

Title: **On the Katowice Problem**

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Abstract: Main focus of this work is the so called Katowice problem, namely whether it is consistent with ZFC, that $\mathcal{P}(\omega)/\text{Fin}$ is isomorphic with $\mathcal{P}(\omega_1)/\text{Fin}$. The core of this work is development of new forcing notions and tools for establishing consistency results related to Katowice problem.

In the first chapter an overview of known results is given. In the second chapter we give an introduction to filter games (a method due to F. Galvin and C. Laflamme) and define a new tower game. We prove that player I has no winning strategy in this tower game if the involved tower generates a non-meager filter. This is a nontrivial strengthening (under CH) of the classical result for p-filter game. This result plays crucial role in the proof of properness of forcing notions from later chapters.

In the third chapter we present a simplification of S. Shelah's result, that existence of only one unique p-point is consistent with ZFC. The fourth chapter deals with strong-Q-sequences in $\mathcal{P}(\omega)/\text{Fin}$. We review results of J. Steprans on this topic and introduce an ${}^\omega\omega$ bounding forcing, which creates a strong-Q-sequence. This enables us to prove consistency of existence of a countable like ideal in $\mathcal{P}(\omega)/\text{Fin}$ and hence answering a weakening of Katowice problem question.

The last chapter focuses on automorphisms of $\mathcal{P}(\omega)/\text{Fin}$. We present a proof of K. P. Hart showing that isomorphism between $\mathcal{P}(\omega)/\text{Fin}$ and $\mathcal{P}(\omega_1)/\text{Fin}$ induces a nontrivial automorphism of $\mathcal{P}(\omega)/\text{Fin}$. A new forcing method for reducing amount of nontrivial automorphisms is also introduced. We are able to use this method to build a model of ZFC where $\mathfrak{d} = \omega_1$ and each automorphism is trivial on some member of each non-meager p-filter. Most ideas behind this method are due to A. Dow.