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Second sound as a tool to study quantum turbulence generated by superflow of He II

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I declare that I carried out this bachelor thesis independently, and only with the cited sources, literature and other professional sources.

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Název práce: Druhý zvuk jako nástroj pro studium kvantové turbulence generované prouděním supratekuté složky He II

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Abstract: Liquid helium bellow 2.17 K at saturated vapour pressure becomes superfluid. Hydrodynamics of such liquid is well described in terms of two-fluid model. Superfluid helium is also capable of developing turbulent flow, called quantum turbulence, through randomization of distribution and orientation of vortex lines. Quantum turbulence in superfluid helium generated by mechanically driven pure superflow was studied by measuring the vortex line density in 10×10mm2 square channel using the attenuation of second sound technique. Agreement with results predicted by Vinen’s equation was found in steady state. For decaying turbulence, agreement with Vinen’s equation holds only for low initial vortex line densities and tends to develop a region where the time dependence of the decaying vortex line density appears exponential. In course of this work new second sound sensors were made and tested.

Keywords: liquid helium, superfluidity, second sound, quantum turbulence
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Introduction

Helium was the last of all gases to be liquefied. This milestone, achieved in 1908 by Heike Kamerlingh Onnes in Leiden, is generally accepted as a start of modern experimental physics, and low temperature physics in particular.

It was soon found that Helium lacks a characteristic common to all other substances, namely a triple point and remains liquid down to the limit of absolute zero unless high pressures are applied.

Liquid helium was and still is used as most valuable tool in low temperature physics to achieve low temperatures.

But Helium itself possesses some interesting properties. At roughly 2.17 K at saturated vapor pressure helium undergoes a second order phase transition, called the lambda transition.

The behaviour of liquid helium bellow this transition changes significantly. It was observed that thermal conductivity rises by about six orders of magnitude becoming the highest of all known substances. Later it was discovered by Pyotr Kapitsa in 1937 that it is capable of flow through very narrow constrictions and seemingly with zero viscosity. His research was awarded by Nobel prize for Physics in 1978.

Several other effects were observed, such as temperature or entropy waves, called second sound or thermomechanical effect.

Later, most of these effects were described theoretically in terms of the phenomenological two-fluid models developed by Tisza and Landau. Together with London’s original idea connecting lambda transition with Bose-Einstein condensation, these theories form the basis of current understanding of hydrodynamics of superfluid helium.

These theories seemingly did not allow for vorticity to exist in superfluid helium. However, Feynman developed the idea, originally due to Onsager, of quantized vortex lines, making circulation quantized around linear defects in the fluid.

This allows for turbulence to exist in superfluid helium. Since the quantized vortices in superfluid helium are better defined than their counterparts in classical fluids, quantum turbulence can be regarded as a prototype of classical turbulence, regarded as a still unresolved problem of classical physics.

Description of experimental technique useful for study of quantum turbulence and results obtained are given in the following.
1. Theoretical part

1.1 Helium and superfluidity

Helium is the second element in periodic table of elements. It has two electrons that form a closed s-orbital. Two stable isotopes of Helium exist in Nature — $^3\text{He}$ and $^4\text{He}$. The nucleus of the $^4\text{He}$ atom is formed by 2 neutrons and 2 protons and that of $^3\text{He}$ is missing one neutron. At high temperatures both isotopes behave like an ideal gas. With critical temperatures around 5.5K for $^4\text{He}$ and 3.3K for $^3\text{He}$, Helium has the lowest boiling temperature of all gases.

Due to Pauli principle, the total spin of $^4\text{He}$ is zero and therefore $^4\text{He}$ is governed by Bose-Einstein quantum statistics. $^3\text{He}$, on the other hand, has an odd number of fermions in nucleus, so its total spin must be odd half-integer, thus it is governed by Fermi-Dirac statistics. All the experiments presented in this work were done in $^4\text{He}$, therefore in the following I discuss only this isotope.

Liquid helium down to approximately 2.17K (at atmospheric pressure) behaves like ordinary Newtonian fluid with low viscosity. At 2.17K a second-order phase transition occurs. Historically, the phase above this transition is referred to as Helium–I and below as Helium–II. Because of the characteristic shape of specific heat discontinuity that resembles a Greek letter $\lambda$ (Figure 1.1), this transition is commonly referred to as $\lambda$–point or $\lambda$–transition.

![Figure 1.1: Lambda-discontinuity in specific heat capacity of liquid helium. Data from [7]](image)

Helium–II possesses a number of remarkable properties, the most prominent of which are apparent zero viscosity, very large thermal conductivity or existence
of peculiar wave processes other than longitudinal pressure waves, i.e. ordinary sound. These properties can be explained in terms of the two-fluid model.

1.2 Two-Fluid model

1.2.1 Equations of motion

Hydrodynamics of Helium–II can be described in terms of the two-fluid model first proposed by Tisza and developed by Landau. According to this model, He–II behaves as a mixture of two fluids: superfluid and normal fluid. Superfluid flows without viscous dissipation, as postulated by Landau, while normal fluid displays a normal viscous flow.

Using this model, we can assign velocity fields \( v_n, v_s \) and densities \( \rho_n, \rho_s \) to the normal and superfluid part respectively. According to Tisza, the superfluid component is associated with the condensate part and the depletion due to interactions while normal component is the gas of thermal excitations.

Total density thus being

\[
\rho = \rho_n + \rho_s, \tag{1.1}
\]

and total flow density

\[
j = j_n + j_s = \rho_n v_n + \rho_s v_s. \tag{1.2}
\]

Equation of continuity holds for the entire fluid

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot j. \tag{1.3}
\]

Since the flow of the superfluid component is dissipationless, it is assumed that only the normal component carries entropy. In what follows I also assume that the flow is adiabatic and we can write sourceless equation of continuity also for the entropy density

\[
\frac{\partial (\rho \sigma)}{\partial t} = -\nabla \cdot (\rho \sigma v_n), \tag{1.4}
\]

where \( \sigma \) is entropy per unit mass of liquid.

Since the superfluid currents flow without dissipation, analogy of these currents with electron orbitals in atoms was drawn by London, since the currents associated with these orbitals also flow without dissipation. He also suggested, that by this analogy, a macroscopic wave function could describe the superfluid part. The wave function is given by

\[
\psi_s = \psi_0 e^{iS(r,t)} \tag{1.5}
\]
$S$ and $\psi_0$ are real functions of the coordinates and time. For ordinary wave-function describing a particle, the squared modulus of the wave function $|\psi|^2$ gives the probability density of location of a particle. When the wave function describes a macroscopic number of particles, the square modulus gives number density of particles as a function of coordinates, that is $\rho_s/m_4$ ($m_4$ is mass of $^4$He atom), thus

$$\psi_0 = \sqrt{\rho_s/m_4}. \quad (1.6)$$

The superfluid density $\rho_s$ depends mostly only on temperature. Because thermal conductivity in He-II is extremely high, temperature is very closely uniform (indeed, as will be shown later, any temperature gradients cause strong mechanical effects). Thus we can neglect variation of $\rho_s$ with coordinates.

