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Logic, form and argument

Logika, forma a argument

D I Z E R T A Č N Í   P R Á C E

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„Prohlašuji, že jsem dizertační/disertační práci napsal(a) samostatně s využitím pouze uvedených a řádně citovaných pramenů a literatury a že práce nebyla využita v rámci jiného vysokoškolského studia či k získání jiného nebo stejného titulu.“

ABSTRAKT

Cílem mé disertace je obhájit a vysvětlit tezi, že tradiční logická analýza není vhodným nástrojem ke zkoumání argumentace v přirozeném jazyce.
Nejčastější kritika formální logiky jako nástroje pro analýzu přirozeného jazyka je obvykle založena na poukázování na podstatné rozdíly mezi strukturou a sémantikou jazyků přirozených a jazyků logických formalismů. V tom však nevidím hlavní zdroj problémů.
Podle mého úsudku je daleko zásadnějším problémem, že tradiční logická analýza často vychází z problematických epistemologických předpokladů, které analytická filosife zdědila z empiristicko-positivistické tradice. Jedná se především o pozitivistickou verzi klasického modelu racionality, jako deduktivního usuzování z nějaké báze bezprostředně ověřitelných a nepochybných poznatků. Přesvědčení, že každou rozumnou argumentaci lze redukovat na dedukci takového druhu je tím, co má ospravedlnit tradiční logickou analýzu.
Můj přínos spočívá především v prokázání toho, že nezměníme-li zásadně tato východiska, pak nám pranic nepomůže, budeme-li zkoušet argumentaci v přirozeném jazyce analyzovat pomocí nových a přesnějších logických formaismů.
Problém tedy není ani tak v samotném nástroji, jako spíše ve způsobu jeho užití. Pokud dostatečně zreflektujeme roli demonstrativního usuzování pro argumentaci jako takovou, můžeme její jisté apsekty zkoumat pomocí standardních logických formalismů mnohem plodněji.

ABSTRACT

The goal of this thesis is to defend and explain the claim that traditional logical analysis is not the best tool for studying natural language argumentation.
The most common critique directed at employment of logical formalisms as tools for analysis of the natural language is usually based on pointing out of differences between structure and semantics of natural languages and languages of logical formalisms. This is not the main issue, I believe.
According to my findings the most fundamental problem of the traditional analysis is that it is based on many problematic epistemological assumptions, which are inherited from empiricist-positivist tradition. Namely the positivist version of the classical model of rationality as deductive reasoning from some basis of immediately verifiable and therefore unquestionable knowledge. The doctrine that every reasonable argumentation is reducible on deductions of such kinds is supposed to justify the traditional analysis of argumentation.
My original contribution is mainly in showing that without abandoning those presuppositions, we cannot hope to arrive at better understanding of natural language argumentation by developing new and more precise logical formalisms.
Logical formalisms are mere tools, which we have to use for the right purpose in the first place. If we can reflect more deeply on the role of deductive reasoning for argumentation as such, we can study some aspects of it more fruitfully, with the aid of standard logical formalisms.

LOGIC, FORM AND ARGUMENT

Svatopluk Nevrkla

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Contents

Preface

Anyone who holds that logic provides useful insights into natural language will often claim that the subject of logic is mostly clarification of *argumentation* or *reasoning*. I will strive to reexamine foundations of modern formal logic, in order to determine whether it really is suitable for such a purpose. In the end I will hopefully establish that its actual subject is much narrower and that we should not expect formal logic to perform too well as the science of argumentation.

Before exploring this doctrine of logic as a science of argumentation or even reviewing alternative concepts of logic, I should perhaps say a few words concerning why I believe this particular concept is of such an importance. It is due to the fact that this understanding of logic is quite widespread among 'lay' people.

Logicians, who aim at educating lays in logic often succesfuly root out the most vulgar psychologistic interpretations of this doctrine in their textbooks, still they too often accept the basic premise that the chief goal of logic lies in clarification of argumentation.

I do not wish to challenge this doctrine entirely. Formal logic can help us to evaluate some interesting arguments after all. However it is necessary, I believe, to demonstrate that the problematic of argumentation is much broader and that formal logic has only limited role in its description.

Personally, I believe the utility of modern formal logic is mostly in mathematics. It is the mathematical discourse for which it has been originally designed and its goal therefore is clarification of *inference*, or *demonstration*, not argument or reasoning in general.

Still, one may object, some insightful and fruitful applications of logic for studying natural language argumentation are possible. I do not intend to deny that claim neither. But again, it is important to see what particular area of natural language argumentation can be meaningfuly described by methods of formal logic and where would be any attempt to apply it out of place.

There can be only little harm if linguists or philosophers carefully employ logic in their respective scientific fields.

This unfortunately is not the case when people with only superficial knowledge of logic and very shallow reflection on its philosophical foundations, often encouraged to do so by logical textbooks, try to muster natural language reasoning to fit this or that logical formalism.

There are many introductory logical textbooks which try to present an alternative to purely formal, symbolic and mathematical account of logic and try to provide some convenient justifications of their subject.

These elementary textbooks are often directed at readers who are not particularly interested neither in philosophy, nor mathematics or computer science. Most often their intended recipients are students of law or humantities, or even preuniversity pupils at high schools. As a result of this, such textbooks differ from those of mathematical logic in two related but different aspects.

First—the relevance of logic has to be justified somehow to the target audience. The argument that logic plays an important role in modern mathematics and philosophy is of no value for future lawyers or students of, say, sociology. Therefore authors of these textbooks often try to come up with such an account of logic to give an impression that it is of a high practical utility. They often picture it as a discipline which is crucial for mastering such an important and highly practical endeavor as is argumentation or rational inquiry in general.

But is it really so? Does knowledge of elementary logic teach people rational argumentation and reasoning?

I find justifications of this claim provided in most of the textbooks of mentioned kind too brief, superficial, unsatisfactory, and sometimes even blatantly incorrect.

The problem of such justifications is that practical utility is being claimed for the discipline of formal logic which has never really been intended to serve as a tool to settle down disputes outside of the narrow field of mathematics, as already noted above.

I personally could hardly find an example of legal or commonsense argument in such textbooks of logic which would require some deep understanding of the principles of formal logic and would not be blatantly artificial at the same time[1].

The sentences we use to form arguments in our language rarely contain more than two quantifiers or more than three propositional connectives. We may use intricate methods of formal logic to resolve situations in which 'Adam will go to the Cinema if Bertie will go and

[1]Anyone hesitant to acknowledge this or who would wish to maintain that logic is truly a chief instrument at understanding argumentation is invited to give purely logical account why my arguments are fallacious or try to apply methods of formal logic on some of the real arguments recorded in a book [**20**].

Cindy will not go.' etc. however such situations rarely occur outside textbooks of logic, or perhaps relational databases.

I will argue in this thesis that modern logic was not intended to explicate natural language reasoning, but rather to provide a completely new language for the sole purpose of expressing intricate ideas of mathematics with sufficient rigor. This new language lacks some expressive power of natural languages, but it can capture functional relations between individuals more clearly by using quantifiers and functions.

Relevance of modern logical languages for the description of natural language is therefore limited and questionable.

Second—the reader's ignorance of mathematics, if not direct aversion to it, is presupposed by the authors. Therefore they often try to avoid technical aspects of the subject as much as possible, which is, concerning the above mentioned purpose of modern logic, quite unfortunate.

Indeed, there is not much left of formal logic that would be worth mentioning when it is stripped off the mathematical apparatus it is built upon and when its applications in mathematics are omitted. This lack of introduction of formal methods disables the opportunity to introduce the most interestng meta-theoretical results of logic, such as completeness and incompleteness theorems to mention just a few because they are too difficult, abstract and technical.

The major problem is that nothing truly substantial is offered in return for giving up these results. Logic has simply been vulgarized so that even those, who would otherwise have not been willing to invest nontrivial efforts into learning its more difficult techniques and results, were exposed to some of its menial fragments. A derogatory term 'baby logic' is often being used for such accounts of logic.

It is most unfortunate that these textbooks perpetuate the myth of usefulness of logical formalisms for understanding of practical argumentation, while often presenting what could be called its caricature.

Studying formal logic may actually improve reasoning abilities. After all studying mathematics in general is believed to do so. Studying baby logic may have zero, if not negative, effect.

I intend to criticize the philosophical assumptions which seemingly justify the teaching of baby logic and raise a question, whether logic really should be taught to people, who do not need to understand foundations of mathematics, analytic philosophy or computer science and neither are willing to undergo certain mathematical training.

But where did this idea that logic is a discipline describing rational argumentation came from? In section 1.5 I will show that its origins can be traced to founder of logic, Aristotle, but in section 1.6 I will argue that the logic somehow departed from this original intention during its evolution.

The philosophical reflection of this development, however, came only very recently. It was no sooner than in nineteenth century that new logic started to emancipate from this predicament.

Until the beginning of the 20th century most of the logical textbooks intended for high school students shared the very same structure, of what M. Jauris calls *traditional logic*: Concept, judgement, inference (deductive and inductive), some basics of scientific methodology and heuristics, including some elementary treatment of argumentation and fallacies particularly in textbooks written in English ([**26**], p.10). This scheme corresponds to the traditional interpretation of Aristotle's logical works. Examples of such textbooks are given in ([**28**], p.343).

The growing influence of so called symbolic or mathematical logic culminated in Quine's textbook *Method of Logic* in which traditional logic is abandoned and is replaced by modern logic which, according to Quine, does not study arguments or inferences but rather relations between statements. To criticize Quine for not providing an adequate account of argument and reasoning is therefore unjustified. It came only after a synthesis of the traditional and modern in Irving Copi's *Introduction to Logic* which became the standard of other logic textbooks, that these two already quite distinct disciplines were confused and mixed into a somehow suspicious hybrid ([**28**], p.344). To quote John Woods:

> It is easy to see that Aristotle, Frege and Turing were pursuing entirely different ends. This is frequently not understood by writers of our logic textbooks and by many of those who teach from them. There is an altogether entrenched disposition to tell students that logic is the theory of argument and/or of deductive reasoning (which is partly true of Aristotle and not true at all of Frege and Turing), and then to give as this logic a version of Frege's logic or (worse) of Turing's. And so we have it: In classrooms the world over Aristotle's targets were being chased down with Frege's ordnance or (worse) with Turing's. Small wonder that in the early 1970's students started voicing their skepticism about the prospects of modern symbolic logic producing anything like a theory of argument, still less of deductive reasoning.

