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Report by Dolors Herbera Espinal of the doctoral thesis: **Modules over Gorenstein rings.**

by RNDr. David Pospíšil

Tilting and cotilting modules are one of the tools that the Representation Theory of finite dimensional algebras has given to Algebra. Its development, for general rings and outside the world of finitely generated modules has been done in the last two decades and the Prague School, leaded by Jan Trlifaj, has had an essential role in this development. This thesis is inside this line of work and has important new contributions to it as it gives a classification of all tilting and cotilting classes over commutative noetherian rings.

Let R be any ring, and let $\mathcal{S}$ be a class of finitely generated modules of projective dimension at most n and having a projective resolution consisting of finitely generated modules. Then

$$\mathcal{S}^{\perp_\infty} = \{M \mid \operatorname{Ext}^i_R(S, M) = 0 \text{ for any } S \in \mathcal{S} \text{ and any } i > 0\}$$

is an n-tilting class. One of the main results in the area states that all n-tilting classes are of this form, that is to say that all *tilting classes are of finite type*. On the other hand,

$$\mathcal{S}^{\top_\infty} = \{M \mid \operatorname{Tor}^R_i(S, M) = 0 \text{ for any } S \in \mathcal{S} \text{ and any } i > 0\}$$

is a cotilting class, but there are examples showing that not all cotilting classes appear this way (the cotilting classes constructed in this way are called of *cofinite type*). Moreover, if $\mathcal{S}_0$ denotes another class of finitely generated modules of projective dimension at most n, having a projective resolution consisting of finitely generated modules, then $\mathcal{S}^{\perp_\infty} = \mathcal{S}_0^{\perp_\infty}$ if and only if $\mathcal{S}^{\top_\infty} = \mathcal{S}_0^{\top_\infty}$ if and only if $\mathcal{S}$ and $\mathcal{S}_0$ have the same resolving closure. Therefore there is a bijective correspondence between n-tilting classes and n-cotilting classes of cofinite type.

From this point of view, over a ring R the classification of tilting classes should follow as a consequence of a classification of the finitely presented

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modules. However, over most of the rings, including the commutative noetherian ones, the category of finitely generated modules is of wild type, hence, alternative tools must be developed to carry out such a classification. In the cotilting case, the problem seems even harder because, as mentioned above, not all cotilting modules are of cofinite type. Therefore, **giving a complete characterization of tilting and cotilting classes over commutative noetherian rings is really a very valuable, highly unexpected and tremendously interesting result.**

The thesis consists of an introduction, and three papers. The first two are already published. The second one is published in the prestigious Journal of Algebra and the third one deserves to be published in a highly reputed journal. Now I briefly summarize the outline of the three papers contained in the thesis:

- The first paper deals with the problem of classifying tilting and cotilting classes over (commutative) Gorenstein rings. It is completely solved for Gorenstein rings of Krull dimension 1 by classifying tilting classes and showing that there is a bijective correspondence between tilting and cotilting classes. Both of them are parametrized by the subsets of the set of maximal ideals of R of height 1.

  This paper also contains explicit constructions of tilting and cotilting modules, and gives some classification results for dimension $n > 1$.

  It is somewhat surprising that the version of the paper included on the thesis does not coincide with the published one. This is not at all a problem because the results in both versions are essentially the same, just some proofs of the main results differ.

- The second paper classifies tilting classes for regular rings of Krull dimension 2. They are parametrized by suitable pairs of subset of the prime spectrum of R. Some results on the structure of these tilting modules are also obtained.

- The third paper gives a complete parametrization of n-tilting and n-cotilting classes over any commutative noetherian ring in terms of suitable sequences of n subsets of $\operatorname{Spec} R$. The result and its proof are very interesting. The key idea seems to be to deal with n-cotilting classes instead of dealing with the tilting ones. As a consequence of the classification, it is shown that all cotilting modules are of cofinite type and then the general theory implies that there is a bijection between tilting and cotilting classes.

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It is important to stress that the study of commutative noetherian rings is specially important in connection with Algebraic Geometry. One of the possible continuations of this work is to investigate the geometric implications of the results obtained in the thesis.

With no doubt, the thesis proves the author's hability for creative scientific work.

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