With this assumption, the superfluid velocity field $v_s$ is given by

$$v_s = \frac{1}{m_4} \hat{p}\psi_s = \frac{\hbar}{m_4} \nabla S(r, t). \quad (1.7)$$

The velocity is proportional to a gradient of a scalar function, that is, the flow of the superfluid part is potential. This also means that in a simply connected region, flow is irrotational.

To obtain the equation of flow we allow both amplitude and phase of the wave function to vary, but still assuming that (1.7) holds, and we solve Schrodinger’s equation

$$i\hbar \frac{\partial \psi_s}{\partial t} = -\frac{\hbar^2}{2m_4} \Delta \psi_s + \mu \psi_s, \quad (1.8)$$

where $\mu$ is the chemical potential. Substitution of (1.5) to (1.8) leads to

$$i\hbar \frac{\partial \psi_0}{\partial t} - \hbar \psi_0 \frac{\partial S}{\partial t} = -\frac{\hbar^2}{2m_4} [\Delta \psi_0 + 2i \nabla \psi_0 \cdot \nabla S + i \psi_0 \Delta S - \psi_0 (\nabla S)^2] + \mu \psi_0. \quad (1.9)$$

Equality must hold for both real and imaginary parts separately. From the real part of (1.9) we get

$$\hbar \frac{\partial S}{\partial t} = \frac{\hbar^2}{2m_4} \Delta \sqrt{\rho_s} - \frac{\hbar^2}{2m_4} (\nabla S)^2 - \mu. \quad (1.10)$$

If we are sufficiently far from walls or vortex cores, we can neglect variation of $\rho_s$ and omit the first term on the right hand side of (1.10). Taking a gradient of (1.10) with this term omitted and using (1.7) we obtain

$$\frac{\partial v_s}{\partial t} + \frac{1}{2} \nabla (v_s^2) = -\frac{1}{m_4} \nabla \mu. \quad (1.11)$$

1Cases where $\rho_s$ does vary with coordinates include very close neighborhood of walls of the enclosing vessel, free surface of the liquid or cores of the vortices (vortices will be discussed later).

2Violation of the condition that the region is simply connected leads to quantized vorticity, as will be discussed later.
Using the identity $\mathbf{v} \times (\text{rot} \mathbf{v}) = \frac{1}{2} \nabla (v^2) - (\mathbf{v} \cdot \nabla) \mathbf{v}$ we can rewrite the last equation

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s + \mathbf{v}_s \times (\text{rot} \mathbf{v}_s) = -\frac{1}{m_4} \nabla \mu. \quad (1.12)$$

When the flow is irrotational ($\text{rot} \mathbf{v}_s = 0$), this equation reduces to Euler’s equation for ideal inviscid fluid, as it was expected.

Using the Gibbs-Duhem relation,

$$d\mu = -s \, dT + v \, dp, \quad (1.13)$$

where $s$ and $v$ are entropy and volume per atom, equation (1.12) (without the rotation term) becomes

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s = -\frac{1}{\rho} \nabla p + \sigma \nabla T, \quad (1.14)$$

which is the required equation for the superfluid.

The imaginary part of (1.9), using (1.7) and (1.6), is

$$\frac{\partial \rho_s}{\partial t} = -\nabla \cdot (\rho_s \mathbf{v}_s), \quad (1.15)$$

which is the equation of continuity for the superfluid part.

By subtracting (1.15) from (1.3) we get the continuity equation also for the normal component of fluid.

It can be shown that due to conservation of momentum, equation of Navier-Stokes type holds for the entire liquid

$$\rho_n \left(\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n\right) + \rho_s \left(\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s\right) = -\nabla p + \eta_n \Delta \mathbf{v}_n \quad (1.16)$$

where $\eta_n$ is the dynamic viscosity of the normal component. Subtracting (1.14), we get the equation of flow for the normal component

$$\rho_n \left(\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n\right) = -\frac{\rho_n}{\rho} \nabla p - \rho_s \sigma \nabla T + \eta_n \Delta \mathbf{v}_n \quad (1.17)$$

However, in this derivation of equations of motion we used one assumption that is not justified at higher flow speeds, namely the Gibbs-Duhem relation, equation (1.13), does not hold in this form at higher speeds. This arises from the fact that the so called counterflow velocity $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$ is another thermodynamic parameter of HeII. It can be shown (2 or 3) that the chemical potential depends on $\mathbf{v}_{ns}$ by

$$\frac{1}{m_4} \mu(p, T, \mathbf{v}_{ns}) = \frac{1}{m_4} \mu(p, T, 0) - \frac{\rho_n}{2\rho} (\mathbf{v}_n - \mathbf{v}_s)^2. \quad (1.18)$$

Using Einstein summation convention and denoting by $\epsilon_{ijk}$ the Levi-Civita tensor:

$$\mathbf{v} \times (\text{rot} \mathbf{v}) = \epsilon_{ijk} e_i v_j \epsilon_{klm} \partial_l v_m = e_i \epsilon_{ijk} \epsilon_{lmk} v_j \partial_l v_m = e_i (\delta_{il} \delta_{jm} - \delta_{im} \delta_{lj}) v_j \partial_l v_m = e_i v_j \partial_l v_i = \frac{1}{2} \nabla (v^2) - (\mathbf{v} \cdot \nabla) \mathbf{v}$$
This leads to the additional term, of the form $\frac{\rho_s\rho_n}{2\rho} \nabla(v_n - v_s)^2$ on the right hand side of (1.14) and minus the same term on the right hand side of (1.17).

