A supervisor report on the PhD thesis of *Mgr. Jiří Lipovský*:

Quantum Graphs and Their Generalizations

Among the subject of interest in mathematical quantum mechanics, the description of particles whose motion is constrained to a metric graph is rather popular, at least in the last two decades. There are various reasons for that. One is of a practical nature, namely that such models can be used to describe numerous systems of submicron size prepared by the experimentalists. The other important reason is of a theoretical nature: the so-called quantum graphs represent a useful laboratory to study properties of quantum systems being rich in structure and at the same time mathematically accessible. The present thesis, which I had the pleasure to supervise, addresses several questions in this area.

From the formal point of view the thesis represents a collection consisting of an extensive introductory chapter containing theoretical foundation of the work and a description of the results, and several appendices with the candidate published papers. The latter are four and they were published in renown journals, two in *Journal of Physics A: Mathematical and Theoretical*, one in *Physics Letters A* and one in *Journal of Mathematical Physics*.

The thesis, including the introductory chapter, is written in English with the Czech and English summaries. The co-authorship of the four papers in question includes myself and in one case also E. Brian Davies from King's College London; I can attest that in all four cases the contribution of Jiří Lipovský to the work was substantial.

The thesis has several main topics. The first of the included papers, *J. Phys. A: Math. Theor.* **43** (2010), 105301, is devoted to study of resonances in quantum graphs. It is shown that the notions of resolvent resonance and scattering resonance are equivalent in this case and the way how resonances can arises as a result of geometric perturbations of graphs with rationally related edges is discussed in detail. In addition the paper contains analysis of two examples which allow one to follow the resonance trajectories globally.

The next two papers are devoted to the behaviour of resonances (and embedded eigenvalues) at high energies. The first one of them, *J. Phys. A: Math. Theor.* **43** (2010), 474013, reflects the surprising observation of E.B. Davies and A. Pushnitski that in some case the number of resonances can be asymptotically smaller than the usual Weyl asymptotics would predict. The paper analyzes graphs with a general vertex coupling and proves a criterion under which this happens; it also shows that the resonance asymptotics is a global property of the graph. In the companion paper, *Phys. Lett.* **A375** (2011), 805–807, the asymptotic behaviour is studied in the situation when the graph is exposed to a magnetic field. It is shown that its presence cannot change the Weyl asymptotics into a non-Weyl one, however, it can change the leading term in the non-Weyl case.

The last included paper, *J. Math. Phys.* **51** (2010), 122107, deals with spectral problem for radial tree graphs. Using a trick belonging to M. Solomyak, J. Breuer and R. Frank showed recently that sparse trees with Kirchhoff coupling at vertices have empty *ac* spectrum. The paper studies the situation with general vertex conditions considering a wide class of them — the most general to which Solomyak' method can be applied — and shows that the result remains generically true, however, that there are exceptional cases when the *ac* spectrum is present or even it represents the only non-empty spectral component.

The papers on which the thesis is based contain original and important results proving the candidate imaginativeness and technical skills. Without any doubt I can recommend to accept the work of Jiří Lipovský and to award the PhD title to its author.

Given in Řež on July 11, 2011

Prof. RNDr. Pavel Exner, DrSc.