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### Report on the PhD-thesis by Norman Gürlebeck

To the members of the Examination committee,

It is a pleasure for me to report on the thesis “Matter models in General Relativity with decreasing number of symmetries” by Norman Gürlebeck.

I would like to start this report by first congratulating the candidate Mr. Norman Gürlebeck and his advisor Prof. Bičák for all the work presented in the submitted thesis. The thesis is composed of several chapters each of which deals with a gravitational system with matter of increasing generality. Some chapters consist of already published articles.

1. A discussion of spherically symmetric charged matter shells, “spherical condensers”. The main result is that there exist concentric charged shells with an electric field present only in the space between them. The inner shell can be made from dust, while the outer shell always has positive pressure. An energy balance using a quasi-local notion of gravitational mass shows that the total (ADM) energy reflects the binding energy due to the electric field.
2. The derivation of jump conditions for the Einstein-Maxwell system across hypersurfaces. Using Israel’s formalism for matching space-times extended to the Einstein-Maxwell case Mr. Gürlebeck formulates the appropriate jump conditions and applies them to Schwarzschild discs. I am wondering whether these junction conditions can be derived also from the familiar ‘pill-box’ argument as it is used in some books on electrodynamics.
3. Properties of axisymmetric and stationary dust solutions. This is an elegant discussion of the local properties of such systems close to the rotation axis without taking infinity into account. Using rigorous methods it is shown that there cannot exist homogeneous dust configurations and compact dust configurations for

which the mass density vanishes on the surface. When reading this I was wondering to what extent these results can be combined with properties of the exterior (vacuum) field in order to exclude the existence of asymptotically flat, regular dust configurations. On physical grounds one would expect that such configurations cannot exist. However, a rigorous proof of this statement has been lacking for a long time and, at least to my knowledge, it is still not in sight.

4. The last two chapters are concerned with Newtonian ellipsoidal solutions and their analogs in a first order post-newtonian approximation. These are certainly the most challenging chapters containing several very interesting results. Among them are the discussion of the effects of gravitomagnetism on the shape of the ellipsoids and the limit to a rod-like structure. There is also the result that one can remove an unphysical singularity in the parameter space, which apparently is due to an unfortunate parameter choice. It might be useful to work out parameters which have a physical interpretation, because then it might be easier to see what happens when the parameters are varied individually.

These results are all very impressive and interesting, covering a wide range of different systems. The thesis is well written and makes good reading. It is clear that Mr. Gürlebeck has acquired a profound knowledge in general relativity and the techniques used to obtain explicit solutions. This thesis is based on results which have already been published in well respected journals. So there is no question that the candidate has shown that he is capable of conducting research on an internationally competitive level. Therefore, I strongly recommend that this work be accepted as a doctoral thesis.

Yours sincerely

A handwritten signature in dark ink, appearing to read 'J. Frauendiener', with a stylized, flowing script.

J. Frauendiener