To recapitulate, all the equations of flow in the two-fluid model are:

- **Equation of continuity of mass**
  \[
  \frac{\partial}{\partial t} (\rho_n + \rho_s) = -\nabla \cdot (\rho_n v_n + \rho_s v_s).
  \] (1.19)

- **Equation of continuity for entropy**
  \[
  \frac{\partial(\rho\sigma)}{\partial t} = -\nabla \cdot (\rho\sigma v_n).
  \] (1.20)

- **Equation for the superfluid component**
  \[
  \rho_s \left( \frac{\partial v_s}{\partial t} + (v_s \cdot \nabla) v_s \right) = -\frac{\rho_s}{\rho} \nabla p + \rho_s\sigma \nabla T + \frac{\rho_s\rho_n}{2\rho} \nabla(v_n - v_s)^2.
  \] (1.21)

- **Equation for the normal component**
  \[
  \rho_n \left( \frac{\partial v_n}{\partial t} + (v_n \cdot \nabla) v_n \right) = -\frac{\rho_n}{\rho} \nabla p - \rho_s\sigma \nabla T + \eta_n \Delta v_n - \frac{\rho_s\rho_n}{2\rho} \nabla(v_n - v_s)^2.
  \] (1.22)

It is apparent from this set of equations that superfluid helium is capable of supporting two different velocity fields. The vector $v_{ns}$ therefore doesn’t have to be zero.

If we neglect the term containing counterflow velocity, which can usually be done for small velocities, the equations of flow are uncoupled and equations of continuity hold for both components separately, which means that the velocity fields are independent.

### 1.2.2 Consequences of two-fluid model

In the following, nonlinear terms on the left hand side of equations of flow, the counterflow term and the viscosity term in equation of flow for the normal component are neglected.

**Fountain effect**

In equations (1.21) or (1.22) if we put $\nabla p = 0$ and establish finite temperature gradient, both components of the liquid will be accelerated – superfluid towards the increasing temperature and normal fluid oppositely. This effect is quite strong and with proper placement of heat sources and flow constrictions it can produce a quite sizable fountain, hence the name.
A similar effect is the thermal-mechanical effect, in which the acceleration of superfluid component is set to zero. In this case the gradient of temperature corresponds to a gradient of pressure.

**Quantization of circulation**

Given equation (1.7) the circulation of superfluid velocity $\Gamma$ around a contour $C$, defined as

$$\Gamma = \oint_C v_s \cdot dl$$

(1.23)

is zero if we are in a simply connected region. If this condition is not satisfied, the only thing that must hold is that the wave function (1.5) must by single-valued, so that after going around any contour the phase $S$ can only change by an integer multiple of $2\pi$. Hence we have

$$\Gamma = n\kappa,$$

(1.24)

where

$$\kappa = \frac{h}{m_4}$$

(1.25)

is called the quantum of circulation. It’s numerical value is approximately $9.97 \times 10^{-8} \text{ m}^2/\text{s}$.

The region of the flow is not simply connected, for example, in the case when the flow is established inside a torus. Another important case is when the defect establishes in the liquid, forming a vortex line. This case will be discussed further later.

**New wave processes**

As it has been said in the introduction, Helium supports more wave processes than just ordinary sound, usually called first sound. Existence of another sound, so called second sound, can be shown from equations (1.19) – (1.22). Here follows a standard derivation of this result, as presented in, e.g., [1].

Adding equations (1.21) and (1.22) together we get (neglecting viscous and nonlinear terms)

$$\frac{\partial j}{\partial t} = -\nabla p.$$  

(1.26)

By differentiating the equation of continuity (1.19) with respect to time and using the last equation we get

$$\frac{\partial^2 \rho}{\partial t^2} = \Delta p.$$  

(1.27)

Multiplying equation (1.21) by $\rho_n/\rho_s$ and subtracting it from (1.22) leads to

$$\rho_n \frac{\partial}{\partial t}(v_n - v_s) = -\rho \sigma \nabla T.$$  

(1.28)
If we further assume that counterflow velocity \( v_n - v_s \) and derivatives of density and entropy per unit mass are small, combining equation (1.19) and (1.20) and neglecting terms nonlinear in small quantities we get

\[
\nabla \cdot (v_n - v_s) = -\frac{\rho}{\sigma \rho_s} \frac{\partial \sigma}{\partial t}.
\]

(1.29)

By combining (1.28) and (1.29) we get

\[
\frac{\partial^2 \sigma}{\partial t^2} = \rho_s \frac{\sigma^2}{\rho_n} \Delta T.
\]

(1.30)

We are looking for equations for \( \rho \) and \( \sigma \), so we must express \( \Delta T \) and \( \Delta p \) as functions of \( \rho \) and \( \sigma \), that is (neglecting terms quadratic in small quantities),

\[
\Delta T = \left( \frac{\partial T}{\partial \rho} \right)_{\sigma} \Delta \rho + \left( \frac{\partial T}{\partial \sigma} \right)_{\rho} \Delta \sigma;
\]

\[
\Delta p = \left( \frac{\partial p}{\partial \rho} \right)_{\sigma} \Delta \rho + \left( \frac{\partial p}{\partial \sigma} \right)_{\rho} \Delta \sigma.
\]

(1.31)

We now seek the solution in form of small oscillations around constant values in the form

\[
\rho = \rho_0 + \rho' \exp \left[ i \omega \left( t - \frac{x}{c} \right) \right], \quad \sigma = \sigma_0 + \sigma' \exp \left[ i \omega \left( t - \frac{x}{c} \right) \right].
\]

(1.32)

Substituting (1.32) and (1.31) to (1.27) and (1.30) we obtain

\[
\rho' \left[ \frac{c^2}{c_1^2} - 1 \right] + \frac{1}{c^2} \left( \frac{\partial p}{\partial \sigma} \right)_{\rho} \sigma' = 0;
\]

\[
\rho' \frac{\rho_s}{\rho_n} \frac{\sigma^2}{c^2} \left( \frac{\partial T}{\partial \rho} \right)_{\sigma} + \left[ \frac{c_2^2}{c^2} - 1 \right] = 0,
\]

(1.33)

where

\[
c_1 = \sqrt{\left( \frac{\partial p}{\partial \rho} \right)_{\sigma}}
\]

(1.34)

and

\[
c_2 = \sigma \sqrt{\frac{\rho_s}{\rho_n}} \left( \frac{\partial T}{\partial \sigma} \right)_{\rho}.
\]

(1.35)

Equations (1.33) have non-zero solution for \( \rho' \) and \( \sigma' \) only if their determinant is zero, which results in

\[
\left[ \frac{c_2^2}{c_1^2} - 1 \right] = \left( \frac{\partial p}{\partial \sigma} \right)_{\rho} \left( \frac{\partial T}{\partial \rho} \right)_{\sigma} \left( \frac{\partial p}{\partial \sigma} \right)_{\rho} \left( \frac{\partial \sigma}{\partial T} \right)_{\rho} = \frac{C_p - C_v}{C_p} \approx 0,
\]

(1.36)

where \( C_v \) and \( C_p \) are heat capacities of helium at constant volume and pressure, respectively. These two quantities are very similar in helium and (1.36) holds with very good approximation.
Equation (1.36) means that there are two solutions. The first solution corresponds to \( c = c_1 \), and therefore \( \sigma' = 0 \). These are density (or pressure) waves while entropy stays constant. This is the first sound, or ordinary sound that exists in all compressible media. In this case superfluid and normal components move in phase.

