that it describes the growth regression in a most efficient way and consists of the following 13 growth determinants: life expectancy, initial GDP per capita level, population growth, primary exports, ethnolinguistic fractionalization, degree of capitalism (EcoOrg), labour force, higher education enrolment, public education share, fraction speaking a foreign language, equipment investments, standard deviation of the black market premium (degree of uncertainty) and debt to GDP ratio.

Compared to the baseline model for the intuitive reason Sub-Saharan dummy, fraction Confucian, Muslim and Protestants are dropped and hence not included. Additionally, in contrast with the baseline model non-equipment investments are not included in the best model though being in the set of the explanatory variables. The same is true for the rule of low, which might be due to the similarities across the countries chosen for the second regression. Specifically, all OECD countries have similar level of the rule of law with two exceptions, this is why rule of law does not generate an explanation for the bigger growth rate. Thus, rule of law is not an important variable for the OECD countries.

It is worthy to note that there are two variables which stress the role of the education in the economic growth. These are the share of the public education and higher education enrolment rate. Those two variables are included in the best model and positively affect economic growth. This result is in line with the endogenous growth theories that human capital positively affects the economic growth. Higher education enrolment rate is used as a proxy of the human capital in Mankiw *et al.* (1992) as noted in the literature review. Additionally, fraction of the foreign language speaking population might be reflecting the level of the education in the country and essentially it is positively related with the economic growth similar to the higher education enrolment rate.

As in the baseline model initial level of the GDP per capita is included in the best model. Therefore the conditional convergence is presented too. Population growth is not included in any top 5 models in the baseline model, this might be due to the large differences among the sample. OECD countries are more similar in terms of the structure of the economy and also they seem to be the most closely representing the neo-classical production function and its features. Specifically, the scale effects are presented for 21 OECD countries.

Another difference is that the best model for OECD countries contains equipment investments excluding non-equipment investments. This result is important since both types of investments are included in the best model for the full sample. The reason might be fact that non-equipment investment does not have high return in the developed countries such as OECD. However, the return is high for the full sample, hence non-equipment investment is included

Figure 6.1: Descriptive Statistics for OECD Countries and Top 5 Models

Mean no. reg	Draws	Burnins	Time	No. models visited	Modelspace 2^K
"13.6297"	"2e+06"	"1e+06"	"7.45532 mins"	"247728"	"5.4e+08"
% visited	% Topmodels	Corr PMP	No. Obs.	Model Prior	g-Prior
"0.046"	"89"	"0.9991"	"21"	"uniform / 14.5"	"BRIC"
Shrinkage-Stats					
"Av=0.9988"					

Model	03465c55	3665455	03665c55	130f8315	03465c5d
Abslat	0	0	0	1	0
Area	0	0	0	0	0
PrScEnroll	0	0	0	0	0
LifeExp	1	1	1	1	1
GDP60	1	1	1	1	1
Mining	0	0	0	0	0
EcoOrg	1	1	1	0	1
YrsOpen	0	1	1	0	0
Age	0	0	0	0	0
Catholic	0	0	0	1	0
EthnoL	1	1	1	1	1
PrExports	1	1	1	1	1
Protestants	0	0	0	1	0
RuleofLaw	0	0	0	1	0
Popg	1	1	1	0	1
WorkPop	0	0	0	0	0
LabForce	1	1	1	0	1
HighEnroll	1	0	1	0	1
PublEdupct	1	1	1	0	1
PolRights	0	0	0	1	0
CivlLib	0	0	0	1	0
English	0	0	0	0	0
Foreign	1	1	1	0	1
RFEXDist	0	0	0	0	0
EquipInv	1	1	1	1	1
NequipInv	0	0	0	0	1
stdBMP	1	1	1	1	1
BlMktPm	0	0	0	0	0
Debt	1	1	1	1	1
PMP (Exact)	0.2390738	0.021245	0.01521576	0.01236061	0.01177836
PMP (MCMC)	0.2484205	0.020871	0.016012	0.0159035	0.012448

in the model for the full sample.

Unlike the full data sample the standard deviation of the black market premium is included in the best model. As noted in chapter four, this variable is a measure of the uncertainty. Hence for the developed countries small differences in the degree of the uncertainty affects the economic growth largely. This result is in line with the international trade theory, where movements of the investments across countries are modelled formally. Debt to GDP ratios is also included in the best model and emphasizes the fact that this fraction is an important determinant of the economic growth in the developed countries.

Figure 6.2 lists the PIPs, posterior means and other descriptive statistics. Similar to the baseline class of models initial level of GDP per capita has the highest PIP and is negatively related with the growth. However, the variable with the second highest PIP is equipment investment. This states that investments in the technology is an important variable. Specifically, an increase in equity investments by one unit generates a rise in the GDP per capita growth by 0.16%. Primary exports are the third variable with high PIP and is negatively related with the GDP growth level, similar to the baseline class of the models. This is in line with the results obtained by Fernandez *et al.* (2001a), though contradicts the results obtained by Sala-i-Martin et al. (2004).

Education proxies have negative estimates but the posterior probability of a coefficient being positive conditional on inclusion (fourth column) is 0.27.8%. Meaning that if the variable is included in the model the probability that it will be positive is nearly 28%. Additionally, posterior standard deviation of both variables are large indicating that the coefficient is not stable.

Both education proxies have to be positive, otherwise there is lack of the theoretical models or intuition supporting the negative sign on the estimates. Figure 6.2 gives posterior standard deviation and posterior probability of the coefficient being positive, which allows for the complete analysis of the coefficient sign.

Our contribution is to examine the new explanatory variable debt to GDP ratio under the BMA framework. The posterior inclusion probability of the debt to GDP ratio is 77%, which is the tenth highest PIP across all variables. As expected the estimate is positively related to the economic growth, since for all countries only with one exception the ratio is smaller than the broadly recognized threshold level 90%. Therefore, increase in the public debt is negatively related with the growth. In other words, countries with higher public debt level up to the threshold level have higher economic growth. This finding

Figure 6.2: Descriptive Statistics of the Posterior Coefficients

Coefficients Under Analytical Likelihoods

	PIP	Post Mean	Post SD	Cond.Pos.Sign	Idx
GDP60	0.99997	-0.01949	0.00572	0.00000	5
EquipInv	0.93596	0.16300	0.06880	0.99778	25
PrExports	0.85987	-0.01612	0.00772	0.00065	12
PublEdupct	0.85737	-0.13083	0.40768	0.27708	19
LabForce	0.82101	0.00000	0.00000	0.99642	17
Popg	0.80624	0.13083	0.55882	0.29018	15
LifeExp	0.80166	0.00129	0.00071	0.98483	4
stdBMP	0.79438	0.00101	0.00068	0.98363	27
EthnoL	0.78983	0.01084	0.00608	0.99112	11
Debt	0.76969	0.00552	0.00439	0.96949	29
HighEnroll	0.73875	-0.04578	0.05939	0.01240	18
Foreign	0.65806	-0.00441	0.00410	0.05769	23
EcoOrg	0.62265	0.00085	0.00072	0.99615	7
Protestants	0.37589	0.00297	0.00451	0.99159	13
PrScEnroll	0.29666	0.04264	0.07638	0.90244	3
NequipInv	0.26394	-0.02384	0.04685	0.05654	26
Abslat	0.21056	-0.00007	0.00017	0.10319	1
RuleofLaw	0.19531	-0.00470	0.01077	0.04308	14
Age	0.19343	-0.00001	0.00002	0.30132	9
CivlLib	0.18886	0.00050	0.00140	0.79996	21
PolRights	0.18592	-0.00160	0.00388	0.12036	20
BlMktPm	0.16880	-0.00054	0.00868	0.33490	28
YrsOpen	0.16201	-0.00168	0.00571	0.06241	8
RFEXDist	0.15454	0.00001	0.00003	0.79815	24
WorkPop	0.15182	0.00215	0.00821	0.74666	16
Catholic	0.13100	-0.00035	0.00133	0.10358	10
English	0.11338	0.00003	0.00107	0.45886	22
Mining	0.08789	-0.00052	0.00975	0.41233	6
Area	0.07579	0.00000	0.00000	0.57769	2

