Institut ekonomických studií / Institute of economic studies
Fakulta sociálních věd /Faculty of social sciences
Univerzita Karlova v Praze / Charles University in Prague
Opletalova 26

Teze doktorské práce[1]
Dissertation thesis summary

Název / Title	Wavelet-based Realized Variation and Covariation Theory
Student	Jozef Baruník
Studijní program / Study program	Ekonomie
Školitel / Advisor	Prof. Ing. Miloslav Vošvrda, CSc.
	Prof. RNDr. Jan Ámos Víšek CSc.
Předseda komise / Committee chairman	
Oponenti / Opponents	1. Prof. Ing. Evžen Kočenda, PhD.
	2. Dr. Tiziana Di Matteo (King's College London)
	3. Dr. David Veredas (ECARES Brussels)
Termín a místo obhajoby / Term and venue of defence	6.10., 17:00 IES

[1]Příloha přihlášky k obhajobě disertační práce. Submit together with the application to the Study Department of the FSV UK.

Abstrakt / Abstract[2]

Kvadratická variace a kovariace se během několika posledních desetiletí staly jedněmi z nejfrekventovanějších, ale také nejúspěšněji studovaných témat ekonometrie časových řad. Tato disertace obsahuje kompletní teorii odhadu realizované variace a kovariace. Tato teorie je zobecněním současného stavu poznání v dané oblasti, přičemž vlastním přínosem je odhad veličin v časově frekvenční doméně. Zatímco první část teorie je věnována jednorozměrnému odhadu realizované variace pomocí waveletů, druhá část přináší vícerozměrný protějšek této teorie: odhad realizované kovariace pomocí waveletů. Konkrétními přínosy k již známým přístupům k realizované variaci je jednak robustifikace šumu, který nově nemusí být nutně Gaussovský a může obsahovat skoky, dále pak možnost měřit realizovanou variaci a kovariaci nejen v časové, ale i frekvenční doméně. Teorie je ověřena pomocí numerické studie zkoumající výkonnost z ní odvozených odhadů na malých vzorcích a srovnávající tyto odhady s ostatními užívanými odhady realizované variace, přičemž odhady jsou srovnány pomocí simulace při různých úrovních šumu a velikosti skoků. Výsledky studie ukazují, že tyto nové odhady dosahují nejlepších výsledků a jsou tedy dobře použitelné pro odhad realizované volatility (druhé odmocniny realizované variace). Za zmínku stojí ještě jedna aplikace v práci dosažených výsledků: rozklad realizované variace a kovariace podle libovolně zvolených investičních horizontů - výsledky práce tedy mohou kromě přesnějšího odhadování sloužit i k lepšímu porozumění dynamiky akciových trhů. V poslední části je zkonstruován model pro predikci volatility, kovariace a korelací. Použití odhadů pomocí naší waveletové realizované teorie zlepší predikční schopnosti. Odhad individuálních skoků a společných skoků pomocí naší teorie dále zlepší predikční schopnosti realizované kovariace.

[2] Pro disertace napsané česky vložte abstrakt anglicky. Pro disertace psané anglicky vložte abstrakt česky. / For thesis in Czech fill in abstract in English. For thesis in English fill in abstract in Czech.

Teze doktorské práce / Dissertation summary

The research of financial market volatility and co-volatility has been progressing rapidly in recent decades. These topics are critical to fundamental risk and asset pricing and have important applications to risk management and portfolio allocation. More generally, research on financial markets is a highly empirical discipline that has become one of the most active fields in the social sciences. Beyond the motivation of high expected profits, or perhaps the high willingness of market participants to pay for this kind of research, there are deep intellectual reasons driving this huge interest in financial markets. Over the past 30 years, financial econometricians have uncovered fascinating properties and regularities of asset returns. The market itself has been reminding us of the crucial properties that help us to understand its behavior: events like sudden stock market crashes are a good example. Black Monday on October 19, 1987, prompted researchers to sit up and notice that the underlying distribution of asset returns is far from normal.