([**72**], pp. 141–142)

The students' critique mentioned by Woods dates back to the1970's and originated in the US. After 1960s, universities in US and Canada often took increasingly political stances and the students, who often actively participated in protests against war in Vietnam, demanded that their lecturers help them to develop their critical attitude to the political establishment. Johnson and Blair quote Howard Kahane, who described his teaching experience of the year 1971 in these words[2]:

> Today's students demand a marriage of theory and practice. That is why so many of them judge introductory courses on logic, fallacy and even rhetoric not relevant to theirs interests.
>
> In class a few years back, while I was going over the (to me) fascinating intricacies of the predicate logic quantifier rules, a student asked in disgust how anything he'd learned all semester long had any bearing whatever on President Johnson's decision to escalate again in Vietnam. I mumbled something about bad logic on Johnson's part, and then stated that Introduction to Logic was not that kind of course. His reply was to ask what courses did take up such matters, and I had to admit that so far I knew none did. He wanted what most students today want, a course relevant to everyday reasoning, a course relevant to the arguments they hear and read about race, pollution, poverty, sex, atomic warfare, the population explosion, and all the other problems faced by the human race in the second half of the twentieth century.

([**28**], pp. 340–341)

The above mentioned critique of modern formal logic being used as theory of argumentation gave rise to a discipline called *informal logic*[3].

Now should prof. Kahane answer to the impertinent student that if he is not capable of making sense of the president's statements and he expects a university course on logic to teach him, he might not be the right person to study at the university in the first place, there would perhaps be no informal logic movement at all. Such conversation could

[2] Johnson and Blair identify three major sources of the critique of formal logic as the theory of argument: *pedagogical critique*, *empirical critique* and *internal critique*. I will elaborate on those critiques and their relevance, once formal logic and its fundamental concepts will be properly introduced in explained in section 2.3.

[3] In Czech republic it was pioneered by Z. Zastávka and M. Jauris (see [**75**]). For an overview of this discipline see [**27**].

truly take place only in the United States and only in late 60's within the American educational system based on a pragmatistic ethos of John Dewey.

Informal logic is therefore often associated with the critical thinking movement in education[4]. According to it, learning formal logic does not cultivate critical thinking abilities, such as the ability to recognize arguments in natural language, to evaluate them, or to form rational arguments and apply them correctly in a reasonable discussion.

However, such skills, as I already suggested, are simply preliminary for any genuinely scholarly attitude to any subject and should be already prerequisites to study of any such subject at the level required by universities. They are essential learning capabilities and are inherently cultivated by studying respective special subjects in an appropriate way. University students should get acquianted with them and no special subject aimed at their cultivation should be needed.

The relevant question therefore is: Can there be a special subject cultivating these skills? And do we actually need such course? More specifically, can informal logic stand up to be a subject of this kind?

The most current accounts of informal logic would perform even more poorly at this task, I believe, than their formal counterparts. Moreover, with its pedagogical ethos, informal logic is mostly developed in various textbooks in quite an unsystematic fashion. Truly scientific monographies containing some systematic treatment of the overarching theory of informal logic are therefore still quite hard to come by.

In fact, it seems as if the only merit of informal logic was that it revived interest in Aristotle's theory of informal fallacies. Douglas Walton, one of major proponents of informal logic, is very critical of this situation in his field. To borrow his words:

> Most logic textbooks include a short section on informal fallacies; the Aristotelian influence is usually dominant, or so it appears on the surface, but the so-called standard treatment consists of a series of 'one-liner' examples with a brief, superficial commentary on the fallacy. Attempts are made to put Aristotle's classifications into a modern context, often with peculiar results. What we end up with is superficial and often erroneous and incoherent treatment of the subject.

([**66**], p. 8).

[4] see [**21**] for a very brief introduction.

Even some of the textbooks of baby logic incorporated the theory of informal fallacies. No need to say that such treatment of the topic inherited all the errors perpetuated by the authors of the textbooks from the informal logic camp.

Moreover, because modern formal logic is an intended subject of those textbooks, some of them were supplementing or even replacing Aristotle's account of formal reasoning, theory of syllogism, by some of the modern formalisms, usually sentential logic and/or some reduct of the logic of quantifiers[5].

My primary aim, however, is not to criticize such accounts or suggest alternative approaches to teaching logic. Neither will I discuss informal logic or critical thinking into great depth and comment on its merits and shortcomings.

The discussion of previous paragraphs was merely intended to show why it is important to investigate carefuly the relation of logic to the argumentation. My primary aim is to investigate the 'traditional' paradigm of modern logic, its historical origins and philosophical motivations and explain, why logic studied within this paradigm is not suitable to be the theory of good arguments.

I will also explore an alternative paradigm of logic and its modern incarnations in so-called nonmonotonic logics. By comparing these two traditions I will hopefuly explain some misunderstandings arising from confusing the two distint yet related motivations of such approaches to logic.

Before I will sumarize all these particular goals into the main thesis, let me make some remarks on my sources first.

This thesis, although it strives to provide relatively self-contained effort, presupposes that a reader is acknowledged with basic notions of formal logic and, to a lesser degree, has an elementary knowledge of history of philosophy. Most of the facts from these areas are presumed throughout the text.

It would be inconvenient if I quoted properly source of each basic fact. I will therefore list the major sources of my thesis in following paragraph without mentioning them in particular places of the text. It is also unavoidable, but somehow desirable, that these sources are mostly works of my tutors, as at this stage of my learning I have still accumulated substantial portion of my knowledge on logic from their lectures.

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[5]Ironically, Aristotles syllogistic often proves to be a more useful tool for representation of arguments of natural language than its modern counterparts.

In the case I do not quote from some of the books I am going to mention here, I do not list them in the bibliography, but that does not mean they are not relevant for understanding this work properly—quite the contrary.

There is a plenty of books on mathematical logic, ranging from basic introductions to the field, to quite exhausting accounts—they mostly vary in pointing out different aspects of the problem.

To give a representatory account of the most distinguished approaches: Quine's *Mathematical logic* or *Methods of logic*, Church's *Introduction to mathematical logic*, Priest's *Logic: a very short introduction*, Kleene's *Mathematical logic* or Lorenzen's *Formal logic.*

For a more recent treatment on specialized areas of logic see classical works *Model theory* by Hodges, *Basic proof theory* by Troelstra and Schwichtenberg, *Classical recursion theory* by Odifreddi and *Computational complexity* by Papadimitriou.

In Czech the most acknowledged publications in the field, both for their rigor, clarity, and range are Sochor's *Klasická matematická Logika* (its briefer and somehow more reader friendly version has just been recently published as *Logika pro všechny ochotné myslet*) and Švejdar's *Logika: Složitost, neúplnost, nutnost.*

Particularly relevant and often quoted in the text are such accounts which relate logic to natural language. Specifically Peregrin's *Logika a logiky*, *Logika a přirozený jazyk* by Svoboda and colleagues and Svoboda's and Peregrin's *Od jazyka k logice.*

A brief, yet exhaustive, survey of most important theorems of nonmonotonic logics which I will discuss in the section 3.3, is provided in Antoniou's *Nonmonotonic Reasoning.* Another summary is provided by *Inteligencia ako výpočet* by Šefránek, whose chief merit lies especially in relating those systems to practical problems of knowledge representation in artificial intelligence. It is also good to consult *Handbook of Logic in Artificial Intelligence* for this purpose. Last but not least, Makinson's *Bridges from Classical to Nonmonotonic Logic* is a useful introduction which not only classifies nonmonotonic logics in a very systematic way and compares theirs properties, but also points to most other important sources in the field.

All my scarce claims about history of philosophy could be hopefuly backed up by Copleston's seminal work *A history of philosophy.* For details from history of logic it is best to consult *Development of Logic* by W. and M. Kneale or *Formal Logik* by Bochenski. A handy introduction to Aristotelian logic, as well as guide for further reading is the book *Úvod do logiky Aristotelské tradice* by Lukáš Novák and Petr Dvořák.

Concerning analytical philosophy in general, the sources I consulted most often were Coffa's *Semantic Tradition: From Kant to Carnap*, Peregrin's *Kapitoly z Analytické filozofie* and Valenta's *Problémy Analytické filozofie*.

Very useful insights into basic problems of philosophy of logic are provided by Haack in *Philosophy of Logics* and in Quine's book *Philosophy of Logic*. Some selected frequently discussed, though not always relevant, topics from philosophy of logic can be found in Read's book *Thinking about logic: an Introduction to Philosophy of Logic*.

This thesis is mostly based on papers I have already published on several occasions. I have stressed the need for better understanding of the nature of logic for pedagogical purposes in the bulletin for education of logic [**40**].

The core of my critique of traditional understanding of formal logic as the science of reasonable arguments (which is based on a radicalized version of Toulmin's critique) is summarized in [**41**]. Its implications and background are explained in greater detail in [**43**].

I have also briefly discussed the foundations of some alternative approaches to the study of argumentation. These include informal logic and accounts which arose quite recently in the field of computer science an artificial intelligence [**38**]. In [**42**] I explore some technical aspects of one of the possible approaches to argumentation based on deductive logic, as defined in [**8**].

The differences between formal and informal approaches to argumentation and their historical origins have been briefly mentioned also in my article [**39**].