is in line with the existing literature. Although, there is a discussion regarding the validity of the results obtained by Reinhart *et al.* (2012). The negative correlation between debt and growth is empirically confirmed if the debt to GDP ratio exceeds 90%. However the causality is not well explored and none of the listed empirical works suggest causal effect which would be credible.

The posterior coefficient has an advantage compared to the single model estimate since the posterior mean is a weighted average across all models. Thus, we believe that debt to GDP ratio is a significant variable in explaining the growth due to the higher credibility of the BMA results compared single model coefficients. Specifically, a country can fasten its economic growth by borrowing money and investing in the various sectors which could increase the profitability of each unit of capital and hence rise in efficiency. The classical example would be public debt used to improve infrastructure which could stimulate economy.

Population growth is positively related with the economic growth and emphasizes that the scale effects are presented for OECD countries. The same is true for life expectancy and ethnolinguistic fractionalization. The rest of the coefficients can be interpreted in the similar fashion.

6.2 Combined MCMC Sampler

To obtain more accurate results we run reversible-jump sampler. Figure 6.3 shows the model description of the under reversible-jump sampler and combined model.The correlation coefficient between analytical and MCMC PMPs is the same (0.999) meaning that the sampler results converge to the "exact". However, the PIP of debt to GDP ratio is higher in case of the combined MCMC sampler. Specifically PIP of the debt to GDP ratio is 0.7841% and ranks eighth with PIP criteria across all growth determinants.

The posterior coefficient is also higher by one decimal, pointing that debt to GDP ratio has a larger size effect on the economic growth, which is positive.

Figure 6.4 illustrates the posterior model inclusion probabilities for top 5000 models with the corresponding sign indicators. The interpretation is similar to the figure 5.3.

6.2.1 Model Size and Prior Analysis

To analyze the effects of the prior model size on the posterior model size we use fixed and random model priors. Unlike the previous case the prior and

Figure 6.3: BMA under Different MCMC Samplers

BMA under Reversible-jump Sampler

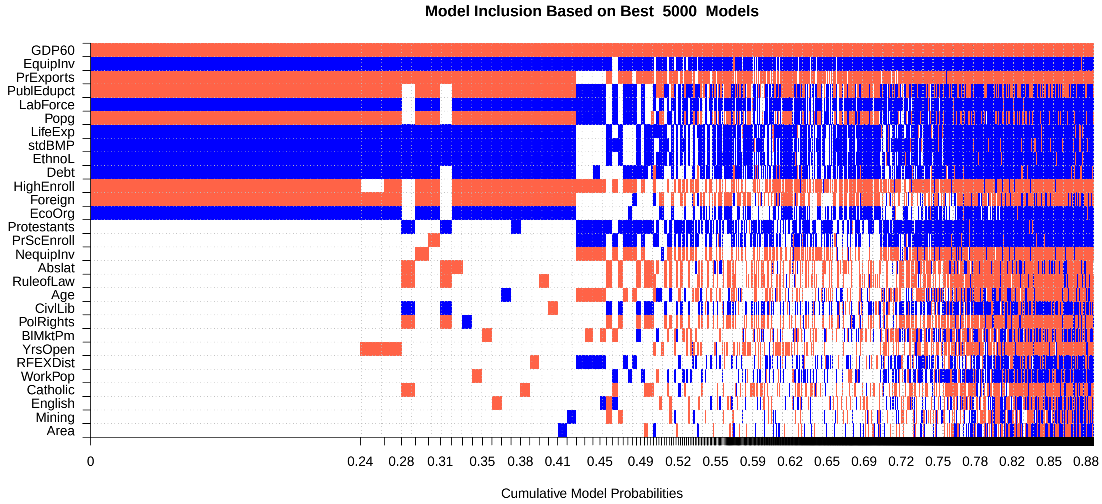
Mean no. regressors	Draws	Burnins	Time
"13.6881"	"2e+06"	"1e+06"	"9.098903 mins"
No. models visited	Modelspace 2^K	% visited	% Topmodels
"199342"	"5.4e+08"	"0.037"	"87"
Corr PMP	No. Obs.	Model Prior	g-Prior
"0.9986"	"21"	"uniform / 14.5"	"BRIC"
Shrinkage-Stats			
"Av=0.9988"			

BMA under Combined Sampler

Mean no. regressors	Draws	Burnins	Time
"13.6589"	"4e+06"	"2e+06"	"16.55422 mins"
No. models visited	Modelspace 2^K	% visited	% Topmodels
"447070"	"5.4e+08"	"0.083"	"88"
Corr PMP	No. Obs.	Model Prior	g-Prior
"0.9991"	"21"	"uniform / 14.5"	"BRIC"
Shrinkage-Stats			
"Av=0.9988"			