The other solution is when \( c = c_2 \) and \( \rho' = 0 \) while \( \sigma' \) being non-zero. In this solution the density stays constant but the entropy (or temperature) changes. In this case the two components move phase-shifted by 180°. This wave process is called second sound and has no analogue in classical fluids.

In this derivation the term in \( v_{ns} \) in the equations of motion was neglected. This term leads to coupling between the two sounds, making them not possible to exist without each other. This coupling is rather weak and can usually be ignored.

There exist other wave processes still, namely third and fourth sounds. The third sound is a surface wave in the film of helium, accompanied with a wave of temperature and the fourth sound exists in porous media or capillaries where the normal component cannot move due to viscosity.

1.2.3 Experiments with rotating He-II and vortex lines

The question arises as to what happens when the vessel containing superfluid helium rotates. If the angular velocity of rotation was \( \Omega \), ordinary liquid would undergo a solid-body rotation and the vorticity of flow \( \omega = \text{rot}v \) would be \( 2\Omega \) everywhere. This rotation would, in gravitational field, result in a meniscus of parabolic shape formed by the liquid surface.

In helium II, the velocity field of superfluid component given by (1.7) is proportional to the gradient of a scalar function, therefore its rotation should be zero and only normal component should participate in creation of the meniscus, which should be flatter than that predicted by classical mechanics if the density were taken to be the density of the entire fluid.

This was, however, not observed. Macroscopically, the rotation of superfluid helium is indistinguishable from the rotation of an ordinary classical fluid. This led to the idea of vortex lines, proposed by Feynman and Onsager. Vortex lines are of diameter of order 1Å where the superfluidity is suppressed. Circulation around these lines is quantized by formula (1.24).

This is equivalent to saying that the angular momentum of the helium atom is quantized around a vortex line, that is (assuming that the vortex line is straight
and that $v_s$ lies in a plane perpendicular to it

$$v_s = \frac{1}{r} n \hbar, \ n \in \mathbb{N},$$

where $r$ is a distance from the core of the vortex line considered.

The energy of a vortex line is given by the mechanical energy of the superfluid flow around it. Thus integrating $\frac{1}{2} \rho_s v_s^2$ with bounds given by the radius of the vortex core and the size of the enclosing vessel we find that this energy is proportional to $n^2$. Energetically the most efficient case is obviously when the lines are singly-quantized. It can also be shown that the energetically most efficient ordering of vortex lines is a triangular lattice (neglecting irregularities at boundaries of the enclosing vessel).

In macroscopic rotation there are usually many lines present. To describe such system, the quantity vortex line density $L$ is defined as total length of all vortex lines in a unit volume, or, equivalently, number of vortex lines crossing a unit surface element perpendicular to the orientation of vortex lines.

The macroscopic average vorticity of the fluid is then given, by analogy with rotating vessel, by

$$\bar{\omega} = \kappa L$$

where $\kappa$ is the quantum of circulation. This relation assumes that vortex lines are fully polarized.

The so-called Feynman rule gives for the mass of helium II undergoing a solid body rotation at angular velocity $\Omega$

$$L = \frac{2\Omega}{\kappa}.$$  

### 1.2.4 Mutual friction force

To investigate the interaction between the two components of the fluid and the vortex lines, Vinen and Hall performed experiments with rotating second sound resonators. Their second sound source was heater made of resistive wire through which they passed AC current (offset so that the current doesn’t change sign, otherwise the detected second sound would have double frequency of the current). The receiver of the second sound was a resistance thermometer.

It was possible to rotate the resonator in such a way that the velocity $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$ of the second sound wave could be perpendicular or parallel to the axis of rotation.

They measured the attenuation of second sound wave as a function of angular velocity of rotation and power input (amplitude of second sound). The value
measured was the effective attenuation constant $\alpha$ defined so that the amplitude of the propagating wave is proportional to $\exp(-\alpha x)$. This constant can be expressed as

$$\alpha = \alpha_0 + \alpha'(\Omega), \quad (1.40)$$

where $\alpha_0$ is due to residual attenuation when the resonator is at rest.

They found that there is additional attenuation of the second sound when the resonator is rotating and that is proportional to the angular velocity, $\Omega$, of the rotation and independent of the amplitude of the second sound. The velocity of the second sound was found to be independent of rotation.

They also found that the attenuation is much weaker when $v_{ns}$ is parallel to the axis of rotation than when it is perpendicular.

From these observed properties, they derived, based on phenomenological model, a formula for mutual friction acting on a unit volume of normal fluid

$$F_{sn} = B\frac{\rho_n \rho_s}{\rho} \Omega \times \left[ \frac{\Omega \times (v_n - v_s)}{|\Omega|} \right] + B'\frac{\rho_n \rho_s}{\rho} \Omega \times (v_n - v_s), \quad (1.41)$$

where $\Omega$ is the vector along the axis of rotation and of magnitude equal to the angular velocity. The coefficient $B$ is related to $\alpha'(\Omega)$ through $B = 2c_2 \alpha' / \Omega$, where $c_2$ is the velocity of second sound, $B$ is of order of 1. The second term with $B'$ is non-dissipative and usually small. Force acting on superfluid is determined by the equilibrium condition, $F_{ns} = -F_{sn}$.

Using the Feynman rule in vector from $L = 2\Omega / \kappa$, where the orientation of $L$ gives orientation of lines at given point, and neglecting all non-dissipative terms (perpendicular to $v_n - v_s$), $(1.41)$ reduces to

$$F_{sn} = -B\kappa \frac{\rho_s \rho_n}{2\rho} L (v_n - v_s) \sin^2 \theta \quad (1.42)$$

where $\theta$ is the angle between $L$ and $v_n - v_s$.

1.2.5 Quantum turbulence

Vortex lines discussed in the previous section had uniform spatial ordering. This does not always need to be so. When the distribution and orientation of vortex lines is significantly complex, we speak of quantum turbulence. This generally accepted picture of quantum turbulence as a random tangle of vortices is due to Vinen and has been confirmed by computer simulations.

Many studies of quantum turbulence utilized a thermal counterflow in a channel. In this technique a channel with one dead end is used, where a heater is
placed. Heat supplied to one end of the channel will result in a counterflow inside the channel.

