Referee report on the doctoral thesis

General Relativity in Higher Dimensions

by Mgr. Tomáš Málek

The thesis represents an interesting and original contribution to the study of exact spacetimes within the context of higher dimensional gravity. Its topic naturally fits into the contemporary research in theoretical physics which is motivated by various attempts to generalize and extend Einstein's general relativity, to quantize it or (hopefully) even to unify it with other forces which occur in nature. Apart from these motivations, rigorous investigation of exact solutions of generalized Einstein gravity equations is also important from purely mathematical point of view. It may elucidate in what sense are four-dimensional spacetimes privileged.

To be specific, the author investigates properties of the family of Kerr-Schild-type spacetimes, both nonexpanding and expanding, in an arbitrary dimension. The original ansatz, which proved to be fruitful in four-dimensional general relativity, is also generalized in various ways: the background need not be just a flat space, but everywhere curved (anti-)de Sitter space of any dimension [generalized Kerr-Schild metrics (GKS) studied in chapter 2], additional terms need not correspond just to a null congruence but "spacelike terms" may also be present [extended Kerr-Schild metrics (xKS) described in chapter 3], or solutions in a wider context of quadratic gravity, whose action contains curvature squared terms, [chapter 4] are considered.

The work contains a lot of new results which were obtained by a skillful combination of various methods (algebraic classification of higher dimensional spacetimes, generalization of the optical scalars and the NP formalism, the Brinkmann warp construction method etc.). The core of the thesis is based on two papers published in top professional journals *Classical and Quantum Gravity* and *Physical Review D*, which by itself manifests the high quality of the work.

The technical quality is excellent. The thesis is well-organized and, in spite of its large length, there are almost none typographical or language mistakes. The list of relevant references is adequate, and relations to previous works are properly included.

To summarize, in my view the submitted thesis is of a high quality. In all aspects it fully satisfies the required conditions and demonstrates the ability of the author to perform independently a creative research. I am thus happy to recommend its acceptation as a doctoral thesis at the Faculty of Mathematics and Physics of the Charles University.

To open a discussion, I would suggest the following three questions:

1) It is often convenient to represent the n-dimensional (anti-)de Sitter space as a hyperboloidal subspace of $(n+1)$-dimensional flat space. This was useful, e.g., in [48] for the construction of photon rocket spacetimes with an arbitrary cosmological constant. Would it be useful to apply such an "embedding method" for derivation of the GKS spacetimes from the KS spacetimes? Technically, this would arise by taking the background metric $\bar{g}_{ab}$ in (2.11) to be a hyperboloidal

subspace of a standard *flat* background with one additional spatial dimension, instead of its explicit coordinate representations (2.9), (2.10).

2) What are the differences between odd and even dimensional Kerr-NUT-(A)dS metrics? In particular, the metric functions H and H_μ in (2.229) and (2.232) seem to be different. Isn't there a typo?

3) Please, clarify the relation between chapters 2,3 and chapter 4. Namely: What are the Kerr-Schild spacetimes in quadratic gravity, and how they differ from those obtained in the theory corresponding to the standard Einstein-Hilbert action?

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