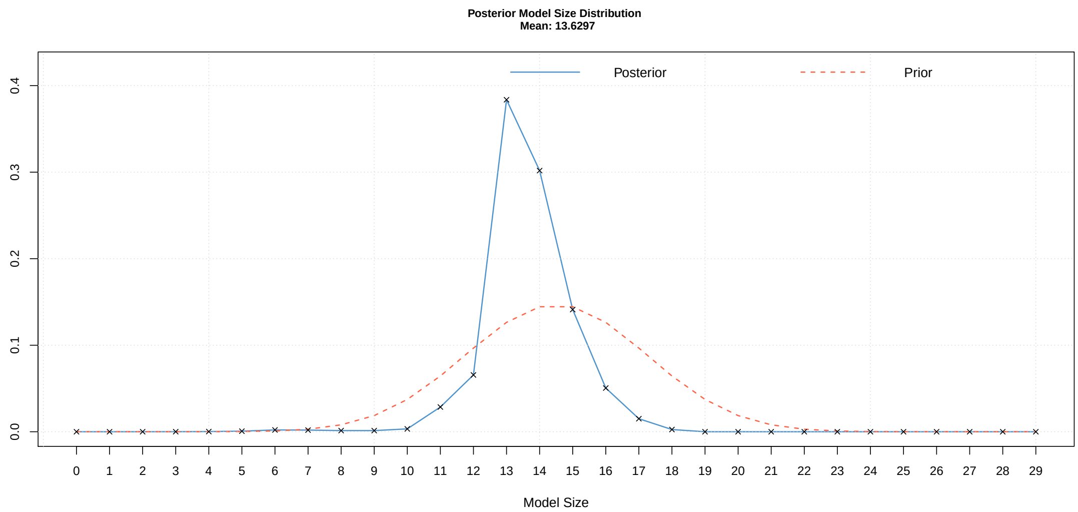
	PIP	Post Mean	Post SD	Cond.Sign	Idx
GDP60	0.974747	-0.01959	0.00680	0.0008831	5
EquipInv	0.936550	0.17261	0.06889	0.9975642	25
PrExports	0.902103	-0.01753	0.00776	0.0043349	12
LifeExp	0.839391	0.00135	0.00069	0.9860169	4
EthnoL	0.835358	0.01125	0.00581	0.9873943	11
PublEdupct	0.823586	-0.18367	0.37924	0.2127264	19
stdBMP	0.805576	0.00100	0.00070	0.9752609	27
Debt	0.784058	0.00677	0.00737	0.9691869	29
LabForce	0.763357	0.00000	0.00000	0.9889521	17
Popg	0.749206	0.08079	0.52029	0.2570755	15
HighEnroll	0.684898	-0.03554	0.05233	0.0286441	18
Foreign	0.661893	-0.00433	0.00436	0.0610435	23
EcoOrg	0.621721	0.00083	0.00079	0.9863243	7
Protestants	0.377688	0.00285	0.00534	0.9337396	13
Abslat	0.272974	-0.00009	0.00019	0.0980306	1
PrScEnroll	0.268966	0.03319	0.07054	0.8614731	3
NequipInv	0.243194	-0.01763	0.04171	0.1268513	26
CivlLib	0.235780	0.00071	0.00274	0.8037064	21
RuleofLaw	0.233947	-0.00495	0.01262	0.0885096	14
PolRights	0.228567	-0.00158	0.00483	0.1687188	20
YrsOpen	0.189726	-0.00133	0.00793	0.1766348	8
BlMktPm	0.188885	0.00124	0.01319	0.5126744	28
Age	0.177743	0.00000	0.00002	0.4675486	9
WorkPop	0.175031	0.00165	0.01087	0.6338143	16
Catholic	0.165816	-0.00074	0.00281	0.0833635	10
RFEXDist	0.150946	0.00001	0.00005	0.6803217	24
Mining	0.138885	0.00397	0.03479	0.5078284	6
English	0.134508	0.00011	0.00205	0.4401401	22
Area	0.093794	0.00000	0.00000	0.5461279	2

Figure 6.4: PMPs for Top 5000 Models



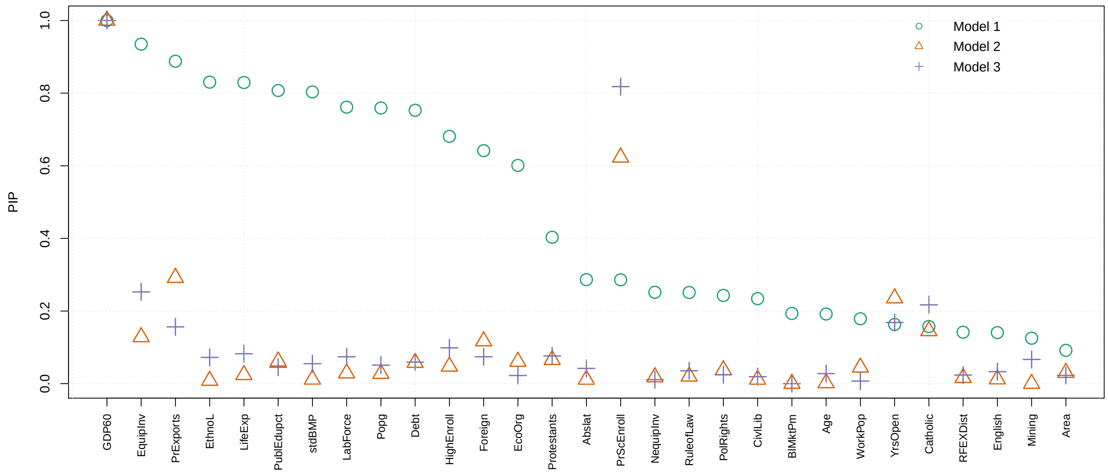
posterior model sizes are close to each other. From figure 6.5 it is visible that means of the model sizes are nearly the same. To detect the differences under

Figure 6.5: Prior and Posterior Model Sizes



different model prior specification we run BMA with fixed and random model priors. Figure 6.6 shows that fixed and random model priors fail to reflect the given data properties and assigns approximately the same PIPs to all variables. In contrast, uniform model prior generates reasonable PIPs. Additionally, the correlation coefficient is smaller by 0.03 for the fixed and random model prior specification pointing that the convergence does not occur as good as in case uniform model priors. Therefore, we only consider uniform model prior due to its tractable properties.

Figure 6.6: Model Comparisons



6.3 Densities of the Selected Coefficients

Some of the posterior coefficients have negative sign which is counter-intuitive. Part of the proof that the negativity of these coefficients might not be the case is provided in the previous section. Observing the densities of the coefficients is another possibility to conclude in which regions of the model space the coefficient is negative or positive.

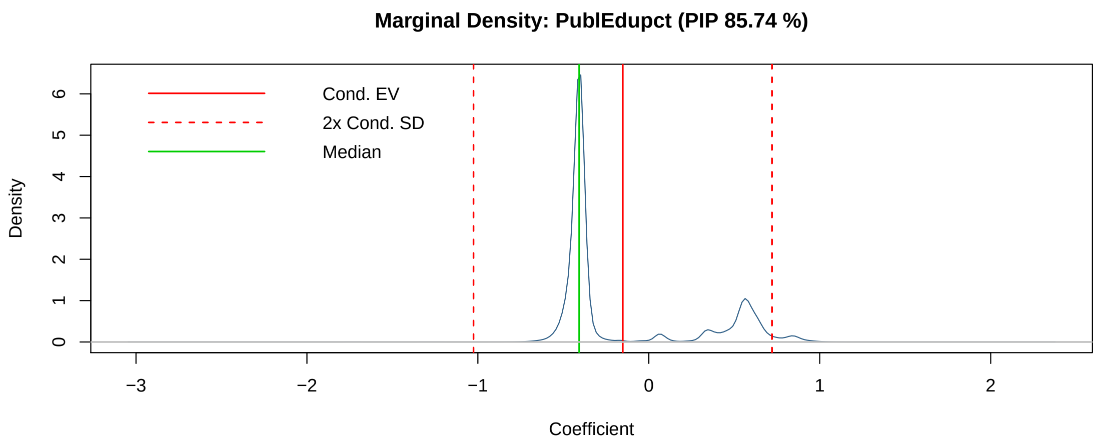
Figure 6.7 shows density of the public education share which has high PIP. Distributions for more variables are given in appendix 1.

6.4 Robustness Checks

6.4.1 GDP and Debt to GDP Ratio

Figure 6.8 shows the scatter plot between averaged GDP per capita growth and debt to GDP ratio. There is one outlier which we dropped and it is not included in the graph. Specifically, Korea has the highest average GDP per capita growth rate not only within the OECD countries, but within the full sample including 72 countries. Hence, we consider Korea as an outlier to see

Figure 6.7: Densities of Selected Coefficients



the real dependence between GDP growth and debt to GDP ratio. The highest GDP growth rate is given in table 4.1 and is .066179, which is nearly 7%. We run the BMA estimation excluding Korea, but the results do not change largely. It is worthy to note that the estimate on debt to GDP ratio is more positive when excluding Korea from the regression.