Regardless of the huge amount of commotion present in the data, the most promising part of the research was the predictability of the second distributional moment of returns. Volatility clustering, persistence and its time variation are directly observable even to random bystanders. Thus, a vast literature emerged on the phenomenon, and time-varying volatility approaches became a traditional benchmark. The most direct way to capture the dynamics of second moments is to write a process for the volatility of returns conditional on past returns and other available information. Robert Engle was the first to propose this approach to volatility modeling by allowing for conditional heteroskedasticity to the autoregressive process of returns (the ARCH model). His seminal contribution, which changed the world of stock market modeling, was recognized in 2003 when he was awarded the Nobel Memorial Prize in Economic Sciences. With this award, interest in financial market volatility increased even further. An alternative approach that has been attracting interest in recent years permits the conditional mean and variance of financial returns to depend on a latent state that cannot be observed directly; this so-called regime-switching approach was pioneered by James Hamilton. The technical difficulty of this approach is that in order to capture the complexity of asset markets one would like to allow for many possible states in volatility, resulting in an inapplicable model with a very large number of parameters to be estimated.

In recent years, the study of high-frequency financial data has brought new development into the field. Explosive growth in the availability of such data has made the unobservable theoretical process of volatility suddenly observable. As noted by Nour Meddahi, Per Mykland and Neil Shephard in the editorial of the special issue of the Journal of Econometrics on realized volatility modeling published in January 2011, the concept of: "Realized Volatility is emblematic of this development, in that it was the earliest estimator which took advantage of the data in a non-parametric fashion." Thus, with the availability of high-frequency data, modeling of the second distributional moment of returns has turned into simple use of nonparametric measures. The important result of the field is the finding that the returns standardized by the realized volatility are asymptotically normally distributed, as implied by the normal-mixture hypothesis. As both the data availability and the theoretical results have grown, it has become possible to find answers to more complex questions, up to the point where "Realized Volatility" is now as much the name of a paradigm as the name of an estimator.

The popularity of realized volatility is mainly due to its two distinct implications for practical estimation and forecasting. The first relates to the measurement of realizations of the latent volatility process without the need for any assumptions about the explicit model. The second brings the possibility of modeling volatility directly through standard time series econometrics with discretely sampled daily data, while effectively extracting information from intraday high-frequency data.

The most fundamental result in realized variation states that it provides a consistent nonparametric estimate of price variability over a given time interval. The formalized theory is presented by Andersen et al. (2003). While these authors provide a unified framework for modeling, Zhou (1996) was one of the first to provide a formal assessment of the relationship between cumulative squared intraday returns and the underlying return variance. The pioneering work by Olsen & Associates on the use of high-frequency data, summarized by Dacorogna et al. (2001), produced milestone results for many of the more recent empirical developments in realized variation. A vast quantity of literature on several aspects of estimating volatility has emerged in the wake of these fundamental contributions. Rather than providing a tedious literature review here, we will introduce the main findings of the literature gradually in the text while discussing the aspects they develop.

The present dissertation contributes to this fascinating research in several ways. It offers a new complete theory generalizing the popular realized volatility as well as covariance measures. The theory introduces innovative estimators robust to noise as well as jumps in the financial stock markets and brings a considerable improvement to estimates of the realized variance-covariance matrix in terms of precision. The theory can also be used to disentangle jumps and co-jumps from the continuous part of the processes. Moreover, the newly developed estimators are able to decompose the realized measures into several investment horizons, providing much general understanding of the stock markets. This last theoretical novelty allows volatility and covariation to be studied in the time-frequency domain. In addition, the dissertation contains a small sample study of the properties of the proposed theory as well as the small sample performance of the estimators in the forecasting exercise under different conditions, confirming the theoretical results. Another notable contribution to the research field lies in the application of the derived theory. General time-frequency estimators yield not only more efficient estimates, but also insights into the decomposed realized variance, covariance or its transformations, correlation and portfolio beta. Thus, the theory helps to improve our understanding of stock market generating processes. The dissertation is divided into two parts presenting the theory and application on univariate as well as multivariate results.

Our work thus builds on the popular Realized Volatility approach, bringing even more insights to the theory. While most time series models are set in the time domain, we enrich the analysis by the frequency domain. This is enabled by the use of the continuous wavelet transform. It is a logical step to take, as the stock markets are believed to be driven by heterogeneous investment horizons. In our work, we ask if wavelet decomposition can improve our understanding of volatility series and hence improve volatility forecasting and risk management.

Another very appealing feature of wavelets is that they can be embedded into stochas- tic processes, as shown by Antoniou and Gustafson (1999). Thus we can conveniently use them to extend the theory of realized measures. One of the issues with the interpretation of wavelets in economic applications is that they behave like a filter. Thus wavelets can hardly be used for forecasting in econometrics. But in the realized measures, we use wavelets only to decompose the daily variation of the returns using intraday information. Moreover, the approach suggests constructing a model from the wavelet decomposition.

