Cosmic black holes can act as agents of particle acceleration. We study properties of a system consisting of a rotating black hole immersed in a large-scale organized magnetic field. Electrically charged particles in the immediate neighborhood of the horizon are influenced by strong gravity acting together with magnetic and induced electric components. We relax several constraints which were often imposed in previous works: the magnetic field does not have to share a common symmetry axis with the spin of the black hole but they can be inclined with respect to each other, thus violating the axial symmetry. Also, the black hole does not have to remain at rest but it can instead perform fast translational motion together with rotation. We demonstrate that the generalization brings new effects. Starting from uniform electro-vacuum fields in the curved spacetime, we find separatrices and identify magnetic neutral points forming in certain circumstances. We suggest that these structures can represent signatures of magnetic reconnection triggered by frame-dragging effects in the ergosphere. We further investigate the motion of charged particles in these black hole magnetospheres. We concentrate on the transition from the regular motion to chaos, and in this context we explore the characteristics of chaos in relativity. For the first time, we apply recurrence plots as a suitable technique to quantify the degree of chaoticness near a black hole.