The major thesis I want to establish is that formal logic alone cannot give us a sufficient tool to decide which kinds of argumentation are reasonable, but can be utilized within a larger theory, that provides more refined account of good argument. This main thesis will be subdivided to following intermediary propositions:

First—I will show that there are reasonable arguments which are not logically valid. The traditional doctrine of Aristotle was that we can transform them into logically valid arguments by adding additional premises. This is certainly correct, but this doctrine is being largely misinterpreted by modern logicians, because they are already working with a different epistemology than Aristotle.

Aristotle acknowledged the category of plausible statements which are often the only ones that we can add to such incomplete arguments.

Modern doctrine requires that all premises of reasonable (sound) argument were true. This is mostly not achievable.

The evolution of Aristotelian philosophy of logic and science to its positivistic descendant will briefly exposed be in order to substantiate this claim.

Second—I claim that the inability to describe argumentation properly is not accidental failure of our logic, but this limitation is inherent to the very notion of formal logic itself. This means we cannot produce any logic in a way non-classical logics are derived from classical one, without giving away its central and defining feature, such as formality of logical truth.

I argue that the formality of logical truth and validity is in fact essential for formal logic which was historically developed to serve as the science of demonstration. This function of formal logic binds it with modern mathematics on the one hand, but prevents it to give an adequate account of valid arguments on the other hand.

Third—I will conclude that this predestination of formal logic is not that much a result of contrast between clarity of mathematical languages on one hand and ambiguities and complexities of natural languages on the other, but rather comes from a greater variety of uses of argument outside mathematical discourse in which only demonstrations are relevant sources of knowledge.

Consequently we may model natural language argumentation using abstract and symbolic model, if we give up the requirement that each reasonable argument must have some generic form.

Let me conclude this preface with some methodical remarks.

My thesis aims to reflect certain rarely questioned assumptions and prejudices about the nature of logic from philosophical and historical perspectives. Consequently it is mostly an account of someone else's accounts of logic.

It is therefore not intended as introductory text that should initiate its reader into the basic concepts and methods of logic. Certain level of knowledge and understanding of the fundamental concept is presupposed, although I introduce certain elementary definitions in places, in order to illuminate those concepts.

Such paragraphs containing very elementary definitions may be regarded as superfluous, however, I include them only to demonstrate some of the fundamental concepts. These expositions of elementary material will be often accompanied by my commentary, relating it to the more substantial material in the text.

Nonetheless, even these few elementary accounts are not intended to provide exhaustive and intelligible picture of the subject. It would be impossible and useless to structure the text as a treatise, where basic concepts and axioms of the field are introduced at the beginning and new concepts defined from them later and as new theorems proved. The thesis is hermeneutical in nature, uses hermeneutical methods and it is therefore the hermeneutical circle that is at its core. The preliminary understanding of the complex matrix of concepts and opinions is presupposed, to be revised in the light of new evidence.

The conclusions that can be drawn from this critique are not unique and unambiguous.

I do not wish to claim that a more appropriate account of argumentation could not be reached by the refinement of the old approaches, such as formal logic and theory of probability, rather than by developing the new ones, such as abstract or dialectical approaches to argumentation. I hope only to illustrate that taking a new start and giving chance to yet undeveloped and somehow naive approaches might sometimes be more reasonable than the stretching of old meritorious theories beyond the point of clarity and lucidity.

CHAPTER 1

Logic and Argument

1.1. Logical evaluation of arguments

So what is an argument and what does logic have to do with it?

An argument is a pair of set of *sentences*, its *premises* and a single sentence designated as its *claim*. It is often required that those are declarative sentences which are unambiguous and have determinate meaning. It is also usually required that their truth-value be clearly determinable and be either true or false[1].

An argument is said to be *valid*, if its claim is somehow *justified* during course of argumentation that is whenever the truth of its premises somehow guarantees or purports the truth of the claim. A claim of a valid argument is said to be a *conclusion* of its premises or we can say it *follows* from the premises[2].

We may also encounter accounts of logic in which a notion of *inference* is used in place of a notion of argument. The reasoning behind such terminological usus, I believe, stems from acknowledging that there are some arguments which cannot be quite well explained in terminology of formal logic. These include some familiar examples I will also mention later on the page 8, such as argument from authority, or argument from analogy etc.

These accounts of logic will avoid most of the criticism aimed at accounts of logic as the theory of arguments. Nonetheless, they lead to a whole new set of problems, such as: What is a relation of inference to argument, rational dialogue and debate? Does inference play any important role in reasoning in natural language, or is it something limited to mathematical reasoning only?

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[1]To find such species is actually a quite difficult task already. Sometimes another kind of truth-bearer is required, for example propositions which are often defined as meanings of sentences. Propositions are either eternally true or eternally false so that all the trouble with the indexical sentences like ‘I am hungry now’. Self-denying, vague, or meaningless or uninformative sentences must be excluded from consideration, or given some other value.

The discussion of truth-bearers is nicely summarized in [**24**]. Practical examples of problematic sentences are in [**59**]. While this problem is important for philosophy of logic, it is one of the kind which should probably be settled by linguistic considerations and is not so relevant for my exposition.

[2]Terms ‘claim’ and ‘conclusion’ are sometimes used as synonyms, as in [**19**], but sometimes the latter is used for claims of valid arguments exclusively. I will follow the second use of the term in this thesis.

We can also say that the premises of valid argument *entail* the claim or alternatively we can say that there exists an *entailment relation* between premises and claim of valid argument. The notion of entailment, however, has been explicated quite precisely by semantic methods of formal logic which I will mention on the page 49, so I will reserve this term for more its mores specific use.

Let me therefore, for the sake of contradiction, presuppose that an inference is the same thing as an argument. The necessity to distinguish those two concepts will become clearer in following sections.

An argument is said to be *sound*, whenever it is valid and all its premises are *true*[3].

The following motivations, although not explicitely mentioned in the book, obviously underlie the above definitions and are usually implicitly presumed in textbooks of logic which aim to provide account of argumentation.

Sound arguments are such arguments which establish their claim. Other than sound arguments, for one reason or another, fail to do so. Their claim still might be acceptable, but we should not accept it just on basis of the given argument. Clearly a logic should help us to distinguish sound arguments from unsound ones, but we will need something more than pure logic to be able to do so[4].

Recognition of sound arguments is something that cannot be done by logical considerations alone because the problem of deciding whether some proposition is justified lies outside the scope of logic. Obviously if we were able to recognize sound arguments by logical considerations alone, we would need nothing else than proficiency in logic to understand, whether an argument presented to us by some expert, say microbiologist or historian of art, is sound or not. Unfortunately for logicians, this evidently is not the case.

Logic is therefore intended to be a science that should describe grounds of validity, not of soundness. Unlike the soundness of arguments which must be evaluated within some specific domain, be it a domain of some particular science or common sense, validity of argument must be evaluated without reference to the methods and discoveries of other sciences and must be delegated to logic.

The validity of an argument alone is usually completely independent of the content of its premises, as logicians often demonstrate on valid

[3]Once more—this definition often excludes arguments such that it is not quite clear, how should their truth be evaluated, or whether it makes sense to speak about their truth at all. Examples of such arguments are moral and aesthetic arguments, as well as arguments containing imperatives or even questions among the premises or in the claim.

To evaluate such arguments according to the traditional approach, it is often necessary to reformulate the definition of soundness or to come up with broader concept of truth. I will not address this problem directly here, although I will argue later (on the page 55) that we need not to have a clear-cut definition of truth before we start to do logic.

[4]This doctrine has been also challenged by many authors. See for example an article of P. Bondy [**12**].

arguments with obviously absurd premises. The acceptability of the premises is required for the soundness of an argument. The goal of logic which is to determine validity of an argument, should therefore be independent of particular sciences[5].

Ideally logic should not only be independent of other sciences, but complement their methods in a substantial way because to determine if an argument is sound we will need to determine if it is valid as well.

So what actually is the role of sound arguments for rational argumentation? Is it to persuade our partner in dialogue about some true sentence?

Suppose we wish to persuade Mr. Smith to accept a certain sentence. Now we may present him an argument, such that its claim is the sentence we wish Mr. Smith to accept, and its premises are sentences that are true.

In such a case, if now Mr. Smith recognizes that our argument is valid and that its premises are all true, we have already achieved our goal. Mr. Smith now has no other choice, but to accept the claim of our argument as well.

Yet this argument of ours can be perfectly valid, but fail to persuade Mr. Smith because it is not persuasive enough or because Mr. Smith is very stubborn. On the other hand Mr. Smith can be persuaded by an argument which does not meet the standards of validity and should be avoided in rational argumentation.

A distinctive feature of valid argument therefore is that Mr. Smith ought to accept its conclusion, once he accepts its premise, if he is to be concerned as rational reasoner. This feature of valid arguments is called *normativity* in [**6**]. The laws governing validity should be normative and inter-subjective.

Logic is therefore not concerned in its first instance with the persuasiveness or rhetorical strength of the argument. Valid arguments are usually used in a *dialogue*, whose purpose is mutual and cooperative 'uncovering of truth', in contrast to a *debate*, whose purpose is

[5]The place of logic in the body of sciences has been always a matter of great controversy within a classical model science which I will introduce in the section 1.6. It seems natural to assume that logic has its own subject, so called second intentions such as concepts, truth, propositions or relations among them or else. Such logic would be a science among sciences.

On the other hand its elusive position among other sciences would suggest it is rather an *ars demonstrandi* than another scientific discipline. This has been quite a prevailing stance from Aristotle to Kant (see [**17, 16**]), but opposite doctrine was mainly adopted by the thinkers of the semantic tradition, with the exception of Wittgenstein.

to persuade the opponent to accept our opinion, using sometimes even 'illegitimate' irrational methods[6].

Rational argumentation, consequently, can be roughly delimited as such kind of dialogue, where only sound arguments count as appropriate. The traditional doctrine is that rational argumentation is simply an exchange of sound arguments. It follows that logic, as the science of valid arguments, is an essential part of such science of argumentation.

We can now someone like a rational reasoner. A rational reasoner should always be persuaded by sound arguments and sound arguments only. He also should not accept anything which simply is not true.