We are not the first to use this idea. Several attempts to use wavelets in the estimation of realized variation have emerged in the past few years. Høg and Lunde (2003) were the first to suggest a wavelet estimator of realized variance. Capobianco (2004), for example, proposes to use a wavelet transform as a comparable estimator of quadratic variation. Subbotin (2008) uses wavelets to decompose volatility into a multi-horizon scale. Next, Nielsen and Frederiksen (2008) compare the finite sample properties of three integrated variance estimators, i.e., realized variance, Fourier and wavelet estimators. They consider several processes generating time series with a long memory, jump processes as well as bid-ask bounce. Gençay et al. (2010) mention the possible use of wavelet multiresolution analysis to decompose realized variance in their paper, while they concentrate on developing much more complicated structures of variance modeling in different regimes through wavelet-domain hidden Markov models.

One remarkable exception which fully completes the current literature on using wavelets in realized variation theory is the work of Fan and Wang (2007), who were the first to use the wavelet-based realized variance estimator and also the methodology for the estimation of jumps from the data. In our work, we generalize the results of Fan and Wang (2007) in several ways. Instead of using the Discrete Wavelet Transform we use the Maximum Overlap Discrete Wavelet Transform, which is a more efficient estimator and is not restricted to sample sizes that are powers of two. We also use the Daubechies family of wavelets instead of the Haar type. Moreover, in the next chapters of this thesis, we will introduce a generalization of this approach to covariation estimation. In the next section, we will define the wavelet-based framework for the estimation of realized variance.

After the necessary introduction to the theory of quadratic variation and realized variation measurement, the first part of the dissertation defines the wavelet-based realized variation theory. Standing on our theoretical results proposing the Wavelet Representation Theorem, which extends the well-known Martingale Representation Theorem, the estimator of wavelet-based realized variation is defined together with its theoretical properties. Using wavelets, the estimator is able to consistently estimate jumps from the price process. It is robust to noise and it generates an unbiased consistent estimator of the true underlying variance. The theoretical part also contains an important discussion of the similarities between wavelet theory and stochastic processes.

To support the theory, a numerical study of the small sample performance of the estimators is carried out. In this study, we compare our estimators to several of the most popular estimators, namely, realized variance, bipower variation, two-scale realized volatility and realized kernels. The wavelet-based estimator proves to have the lowest bias of all the estimators in the jump-diffusion model with stochastic volatility as well as the fractional stochastic volatility model simulated with different levels of noise and numbers of jumps. While all the other estimators suffer from substantial bias caused either by jumps or by noise, our theory proves to hold its properties. As predictability of volatility is of interest to researchers as well as practitioners, a numerical study of the behavior of the forecasts is also carried out. Again, our theory proves to be the most powerful in forecasting volatility under the different simulation settings.

While the first chapters of Part I derive the theory and show its power on a small sample study, the last chapter uses the theory to decompose the empirical volatility. By studying the statistical properties of unconditional daily log-return distributions standardized by volatility estimated using the different estimators we find that standardization by our wavelet-based estimator brings the returns close to the Gaussian normal distribution. All the other estimators are affected by the presence of jumps in the data. The differences are economically significant, as we find that the average volatility estimated using our wavelet-based theory is 6.34% lower than the volatility estimated with the standard estimator.

Furthermore, we decompose the realized volatility into several intraday horizons. Here we note that the theory is able to decompose the realized measures into any arbitrary investment horizon, i.e., from 1-minute up to 1-month, when estimating monthly measures. In our analysis performed on forex data, we limit ourselves to illustrating the theory on the decomposition of daily realized measures. Specifically, we decompose the realized volatility into investment horizons of 5-10 minutes, 10-20 minutes, 20-40 minutes and 40-80 minutes, and the rest (80 minutes up to 1 day). The analysis uncovers interesting dynamics. Most of the action in the stock markets comes from higher frequencies. We find that on average, about 52% of the volatility of the forex markets examined is created on the 5-10 minute investment horizon, approximately 25% comes from the 10-20 minute investment horizon, and only 12%, 6% and 5% correspond to the horizons of 20-40 minutes, 40-80 minutes and the rest (80 minutes up to 1 day), respectively. Note that by adding the contributions of the different investment horizons we always get 100%.