To describe turbulence in a thermal counterflow, Vinen derived in [6], on the basis of phenomenological model, equations describing the behavior of vortex line density $L$ of homogeneous isotropic vortex tangle as a function of time and counterflow velocity $v_{ns}$ inside the channel:

$$\frac{dL}{dt} = \kappa_1 \frac{B \rho_n}{2\rho} v_{ns} L^{3/2} - \kappa_2 \frac{\hbar}{m} L^2 + g(v_{ns}), \quad (1.43)$$

where $g(v_{ns})$ is an unknown function related to the fact that turbulence sets in only above some critical velocity $v_c$. Above this velocity this term can be ignored. $\kappa_1$ and $\kappa_2$ are adjustable parameters.

In the case of steady state ($dL/dt = 0$) (1.43) leads to

$$L^{1/2} = \gamma(T) (v_{ns} - v_c). \quad (1.44)$$

All these results were obtained for counterflow, where superfluid and normal fluid both moved in opposite directions, but in the experiments discussed in this thesis the flow was pure superflow, where only the superfluid component flowed in the channel while the normal component was held stationary by superleaks at both ends of channel. Derivation of these results, however, didn’t use any special property of the thermally driven counterflow, so the difference with the pure superflow should only be in change of reference frame to one in which the normal fluid is stationary. Obviously, at least for an unbounded system, the choice of reference frame should not affect the physics involved and equations (1.43) and (1.44) should still hold.

1.2.6 Attenuation of second sound due to presence of vortex lines and theoretical principle of experimental technique

Mutual friction leads to attenuation of second sound. It enters equations of motion, and therefore equation (1.28) in the derivation of second sound is modified as follows:

$$\rho_n \frac{\partial \mathbf{v}_{ns}}{\partial t} = -\rho \sigma \nabla T + \frac{\rho}{\rho_s} \mathbf{F}_{sn}. \quad (1.45)$$

The equations of continuity for both entropy and density stay the same and (1.29) is unchanged. However, now we only look for a second sound wave in the form of small perturbations $\sigma'$ and $T'$ to constant values of $\sigma_0$ and $T_0$ of
entropy and temperature while leaving pressure and density constant. With these simplifications (1.29) can be written as

$$\nabla \cdot \mathbf{v}_{ns} = -\frac{\rho \sigma}{\rho_n c_2^2} \frac{\partial T}{\partial t},$$

(1.46)

where $c_2$ is velocity of second sound given by (1.35).

Substituting (1.42) to (1.45) and solving for $\nabla T$ leads to

$$\nabla T = -\frac{\rho_n}{\sigma \rho} \left( \frac{B\kappa L}{2} (\sin^2 \theta) v_{ns} + \frac{\partial v_{ns}}{\partial t} \right).$$

(1.47)

Taking a gradient of (1.46) and interchanging time and space derivatives we obtain

$$\nabla (\nabla \cdot \mathbf{v}_{ns}) = \frac{1}{c_2^2} \left( \frac{B\kappa L}{2} (\sin^2 \theta) \frac{\partial^2 v_{ns}}{\partial t^2} + \frac{\partial^2 v_{ns}}{\partial t^2} \right).$$

(1.48)

We now consider a second sound plane wave traveling along the $z$ direction with angular frequency $\omega$ and wave vector $k \mathbf{e}_z$.

$$\mathbf{v}_{ns} = e_z v_{ns0} e^{i(\omega t - kz)}.$$  

(1.49)

Substituting this to (1.48) we obtain

$$k = \pm \frac{\omega}{c_2} \sqrt{1 - i \frac{B\kappa L}{2\omega} \sin^2 \theta}.$$  

(1.50)

Vortex line density measured in experiments is usually of order $10^5$ cm$^{-2}$, making $\kappa L$ of order of 10Hz. The frequency of second sound used in experiments is usually around 1kHz (if 1st harmonic is used). We can therefore use first two terms of Taylor expansion of the square root. Physically meaningful is only the solution with minus sign

$$k = -\frac{\omega}{c_2} \left( 1 - i \frac{B\kappa L}{4\omega} \sin^2 \theta \right).$$  

(1.51)

Substituting this back to (1.49) we get the propagation of an attenuated wave

$$\mathbf{v}_{ns} = e_z v_{ns0} \exp \left[ i \left( \omega t - \frac{\omega}{c_2} z \right) - \alpha z \right],$$

(1.52)

where the attenuation constant is

$$\alpha = \frac{B\kappa L}{4c_2} \sin^2 \theta.$$  

(1.53)

Note that, apart from $\sin^2 \theta$ this is the same as $\alpha'$ defined in the previous section on mutual friction.
To get attenuation constant for randomly oriented vortices, we replace \( \sin^2 \theta \) by its mean value. Choosing a coordinate system so that \( \mathbf{v}_{ns} \) always lies on \( z \) axis we get

\[
\langle \sin^2 \theta \rangle = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi (\sin^3 \theta) \, d\theta \, d\phi = \frac{2}{3}
\]

and thus the attenuation constant for randomly oriented vortex lines is

\[
\alpha = \frac{B\kappa L}{6c^2}
\]

Note that if the tangle were completely polarized in the direction perpendicular to the propagation of second sound, the result in (1.54) would be 1/2. So if the assumption of isotropy were completely wrong, (1.55) would be at worst 25% lower than the true value. If the polarization was in direction parallel with propagation of second sound, \( \langle \sin^2 \theta \rangle \) would be zero.

Mutual friction, however, is not the only source of attenuation. To account for this, we decompose, similarly as Hall and Vinen did in [4], the attenuation constant to a part independent of flow and one given by (1.55):

\[
\tilde{\alpha} = \alpha_0 + \alpha
\]

The attenuation constant is related to a quality factor of a resonator \( Q \), defined as \( Q = \frac{f_0}{\Delta f} \) where \( f_0 \) is the resonance frequency and \( \Delta f \) is full the width at half maximum of resonance the curve, through

\[
\tilde{\alpha} = \frac{\pi}{\tilde{\lambda}Q} = \frac{\pi \Delta f}{c_2}.
\]

Here \( \tilde{\lambda} \) is an integer multiple of the resonant wavelength, depending on which harmonic is used in measurements.

When the second sound is excited at resonant frequency the waves reflected at the receiver will constructively interfere with the waves emitted from the transducer, giving the total measured amplitude

\[
A = A_e \sum_{n=1}^{\infty} e^{-\tilde{\alpha}D} = \frac{A_e}{e^{\tilde{\alpha}D} - 1} \approx \frac{A_e}{\tilde{\alpha}D}
\]

where \( D \) is channel width and \( A_e \) is amplitude of the wave excited by the transducer.

