

Topological and Geometrical Combinatorics

Martin Tancer

Abstract

The task of the thesis is to present several new results on topological methods in combinatorics. The results can be split into two main streams.

The first stream regards intersection patterns of convex sets. It is shown in the thesis that finite projective planes cannot be intersection patterns of convex sets of fixed dimension which answers a question of Alon, Kalai, Matoušek and Meshulam. Another result shows that d -collapsibility (a necessary condition on properties of intersection patterns of convex sets in dimension d) is NP-complete for recognition if $d \geq 4$. In addition it is shown that d -collapsibility is not a necessary condition on properties of intersection patterns of good covers, which disproves a conjecture of G. Wegner from 1975.

The second stream considers algorithmic hardness of recognition of simplicial complexes embeddable into \mathbb{R}^d . The following results are proved: It is algorithmically undecidable whether a k -dimensional simplicial complex piecewise-linearly embeds into \mathbb{R}^d for $d \geq 5$ and $k \in \{d-1, d\}$; and this problem is NP-hard if $d \geq 4$ and $d \geq k \geq \frac{2d-2}{3}$.