Figure 6.8 confirms that there is a strong positive correlation between the ratio and GDP growth. However, as emphasize in chapter two we get credible empirical evidence on the causality issue, since ratio gets limited values (see the descriptive statistics of the ratio - table 4.2).

6.4.2 Testing for Non-Linearities

We include debt squared to test for non-linear relation between the ratio and GDP growth. Taking into account the fact that all the ratios are between zero and one, by squaring the ratio we get a set of the values which are smaller than the ratio itself. In other words, by squaring the debt to GDP ratio we generate artificial set of the values which in reality corresponds to a smaller government debt. Following the abovementioned empirical works and our reasoning the estimate on the square of the ratio must be positive. We include this additional regrressor and obtain results which are presented in figure 6.9.

Debt.sqr denotes the square of the debt to GDP ratio. The estimate on the debt to GDP ratio squared is positive and PIP is higher than the PIP of the ratio itself. The same is true for the posterior means. This result is in line Reinhart *et al.* (2012) since the magnitude of the posterior mean is

Figure 6.8: Scatter Plot between GDP growth and Debt to GDP ratio

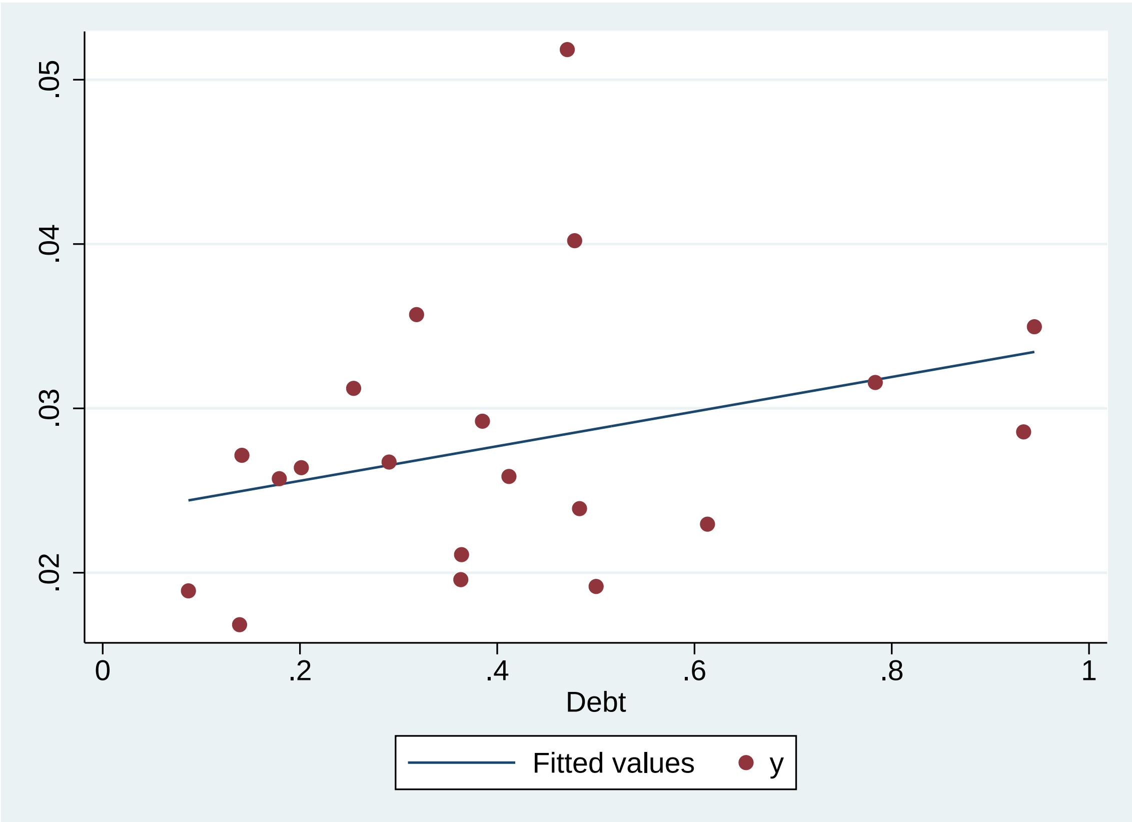


Figure 6.9: Descriptive Statistics for Debt to GDP Ratio Squared

Mean no. regressors	Draws	Burnins	Time
"13.1079"	"2e+06"	"1e+06"	"8.484885 mins"
No. models visited	Modelspace 2^K	% visited	% Topmodels
"298986"	"1.1e+09"	"0.028"	"90"
Corr PMP	No. Obs.	Model Prior	g-Prior
"0.9976"	"21"	"uniform / 15"	"BRIC"
Shrinkage-Stats			
"Av=0.9989"			

	PIP	Post Mean	Post SD	Cond.Pos.	Idx
GDP60	0.99978515	-0.02007	0.0024	0.000	5
EquipInv	0.98710989	0.19094	0.0330	1.000	25
PrExports	0.98102884	-0.02207	0.0044	0.000	12
PublEdupct	0.97233707	-0.42878	0.2177	0.038	19
EthnoL	0.96572688	0.01445	0.0032	0.999	11
LifeExp	0.96474173	0.00158	0.0004	0.999	4
stdBMP	0.95070644	0.00115	0.0003	0.998	27
Foreign	0.9322772	-0.00526	0.0020	0.008	23
Debt.sqr	0.86943172	0.00847	0.0042	0.989	30
Catholic	0.82976545	-0.00341	0.0020	0.002	10
LabForce	0.80269548	0.00000	0.0000	0.999	17
Age	0.37255045	0.00000	0.0000	0.939	9
Popg	0.34063647	-0.00020	0.2362	0.146	15
HighEnroll	0.33180285	-0.00380	0.0316	0.538	18
Debt	0.1779583	0.00061	0.0031	0.671	29
EcoOrg	0.156623	0.00013	0.0005	0.838	7
NequipInv	0.15511011	-0.00496	0.0205	0.019	26
BlMktPm	0.12152631	0.00054	0.0053	0.850	28
Protestants	0.10409394	0.00058	0.0022	0.867	13
Abslat	0.10005932	-0.00001	0.0001	0.568	1
PrScEnroll	0.09818493	0.00721	0.0352	0.739	3
YrsOpen	0.08821631	-0.00055	0.0036	0.157	8
CivlLib	0.07998224	0.00010	0.0006	0.764	21
English	0.07970128	-0.00003	0.0009	0.203	22
RuleofLaw	0.07651209	-0.00092	0.0051	0.090	14
WorkPop	0.075266	0.00014	0.0032	0.593	16
PolRights	0.07050122	-0.00028	0.0017	0.206	20
RFEXDist	0.06640243	0.00000	0.0000	0.346	24
Area	0.04009088	0.00000	0.0000	0.372	2
Mining	0.03925748	-0.00008	0.0034	0.364	6

0.00847 which is bigger than the posterior coefficient on the debt to GDP ratio excluding the square of the ratio. Specifically, with birth-death sampler the value of the posterior mean is 0.00552 (Figure 6.2) and with combined birth-death and reversible-jump sampler the posterior mean is 0.00677 (Figure 6.3). Therefore, smaller debt (we refer newly constructed variable - ratio-squared) generates higher posterior mean. In other words, given the lower debt to GDP ratio, increase in the ratio generates higher GDP growth, which is a consistent result. Additionally, we see that the PIP of the ratio is smaller when we include the square of the ratio.