Using (1.56) and (1.57) we arrive at

\[
\alpha = \frac{\pi \Delta f}{c_2} \left( \frac{A_0}{A} - 1 \right).
\]

Here \( A_0 \) is the amplitude when there is no flow in the channel and \( A \) is the amplitude with the flow.
Finally, using (1.55) we obtain

\[ L = \frac{6\pi \Delta f}{B_K} \left( \frac{A_0}{A} - 1 \right). \]  

(1.60)

This is the required formula that gives vortex line density as a function of directly measurable quantities.
2. Experimental setup and analysis

The main goal of work reported in this thesis was the study of quantum turbulence generated by flow in a channel. In this case, the flow was generated by a mechanical bellows. A sketch of the experiment is shown in Figure 2.1. The used channel had a square cross section of dimensions 10 × 10 mm. The second sound sensors (described in more detail in the next section) were at opposite sides of the channel.

![Figure 2.1: Sketch of the experiment. The bellows is squeezed by a linear motor located outside of the cryostat at room temperature. All joints that were not connected permanently (i.e., welded) were sealed against superfluid leaks by indium. Sensors were located at opposing sides of channel of square cross-section.](image)

Bellows was squeezed using a computer-controlled linear motor located on the top of the cryostat at room temperature. The motor was connected to the bellows by a shaft shown in the picture.

In addition to the second sound response, the simultaneously measured quantities were: vapor pressure, temperature of the bath, temperature of the helium inside the bellows and position of the bellows, all as a function of time. Temperature control was achieved by PID temperature controller. Stability of temperature
was within few mK of the set value.

In case where full spectrum around the resonance was measured, amplitude was measured as a function of excitation frequency while the speed of the bellows was set to a constant.

Measurements were performed in several modes, which will be described later in this chapter.

During experiments, the position of the bellows was measured as a function of time. The speed of the motor pushing the bellows could be set and this value was found to agree with the measured one. The volume of the bellows was calibrated against the position reported by the motor, so via the known dimensions of the channel (10 × 10 mm cross-section) and assuming negligible compressibility, flow speed in the channel can be deduced.

Both ends of the channel were blocked by superleaks made of 2 mm thick plates of silver powder sintered in situ. The filling factor is roughly 50%. The superleak has many narrow channels (of a µm size) that allow flow of the superfluid component but is impermeable to the normal component due to viscosity.

All measurements were computer-controlled through program created in LabVIEW© environment. All analysis was done using programs written by myself using GNU Octave[1] and C programming language with GNU Scientific Library[2].

### 2.1 Second-sound sensors

The sensors are essentially capacitors where one electrode is a Nucleopore membrane gold-plated on one side. The pores in the membrane are of size of a µm, making them impermeable to normal fluid but posing no obstruction for the superfluid. Electrically, the gold plated side is connected with the brass channel body while the other electrode is insulated and connected to the Agilent 33250A wave generator when the sensor is used as a transducer or to the lock-in amplifier when as a receiver. The sensors themselves are equivalent and their roles were switched several times during the measurements.

When the membrane oscillates, it moves only the normal component and this creates longitudinal waves of oscillating normal fluid and (through equation of continuity) superfluid densities, thus creating waves of temperature or second sound.

In the course of the work reported here, I prepared a pair of these sensors.

The membrane is held on a Delrin ring with the gold-coated side facing inside of the channel (Figure 2.2). Other electrode is made of small brass piece held closely against the back side of the membrane on a spring. The sensor is covered by a lid, which closes the openings for the sensors on sides of the channel.

Figure 2.2: Second sound sensors assembly. The gold-plated Nucleopore membrane is held by a Delrin ring close to a brass electrode which is held by a spring connected to the lid. The lid (brass plate) and membranes held by Delrin rings (white plastic) are visible.

Response measured with these sensors is shown in figure 2.3. The measured response is the real component of complex voltage measured by the phase sensitive lock-in amplifier. The phase difference between the reference and the measured signal was inputted manually and optimized so that the resonance peak looks symmetric and has a Lorentzian shape. From experience, the quality of the resonance peak strongly depends on how well the membrane is stretched.

It should be noted that these sensors were not used in the actual measurements, as they were made only after the experiments were completed. The deterioration of the sensors used became apparent only towards the end of the experiment. Their examination after the experiments revealed that membrane was somewhat wrinkled. This probably led to noise which will be seen in the data and the peak was also more irregular at certain temperatures. This, although inconvenient, does not make the data any less valid, as the observed resonance peak had all the properties needed for measuring vortex line density, namely the shape was that of Lorentzian function and the central frequency and background signal were not changing.
2.2 Modes of measurement

To focus on certain aspect of the studied problem, several modes of measurements were employed, which will be discussed in some detail in the following.

2.2.1 Full-spectra mode

In this mode, the entire resonance peak was measured. This was used to characterize the resonance, that is, to obtain resonance width and background signal to allow calculating the amplitude of the response, as required by formula (1.60). Examples of these measurements are in Fig. 2.4. The red line is a fit of Lorentzian curve, eq. (2.1), or a sum of two of them when necessary, plus a constant background:

\[
y(x) = y_0 + \frac{A}{2\pi} \frac{w}{w^2 + (x - x_0)^2};
\]

(2.1)

where \(y_0\), \(A\), \(w\) and \(x_0\) are adjustable parameters. The amplitude is then given by \(A/2\pi w\) and the width is \(w\).

When there was more than one peak in the spectrum, the one whose resonant frequency didn’t change with the flow was used for measurements. This is in accordance with the observed fact that the second sound velocity doesn’t change.

Figure 2.3: Second-sound resonance peak, 2nd harmonic.
Figure 2.4: Second sound peaks sample at two different temperatures (1.35K on the left and 1.75K on the right). The flow was not changing during this measurement. Sometimes, the sum of two Lorentzian peaks was necessary to describe the observed spectrum. Note that the curves don’t drop to zero as they get further from the peaks.

in the presence of vortex lines([4]). The background signal, $y_0$, is necessary to determine the amplitude in the mode discussed next.

During these measurements the lock-in time constant (time during which each reading was averaged) was 100 ms.

### 2.2.2 Constant frequency mode

In this mode, the frequency was held constant at resonant frequency and changes in amplitude were observed as the flow changed. Note that to observe actual change in amplitude at resonance, the resonant frequency must not change, hence the choice of peak with non-changing central frequency in previous section.

A typical constant frequency mode measurement is shown in Fig. 2.5. The arrows indicate beginning and end of movement of the bellows or the flow through the channel.