The rational argumentation can only fulfill its purpose, if its participants are rational. By presenting sound arguments, one to another, the participants of rational argumentation persuade their partners to accept propositions which are also guaranteed to be true.

So what if Mr. Smith refuses a claim of a sound argument? What if Mr. Smith simply fails to recognize the truth of the premises or the validity of the argument? Can we just dismiss him as irrational?

This problem is often addressed quite unproblematically and uniformly: each valid argument can be broken down into a chain of valid arguments, such that the premises of each argument are already included among the premises of the original arguments, or have been established as claims of the former arguments in the chain.

Such a chain of arguments leading to one final particular claim c from some original set of premises P is usually called a *demonstration* or *proof* of this claim c from premises P.

Each of the intermediary arguments in the proof should be already obviously valid for the rational reasoner. Moreover, such theory of demonstrations is often a crucial part of logic. By means of proof theory we are not only able to establish to ourselves that some argument is valid, but also demonstrate it to our partner in discussion.

There may be some other methods of logic how to establish that an argument is valid beside constructing such chains of arguments which are traditionally not considered to be proofs, some of them will be mentioned later in section 2.3.

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[6]If we adopt Aristotle's ethos we would have to condemn all who would engage in unfair, manipulative, and irrational practice of debate. Such people must be either greedy and dishonest sofists seeking only personal gain or mentally, or socially inept beings (see [**19**]), who were not told how to argue properly.

Walton [**66**] takes a more unbiased approach classifying eristic debate as one of many possible modes of dialogue, with its specific rules and goals. He also offers much finer refinement of this classical distinction.

However, we still may apply such methods and procedure to demonstrate the validity of arguments and therefore they can serve the same purpose as proofs in dialectical argumentation.

The value of logic is in the fact that it provides us with such rules of derivation or other methods of demonstrations. It gives us useful and practical tools how to recognize valid arguments. Moreover, it can be used to demonstrate that a certain argument is valid to other rational person.

And what if Mr. Smith simply refuses to acknowledge the premises of our chain of arguments? This concern is usually dismissed. After all, we have already observed that logic can help us only to determine the validity of an argument. To demonstrate its soundness is something for which we will also often need to use methods of other sciences which can demonstrate truth of the particular premises in question.

I have presented, in a nutshell, fundaments of the common doctrine which justifies usefulness of logic for argumentation. It is presumed that goal of a rational argumentation is to persuade our opponent about verity of some thesis using only rational tools. That means we are obliged to demonstrate to him that this thesis follows from a certain set of premises by constructing such demonstration, usually a chain of obviously valid arguments.

Clearly this is a common practice in mathematics. We begin with a set of axioms and premises and construct proofs. The axioms must be accepted by our partner in dialogue, so that we could proceed.

According to a certain understanding of science which I will discuss in detail in the section 1.6, other sciences should function in a similar fashion— so to say—*more geometrico*. The difference between mathematical and generally scientific argumentation would consist in a nature of the premises of admissible arguments.

At least since Renaissance it is commonly held that science should be based mainly on empirical observations and later this has been contrasted with the nature of mathematical axioms which are nowadays almost exclusively considered to be prior to any experience.

The utility of logic outside scientific reasoning, so important for baby concepts of logic, is far more problematic issue. It is not at all obvious why every rational debate should follow this scientific model and why eristic argumentation should always be considered irrational. This often contrasts with the argumentation practice we experience.

Also the importance of selection of proper set of premises for our argument will be more important than in scientific case where a certain agreement on basic principles of the science is usually presupposed prior to any argument. The selection of proper premises will often be the most crucial part of persuading our opponent in common argumentation. Knowledge of proper way to construct demonstrations will play only minor role. This claim is certainly sufficiently backed by our daily observations of argumentation practice.

More importantly though, demonstration is not always the chief instrument of rational argumentation. It is quite common that arguments that would not normally be clasified as logically valid are used within an argumentation without their validity being questioned. Usually logicians admit that there are kinds of validity beyond that recognized by logic.

Various logicians differ in their explanation of how do these valid argument relate to arguments valid according to logic. In the following sections I will give two different classifications of valid arguments and compare their motivations and relation of one to another.

1.2. Deduction and its limits

Valid arguments are usually divided into those which are *deductively valid* and those which are *inductively cogent*[7].

Deductively valid arguments are such that their claim *necessarily* follows from their premises—that is—in all cases in which all of their premises are true, so is their claim. In other words—it cannot be the case that all of the premises of deductively valid argument would be true, while its claim would be false. Beall and Restall [**6**] call this feature of arguments *necessity* and consider it to be one of the fundamental properties of valid arguments described by logic[8].

Inductively cogent arguments, on the other hand, establish their claim merely in most typical cases. Their premises present a good and

[7]See for example [**19**].

[8]The other two being normativity which I have already mentioned and formality which I will mention in section 2.2.

Beall and Restall, in my opinion, sufficiently establish that all arguments described as valid by methods of modern formal logic indeed have all these features, but it is not clear whether also each argument with all or some of these features will be recognized as valid by logical methods.

There will be, it seems to me, many arguments which will have some of those attributes and not others. Inductively cogent arguments, for example, seem to be normative, but not necessary. They are most often not formal neither as i will conclude in the section 2.6. As a consequence of this they fall out of the scope of formal logic.

Later (section 2.6) I will argue that formality implies necessity, but the related question whether all necessary arguments are also formal and vice versa is extremely complex and would require a thesis of its own. In section 1.3 I will also conclude that not all normative arguments are formal.

strong evidence that supports their claim, but it might be the case (although this case would be somehow exceptional or uncommon) that all of their premises would be true, while their claim would be false.

Inductively cogent arguments can often be listed under one of the following headings: *probabilistic arguments*, *causal arguments*, *predictive arguments*, *arguments of analogy*, *arguments of appearance*, *arguments of authority*, *arguments of ignorance*, *moral arguments*, *aesthetic arguments* etc[9].

At this point a word of explanation is required. The term 'induction' is desperately ambiguous and used for various different kinds of reasoning.

One may, for example, often encounter accounts of deductive reasoning as reasoning from general premises to specific conclusions and of inductive reasoning as reasoning from specific premises to general conclusions. This is not a very useful though, unless we explain which premises are general and which are specific. In the case of most of the arguments, it is not quite evident what would that mean[10].

Originally this term was used to translate Aristotle's term *epagoge* which loosely corresponded to reasoning common in geometry, that is making conclusions about some infinite class of figures on basis of some particular, yet sufficiently general, representations of these figures.

Nowadays a term 'mathematical induction' would probably be used for such kind of reasoning[11]. I am briefly mentioning historical background of such distinction on the page 30.

This is often contrasted with induction in empirical sciences which makes claims about whole set of objects on basis of some facts about relatively large subset of these sets[12].

Actually it is not quite uncommon to simply identify inductive arguments with such incomplete generalizations. The remaining kinds of inductive arguments from our list are either reduced to arguments

[9]See for example [**19**].

[10]These reservations towards deductive-inductive distinctions and attempts to explain them using notions like 'information content' have been expressed also in [**24**].

[11]Critical account of classical theories of deductive and inductive arguments can also be found in [**73, 74**].

[12]In some modern models of the science, Walton mentions ([**67**]) that of Peirce, yet another kind of reasoning is acknowledged—that is *abduction*. It is particularly important, that both induction and abduction can be explained in logical terms (see [**62**]).

In fact in the section 3.4 I will present Poole's account of abduction, in order to demonstrate that logic can be used to analyze abductive reasoning as well.

of this type (predictive and probabilistic arguments are the most obvious candidates for such reductions) or considered to be invalid and fallacious straight away. Consequently some kind of inductive or probability logics is offered as an adequate theory of all inductively cogent arguments[13].

Authors of [**28**] (p. 349) mention criticism of such division of valid arguments into deductively valid or reducable to an argument from incomplete generalization as the ‘positivistic’ doctrine. Indeed there are many problems with such reductions but to evaluate all of them would deserve its own book.

My focus, though, will be on criticizing an even stronger doctrine, referred to as ‘deductivism’ (ibid.). According to this doctrine all valid arguments are essentialy reducible to deductively valid arguments. I will explore this doctrine and analytical ideal behind it in greater detail in the section 1.3.

In this text inductively cogent (or cogent, for short) argument will simply be any argument which is valid, yet does not establish its conclusion with necessity. This will include some incomplete generalizations, but will not be reducible to them.

Cogent arguments, as we have observed, often involve uncertain, ambiguous, conflitcting claims, or make predictions about future. The attempts to provide formal logical semantics for notions of probability, uncertainty, vagueness, temporality and modality are the most common attempts to remedy genuinely cogent arguments from the formalists camp.

What is meant by this reduction is what will be demonstrated on the second classification of arguments in [**59**]. In their account valid arguments are further divided into three classes according to what additional resources besides logic we need to employ to evaluate them. The valid arguments are valid: *logically*, *analytically* and *factually*.

This is probably a refinement of classification of W. Sellars [**58**], who divides arguments into *formally* (logically and analytically) valid and *materially* (factually) valid. There are numerous problems with such classifications and authors of [**59**] admit that the boundaries between those categories are fuzzy and vague[14].

For my current purpose—to compare this classification with that of classyfing arguments as deductively and inductively valid—it suffices to presen it only in very brief and imprecise outline.

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[13]See [**69**].

[14]I will discuss motivations for such classifications and its problematic nature in section 2.6.

Following arguments are examples of logically, analytically and factually valid arguments respectively.

A1:

Premise 1: If Barney is a dog, then Barney is an animal.

Premise 2: Barney is a dog.

THEREFORE

Claim: Barney is an animal.

A2:

Premise: Barney is a dog.

THEREFORE

Claim: Barney is an animal.

A3:

Premise: Barney is a dog.

THEREFORE

Claim: Barney cannot fly.

