Referee report

For the doctoral thesis: *General relativity in higher dimensions* by Tomas Malek
Referee: Joris Raeymaekers

This thesis deals with higher dimensional general relativity in the presence of a cosmological constant. The first part discusses the general properties of the Kerr-Schild type metrics, in particular where they fit in the classification of higher dimensional metrics according the algebraic type of the Weyl tensor. The second part deals with higher derivative extensions of general relativity, and algebraically special solutions to their equations of motion. Let me first summarize the results presented in the thesis and then comment on originality and applicability of those results.

In chapter 1, the author gives a particularly nice and clear review of the algebraic classification of spacetimes in general dimensions according to the properties of the Weyl tensor. He also discusses a solution generating technique (called the Brinkmann warp product) product which generates Einstein manifolds from a lower dimensional Einstein seed metric. This technique enters in many branches of higher dimensional physics, e.g. in string theory in the construction of a Ricci flat Kahler manifold as a cone over a Sasaki-Einstein manifold.

Chapter 2 explores the properties of higher dimensional Kerr-Schild metrics with (A)dS backgrounds, in particular their algebraic properties. Attention is focused on the case where the Kerr-Schild vector is geodetic, in which case the spacetime is of type II or more special. For Einstein spaces where the geodesic congruence is nonexpanding, it is shown that the spacetime is of algebraic type N, and belongs to the Kundt class. When it is nonexpanding, it is shown that it must be of type II or D. Furthermore, an interesting result is derived on the form of the optical matrix, which generalizes the 4D Goldberg-Sachs theorem to higher dimensional metrics of the Kerr-Schild form. The author also discusses the issue of curvature singularities and provides examples.

In chapter 3, a similar analysis is initiated for metrics of an extended Kerr-Schild form, involving and additional spacelike vector field. When the null congruence is geodetic, such spacetimes are shown to be of type I or more special. If the metric is furthermore of the Kundt class, conditions are found under which it is of type II or more special, and examples are given. In the expanding case, attention is restricted to the CCLP black hole solution, and its algebraic type and optical matrix are computed.

Chapter 4 deals with quadratic gravity, which adds generic curvature squared corrections to the Einstein action which arise, for example, as low energy effective actions in string theory in string theory. First, the author investigates under which conditions Einstein spaces are solutions of these extended theories. It turns out that all type N Einstein spaces solve the equations of motion (for appropriately chosen value of the curvature radius). For type III Einstein spaces, a necessary and sufficient condition is derived under which they are solutions, and examples are given. The analysis is then extended to include a null radiation term in the Ricci tensor, and conditions are found under which spaces of type II, D and O are soutions, and further examples are provided.

Let me now discuss the originality and applicability of the results. The main new ingredient in chapter II is the inclusion of a cosmological constant, the zero cosmological constant case having been worked out in ref. [27]. Although the final results about the algebraic type and the optical matrix are the same as in the vanishing cosmological constant case, their derivation is nontrivial and doesn't follow from earlier work. I think these general results will certainly prove useful for the construction of algebraically special solutions. The results on the form of the optical matrix are especially nice since they hint at the higher dimensional generalization of the Goldberg-Sachs theorem, which as far as I know remains an open problem. Some of the concrete examples discussed and their properties were known before and hence serve more as a check of the formalism, although I believe some of the ones presented in section 2.5.2 are indeed new. Chapter III contains original and unpublished results for extended Kerr-Schild spacetimes, although the results are not yet as complete as for the Kerr-Schild ones in chapter II. Nevertheless the result on the algebraic properties of this class should prove useful. Unfortunately the approach has not yet yielded new solutions as far as I can see, although interesting existing ones do fall into this class. Finally, chapter 4 presents new general results on solutions to quadratic gravity and various new examples. These should certainly prove useful, for example to the string theory community.

In all, I think this is a well-written and clearly presented thesis containing, as far as I can see, careful and solid research. It contains sufficient new results which should be useful within the field of general relativity but also in neighboring areas such as supergravity, string theory and especially the AdS/CFT correspondence. The only small criticism I have is that I missed some discussion of the physics, especially for the new solutions presented. This could be partly due to the fact that the algebraic classification technique in higher dimensions is relatively new, and has not yet generated as much interesting physics as it has in four dimensions. Yet, with with the help of solid mathematical results as the ones presented in this thesis I am hopeful that this will be remedied in the future.

Joris Raeymaekers

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