The last part of the univariate empirical analysis is devoted to the forecasting of realized volatility. One of the issues with the interpretation of wavelets in economic applications is that they behave like a filter. Thus wavelets can hardly be used for forecasting economic time series most of the time. But in the realized measures, we only use wavelets to decompose the daily variation of the returns using intraday information, while forecasting daily volatility.

We build a new forecasting model based on an ARFIMA type model using the decomposition provided by our theory. In-sample as well as more important out-of-sample forecasts show that our theory is able to forecast volatility with the lowest error. Concluding the empirical findings, we show that our wavelet-based theory brings a significant improvement to volatility estimation and forecasting. It also offers a new method of time-frequency modeling of realized volatility which helps us to better understand the dynamics of stock market behavior. Specifically, our theory uncovers that most of the volatility is created on higher frequencies.

One of the most fundamental issues in finance is research of the covariance generating process between asset returns. Demand for accurate covariance estimation is becoming more important for risk measurement and portfolio optimization than ever before. The increasing availability of high-frequency data for a wide range of securities has allowed a shift from parametric conditional covariance estimation based on daily data toward the model-free measurement of so-called "realized quantities" on intraday data. Using a seminal result in semi-martingale process theory, Andersen et al. (2003) show that realized variance becomes a consistent estimator of integrated variance with increasing sampling frequency under the assumption of zero microstructure noise. Barndorff-Nielsen and Shephard (2004a) generalize the idea to a multivariate setting of so-called "realized covariation" and provide an asymptotic distribution theory for covariance (and correlation) analysis – again with the assumption of zero microstructure noise.

Although the theory is very appealing and intuitive, it assumes that the observed high-frequency data are the true underlying process. But real-world data are contaminated with microstructure noise and jumps, which makes statistical inference difficult. Realized measures suffer from large bias and inconsistency with the presence of noise and jumps in the observed data. The first approach to dealing with noise actually throws away a large amount of data. While this may not seem to be a logical step, the reason can be found quickly when one looks at the data at various sampling frequencies. The higher the frequency of the data we use (i.e., 1 second, 1 tick), the more microstructure noise they contain and the more biased the estimator is. Thus, a lot of researchers use lower frequencies (i.e., 5 minutes), which results in the throwing away of a very large amount of data directly. This is not an appropriate solution for a statistician to use. In the recent literature, a number of ways have been proposed to restore consistency through subsampling, for example Zhang et al. (2005)'s two-scale realized volatility estimator described in the previous part of this thesis. Zhang (2011) generalizes these ideas to a multivariate setting and defines a two-scale covariance estimator. Barndorff-Nielsen et al. (2011) achieve positive semi-definiteness of the variance-covariance matrix using multivariate kernel-based estimation.

While inference under noise and jumps in realized variation theory has been widely studied in recent contributions, its generalization to covariation theory is only now emerging in the literature. Together with important contributions by Zhang (2011) and Barndorff-Nielsen et al. (2011), Griffin and Oomen (2011) and A¨ıt-Sahalia et al. (2010) deal with microstructure noise and non-synchronous trading and propose a consistent and efficient estimator of realized covariance. Audrino and Corsi (2010) propose a fore- casting model for realized correlations. This research is becoming very active and stands at the frontier of current research in financial econometrics.

The second part of this dissertation follows the structure of the first part closely. After the necessary introduction of the generalized multivariate framework for modeling the covariation structure between processes, we build a new, wavelet-based realized covariation theory by extending the findings from the univariate part, and we define the wavelet-based realized estimator of covariance together with its properties. We use wavelets to disentangle jumps from co-jumps, which is crucial in the study of multivariate dependencies. Having defined the estimators of variance and covariance, we also define the transformations of interest for portfolio theory: the wavelet-based realized correlation measure and the wavelet-based realized beta. Similarly to the univariate findings, the presented theory provides a new type of multivariate estimators in the time-frequency domain.

To support the theoretical results, we again run a numerical study of the small sample behavior of the estimators. In the study, we simulate prices using a bivariate jump-diffusion stochastic volatility process and compare the performance of the wavelet-based realized covariation and correlation estimators with the popular realized covariance, bipower realized covariance, two-scale realized covariance and multivariate kernels estimators. The study proves that in this generalized setting as well, our wavelet-based realized theory is able to outperform other methods of estimation, as it displays the lowest bias under different amounts of simulated noise and jumps in the bivariate process. It is interesting to note that the realized covariance is significantly affected by co-jumps in the processes, while individual jumps do not have such an effect. The realized correlation is hugely biased when a single large jump is put to the series independently. On the other hand, co-jumps cause slight positive bias to the correlation. Our JWTSCV estimator may thus be very important for portfolio theory, as it is able to consistently estimate the true covariation and correlation of the processes as well as all jumps and co-jumps.