According to the most common interpretation in so-called semantic tradition (It will be introduced in the section 1.6), only logically and analytically valid arguments establish their conclusion with necessity and are deductively valid in the sense of deductive validity explained above. Moreover, it is also presupposed within this tradition that all deductively valid arguments are formally (that is logically or analytically) valid.

Provided this was true and class of deductively valid arguments was coextensive with the class of formally valid arguments and provided that the above classification gives an exhaustive description of all valid arguments, it would follow that inductively valid arguments, to the extent they are valid, would be at most factually valid.

But is this truly so? Are all logically and analytically valid arguments deductively valid?

There are various different explanations why claims of logically and analytically valid arguments follow inevitably from their premises indeed. Claims of arguments **A2** and **A3** are justified purely on basis of the premises of those arguments, provided we give certain key terms in these premises and claim correct meaning.

In case of argument **A1** we only need to understand the meaning of the phrase 'IF ... THEN' to immediately grasp that the claim is inevitable. In case of argument **A2** we only need to understand terms 'dog' and 'animal' to the extent to realize that every dog is also an animal, so to say by definition. So we are entitled to conclude that a claim of certain argument follows from its premises with necessity because we know that certain concepts of the argument in question have fixed meanings.

The reason to come up with two categories for formally valid arguments is motivated by internal semantic concerns. If we truly wish to maintain that logic is indeed an universal science, its laws should not rest on meanings of, say, zoological concepts, such as 'dog' or 'animal'. Logically valid arguments, we are told, are valid due to the meaning of logical concepts only. Just what are these logical concepts is debatable, but following motivation of such classification, it is perhaps safe to adopt G. Ryle's viewpoint [**56**] according to which logical concepts are these concepts which are *topic-neutral*. What underlies logical laws is a meaning of logical terms[15].

Clearly the semantical explanation could be challenged. Quine [**50**] asks why should be meanings of certain concepts considered more invariant than, to relate his critique to our example, generalizations about behaviour of dogs, or say, laws of thermodynamics.

Linguists often describe changes of meanings of concepts within a language. Validity of each argument is therefore merely conditional. It always rests on a certain interpretation of its premises and claim. This problem will be discussed only briefly in section 2.6, however, more fundamental problems of this classification will be pointed out in that section.

Therefore I will not elaborate on this well-known problematic right now and will rather concentrate on a critique of the presumption that all valid arguments that are not deductively valid, must be valid factually.

However there is a crucial problem with body of doctrines presented so far which willl be criticized thoroughly. It is the following dilemma: If we define valid arguments rather broadly as arguments that can be legitimately used in all kinds of rational dialogues and inquiries, we will need other than jusf logically, analytically and factually valid arguments. Some reasonable sound arguments can never be deductively sound, neither logically, nor analytically, nor factually.

Let me now explain why. It is very easy to turn any valid argument into logically valid argument by adding a single premise: 'Whenever $(P_1 \ldots P_n)$ are true, so is C', where $(P_1 \ldots P_n)$ are premises of the original argument and C its claim respectively.

[15]This explanation is not the only possible explanation of deductive validity philosophers have proposed. Whether logical laws are actually ontolological, epistemological or merely semantical is debatable philosophical issue. It is not quite relevant for my exposition, so I will simply accept this explanation as unproblematic.

However, there is no guarantee that its premises will also be true. Consequently there is no guarantee that the new argument will remain sound, provided the original argument was sound in the first place, of course.

Obviously, a more refined completion of the argument might be provided. Still there will be arguments which are sound without any chance that we could turn them into logically sound arguments by adding any premises.

Suppose I know Barney is a dog. I am therefore entitled to claim that Barney likes bones on the basis of the following argument:

A4:
Premise: Barney is a dog.
THEREFORE
Claim: Barney likes bones.

But what if, for some reason, the conclusion of this argument will come out as false? Would that mean my reasoning was invalid? Clearly not, I could not *reasonably presume* this will be the case because dogs usually do like bones.

So we have the following situation. Argument **A4** is valid (according to the definition of [**59**]), its premise is true and therefore it is sound. Yet its conclusion is false. Therefore there can be no way how to add additional true premises to the argument so that it was logically sound because else its conlusion would necessarily have to be true, while it actually is not.

The argument is not valid deductively, but in case Barney actually does not like bones, it is not valid factually neither. There are no facts that would guarantee the truth of its claim because its premises happen to be true and its conclusion is false.

We accept the claim of the argument even if we are ignorant about additional facts, that would make it factually valid or invalid. In case such argument is factually valid, we accept it as valid independently of this fact. In case it is not valid in any of the sense described in [**59**], we still accept it despite the facts, that would prove us that the conclusion will actually be wrong. So whether **A4** is or it is not factually valid is therefore completely irrelevant for us to accept it as valid.

1.3. Principal irreducibility of substantial arguments

To explain an alternative approach, I will confront this theory of arguments with that of the influential Toulmin's book *The uses of Argument* [**65**]. Toulmin's book inspired, directly or indirectly, alternative pragmatic treatments of arguments within the fields of informal logic (see section 3.2) and artificial intelligence (see section 3.3).

For Toulmin [**65**] the primary function of each argument is to support a certain claim which belongs to a specific field on the basis of the particular data of that *field*. The claim can often be spelled out in terms of impossibility, or impropriety, as in the following examples:

(1) You cannot lift one hundred tons with your bare hands!
(2) You cannot smoke here!
(3) You cannot be rude to elderly people!
(4) You cannot call Byron a greater poet than Pushkin!
(5) You cannot call high school teachers 'professor'!
(6) You cannot find the biggest prime!
(7) You cannot affirm "It is red and it is square" and ''It is not red" at the same time!

These improprieties are all of different kind. They range from logical improprieties through analytical, towards factual, but also including improprieties and specific regulative principles of the respective fields from which a given argument is taken.

Toulmin suggests a more refined model of an argument. His argument does not consist merely of premises which he calls *data*, and a claim which he calls *conclusion*, but also includes a *warrant* which justifies the inference from the data to the conclusion, and its *backing*, some external data that justify the use of the warrant in question.

The claim can sometimes be accompanied by a *qualifer* which informs us about the degree of force which we should assign to the claim. Examples of such qualifiers are 'necessarily', 'presumably', 'certainly'.

Toulin also introduces a concept of *rebuttal* which is an information could possibly invalidate the qualification and therefore the whole argument.

Let us demonstrate this model on a practical situation. A clerk comes to an office and encounters a man smoking a cigar. She is disgruntled and shouts: 'Sir, you cannot smoke here! You will necessarily have to go outside.'

'Can't I?', replies the gentleman in a cold blood.

'I am sorry, but smoking is prohibited by the owner of the company, Mr. Smith. Look, behind you! There is a no-smoking sign hanging on the wall just above your head'.

'That is alright.', answers the man, 'I am Mr. Smith.'. 'Oh, excuse me!', replies the lady surprised, 'I did not recognize you, Mr. Smith.'

'That is alright.", says Mr. Smith, 'You could not have recognized me as I hired you quite recently, and not even could you suspect I would have come here. I really do not like if my employees and customers smoke in the offices and you were right to presume I was one of them. You are doing a good job!'

The argument is 'You cannot smoke here and stay inside!' the claim of the argument is 'You will have to go outside!', the data is represented by the fact that the gentleman is smoking. Warrant of the argument is the smoking-prohibiting regulation which can be backed up by the sign on the wall.

An exception to the rules applies to the owner of the company. Therefore, the qualified conclusion of the argument is not warranted by the data—it is not the case that anyone smoking does necessarily have to go outside.

Nonetheless, if the clerk did not use this qualifier the argument would be good after all—the warrant and its backing were sufficient to qualify the inference from the data to the conclusion—as Mr. Smith actually willingly acknowledged himself.

Still, its conclusion will not hold. Perhaps a qualifier 'presumably' or 'probably' should have been used instead to make the argument even more accurate, still in this case the unqualified conclusion is warranted as well as practical use of arguments of such kind in similar situations contest[16].

But are not warrants part of the data?

Warrants differ from data, according to Toulmin, mainly in that aspect that data are given explicitly and warrants are assumed implicitly, but whenever asked for they can be explicitly formulated. Also, data are particular and warrants are in some sense or another general principles applicable in all arguments of some given type.

He gives an example of the following argument: 'Peterson is a Swede, therefore he is most likely not a Roman Catholic.' The warrant of this argument which enables us to move from the data 'Peterson is Swede.' to the conclusion 'Peterson is most likely not a Roman Catholic.', can be expressed by the sentence 'A Swede can be taken almost certainly not to be Roman Catholic.' Is this statement of fact or of meaning? Or a description of admissible reasoning in such particular case. What would it mean to say this sentence is true?

[16]Toulmin does not provide any theory of good and bad arguments which would be alternative to classical definitions of 'sound' and 'unsound', or 'valid' and 'invalid' arguments. His work is mainly critical to the traditional approaches.

I will use the adjective 'good' only in intuitive pre-theoretic sense to refer to arguments which succesfully establish theirs conclusions.

Fully developed abstract theories of good arguments based on Toulmin's insights will be described in section 3.3.

This warrant is not self-authenticating, as Toulmin puts it. Rather it is a merely plausible principle which does have exceptions. Most laws actually admit exceptions, including for example civil laws or laws of physics and are therefore merely plausible.

Contrast it with another example:

A5:

Premise: Anne is one of Jack's sisters.
THEREFORE
Conclusion: Anne has red hair.

This is a factually valid argument, in case it is a matter of fact that all Jack's sisters have red hair. This fact is what warrants the move from the premise of **A5** towards its conclusion.

But what is its backing? We may back this warrant only by checking individually that each one of Jack's sisters actually has red hair. But by doing so we have actually checked also the color of Anne's hair. So we can actually add this warrant as an additional premise and obtain a logically valid argument.

Toulmin remarks that most of the arguments we use on practical ocassions are not of this kind. For example we often make predictions about future and back them up by reference to our experience of how things have gone in the past or we make claims about somebody's feelings and back them up by reference to his utterances and gestures.