Figure 6.10: Model Inclusion for the Best 5000 Models

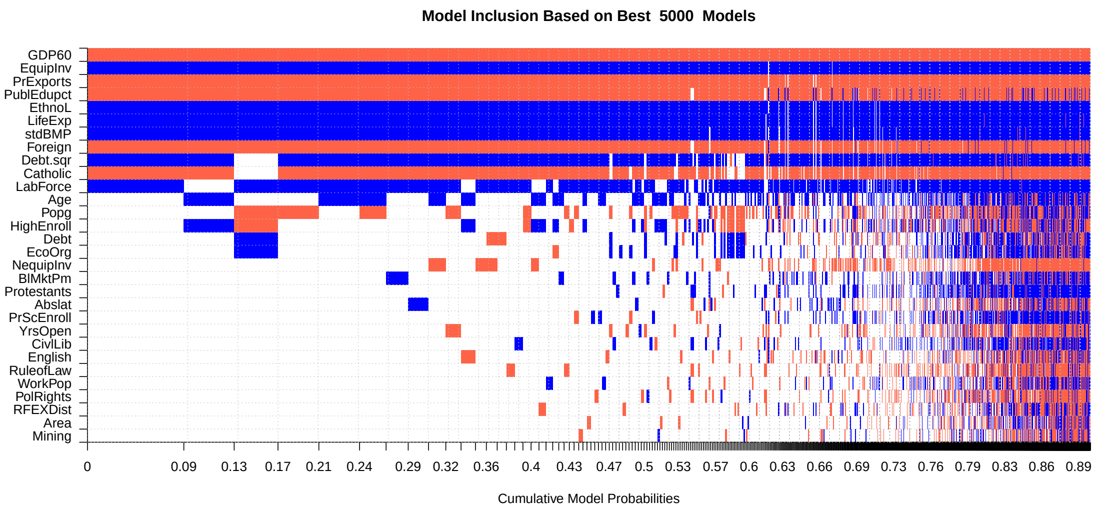


Figure 6.10 shows the similar chart for the best models. The sign of the coefficients and PIPs are interpreted in the same fashion as described in chapter 5. Figures 6.11 and 6.12 illustrate the posterior and prior model sizes and marginal density of the newly constructed variable - debt to GDP ratio squared. 6.12 confirms that the posterior mean of the ratio squared is positive for all model space.

6.5 Instrumental Variable BMA

In order to control for the possible endogeneity we follow Durlauf *et al.* (2008) and instrument debt. Then we include the fitted value of the debt explained by the other regressors in the main regression. In this way we avoid the endogeneity bias. By comparing results (PIPs and PMPs) one can conclude how robust

Figure 6.11: Posterior Model Size Distribution for OECD Countries

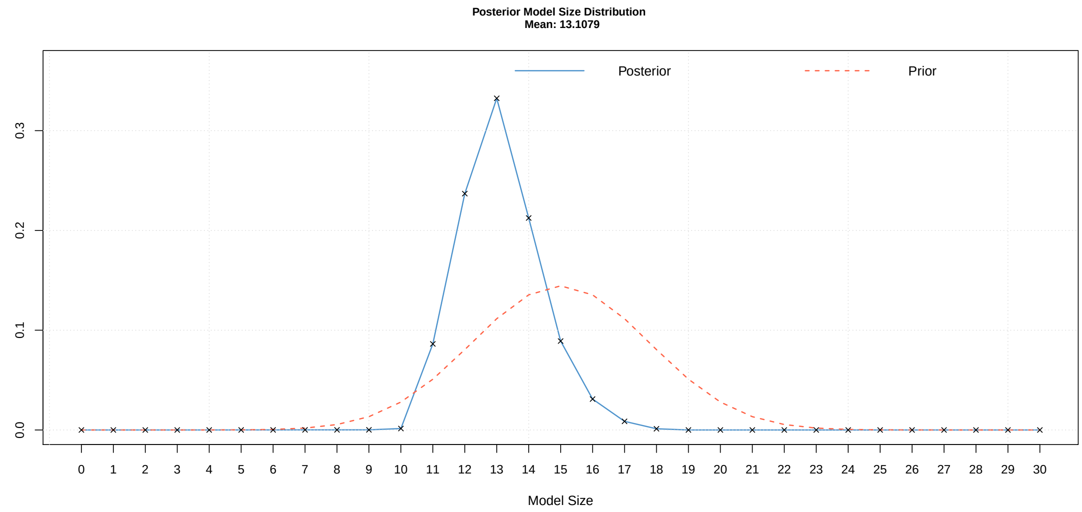
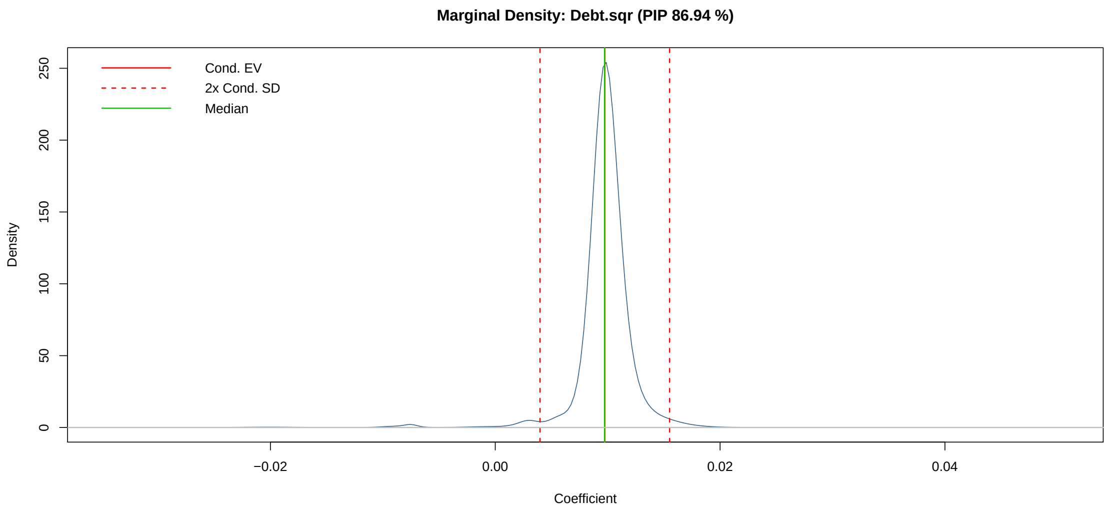


Figure 6.12: Marginal Density for OECD Countries



is the debt to GDP ratio. Due to the assumptions on the set of regressors mentioned in chapter four we do not include all explanatory variables when we estimate fitted values of the ratio. R-squared from the regression is 0.7816 including only 14 regressors and constant. F-value is small (1.53) meaning that we fail to reject that all variables are insignificant. Additionally adjusted R-squared is less than 0.3. Therefore we need additional variables to instrument debt better. Predicting fitted values and running BMA on the fitted debt to GDP ratio generates smaller PIP for debt to GDP ratio. But the estimate is still positive and inclusion probability is around 20%. The implications of the smaller PIP is that more explanatory variables are needed to instrument debt to GDP ratio.

