The compressible Euler equations describe the motion of compressible inviscid fluids. They are used in many areas ranging from aerospace, automotive, and nuclear engineering to chemistry, ecology, climatology, and others. Mathematically, the compressible Euler equations represent a hyperbolic system consisting of several nonlinear partial differential equations (conservation laws). These equations are solved most frequently by means of Finite Volume Methods (FVM) and low-order Finite Element Methods (FEM). However, both these approaches are lacking higher order accuracy and moreover, it is well known that conforming FEM is not the optimal tool for the discretization of first-order equations. The most promissing approach to the approximate solution of the compressible Euler equations is the discontinuous Galerkin method that combines the stability of FVM, with excellent approximation properties of higher-order FEM. The objective of this Master Thesis was to develop, implement and test new adaptive algorithms for the nonstationary compressible Euler equations based on higher-order discontinuous Galerkin (hp-DG) methods. The basis for the new methods were the discontinuous Galerkin methods and space-time adaptive hp-FEM algorithms on dynamical meshes for nonstationary second-order problems. The new algorithms were implemented and tested in the framework of the open source library Hermes.