The last chapter applies the multivariate theory and studies the decomposition of integrated covariation, correlation and beta on the forex markets. Our estimator is able to separate jumps, co-jumps and true covariation from the data. It is also robust to the Epps effect caused by noise in the data. The results suggest that understanding jumps and co-jumps in a multivariate setting may be crucial for studying dependencies. While individual jumps bring some bias to the covariance, co-jumps introduce large bias into the covariation measure. The impact on correlation is even more crucial. Individual jumps in the processes bring large downward bias to the correlation measure, while co-jumps introduce upward bias with a smaller magnitude.

The empirical part also contains an interesting study of multivariate unconditional volatility distributions and their decomposition into several investment horizons. While the multivariate volatilities show strong dependence, a volatility-in-correlation effect suggests that the standard mean-variance efficiency calculations based on constant correlations are misguided. Our results have significant economic value, as a wrong assumption about the dependence process will have a direct impact on the portfolio valuation. The dynamics of the decomposed dependencies reveal interesting results as well. Our wavelet-based realized theory generates a more precise correlation measure with narrower confidence intervals than the standard realized correlations. A study of the temporal dependence in the decomposed correlations reveals that a similar heterogeneous autoregression type model as in the univariate case should be used for forecasting.

Similarly to the univariate part, therefore, we build a forecasting model for covariation and correlation based on wavelet decomposition. Our model again outperforms all other models in-sample as well as out-of-sample. Forecasting study also reveals interesting results as it shows that jumps as well as co-jumps carry important information which may help to forecast the realized covariation. We also decompose the realized covariance into the several investment horizons, individual jumps and co-jumps, and build an ARFIMA model for each of these components. Results confirms that wavelet decomposition contain important information for forecasts of realized covariance as well. Finally, we repeat the exercise and forecast the realized correlations. Again, our wavelet-based estimator of realized correlation help to improve the forecasts, mainly due to the decomposition to jumps and co-jumps. Corresponding to the small sample simulation study we can see that individual jumps cause bias to the realized correlations. Thus when using an estimator which is not able to disentangle jumps from the realized variance and covariance, one ends with biased estimate of correlation.

Finally, the wavelet-based realized beta estimator proves to be more precise, with narrower confidence intervals than the realized beta. All these results shed more light on the dependence, thus improving the applications in portfolio theory.

In conclusion, this dissertation presents a new theoretical framework generalizing the popular concept of realized variance and covariance. The work also contributes to the literature by providing interesting empirical findings from our time-frequency realized measures.

Výběrový seznam literatury a pramenů / Selective list of references

A ̈ıt-Sahalia, Y., J. Fan, and D. Xiu (2010). High frequency covariance estimates with noisy and asynchronous financial data. Journal of the American Statistical Associa- tion 105, 1504–1517.

A ̈ıt-Sahalia, Y. and J. Jacod (2009). Testing for jumps in a discretely observed process. The Annals of Statistics 37(1), 184–222.

A ̈ıt-Sahalia, Y. and L. Mancini (2008). Out of sample forecasts of quadratic variation. Journal of Econometrics (147), 17–33.

Andersen, T. and L. Benzoni (2007). Realized volatility. In T. Andersen, R. Davis, J. Kreiss, and T. Mikosch (Eds.), Handbook of Financial Time Series. Springer Verlag.

Andersen, T. and T. Bollerslev (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International Economic Review (39).

Andersen, T., T. Bollerslev, F. Diebold, and H. Ebens (2001). The distribution of realized stock return volatility. Journal of Financial Economics 61, 43–76.

Andersen, T., T. Bollerslev, F. Diebold, and P. Labys (2001). The distribution of realized exchange rate volatility. Journal of the American Statistical Association (96), 42–55.

Andersen, T., T. Bollerslev, F. Diebold, and P. Labys (2003). Modeling and forecasting realized volatility. Econometrica (71), 579–625.

Andersen, T., T. Bollerslev, and X. Huang (2011). A reduced form framework for mod- eling volatility of speculative prices based on realized variation measures. Journal of Econometrics 160(1), 176–189.