Even though there is a smaller PIP for the debt to GDP ratio, the posterior mean is positive emphasizing the positive correlation. To test wheter

Figure 6.13: Posterior Coefficients, PMPs and PIPs for OECD Countries

	PIP	Post Mean	Post SD	Cond.Pos.	Idx
GDP60	0.9534	-0.0229	0.0096	0.0013	5
Protestan	0.8293	0.0074	0.0051	0.9855	13
EquipInv	0.8035	0.0895	0.0667	0.9887	25
PrExports	0.7063	-0.0153	0.0124	0.0041	12
PrScEnroll	0.6215	0.1086	0.1011	0.9830	3
stdBMP	0.6013	0.0002	0.0009	0.8239	27
LifeExp	0.5883	0.0006	0.0013	0.8577	4
Popg	0.5816	0.6558	0.8119	0.9362	15
EthnoL	0.5720	0.0071	0.0078	0.9426	11
PublEdup	0.5440	0.2390	0.3286	0.9136	19
RuleofLaw	0.5323	-0.0091	0.0227	0.1457	14
Abslat	0.5288	-0.0001	0.0003	0.1889	1
LabForce	0.5257	0.0000	0.0000	0.9632	17
HighEnroll	0.5225	-0.0749	0.0838	0.0468	18
NequipInv	0.5211	-0.0595	0.0706	0.0293	26
PolRights	0.5172	-0.0042	0.0062	0.0958	20
CivlLib	0.4952	0.0011	0.0027	0.8227	21
BlMktPm	0.3381	-0.0029	0.0181	0.2651	28
Age	0.3216	0.0000	0.0000	0.1160	9
RFEXDist	0.3203	0.0000	0.0001	0.7529	24
English	0.3111	0.0007	0.0037	0.6275	22
WorkPop	0.3015	0.0057	0.0198	0.8110	16
Mining	0.2854	-0.0039	0.0280	0.2081	6
Catholic	0.2751	-0.0009	0.0083	0.1484	10
YrsOpen	0.2642	-0.0053	0.0129	0.0582	8
Foreign	0.2180	-0.0012	0.0072	0.3074	23
debt	0.1847	0.0002	0.0102	0.8649	29
Area	0.1801	0.0000	0.0000	0.4421	2
EcoOrg	0.1337	0.0000	0.0026	0.6286	7

Chapter 7

Conclusion

We estimate growth determinants based on two different samples. The first sample consists of 72 countries and 41 explanatory variables. In line with the conditional convergence notion in the exogenous growth theories the initial level of GDP is strongly negatively related with the GDP per capita growth rate. Additionally, equipment investments and life expectancy are important long term growth determinants. The geographical and religion dummies have signs which are standard in macroeconomics. For example countries located in the Sub-Sahara have a lower economic growth compared to the other countries. Similarly, fraction Confucian is positively related with the growth.

The second sample consists of 21 OECD countries and 29 explanatory variables. Public debt to GDP ratio is one of them, which we construct. Running over half million models, we find that the debt to GDP ratio is an important variable in explaining growth. This result is in line with Reinhart *et al.* (2012). Unlike the other papers we include the debt to GDP ratio among 29 variables and test for any possible model specification. The posterior inclusion probability varies from 0.7 to 0.9 for different number of burn-ins and draws.

However, PIPs for the fitted debt to GDP ratio fall once debt is instrumented - stressing that the set of the explanatory variable may not be full. Additionally, results might be due to the measurement issues: some variables are given in levels in 1960, while the dependent variable - GDP per capita growth is an average for the period 1960-1992 similar to the debt to GDP ratio.

Due to the shortage of the data we do not address the causality between debt and GDP growth in this paper. Further research is needed to study the causality between debt and GDP growth. As mentioned in the prominent works in the second chapter there is no empirical evidence for this causality.

However, for sufficiently big number of iterations such as one million burn-ins and two million iterations, PIP for the debt to GDP ratio is around 0.8. The posterior mean is 0.0055 meaning that countries with one more percent of debt have 0.006 % higher growth rate. This could be suggestive for the policy makers to optimize the public debt level to attain high economic growth. We do not consider debt to GDP ratio more than threshold level - 90%. In contrast the ratio more than the threshold level is expected to be negatively related with the growth rate. This needs further research under the BMA framework, to make sure that the estimate is not based on the single model, rather it is a weighted average across all model space.

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Appendix A

Description of Variables and Countries

Figure A.1: List of Countries

Full Sample						OECD countries	
#	Country	code	#				
1	Algeria	DZ	37	Kenya	KE	Australia	AU
2	Argentina	AR	38	Korea	KR	Austria	AT
3	Australia	AU	39	Madagascar	MG	Belgium	BE
4	Austria	AT	40	Malawi	MW	Canada	CA
5	Belgium	BE	41	Malaysia	MY	Denmark	DK
6	Bolivia	BO	42	Mexico	MX	Finland	FI
7	Botswana	BW	43	Morocco	MA	France	FR
8	Brazil	BR	44	Netherlands	NL	Germany West	DE
9	Cameroon	CM	45	Nicaragua	NI	Ireland	IE
10	Canada	CA	46	Nigeria	NG	Italy	IT
11	Chile	CL	47	Norway	NO	Japan	JP
12	Colombia	CO	48	Pakistan	PK	Korea	KR
13	Congo	CG	49	Panama	PA	Mexico	MX
14	Costa Rica	CR	50	Paraguay	PY	Netherlands	NL
15	Cyprus	CY	51	Peru	PE	Norway	NO
16	Denmark	DK	52	Philippines	PH	Portugal	PT
17	Dom	DO	53	Portugal	PT	Spain	ES
18	Ecuador	EC	54	Senegal	SN	Sweden	SE
19	El Salvador	SV	55	Singapore	SG	Switzerland	CH
20	Ethiopia	ET	56	Spain	ES	Turkey	TR
21	Finland	FI	57	Sri Lanka	LK	United States	US
22	France	FR	58	Sweden	SE		
23	Germany West	DE	59	Switzerland	CH		
24	Ghana	GH	60	Taiwan	TW		
25	Greece	GR	61	Tanzania	TZ		
26	Guatemala	GT	62	Thailand	TH		
27	Haiti	HT	63	Tunisia	TN		
28	Honduras	HN	64	Turkey	TR		
29	HongKong	HK	65	Uganda	UG		
30	India	IN	66	United Kingdom	UK		
31	Ireland	IE	67	United States	US		
32	Israel	IL	68	Uruguay	UY		
33	Italy	IT	69	Venezuela	VE		
34	Jamaica	JM	70	Zaire	ZR		
35	Japan	JP	71	Zambia	ZM		
36	Jordan	JO	72	Zimbabwe	ZW		

