**Report of supervisor**

The thesis of Martin Zdráhal is devoted to the analysis of the low energy structure of the pseudoscalar meson amplitudes for the processes $2 \to 2$ in general and (via crossing symmetry) to the applications of the general results to the decays $P \to 3\pi$, where $P$ stays for eta meson and kaons. At present the latter processes are in center of interest thanks to an extensive increase of the corresponding experimental data samples during recent years and due to further expected improvements in the near future. The higher statistics has enabled to measure precisely the detailed structure of the Dalitz plot (let us mention here e.g. a considerable increase of precision in the determination of the Dalitz plot parameters of the $\eta \to 3\pi$ decays obtained by KLOE and a detailed study of the cusp structure due to the $\pi^+\pi^- \to \pi^0\pi^0$ re-scattering in the final state in the $K^\pm \to \pi^\pm\pi^0\pi^0$ and $K_L \to 3\pi^0$ decays performed by NA48 and KTeV collaborations). This improvement of the experimental knowledge has stimulated intensive theoretical interest mainly because these processes provide a direct access to the isospin breaking parameter $R = (m_s - \widehat{m})/(m_d - m_u)$ in the case of the decay $\eta \to 3\pi$, which is forbidden in the isospin limit, and to the precise determination of the $\pi\pi$ scattering lengths from the cusp effects.

In the thesis, the most general parametrization of the amplitudes for the processes mentioned above are constructed using very general assumptions of analyticity, unitarity, crossing symmetry and the existence of the chiral expansion in terms of the powers (and logs) of the momenta and light quark masses. The method corresponds to a generalization of the dispersive approach developed by Jan Stern and collaborators (known as the "reconstruction theorem" ) which was originally used for the analysis of $\pi\pi$ scattering in the isospin limit. Within the thesis this framework is adapted to the case of different masses of the particles under consideration and to the implementation of the isospin breaking effects.

The thesis consists of five parts (which are further divided into ten chapters) and twelve appendices. The first part deals with the general aspects of the reconstruction theorem in the case of general $2 \to 2$ process including particles with different masses and discusses in detail the construction of the amplitudes by means of the iteration of the two-particle unitarity. As an example of the application of the general procedure the most general parametrization of all the independent amplitudes of the scattering processes with the pseudoscalar octet in the isospin limit is constructed to the order $O(p^4)$ in the chiral expansion.

The second part is devoted to application of the general recipe to the $\pi\pi$ scattering. The general amplitude including strong isospin breaking effects due to the different masses of the neutral and charged pion is constructed and a parametrization in terms of the scattering lengths suitable for the studies of the cusp effects is developed to the order $O(p^6)$. Also the parametrization much suitable for the subsequent construction of the $\eta$ decay amplitude is found.

The $P \to 3\pi$ decays are discussed in the third part. Here the results concerning both the equal pion mass case (i.e. isospin limit for $K$ decays and leading order in the isospin breaking for the $\eta$ decay) as well as the isospin breaking case (to the $O(p^4)$ in the general and to the $O(p^6)$ for $P^0 \to 3\pi^0$ decays) are presented in detail.

The main phenomenological application of the above general method, namely the precise determination of the isospin breaking parameter $R$ is presented in the fourth part of the thesis. The method is based on two independent ways how to fit the free parameters of

the amplitude of the $\eta \to 3\pi$ decay (to the first order in the isospin breaking) from the available experimental data obtained by experiment KLOE and subsequent matching with chiral perturbation theory.

The necessary technicalities as well as complete list of the (very long) formulae for the $O(p^4)$ partial waves and polynomials which are the part of the complete $O(p^6)$ amplitudes are postponed to the appendices.

The content of the thesis summaries and presents further details and enlargements of the several years of research performed within a group of four collaborators from three institutions (IPNP Prague, CPT Marseilles and Lund University). The results have been referred by Martin Zdráhal or other members of the team in several conferences and published in two journal articles and six conference proceedings written either with co-authors or independently. Further two papers are being prepared for publication.

Martin Zdráhal proved to be a very valuable member of the team. He contributed to the final results either doing independent paralell calculations and checks, or adding his own original calculations and numerical analysis. He calculated e.g. the complete set of the $O(p^4)$ pseudoscalar meson $2 \to 2$ amplitudes in the isospin limit, developed the parametrization of the $\pi\pi$ amplitude by means of the scattering lengths as an alternative to the subthreshold parametrization used sofar, completed the final general form of the isospin breaking $O(p^6)$ amplitude for the $\pi\pi$ scattering and for isospin breaking amplitude for $P^0 \to 3\pi^0$ decays, independently checked the numerical analysis leading to the determination of the isospin breaking parameter $R$ and independently studied the influence of the $\pi\pi$ scattering inputs to this result. He also carefully analyzed the impact of the result for $R$ to the determination of the $u$ and $d$ quark masses and studied in detail some important theoretical aspect of the method like the proof of the existence of the dispersion relations from axiomatic field theory and some aspects of the analytical properties of the relevant two loop graphs. Last but not least, he also collected and carefully administrated all the relevant Mathematica files necessary for the analysis, which are now thanks to him prepared for any further practical use.

During the work on the topic of the thesis Martin Zdráhal has shown both enthusiasm and enormous diligence as well as talent and invention. He clearly proved his ability to work as an independent researcher and therefore I strongly recommend to award to Martin Zdráhal the PhD degree.

In Prague 26th February 2012

RNDr. Jiří Novotný, CSc.,
supervisor of the thesis