Andersen, T. G., T. Bollerslev, and F. X. Diebold (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. Review of Economics and Statistics (4), 701–720.

Andersen, T. G., T. Bollerslev, and D. Dobrev (2007). No-arbitrage semi- martingale re- strictions for continuous-time volatility models subject to leverage effects, jumps and i.i.d. noise: Theory and testable distributional implications. Journal of Economet- rics (138), 125–180.

Andersen, T. G., D. Dobrev, and E. Schaumburg (2009). Jump-robust volatility estima- tion using nearest neighbor truncation. Working paper, National Bureau of Economic Research .

Antoniou, I. and K. Gustafson (1999). Wavelets and stochastic processes. Mathematics and Computers in Simulation 49, 81–104.

Arneodo, A., J. Muzy, and D. Sornette (1998). Casual cascade in stock market from the “infrared” to the “ultraviolet”. European Physical Journal B (2), 277–282.

Audrino, F. and F. Corsi (2010). Modeling tick-by-tick realized correlations. Computa- tional Statistics and Data Analysis 54, 2372–2382.

Back, K. (1991). Asset prices for general processes. Journal of Mathematical Eco- nomics (20), 317–395.

Bandi, F. and J. Russell (2006a). Separating microstructure noise from volatility. Journal of Financial Economics (79), 655–692.

Bandi, F. and J. Russell (2006b). Volatility. In J. Birge and V. Linetsky (Eds.), Handbook of Financial Engineering. Elsevier.

Barndorff-Nielsen, O., P. Hansen, A. Lunde, and N. Shephard (2008). Designing realized kernels to measure the ex-post variation of equity prices in the presence of noise. Econometrica 76(6), 1481–1536.

Barndorff-Nielsen, O., P. Hansen, A. Lunde, and N. Shephard (2011). Multivariate realised kernels: Consistent positive semi-definite estimators of the covariation of equity prices with noise and non-synchronous trading. Journal of Econometrics 162 (2), 149– 169.

Barndorff-Nielsen, O. and N. Shephard (2001). Non-gaussian ornstein-uhlenbeck-based models and some of their uses in financial economics. Journal of the Royal Statistical Society, Series B (63), 167–241.

Barndorff-Nielsen, O. and N. Shephard (2002a). Econometric analysis of realised volatil- ity and its use in estimating stochastic volatility models. Journal of the Royal Statistical Society, Series B (64), 253–280.

Barndorff-Nielsen, O. and N. Shephard (2002b). Estimating quadratic variation using realized variance. Journal of Applied Econometrics (17), 457–477.

Barndorff-Nielsen, O. and N. Shephard (2004a). Econometric analysis of realized co-variation: High frequency based covariance, regression, and correlation in financial economics. Econometrica 72(3), 885–925.

Barndorff-Nielsen, O. and N. Shephard (2004b). Power and bipower variation with stochastic volatility and jumps. Journal of Financial Econometrics (2), 1–48.

Barndorff-Nielsen, O. and N. Shephard (2006). Econometrics of testing for jumps in financial economics using bipower variation. Journal of Financial Econometrics (4), 1–30.

Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. Journal of Political Economy (81), 637–654.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31(3), 307–327.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. Review of Economic and Statistics 69, 542–547.

Brillinger, D. R. (1981). Time Series: Data analysis and Theory. Holden-Day Series in Time Series Analysis. San Francisco: Holden-Day.

Calderón, A. P. (1964). Intermediate spaces and interpolation, the complex method. Studia Math. 24, 113–190.

Capobianco, E. (2004). Multiscale stochastic dynamics in finance. Physica A (344), 122–127.

Chui, C. (1992). An Inroduction to Wavelets. Academic Press, New York. Comte, F. and E.

Renault (1999). Long memory in continuous-time stochastic volatility models. Mathematical Finance (8), 291–323. Corsi, F. (2009). A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics 7(2), 174–196.

Corsi, F., D. Pirino, and R. Renò (2010). Threshold bipower variation and the impactof jumps on volatility forecasting. Journal of Econometrics (2), 276–288.

Dacorogna, M., U. Müller, R. Nagler, R. Olsen, and O. Pictet (2001). An Introduction to High-Frequency Finance. Academic Press, San Diego. Daubechies, I. (1988). Orthonormal bases of compactly supported wavelets. Communications on Pure and Applied Mathematics 41, 909–996.