Figure A.2: List of full set of Variables

Variable	Full Name	Source
y	Averaged economic growth 1960-1992	Penn World Tables Rev 6.0
Abslat	Absolute latitude	Barro (1996)
Spanish	Spanish colony dummy	Barro (1996)
French	French colony dummy	Barro (1996)
Brit	British colony dummy	Barro (1996)
WarDummy	War dummy	Barro and Lee (1995)
LatAmerica	Dummy for Latin American countries	
SubSahara	Dummy for Sub-Sahara African countries	
OutwarOr	Outward orientation	Levine and Renelt (1992)
Area	Area surface	Barro and Lee (1996)
PrScEnroll	Primary school enrolment	Barro and Lee (1996)
LifeExp	Life Expectancy	Barro and Lee (1996)
GDP60	Initial GDP in 1960	Barro and Lee (1996)
Mining	Fraction of GDP in mining	Hall and Jones (1996)
EcoOrg	Degree of capitalism	Hall and Jones (1996)
YrsOpen	Number of years having an open economy	Sachs and Warner (1996)
Age	Average age of population	Barro and Lee (1996)
Buddha	Fraction Buddhist	Barro (1996)
Catholic	Fraction Catholic	Barro (1996)
Confucian	Fraction Confucian	Barro (1996)
EthnoL	Ethnolinguistic fractionalization	Easterly and Levine (1996)
Hindu	Fraction Hindu	Barro (1996)
Jewish	Fraction Jewish	Barro (1996)
Muslim	Fraction Muslim	Barro (1996)
PrExports	Primary exports 1960	Sachs and Warner (1996)
Protestants	Fraction Protestants	Barro (1996)
RuleofLaw	Rule of Law	Barro (1996)
Popg	Average population growth 1960-1992	Barro and Lee (1995)
WorkPop	Ratio workers to population	Barro and Lee (1995)
LabForce	Size of labor force	Barro and Lee (1995)
HighEnroll	Higher education enrollment	Barro and Lee (1995)
PublEdupct	Public education spending (fraction of GDP)	Barro and Lee (1995)
RevnCoup	Number of revolutions and military coups	Barro and Lee (1995)
PolRights	Political rights	Barro (1996)
CivlLib	Civil liverties	Knack and Keefer (1995)
English	Fraction of population speaking English	Hall and Jones (1996)
Foreign	Fraction speaking foreign language	Hall and Jones (1996)
RFEXDist	Exchange rate distortions	Barro and Lee (1995)
EquipInv	Equipment investment	Delong and Summers (1991)
NequipInv	Non-equipment investment	Delong and Summers (1991)
stdBMP	Standard deviation of black market premium	Levine and Renelt (1992)
BlMktPm	Logarithm of (1+black market premium)	Barro and Lee (1995)

Figure A.3: List of Variables for OECD Countries

Variable	Full Name	Source
y	Averaged economic growth 1960-1992	Penn World Tables Rev 6.0
Abslat	Absolute latitude	Barro (1996)
Area	Area surface	Barro and Lee (1996)
PrScEnroll	Primary school enrolment	Barro and Lee (1996)
LifeExp	Life Expectancy	Barro and Lee (1996)
GDP60	Initial GDP in 1960	Barro and Lee (1996)
Mining	Fraction of GDP in mining	Hall and Jones (1996)
EcoOrg	Degree of capitalism	Hall and Jones (1996)
YrsOpen	Number of years having an open economy	Sachs and Warner (1996)
Age	Average age of population	Barro and Lee (1996)
Catholic	Fraction Catholic	Barro (1996)
EthnoL	Ethnolinguistic fractionalization	Easterly and Levine (1996)
PrExports	Primary exports 1960	Sachs and Warner (1996)
Protestants	Fraction Protestants	Barro (1996)
RuleofLaw	Rule of Law	Barro (1996)
Popg	Average population growth 1960-1992	Barro and Lee (1995)
WorkPop	Ratio workers to population	Barro and Lee (1995)
LabForce	Size of labor force	Barro and Lee (1995)
HighEnroll	Higher education enrollment	Barro and Lee (1995)
PublEdupct	Public education spending (fraction of GDP)	Barro and Lee (1995)
PolRights	Political rights	Barro (1996)
CivlLib	Civil liverties	Knack and Keefer (1995)
English	Fraction of population speaking English	Hall and Jones (1996)
Foreign	Fraction speaking foreign language	Hall and Jones (1996)
RFEXDist	Exchange rate distortions	Barro and Lee (1995)
EquipInv	Equipment investment	Delong and Summers (1991)
NequipInv	Non-equipment investment	Delong and Summers (1991)
stdBMP	Standard deviation of black market premium	Levine and Renelt (1992)
BlMktPm	Logarithm of (1+black market premium)	Barro and Lee (1995)
Debt	Debt to GDP Ratio	Own calculations

Figure A.4: Coefficients Based on the Analytical Likelihoods - Full Sample

	PIP	Post Mean	Post SD	Cond.Pos.Sign	Idx
GDP60	0.99987	-0.01619	0.00297	0.00000	12
Confucian	0.99859	0.05632	0.01280	1.00000	19
EquipInv	0.95487	0.16442	0.06165	1.00000	38
LifeExp	0.95473	0.00084	0.00031	1.00000	11
SubSahara	0.77570	-0.01196	0.00794	0.00000	7
Muslim	0.67043	0.00872	0.00721	0.99980	23
RuleofLaw	0.54516	0.00836	0.00849	1.00000	26
EcoOrg	0.49156	0.00134	0.00149	1.00000	14
YrsOpen	0.48901	0.00713	0.00804	1.00000	15
Protestants	0.47059	-0.00592	0.00707	0.00000	25
NequipInv	0.43631	0.02541	0.03200	1.00000	39
Mining	0.42142	0.01705	0.02242	1.00000	13
LatAmerica	0.16859	-0.00146	0.00365	0.02633	6
PrScEnroll	0.16292	0.00346	0.00872	0.99615	10
Buddha	0.14115	0.00180	0.00499	1.00000	17
BlMktPm	0.13367	-0.00105	0.00298	0.00000	41
Catholic	0.09707	-0.00038	0.00273	0.26558	18
Hindu	0.08186	-0.00199	0.00786	0.02325	21
CivlLib	0.07719	-0.00018	0.00071	0.00068	34
PrExports	0.05228	-0.00053	0.00260	0.00000	24
PolRights	0.04974	-0.00008	0.00041	0.00245	33
RFEXDist	0.04648	0.00000	0.00001	0.01643	37
Age	0.03680	0.00000	0.00001	0.00000	16
WarDummy	0.03247	-0.00013	0.00082	0.00000	5
Foreign	0.03128	0.00015	0.00103	0.95459	36
English	0.03064	-0.00019	0.00132	0.00000	35
LabForce	0.02750	0.00000	0.00000	0.81745	29
EthnoL	0.02051	0.00012	0.00108	0.96830	20
stdBMP	0.01989	0.00000	0.00000	0.00977	40
Spanish	0.01970	0.00008	0.00089	0.89378	2
French	0.01854	0.00008	0.00074	0.99647	3
Abslat	0.01458	0.00000	0.00002	0.41235	1
WorkPop	0.01271	-0.00008	0.00116	0.10208	28
Popg	0.01247	0.00203	0.02667	0.96645	27
HighEnroll	0.01184	-0.00037	0.00499	0.00000	30
Brit	0.01164	-0.00002	0.00034	0.03125	4
OutwarOr	0.01061	-0.00002	0.00030	0.09043	8
Jewish	0.01029	-0.00008	0.00151	0.16786	22
RevnCoup	0.00914	0.00000	0.00056	0.60309	32
PublEdupct	0.00852	0.00025	0.01299	0.59625	31
Area	0.00771	0.00000	0.00000	0.15533	9