Unattenuated amplitude entering eq. (1.60) was determined from the mean of the region indicated by 1, attenuated from 2 (some parts of the signal were discarded to avoid averaging over transient effects such as decay of vortex line density or the initial buildup). These two yielded values $A_1$ and $A_2$. The vortex line density is then

$$L = \frac{6\pi \Delta f}{B\kappa} \left( \frac{A_1 - y_0}{A_2 - y_0} - 1 \right).$$  \hspace{1cm} (2.2)

The resonance width $\Delta f$ and the background signal $y_0$ were determined by scanning the whole peak beforehand. The lock-in time constant (that is, time taken
Figure 2.5: An example of constant frequency mode measurements. The drive frequency was set to resonance and response was measured as a function of time with changing flow. The arrows indicate start (upper arrow) and stop (lower arrow) of movement of the bellows. Intervals labeled 1 and 2 indicate intervals of averaging for obtaining non-attenuated and attenuated signal.

to for integration of the signal) was 100 ms.

Measurements of the decays of vortex line density were essentially the same, only with higher acquisition rate, which was achieved by reading only the amplitude and position of the bellows. This allowed for elimination of waiting time required for synchronization of communication with the instruments over single GPIB bus. The lock-in time constant in this case was 10 ms. In this case, $L$ is usually plotted as a function of time.

### 2.2.3 Critical velocities mode

This mode was focused on determining the onset of quantum turbulence. The measurements were exactly the same as with constant mode. The response of the amplitude could be watched in real time as the flow velocity was changed, and with this I looked for the velocity when the change in amplitude became visible. To reduce the effect of the noise, amplitudes were extracted in the same way as
with constant mode and the relative change was plotted as a function of the flow velocity, see example in Fig. 2.6. From this graph critical velocity was deduced. It should be noted that this determination is somewhat subjective.

Figure 2.6: Determination of critical velocity. The arrow indicates the onset of quantum turbulence, subjectively chosen by the author. Measurements were repeated several times for every velocity. The spread of the points at every velocity is probably the result of temperature fluctuations.
3. Experimental results

3.1 Steady-state

The steady state of turbulence, characterized by vortex line density $L$, during constant flow through the channel was measured as a function of the flow velocity.

Overview of all measured full spectra is shown in figures 3.1—3.5 for temperatures 1.35 K, 1.65K, 1.75K, 1.95K and 2.05K. In all cases the “flatter” curves correspond to a faster flow.

![Figure 3.1: Response of the second sound sensors as a function of drive frequency at 1.35K.](image)

Vortex line densities extracted from these measurements are shown in Fig. 3.6. The data taken at the highest temperature, 2.05 K, are not shown. That is because here the full scan of the resonance peak was used mostly for assurance that the resonant frequency does not change and no velocity dependence was measured. Data from 1.95 K don’t seem to accurately display a linear behavior in the measured range, so no line was fitted through them.

To analyze the data from the constant frequency mode measurements, the width and the background signal was used from the fit of the Lorentzian curve
Figure 3.2: Response of the second sound sensors as a function of drive frequency at 1.65K.

through the resonance peak when there was no flow. Data thus obtained are displayed in Fig. 3.7.

Both ways of measurement display a departure from linearity in $\sqrt{L}$ vs. $t$ plot for higher velocities. The physical significance of this is not clear and might perhaps be rather regarded as a limit of this experimental technique. For the full-spectra measurements at these flows, as can be seen in figures 3.1–3.4, the peak was almost completely attenuated. The fits were therefore becoming uncertain, and in some cases, the fits didn’t converge (these points are not shown on plots).

There is also a potential systematic error in constant mode measurements. The background of the Lorentzian curve fitted in full-spectrum mode does slightly change with increasing flow velocity. The change is small (usually within 2%), but for the strongly attenuated cases, when the amplitude was comparable with the background, this can produce a significant error (hence the large error bars in Fig. 3.7). However, close examination of the resonance curves shows that the scanning range was too narrow for the fit to provide a reliable estimate of the background, as the ends of the curves were still decreasing; or sometimes the peak became slightly asymmetric when strongly attenuated. Both of these sources of
error tend to increase the background signal.

For this reason, resonance characteristics were used from the spectrum measured without flow and no correction was done for the weakly changing background. This could lead to systematic error. However, the pronunciation of this error should increase as the amplitude gets closer to the background, i.e., gets more attenuated, while the observed dependence remains linear up to the point when the attenuation is so strong that faster flow fails to produce stronger attenuation, that is, the natural limit of this experimental technique.

From this data, coefficient $\gamma(T)$, from eq. (1.44), can be obtained, which is commonly used to describe the steady state of quantum turbulence. Plot of $\gamma$ versus temperature is shown in Fig. 3.8. In the linear fits, inverse of error was used as a weight.

The slight difference between the two ways of obtaining $\gamma$ could possibly lie in the error discussed above.
3.2 Critical velocity

It can be seen from Fig. 3.7 and 3.6 that the intercepts of the lines fitted on steady state data are non-zero. Turbulence thus starts to build up only upon exceeding some critical velocity, $v_c$. The estimation of this velocity was obtained in a way described in previous chapter. Results are in Fig. 3.9. This figure also contains the intercepts of the steady state lines, which can be used as an estimate of the critical velocity. It should be noted, however, that the shape of $\sqrt{L}(v)$ is usually not linear just above $v_c$, so this value should be taken only as a rough estimate. Errors in direct observations were estimated as half of the difference between neighboring velocities.

3.3 Decay of Quantum Turbulence

Time dependence of vortex line density in decaying turbulence generated by counterflow can be described by Vinen equation without the creation term, that is

$$\frac{dL}{dt} = -\frac{\kappa}{2\pi} \chi_2 L^2$$

(3.1)
where $\kappa = h/m_4$ is quantum of circulation and $\chi_2$ is dimensionless parameter. Integrating this we get

$$L(t) = \frac{1}{\beta(t - t_0) + \frac{1}{L_0}} \tag{3.2}$$

Where $\beta = \frac{\kappa \chi_2}{2\pi}$, $t_0$ is the time when the decay starts and $L_0$ is the steady state vortex line density.

During measurements the vortex line density didn’t drop to zero, so the actual function that was fitted on the data was

$$L(t) = \frac{1}{\beta(t - t_0) + \frac{1}{L_0}} + L_r \tag{3.3}$$

where $L_r$ denotes remnant vortex line density. The origin of this is that at the scale of vortex line diameter (of the order of $\AA$) all surfaces are rough. This leads to pinning of the vortices to the walls. Note that this approach assumes that the quantum turbulence is approximately isothermal, otherwise the thermal gradient would produce additional flow and thermal decay would have to be considered.

The experiments were performed at 5 different temperatures: 1.35 K, 1.65 K, 1.75 K, 1.95 K and 2.05 K at several different flow velocities at each temperature. In all cases the turbulence was allowed to develop for 20 seconds, while the bellows
were pushed at constant velocity, and then the bellows was suddenly stopped within less than 10 ms.