None of these backings is sufficient to support the conclusion of the argument with the explicit data alone. The argument step from the data and warrant to conclusion is a substantial one.

We claim something that cannot be known merely from knowing the data and backing of the warrant alone. Toulmin calls arguments of this kind *substantial* and the arguments of the above mentioned kind *analytic*[17].

Analytic arguments can therefore be presented in the following manner: Data,'in other words': Conclusion (because of warrant which I can back either by logic, linguistic meaning, or facts).

[17]The above example shows that Toulmin's category of substantial arguments is not identical to the category of factually valid arguments, as authors of [**59**] claim (p 43. fn 23).

All three categories of valid arguments studied in this book would actually be analytic in Toulmin's classification because by establishing that the data *and* the warrant are correct (true), we have already, albeit indirectly and implicitly, also established their conclusion.

So let's not confuse analytically valid arguments with analytical arguments, the former is a subclass of the latter. Toulmin is not concerned with specification of conditions of validity and automatically classifies valid arguments only.

That is why the conclusion of analytic arguments can never be rebutted, it just reformulates what has been already expressed in the premises and presupposed in the warrant.

This is generally not the case of substantial arguments. Their conclusions are merely plausible and can therefore be rebutted. The conclusion of substantial arguments must therefore not be necessarily true every time when the data are all true and warrant is true on the grounds of its backing (logically, analytically or factually).

Conclusion of such argument actually even does not have to be true. It suffices it is plausible with respect to a certain body of knowledge.

It is exactly this notion of plausibility which is, according to my understanding, crucial to proper understanding of Toulmin's account of arguments and which we will also encounter in section 1.5. Without this concept the crucial difference between substantial and analytic arguments cannot be drawn.

Authors of [**59**] try to accomodate this model of arguments with theirs in the following manner. Their categorization of arguments into logically, analytically, and factually valid is supposed to correspond to Toulmin's categorization according to warrants of respective arguments.

In the first case we are warranted to move from data to conclusion by meanings of logical constants, in the second case by meanings of some extralogical terms, in the third case by some true facts. The backings of such warrants will therefore refer to certain laws of logic, linguistic practice, or some observable facts. That is certainly correct, but not exhaustive classification of warrants.

Logically and analytically valid arguments are clearly analytic in Toulmin's classification. But so are factually valid arguments as well because by checking the backing of the warranting fact and that of the data, we have already checked the conclusion as well.

Clearly, implicitly assumed warrant does not always have to take the form of a statement of meaning or fact. It may as well be some general principle of, say, aesthetics or ethics. Toulmin considers much larger group of warrants and not all of them trivial enough to be immediately verifiable as true.

The two classifications of analytical arguments therefore do not correspond very well because of their different motivations. While Sellars and the authors of [**59**] are primarily interested in semantics and the way formal or material inferences (or arguments) contribute to constitution of meanings, Toulmin's interest is mainly epistemological. His concern is how we can back warrants and with what force do data truly establish some claim.

Sellars [**58**], as well as Toulmin, notes the tradition to explain other than logically valid arguments as enthymemes, that is incomplete logically valid arguments with missing premises. He gives a classification of six possible philosophical approaches to questions, whether material arguments are reducible to formal ones, indispensable to language, and whether they contribute to meaning or not.

Unlike for Toulmin, who does not question that a formal counterpart to a factually valid argument can always be constructed, for Sellars such transformation is always possible, although it is not a genuine reduction of the former argument because we are making this argument on different grounds.

In this respect he agrees with Toulmin, but there is no sign that he would consider the arguments whose claim is established with different force than that of analytical arguments as valid.

Toulmin, as an epistemologist, is not concerned with the validity of arguments only, but with their soundness as well. He is concerned in the fact whether an argument truly helps us to determine acceptability or truth of its claim, or not. His substantially sound arguments are arguments which cannot be turned into analytically valid arguments in any way.

The above mentioned concept of plausibility is of extreme importance indeed, especially with connection to rebuttals. They are also substantial to Toulmin's account so once more I will demonstrate that they cannot be properly reduced.

Authors of [**59**] consider negated rebuttals to be simply a part of the warrant. According to them, negations of rebuttals have to be added to premises (along with a warrant) so that the resulting argument would be logically valid. It follows that all sentences which might be used to rebute the argument must be false so that this argument was factually valid.

But it does not follow from Toulmin's account at all that to be able to draw conclusion of a certain argument, falsity of all its rebuttals must be known. All evidence we need to justify the conclusion is the evidence that would justify our warrant (data should always be unquestionably evident).

True, the argument is invalidated if one of its rebuttals is shown to be true. But in order to be entitled to make the conclusion it is simply enough that this is not yet the case. Again, warrants need to be merely plausible and analogically rebuttals must be merely implausible.

Remember argument **A4**. It is good and reputable argument independently of whether the sentence 'Barney is sick and can eat only fresh meet.' is false or even known to be false.

There may be infinitely many other rebuttals to this argument, but we may not know anything about their truth or falsity. To conclude the claim we merely need to consider them implausible, independently of their actual truth-value. The conclusion is warranted by the fact that dogs usually like bones (it suffices only to know this is the usuall case) which can be sufficiently backed by our previous evidence about behavior of dogs.

1.4. Quest for general approach

Toulmin calls a doctrine that each valid argument can be transformed into an analytic argument an *analytical ideal*. Due to the analytical ideal there are no genuinely valid substantial arguments—all valid arguments are either analytical arguments or incomplete analytical arguments in disguise.

The analytical ideal, Toulmin tells us, forces us to conclude that the validity of arguments is something that is field-independent and is determined only by the universal laws of logic which hold under any circumstances. Logic is therefore often not understood not as a jurisprudence of arguments, but as a system of eternal truths[18].

Substantial arguments are not valid, as they lack some substantial explicit premises which would justify the conclusion. Some substantial arguments can be redeemed though, if the missing justifications, together with a linking premise, are added, and so are these arguments turned into analytic ones.

Each analytic argument that is not logically valid can be turned into logically valid argument by adding the linking premise. Therefore each valid argument can be turned into a logical one. All the remaining arguments which cannot be turned into logically valid arguments must be disregarded.

Toulmin identifies three kinds of epistemologies which result from the adherence to the analytical ideal: *transcendentalism*, *phenomenalism*, and *scepticism*.

[18]This objection is not completely justified because logic can be interpreted as a mere tool of scientific reasoning.

Still, even under such interpretation it is quite a different discipline than general theory of arguments, unless we wish to embrace the most radical scienticism.

Moreover, this understanding of logic also presupposes certain understanding of science in general, as will become clearer in section 1.6.

Either way—neither system of eternal truths, nor a tool for scientific demonstrations is particularly useful for description of common argumentation. That, I believe, we can conclude from Toulmin's account undoubtedly.

While the first two approaches aim at bridging the 'logical gulf' between premises and the conclusion of substantial argument, the sceptical approach condemns all substantial arguments as invalid, arguments that we may accept for various reason, but which are nonetheless not normative.

Transcendentalists turn substantial arguments into analytic ones by interpreting the backing of warrants as background data which we obtain thanks to some non-empirical faculties of cognition. In the past philosophers summoned such sources of knowledge as a divine revelation, or pure reason which provided us supposedly with so-called truths of reason, or some kind of intuition.

These truths of reason are used as a linking premise. Therefore we complete seemingly substantial arguments with major linking premises, often in the form of some general statement or implication and turn them into analytic ones.

Thus for example general or predictive scientific judgements are justified, according to transcendentalists because we are endowed with knowledge of eternal truths of reason. Some other faculties of cognition can be summoned, for example our conscience when we are confronted with moral arguments.

The phenomenalist (positivist, reductionist) approach, on the other hand, turns the claim of the argument into something which in fact is contained in our sensual data. Therefore claims about objects are in fact disguised claims about our recent collections of sensual data.

Toulmin gives an example of application of the reductionist approach to arguments about other people's mental states in behaviorism. Behaviorists claim that statements about a person's feelings, emotions, thoughts are in fact statements about his observable behavior[19].

Scepticism just dooms all substantial arguments as invalid.

All those three approaches are unacceptable for Toulmin. According to him they induce epistemological costs, he is is most likely not willing to pay in order to maintain the analytical ideal. What is it worth anyway?

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[19]Positivistic approach of [**59**] would likely fall into this category, with the qualification that authors of this book also acknowledge that truths of facts and truths of meaning originate in various fields of human knowledge, rather than in universal and immutable 'reality' of language and totality of sensual phenomena, and consequently analytically valid and factually valid arguments therefore are field-dependent in their account.

Despite this they postulate category of logically valid arguments which are not field-dependant. This division is, however, very problematic as already mentioned.

A logician who swears to an analytic ideal just epistemology just wants to be credited for deciding validity of all arguments, without doing the difficult part. When faced with a truly interesting (because) substantial argument, he transforms it into a rather dull and simple case of barbara or modus ponens and then tells us that in order to verify that its conclusion is really acceptable, we now 'only' need to check some wild additional premises that he generated (ad hoc) during his analysis. This results in epistemology spammed with such logical 'leftovers'.

Would it not be easier just to deal with substantial arguments directly, without performing such dire transformations?

Despite his criticism of analytical ideal, Toulmin does not give up on the idea of some general and universal approach to argumentation. In his understanding, such approach to evaluating arguments would resemble jurisprudence, rather than scientific theory with its own eternal and universal truths.

Logic as the jurisprudence of scientific arguments would probably not insult him, but would be quite underwhelming according to his expectations. He believes some general theory of all arguments is achievable.

However it is still questionable whether such theory providing us with universal criteria for validity can be formulated in some scientific manner, or whether recognition of rational arguments isn't rather some kind of art than a matter of science, as Toulmin suggests by using his analogy with jurisprudence.

If logic is indeed supposed to be a science about arguments, it should not evaluate just singular tokens of arguments, but rather classify them first and then evaluate those classes of arguments instead.

