Report of the supervisor on the PhD thesis by

Norman Gürlebeck

Matter Models in General Relativity with a Decreasing Number of Symmetries

Norman Gürlebeck finished his master degree at the Institute of Theoretical Physics of the Friedrich-Schiller University in Jena, with which we have had long scientific interactions for many years. Due to a very high level of the relativity group there he was well prepared to proceed with research in general relativity in our group.

In fact, one of the problems included in the thesis, is concerned with the axially symmetric, stationary and rigidly rotating dust configurations, which he started to work on in Jena already but finished it and wrote the paper at the beginning of his studies in Prague. By elegant mathematical methods he has shown that all such configurations are fully determined by giving the density along the rotation axis. The work contains also useful discussion of the Newtonian case, treats in detail the cylindrically symmetric situation and proves the non-existence of homogeneous rigidly rotating dust configurations. Several interesting open questions about possible generalizations and study of the external regions are mentioned at the end of the corresponding part of the thesis.

Because of lower number of symmetries, the previous work forms Chapter 4 of the thesis. After a brief Introduction (Chapter 1), which motivates the problems tackled in the whole thesis, and after the useful summary of the results obtained, the author turns to the problem with highest number of symmetries---the construction of spherical gravitating condensers made from charged perfect fluids (Chapter 2). The emphasis here is to satisfy various types of the energy conditions so that the system is physically plausible. Remarkable possibilities are pointed out like the existence of a gravitational field (curvature) between the shells only, or the condenser with the inner shell below the inner horizon of an associated Reissner-Nordström geometry.

Chapter 3 deals with the surface layers made of electromagnetic monopoles or dipoles, located in a given (background) spacetime. The jump conditions are carefully analyzed in general backgrounds, in case of dipoles this study is the first one ever undertaken, as far as we are aware of. The method is then applied to some specific cases like the Schwarzschild thin discs with various dipole densities and the results are physically interpreted by referring to the so-called membrane paradigm which has played a significant role in the black-hole electrodynamics. A more direct approach is considered for spherical shells in the Schwarzschild geometry, without assuming that the charges/dipoles are distributed in a spherical manner. There are two papers dealing with these problems, both accepted for publication, one appeared in Physical Review D already, the other one will appear in the Festschrift number of General Relativity and Gravitation dedicated to prof. Joshua Goldberg.

In the last, most extended and complicated part of the thesis the author investigates the Dedekind ellipsoids. At the beginning of his PhD studies I suggested a problem of trying to find configurations in general relativity which are made of dust or perfect fluid, are not axially symmetric but stationary, so non-radiative. Dedekind ellipsoids were natural candidates, although it was clear from the very beginning that one would have to use an approximation method to treat such a problem. During one of his visits of Jena, Norman Gürlebeck found out that a similar question was asked by David Petroff from Theoretical Physics Institute in Jena, whom he knew from his studies. Both joined their efforts together. Since Dr Petroff eventually moved to Leipzig and away from general relativity, recently the main contributor to the research on this problem was Norman Gürlebeck. Three main new important conclusions in the work on Dedekind's ellipsoids,
which are not contained in the pioneering work on the subject by Chandrasekhar and Elbert, are concerned with the following:

(i) the existence of the limit in which the Dedekind ellipsoids go over into Maclaurin axisymmetric spheroids;

(ii) the existence of the limit in which the Dedekind ellipsoids degenerate into a rod (this result is new even within the classical Newtonian theory);

(iii) development of the method showing how singularities encountered by Chandrasekhar and Elbert in the Dedekind sequences can be removed.

A number of useful but very lengthy expressions for special functions, symbols and formulas employed in the studies of the Dedekind ellipsoids are contained in four Appendices at the end of the thesis. Some of them are original, they appear in the thesis for the first time.

Both physical and pedagogical depth of the thesis is considerably increased by the fact that, together with relativistic analyzes, the analogous Newtonian cases are discussed. This is the case in essentially all six chapters. The thesis is written in a nice style and should be useful for both future students and researchers interested in these problems.

Norman Gürlebeck has exhibited a considerable talent, ability and zest to do a creative work in theoretical and mathematical physics, achieved outstanding results and wrote a high-quality thesis, based on several papers published in the leading journals. I have no doubts that the thesis deserves to be acknowledged as a PhD thesis at the Faculty of Mathematics and Physics of the Charles University in Prague or at any other academic institution in the world.

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