Lock-in time constant was set to 10 ms and the mean data acquisition rate was around 45-50 Hz.

In the fit, all parameters of (3.3) were allowed to be adjusted. I selected the initial values of $L$ and $t_0$ by hand near the “edge of the steady state plateau” and a interpolated value from data in [8] was used for initial value of $\beta$. All parameters were allowed to vary within 20% of their initial values (10% in each direction).

Data for $T=1.35$ K are plotted on Figure 3.10. The departure from Vinen decay starts to be visible at $v = 3.4$ cm/s. To give an idea of reproducibility of the observed curve at this velocity, plots of all measured curves are in Figure 3.11. Curves measured here, except for one (drawn in purple), appear to fall into two categories with different agreement to (3.3) as can be seen in more detail in Figure 3.12. In this figure, the linear part is visible. Since the plot is in log-linear scales, this would suggest that decay obeys some exponential function. The same is visible at highest speed in Figure 3.10.

Data from $T=1.65$ K and 1.75 K are in Figure 3.13 and Figure 3.14.
situation is similar to 1.35K, the only difference is that the disagreement with (3.3) starts sooner, i.e. at lower steady state velocities. Nevertheless, even though that the agreement isn’t perfect, observed dependence is described by (3.3) rather well.

At $T=1.95\text{K}$, Figure 3.15 and at $2.05\text{K}$, Figure 3.16 we see that the Vinen-type decay holds only for the lowest velocities and the measured decay is significantly different in shape from the functional dependence given by (3.3).

In Figure 3.17 it is shown, that unlike at $1.35\text{K}$ at high speed (Figure 3.12), the exponential-like part is apparently universal.

At 2.05K, Figure 3.16 we get partial agreement with (3.3) only at the lowest velocity. The early fast decay appears to be gone completely. For a more clear view, one of the curves in Figure 3.16 is reproduced in Figure 3.20. The oscillations in plots at 2.05K are probably due to poor temperature control and strong dependence of second sound velocity on temperature in this region.

The agreement of data with (3.3) varies with temperature and flow velocity. In general, the Vinen equation appears to hold for low temperatures and velocities. As the temperature or flow velocity increase, the decay develops a regime in which
Figure 3.8: Slope of curve $\sqrt{L(v)}$ as a function of temperature. The red “+” symbols are from [10] and were obtained for thermal counterflow.

$L(t)$ appears to be exponential and this regime eventually dominates the entire range where the decay is visible. The exponential decay is thought ([9]) to be connected to large-scale toroidal motion inside the channel.

To compare how the fit improves by adding an exponential term to (3.3), that is fitting the function

$$L(t) = \frac{1}{\beta(t-t_0)} + \frac{1}{L_0} \exp \left( -\frac{t-t_0}{\tau} \right) + L_r \tag{3.4}$$

figures 3.18, 3.19 and 3.20 show fits of both functions to few measured decays where the disagreement with (3.3) was significant. In cases where the Vinen decay itself fits data well the $L_0'$ is found to be very small, as expected, and the plots do not differ from the previous ones. The fit improves, but as can be seen from Figure 3.19, not universally.

It should be noted, however, that the description of $L(t)$ simply as a sum of exponential term and Vinen-like decay refers only to the shape of the measured dependence. The physical significance of this might not be clear and the reason why this is seen might be just a coincidence caused by other effects.
Figure 3.9: Critical velocities. Direct observation and intercepts of the fit.
Figure 3.10: Decay of turbulence at $T = 1.35$ K. Bottom picture shows the detail on early part of the decay, the curves are the same as in the top one. In the upper picture the curves are artificially shifted in both directions, in the bottom they are shifted only in time.
Figure 3.11: Reproducibility of decay at $T=1.35$ K and $v=3.4$ cm/s
Figure 3.12: Different agreement with (3.3) at single speed. The red curve is artificially shifted (multiplied by a factor of 10).
Figure 3.13: Decay of turbulence at $T = 1.65\,\text{K}$. Bottom picture shows the detail on early part of the decay, the curves are the same as in the top one. Curves are artificially shifted in time.
Figure 3.14: Decay of turbulence at $T = 1.75\text{K}$. Bottom picture shows the detail on early part of the decay, the curves are the same as in the top one. Curves are artificially shifted in time.
Figure 3.15: Decay of turbulence at $T = 1.95\text{K}$. Bottom picture shows the detail on early part of the decay, the curves are the same as in the top one.
Figure 3.16: Decays of turbulence at $T=2.05\text{K}$. Curves are artificially shifted in time.
Figure 3.17: Decay at $T=1.95\text{K}$, $v=1.7\text{ cm/s}$. Curves are artificially shifted in time.
Figure 3.18: $T=1.35\text{K}$, exponential fit

Figure 3.19: $T=1.65\text{K}$, exponential fit
Figure 3.20: $T=2.05 \text{K}$, exponential fit
4. Conclusions

Quantum turbulence is often studied in the limit $T \rightarrow 0$ K, which allows the study of almost purely superfluid system. However, studying quantum turbulence at finite temperature, that is, in the temperature range $1 \text{ K} < T < 2.17$ K, when both components are present is equally important. The experimental method of attenuation of second sound is particularly sensitive in this range and provides a unique probe into nature of quantum turbulence, and, by extension, into turbulence in general. The main objective of this thesis was to apply this experimental technique to a mechanically generated pure superflow in a channel.

Even at strong flows through the channel, the propagation and velocity of second sound is not changed, as can be seen from the fact that the central frequency of the second sound resonance peak is not changing. This is in agreement with the generally known fact first observed by Vinen and Hall [4].

The observed steady state of turbulence is in agreement with dependence predicted by Vinen’s equation for homogeneous and isotropic turbulence, up to the apparent natural limit of this experimental technique. The observed slopes $\gamma(T)$ are in good agreement with values measured for thermal counterflow [10]. This supports the idea presented in theoretical part that the only difference between these two experiments is the choice of reference frame and that the essential physics is not changed.

The critical velocities observed are around 0.2 cm/s, which is in agreement with previous measurements done in similar channel, where the flow was also generated mechanically [11]. It should be noted, however, that for thermal counterflow the critical velocities tend to be a order of magnitude higher ([10]). This difference is currently not understood.

The behavior of decays is more complicated. At low initial vortex line densities, the prediction of Vinen’s equation appears to hold. This agreement gets worse as the initial vortex line density increases suggesting that there are additional mechanisms that come into play when the turbulence decays. So far, complete understanding of decaying quantum turbulence is not available.

The measurements are of reasonable quality, although they are affected by the use of aged sensors causing significant noise in the data. New sensors were prepared and tested and these will probably be used in similar experiment in the future.
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