Botanists also do not describe every single plant by listing its attributes. By observing particular plants they make up general concepts and classes. Then they categorize particular plants into classes by their attributes and they further study those classes and their relations to each other in the whole taxonomy.

Later (in section 2.2) we will see that arguments can be classified according to their logical form. Formal logicians also create concepts of logical forms (designing new logical formalisms—see section 2.4), categorize particular arguments into those forms (formalization of natural language arguments—see section 3.1), and ascribe attributes to all the arguments sharing the same form (description of mathematical properties of logical formalisms).

Such approach has its limits, as it can recognize only logically valid, i.e. formally and therefore deductively valid arguments. Relation of all those categories should be illuminated in section 2.6.

Some logicians ([**19**]) also aim to arrive at some kind of classification of arguments into argument schemes, despite the fact that they cannot provide any uniform treatment of arguments within the same class and despite that it is often unclear to which argumentation scheme should a particular argument conform.

Such classification must be carried out without referring to the context of use of those arguments and once again does not help us in most cases to distinguish valid arguments from invalid ones.

Feldman [**19**], for example, acknowledges the existence of genuine inductive (i.e. substantial) arguments, however, he hopes to give at least approximate guidelines for their evaluation. He divides them into several categories, such as *arguments of authority*, *causal arguments* etc. and then lists certain conditions we must check for reasonableness in order to accept the argument.

So for example, in order to accept an argument of authority, we must verify that the authority is a true authority in the field to which the argument belongs and he does not have any reasons to lie to us, etc.

Feldman does not pretend that whenever the premises of this argument and the additional conditions will be true, so will necessarily be the claim, he just believes this is the most rational attitude available and in fact the best we can hope for.

This approach, so common to informal logicians steming from the critical thinking theory, is not very helpful. True, it can be helpful to pinpoint several critical errors we often do in practical argumentation.

However, it does not tell us anything about how to do it and so it is of limited practical value. It provides us with no deep justifications of such approach neither and so it is of no theoretical interest. It suffers from certain arbitrariness and lack of coherent backing in a more general theory.

It is this approach that has been criticized by Walton. Such valid argument schemes are usually one step from classical categories of fallacies, such as *argumentum ad ignorantiam* or *argumentum ad verecundiam* which informal logicians also vividly list, reinterpret and reproduce.

It is of crucial importance to tell a solid cogent argument from fallacious argument, that merely resembles a cogent one. Informal logicians often point to this fuzzy border but fail to draw it with greater clarity than we would obtain simply from our intuition.

So it seems that we have a Scylla of formal logic on one side and Charybdis of informal logic on the other. The first one mercilessly ravishes all but firmest formally valid arguments from the board, while the second one drags the whole ship into the whirlpool of subjective opinions and random heuristics.

Is therefore a science of arguments not possible? I am sure there can be something as a general science of arguments, but not as a science that tells us what a valid arguments is, on the basis of the argument alone.

While arguments can certainly be deductively valid, and their validity can be surely recognized with highly scientific methods of formal logic, the notion of cogent argument per se should be abandoned for good.

We should rather concentrate on the argumentation as a process and evaluate arguments with respect to the argumentation in which they occur.

Is their use at such and such place of such and such discussion legit and does it obey the rules of such discussion? Do they actually help their proponent to achieve his goal in this argumentation, whether it is to establish some conclusion, explain something to a partner, persuade an opponent or achieve an agreement on a certain issue?

To be able to tell whether an argument is good, we need to know whether it fulfills its purpose and for that we need to know how it has been used, as Walton stresses [**66**].

Toulmin's observation that arguments validity is in most common cases field-dependent is analogous. It is the use of arguments indeed which is crucial to their evaluation.

Authors of [**59**] dismiss understanding the uses of argument as unimporant for evaluating their validity. According to them, knowing how to play chess is superstructure to knowing the rules of chess.

But this remark is valid only because they already aim at describing just one particular kind of game to be played with arguments, that is demonstrations. Later I will show that formal logic which is described in [**59**], is the science of demonstrations[20].

But to understand that a certain move with a king is valid, we do need to know first that it is chess we are playing, and not, say, checkers with chess pieces.

But this chess analogy speaks against such reductionist account of argumentation.

[20]This was also, according to [**28**], outcome of Perelman and Olbrechts-Tyteca in their *Rhetoric*, but this source has unfortunately not been available to me.

That castling is a proper move in chess we cannot tell only from the fact that the first piece is a king and the other a tower. We need also to know they are in the proper position.

Further we need to know there are no opponent's pieces that would check the king on his way into the tower. Last but not least—we must know we have not yet played this move[21].

Accordingly, the statement that an argument is valid may not only depend on what argument it is (what are its premises and its conclusion), but also whether it has already been used, or whether some other arguments which would defeat it are still 'on board'.

We may be able to discover some feature common to all cases of valid argumentation. We just need to concentrate on the argumentation as a whole and design our theory of argumentation with this new paradigm of argumentation in mind.

Before developing those insights, let us analyze backgrounds of formal logic to understand better why it is not fit to provide a general theory of all valid arguments.

1.5. Two sources of Aristotle's dialectic

In order to understand the origins of the current philosophies of logic, it is instructive to look at their historical development.

The duality of the intended utility of modern logic is already present in its ancient predecessor, Aristotle's *dialectic*. Aristotle designed his dialectic with two different purposes in mind: it should serve both as a universal scientific method and as a general theory of everyday argumentation—that is as an instrument of all scientific reasoning—*organon*.

But it should also serve as a general theory of rational debate and argumentation, including bargaining at the market or political debates. In this second role it should be directed particularly against alleged sophistical abuse of language used in common disputes.

In his *Topics* Aristotle draws a clear distinction between *demonstrative* and *dialectical* deductions.

> Now a deduction is a an argument in which, certain things being laid down, something other than these necessarily comes about through them. It is a demonstration, when the premises from which the deduction starts are true and primitive, or are such that our knowledge of them has originally come through

[21]This analogy is quite strong, indeed. Analogous conditions will have to be met in the dialogue game of that will be described in the section 3.5.

> premises which are primitive and true; and it is a dialectical deduction, if it reasons from reputable opinions. [**5**], p.167 (100a25–100a27))

The first kind of argument therefore differs from the second one merely in the respect that it proceeds from certain and known principles (*episteme*) and produces new items of knowledge, while the second one proceeds from questionable and uncertain commonly held opinions (*endoxa*)[22].

His definition of *endoxa* is the following:

> On the other hand, those opinions are reputable (endoxa) which are accepted by everyone or by majority or by the wise—i.e. by all, or by the majority, or by the most notable and reputable (endoxoi) of them. [**5**], p. 167 (100b21–100b23)

Aristotle's definition of episteme is quite interesting and important. A premise of a demonstrative argument must be not merely *true*, but also *known* and in some sense *fundamental* or *principal*.

I do not suggest that we should interpret Aristotle as if he was anticipating here some sort of anti-realism, identifying truth with what can be known in principle. It is just instructive to keep this definition in mind as it avoids problems some modern versions of Aristotle's theory inevitably face, when interpreted in light of naive realism[23].

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[22]Actually Aristotle recognizes at least three other kinds of arguments, to be used in discussions beyond dialectical ones, namely *didactic*, *contentious* and *examinational* arguments ([**4**], p.279 (165a38–165b12)).

This classification is based solely on the intended purpose of these arguments and the nature of premises needed to achieve their goal, and apparently was motivated by various types of discussions of Plato's dialogues in which Socrates participated.

We will further need only to distinguish between arguments which are based on knowledge and those which are based on opinions.

[23]This is a case of Feldman [**19**] whio struggles with the problem that sound arguments are required to have true premises, but we often cannot know this is the case. We therefore often cannot recognize a sound argument when we see one and it is of no use to use logic in order to recognize it is valid.

He therefore embraces fallibilism and introduces a notion of soundness relative to a person. An argument is sound for a person if and only if it is either deductively valid or inductively cogent and if it is reasonable for this person to believe all its premises.

Whether such arguments truly establish their claims normatively and what is a problem Feldman does not address, although he is very strict about making difference between arguments that are merely persuasive and those that are rational.

Such approach inevitably poses a problem, however, as it is not quite clear why should this notion of relative reasonability fulfill Tarski's truth criteria mentioned

It is important also to note, that both demonstration and dialectical deduction are deductions. Deductions are arguments which bring necesary connection from what has been stated to some new evidence. His concept of *deduction* (*syllogismos*) is also explained in his first Analytics.

> A deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. [**2**], p.40 (24b18–24b20))

There are different interpretations of the following definitions.

It is now a widely accepted interpretation of Aristotle's intention that both kinds of reasoning had to be fulfilled by the very same theory, his theory of deductions that is described in his analytics.

This opinion is dominant thanks to the interpretation of Aristotle in influential Kneale's work on history of logic ([**30**], pp. 1–2) quoting the definition given above.

True, Aristotle's definition clearly states that dialectical arguments differ from demonstrative ones merely by the strength of evidence for their premises. It is clear that both kinds of arguments are still deductions.

However, it does not necessarilly follow that Aristotle's theory of deductive reasoning, syllogistic for short is the only relevant method of Aristotle's logic.

John Woods, for example, believes that Aristotle's goal was a wholly general theory of argument. His theory of deductive reasoning was only intended to serve as a logical core of the larger theory, including among many other things, a theory of refutations which would discipline the distinction between good refutations and merely good-looking refutations. Woods claims that such theory stands apart from the syllogistic and does so in a systematic way, being its *nonconservative extension*([**72**], pp. 139–140).

Aristotle, according to Woods ([**73**], pp. 44–45), originated his dialectic in the context of richly developed argumentative practice nourished by two main arteries. First, there were a Greek mathematics with concepts of demonstration and reduction to absurdity. Second, there was the rhetorical tradition of Sophists. The second of those traditions was developed into the dialectic tradition by Plato.

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on the page 55 and consequnetly raises doubts why should logically valid arguments preserve this relative rationality.

Woods explains the concept of dialectic as follows: The central concept of Plato's dialectic is *elenchis*, that is a reduction, but of a different kind than the one used in mathematics.