Daubechies, I. (1992). Ten lectures on wavelets. SIAM.

Deo, R. S., C. M. Hurvich, and Y. Lu (2006). Forecasting realized volatility using a long memory stochastic volatility model: Estimation, prediction and seasonal adjustment. Journal of Econometrics 131(1-2), 29–58.

Di Matteo, T., T. Aste, and M. Dacorogna (2003). Scaling behaviors in differently developed markets. Physica A (324).

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. Econometrica 50 (4), 987–1007.

Epps, T. W. (1979). Comovements in stock prices in the very short run. Journal of the American Statistical Association (74), 291–296.

Fama, E. (1965). The behavior of stock market prices. Journal of Business 38, 34–105. Fama, E. and K. French (1993). Common risk factors in the returns on stocks and bonds.

Journal of Financial Economics (33), 3–56. Fan, J. and Y. Wang (2007). Multi-scale jump and volatility analysis for high-frequency

financial data. Journal of the American Statistical Association (102), 1349–1362. Fleming, J. and B. Paye (2011). High-frequency returns, jumps and the mixture of

normals hypothesis. Journal of Econometrics 160(1), 119–128. French, K., G. Schwert, and R. Stambaugh (1987). Expected stock returns and volatility.

Journal of Financial Economics (19), 3–29. Gen ̧cay, R., F. Sel ̧cuk, and B. Whitcher (2002). An Introduction to Wavelets and Other Filtering Methods in Finance and Economics. Academic Press.

Gen ̧cay, R., N. Gradojevic, F. Sel ̧cuk, and B. Whitcher (2010). Asymmetry of information flow between volatilities across time scales. Quantitative Finance, 1–21.

Granger, C. W. J. (1980). Long memory relationships and the aggregation of dynamic models. Journal of Econometrics 14(2), 227–238.

Granger, C. W. J. and R. Joyeux (1980). An introduction to long-memory time series models and fractional differencing. Journal of Time Series Analysis (1), 15–29.

Griffin, J. and R. Oomen (2011). Covariance measurement in the presence of non-synchronous trading and market microstructure noise. Journal of Economet- rics 160(1), 58–68.

Hansen, P. and A. Lunde (2006). Realized variance and market microstructure noise. Journal of Business and Economic Statistics 24 (2), 127–161.

Harris, F., T. McInish, G. Shoesmith, and R. Wood (1995). Cointegration, error cor- rection and price discovery on informationally-linked security markets. Journal of Financial and Quantitative Analysis 30, 563–581.

Haslett, J. and A. E. Raftery (1989). Space-time modelling with long-memory depen- dence: Assessing ireland's wind power resource. Applied Statistics 38, 1–50.

Hayashi, T. and N. Yoshida (2005). On covariance estimation of non-synchronously observed diffusion processes. Bernoulli 11, 359–379.

Hø g, E. and A. Lunde (2003). Wavelet estimation of integrated volatility. Working Paper. Aarhus School of Business..

Holschneider, M., R. Kronland-Martinet, J. Morlet, and P. Tchamitchian (1989). Areal- time algorithm for signal analysis with the help of the wavelet transform. In Wavelets, Time-Frequency Methods and Phase Space, pp. 289–297. Springer-Verlag.

Huang, X. and G. Tauchen (2005). The relative contribution of jumps to total price variance. Journal of Financial Econometrics 3, 456–499.

Igari, S. (1998). Real analysis - With an Introduction to Wavelet Theory, Volume 177 of Mathematical Monographs. AMS.

Jiang, G, J. and C. Oomen, R (2008). Testing for jumps when asset prices are observed with noise - a swap variance approach. Journal of Econometrics (144), 352–370.

Lanne, M. (2007). Forecasting realized exchange rate volatility by decomposition. International Journal of Forecasting (23), 307–320.

LeBaron, B. (2001). Stochastic volatility as a simple generator of financial power laws and long memory. Quantitative Finance (1), 621–631.

Lee, S. and P. A. Mykland (2008). Jumps in financial markets: a new nonpara- metric test and jump dynamics. The Review of Financial Studies 21, 2525–2563.

Lee, S. S. and J. Hannig (2010). Detecting jumps from l`evy jump diffusion processes. Journal of Financial Economics 96(2), 271–290.

Lindsay, R., D. Percival, and D. Rothrock (1996). The discrete wavelet transform and the scale analysis of the surface properties of sea ice. Transactions on Geoscience and Remote Sensing 34, 771–787.