Figure A.5: Posterior Model Size - Full Sample

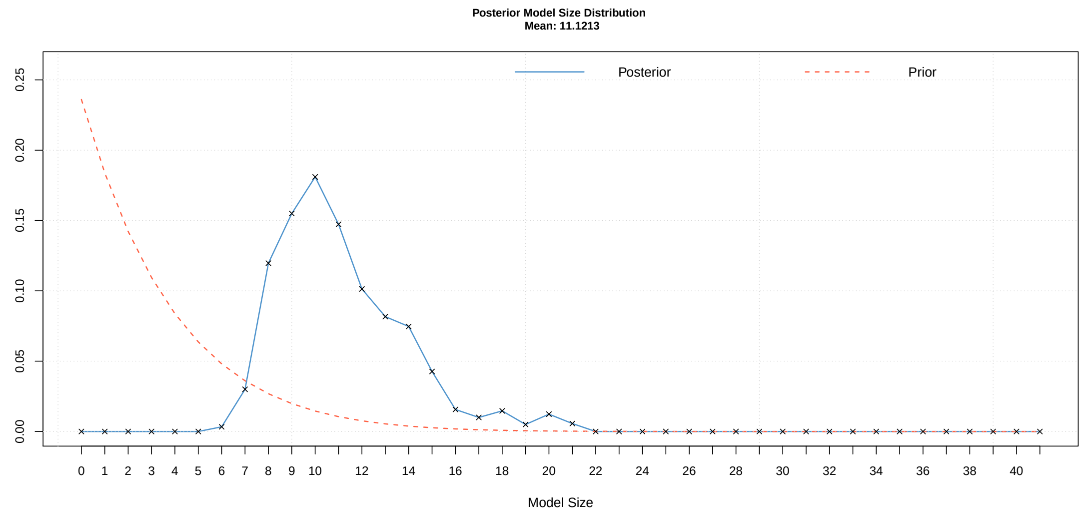

Figure A.6: Explanation-Based Learning (EBL) g-prior - Full Sample

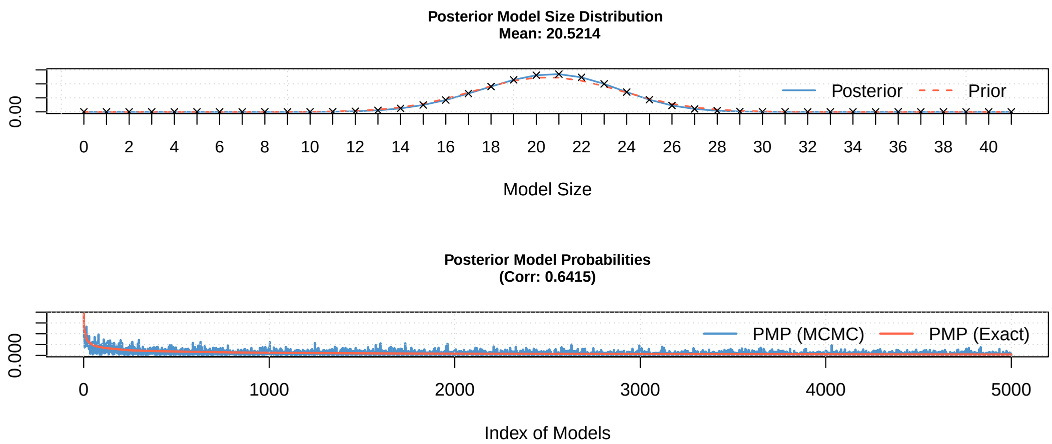

Figure A.7: Marginal Densities - OECD Countries

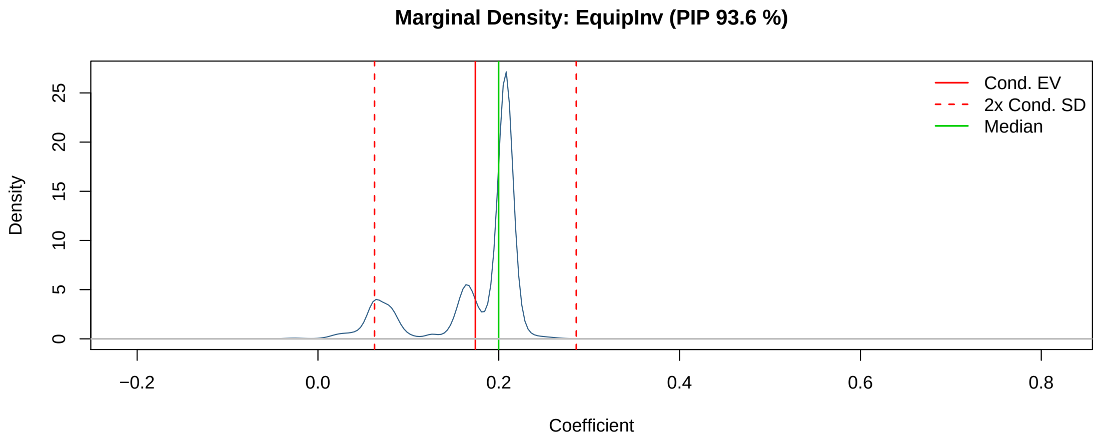

Figure A.8: Marginal Densities - OECD Countries

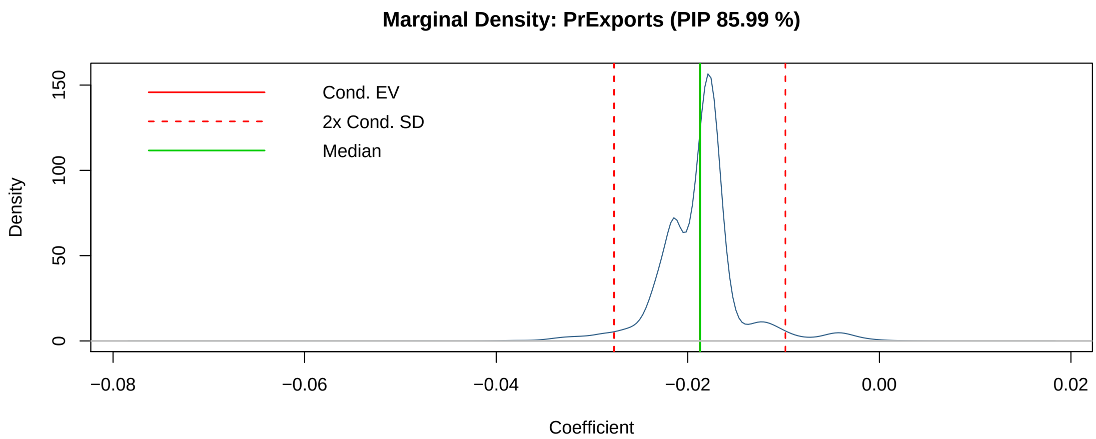

Figure A.9: Marginal Densities - OECD Countries