Title: Collections of compact sets in descriptive set theory
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Abstract: This work consists of three articles.

In Chapter 2, we dissert on the connections between complexity of a function f from a Polish space X to a Polish space Y and complexity of the set $C(f) = \{K \in \mathcal{K}(X);\ f\restriction_K \text{ is continuous}\}$, where $\mathcal{K}(X)$ denotes the space of all compact subsets of X equipped with the Vietoris topology. We prove that if $C(f)$ is analytic, then f is Borel; and assuming $\mathbf{\Delta}^1_2$-Determinacy we show that f is Borel if and only if $C(f)$ is coanalytic. Similar results for projective classes are also presented.

In Chapter 3, we continue in our investigation of collection $C(f)$ and also study its restriction on convergent sequences ($\widetilde{C}(f)$). We prove that $\widetilde{C}(f)$ is Borel if and only if f is Borel. Similar results for projective classes are also presented.

The Chapter 4 disserts on H^N-sets, which form an important subclass of the class of sets of uniqueness for trigonometric series. We investigate the size of these classes which is reflected by the family of measures called polar which annihilate all the sets belonging to the given class. The main aim of this chapter is to answer in the negative the question stated by Lyons, whether the polars of the classes of H^N-sets are same.

Keywords: Descriptive set theory, compact sets, continuity, harmonical analysis, H^N-sets, sets of uniqueness.