Lintner, J. (1965). Security prices, risk and maximal gains from diversification. Journal of Finance (20), 587–615.

Lynch, P. and G. Zumbach. (2003). Market heterogeneities and the causal structure of volatility. Quantitative Finance (3), 320–331.

Mallat, S. (1998). A wavelet tour of signal processing. Academic Press. Mancini, C. (2009). Non-parametric threshold estimation for models with stochastic diffusion coefficient and jumps. Scandinavian Journal of Statistics 36, 270–296.

Mandelbrot, B. (1963). The variation of certain speculative prices. Journal of Busi-ness 36, 394–419.

Marinucci, D. and P. Robinson (1999). Alternative forms of fractional brownian motion. Journal of Statistical Planning and Inference (80), 111–122.

Martens, M. and J. Zein (2004). Predicting financial volatility: High-frequency time-series forecasts vis–vis implied volatility. The Journal of Futures Markets (11), 1005–1028.

McAleer M, M. M. (2008). Realized volatility: A review. Econometric Reviews (27), 10–45.

Merton, R. (1976a). Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics (3), 125–144.

Merton, R. C. (1976b). Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics, 125–144.

Mincer, J. and V. Zarnowitz (1969). The evaluation of economic forecasts. New York: National Bureau of Economic Research.

Müller, U., M. Dacorogna, R. Dav, R. Olsen, O. Pictet, and J. von Weizsacker (1997). Volatilities of different time resolutions - analysing the dynamics of market components. ournal of Empirical Finance (4), 213–239.

Müller, U., M. Dacorogna, R. Dav, O. Pictet, R. Olsen, and J. Ward (1993). Fractals and intrinsic time – a challenge to econometricians. In 39th International AEA Conference on Real Time Econometrics.

Najzar, K. (2004). Základy teorie waveletů. Učební texty Univerzity Karlovy v Praze. Karolinum.

Nielsen, M. and P. Frederiksen (2008). Finite sample accuracy and choice of sampling frequency in integrated volatility estimation. Journal of Empirical Finance (15), 265– 286.

Percival, D. B. (1995). On estimation of the wavelet variance. Biometrika 82, 619–631.

Percival, D. B. and H. Mofjeld (1997). Analysis of subtidal coastal sea level fluctuations using wavelets. Journal of the American Statistical Association 92 (439), 886–880.

Percival, D. B. and A. T. Walden (2000). Wavelet Methods for Time series Analysis. Cambridge University Press.

Protter, P. (1992). Stochastic integration and differential equations: A new approach. New York: Springer-Verlag.

Raimondo, M. (1998). Minimax estimation of sharp change points. The Annals of Statistics (26), 1379–1397.

Serroukh, A., A. T. Walden, and D. B. Percival (2000). Statistical properties and uses of the wavelet variance estimator for the scale analysis of time series. Journal of the American Statistical Association 95, 184–196.

Subbotin, A. (2008). A multi-horizon scale for volatility. Technical report, Documents de travail du Centre d'Economie de la Sorbonne, Université Panthéon-Sorbonne (Paris 1).

Voev, V. and A. Lunde (2007). Integrated covariance estimation using high-frequency data in the presence of noise. Journal of Financial Econometrics 5, 68–104.

Wang, Y. (1995). Jump and sharp cusp detection via wavelets. Biometrika (82), 385–397.

Whitcher, B., P. Guttorp, and D. B. Percival (1999). Mathematical background for wavelets estimators for cross covariance and cross correlation. Tech. Rep. 38, Natl. Res. Cent. for stat. and the Environ..

Whitcher, B., P. Guttorp, and D. B. Percival (2000). Wavelet analysis of covariance with application to atmosferic time series. Journal of Geophysical Research 105, 941–962.

Zhang, L. (2006). Efficient estimation of stochastic volatility using noisy observations: A multi-scale approach. Bernoulli (12), 1019–1043.

Zhang, L. (2011). Estimating covariation: Epps effect, microstructure noise. Journal of Econometrics 160(1), 33–47.

Zhang, L., P. Mykland, and Y. Aït-Sahalia (2005). A tale of two time scales: Determining integrated volatility with noisy high frequency data. Journal of the American Statistical Association (100), 1394–1411.

Zhou, B. (1996). High-frequency data and volatility in foreign-exchange rates. Journal of Business and Economic Statistics 14, 45–52.