It is used to cause the opponent to make claims inconsistent with his original thesis and thus disprove it. It is not necessary (and in fact desirable) that this claim is absurd by itself. It must merely be in some way inconsistent with the original thesis of our opponent.

The role of deductions in dialectical reasoning is just this—to bring our opponent from his original thesis to something that contradicts it.

Douglas Walton ([**66**], pp. 14–15) divides Aristotle's work into two parts—the newer one, developed in his *Prior Analytics* and *Posterior Analytics* providing his theory of syllogisms and the older one, represented by *Topics* and *Sophistical Refutations*, posing the fundamental problems which are only partially solved in the newer writings.

Walton demonstrates the need for a different method for evaluation of arguments with merely plausible premises on the following example of a non-syllogistic argument which he takes from Aristotle's rhetoric ([**3**], p.2235 (1402a11)).

It is an argument used to defend someone accused of murder by using a premise that he is too weak to assault and kill a man so much stronger than he is.

He explains that the premise used for the argument in the defense of the accused man which would Aristotle call an opinion commonly held by wise men would be nowadays probably called *probabilistic*, *inductive*, or more accurately a *plausible* statement.

A plausible statement, unlike a statement which is known or demonstrable as true, should be accepted only tentatively as it still may be refuted by some future evidence[24].

The legal argument against the guilt of the weaker man clearly does not have a form of syllogism, but according to Aristotle it falls into a certain category of incomplete syllogisms which he calls topic (*topos*). By filling in some missing premise, so called *enthymeme*, these incomplete arguments would become proper syllogisms ([**3**], p.2237 (1403a18–1403a19)).

Such incomplete deductions would include factually or even analytically valid arguments. They would include substantially valid arguments, but the missing premises would be merely plausible in this case.

[24]Aristotles concept of endoxa and plausible statements is explained in detail in [**55**] and some references can be found in [**53**]. I will explore its various modern elaborations in the section 3.3.

Aristotle himself does not give many clear examples of such incomplete syllogisms and does not describe any unambiguous and definite methods how to complete them. However, Aristotle's own opinions are not what we wish to reconstruct exactly, but rather their contemporary interpretations.

Aristotle's writings suggest that in order to tell whether some argument of the above kind is good or not, we need not only to know whether we can transform it into a valid syllogism and how, but also whether its premises have or have not been refuted by any of the prosecutor's counterstatements. For this we would need some theory of refutations as Woods suggests.

While nowadays we may classify the first problem of finding a correct enthymeme as a problem of methodology and the second problem of checking its refutation as a problem of epistemology, it makes no sense to distinguish these two disciplines from Aristotle's syllogistic, as all these problems were addressed in mutual relations within the whole corpus of Aristotle's logical works.

1.6. Classical Model of Science

We are now used to make distinction between logical, epistemological and methodical part of argument evaluation. This is partly because the dialectical part of Aristotle's project gradually atrophied into obscurity as there seemed to be no urge to reflect on the epistemic nature of premises of arguments in logic.

The theory of syllogisms, on the other hand, flourished and slowly became to be interpreted as a central topic of Aristotle's logic, rather than means to a much larger end ([**66**], p. 14).

Some proponents of informal logic complain that this doctrine became so dominant that even Kneale's seminal book on history of logic [**30**] completely omitted any account of the first part of logic (See [**28**], p. 348)[25].

This trend is much older, however, as Walton observes.

There are, of course, reasons why this has happened. While the most dominant of those reasons might be that Aristotle's account of formal demonstrative reasoning is fairly more technically developed in comparison to his work on dialectic, historical development in philosophy of science surely also played its role.

The theory of demonstrative reasoning was a necessary tool of a certain kind of science which Aristotle aimed to develop and which

[25] Another influential historiography of logic, Bochenki's *Formale Logik*, has already quite a self-explanatory name and can hardly be blamed for some negligience.

W.R. de Jong and A. Betti ([**16**], p 186.) call *Classical Model of Science (CMS)*[26]. They call a Classical Model of Science any system S which satisfies all the following points:

(1) All propositions and all concepts (or terms) of S concern a *specific set of objects* or are about a *certain domain of being(s)*.
(2) (a) There is in S a number of so-called *fundamental concepts* (or terms).
 (b) All other concepts (or terms) occuring in S are *composed of* (or are *definable from*) these fundamental concepts (or terms).
(3) (a) There is in S a number of so-called *fundamental propositions*.
 (b) All other propositions of S *follow from* or are *grounded in* (or are *provable* or *demonstrable from*) these fundamental propositions.
(4) All propositions of S are *true*.
(5) All propositions of S are *universal* and *necessary* in some sense or another.
(6) All propositions of S are *known to be true*. A non-fundamental proposition is known to be true through its *proof* in S.
(7) All concepts or terms of S are *adequately known*.
 A non-fundamental concept is adequately known through its composition (or definition).

The theory of syllogism was clearly intended to provide the organon for demonstrating propositions of S from the fundamental ones.

This model had a great influence on western philosophy and science, as De Jong and Betti demonstrate. No wonder that the analysis of the general discourse seemed less important, especially in the setting of medieval universities. Medieval scholastics could not possibly be that concerned with discussions of common men as ancient Greeks[27].

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[26]Authors of [**16**] do not attribute origins of this model to Aristotle himself. They recognize that Aristotle's understanding of science was different, but this model was a result of classical interpretation of his works during later periods.

In his master thesis [**53**], J. Ráliš even argues that Aristotle's apodeictic (based on deductions and often contrasted with epagogic) method was intended as a method of teaching someone about science, not a scientific method itself.

Either way, it is classical interpretation of Aristotle's works and its influence on modern accounts of logic and science that interests us, not Aristotle's philosophy itself, provided we are willing to believe that it is possible to reconstruct it from fragments of his work without substantial import of our own interpretation.

[27]The importance of Aristotelian logic in the role of the *ars dispustandum* during medieval times is illustrated by [**10**] or [**57**].

This model has had such an influence on western thought that the discussions of Aristotle's theory of deductive reasoning prevailed and remaining parts of Aristotle's work slowly faded into oblivion.

It is clear that the demonstrative deductions are primarily meant to construct proof of sentences of S from its fundamental propositions, as in point (3b).

Their premises are all true (4), known to be true (6), and necessary (5). Hence, according to the definition of deduction, also their claim will be necessarily true and known to be so due to the deductions demonstrating them. They will also be members of S consequently.

A great deal of western philosophy could be seen as an effort to provide different explanations and elaborations on various points of the model.

For example, ontological questions about the nature (essence) of the objects (things, beings) we are talking about become especially urgent and complicated when the 'talking' is done within some abstract science, especially mathematics or physics. The domain of objects in ordinary discourses does not seem to be so mysterious and so in need of description and explanation, as that of real numbers, sets, functions, forces, or basic components of matter.

Points 2)–7) which are dealing with concepts, sentences, truth and demonstrations, were traditionally associated with Aristotle's logic. Aristotle's theory of definitions was intended to serve as a description of the ways we form concepts (2) and his theory of syllogisms was clearly intended to describe (if only partially) the methods of demonstrations mentioned in (3b).

Nonetheless, it was only after revision of points 4)–6), that Aristotle's logic was put into radically different use than it was originally intended. How did that happen?

The rise of modern empirical sciences in the Renaissance resulted in questioning the relevance of speculative metaphysical reasoning which relied on deductive syllogistic reasoning heavily[28].

The deductive reasoning would not be that suspicious as such, if its major subject weren't necessarily true sentences. For this reason it was of little utility for new science based on particular observations.

The new building blocks of science were results of experiments and observations that is empirical truths that were not necessary by the ancient standard. Deductions with merely contingently true premises could serve science as well.

[28]Roger Bacon, one of apostles of the new sciences, aptly named his major writing *Novum Organon*.

The situation in geometry, arithmetics, and algebra was even more complicated.

Within mathematics two kinds of proof have been traditionally identified, *apagogical*—indirect proof by means of a chain of symbolic transformations which loosely corresponds to deductive reasoning, and *epagogical* proof which rests on intuitive grasping of general features of certain mathematical objects, say geometrical figures such as triangles, by presenting them on an arbitrary, yet particular representant of this possibly infinite domain[29].

The importance of epagogical reasoning for mathematics was being stressed by rationalist philosophers and contrasted with uninsightful verbal finesse of syllogistic which was associated with plain scholastic arguments. It was also not clear entirely how could Aristotle's syllogistic help us to justify such general statements—no general statements can be justified from particular statements via chain of demonstrations.

According to Lorenzen ([**34**], p.175f), the most prominent and outspoken critic of the strictly deductive approach to science was Pascal. But it was also Pascal who greatly inspired logic of Port-Royal in which CMS received perhaps the first and most clear explicit articulation ([**16**], p. 188). Pascal's and Decartes' critique of Aristotle's logic was therefore all but critique of his ideal of science. They took the model very seriously, which is why they tried to improve it radically in the first place.

Pascal's critique of Aristotle mainly targeted his theory of dialectical reasoning. Pascal strived to replace plausible casuistic reasoning of Jesuits with his strictly scientific treatment of the subject—his mathematical theory of *probability*.

His theory of probability slowly became recognized as the paradigm of the only scientific alternative to deductive logic as a discipline concerned with reasoning. It became somehow uncritically accepted that for describing those kinds of reasoning which cannot be faithfuly described by logic we should employ probabilistic terms and methods[30].

Consequently, the word inductive was coined to mesh together epagogical, dialectical, and probabilistic reasoning alike, as well as incomplete generalisations of empirical science. I do not dare to judge how

[29]For clarification of epagogical and apagogical reasoning and their role in mathematics see for example [**32**] or [**53**].

It is most important to realize in the first place, that epagogical reasoning is something completely different from inductive reasoning and why mathematical induction has little to do with ordinary induction is also explained there.

[30]For an exhaustive treatment of possible applications of theory of probability as the theory of inductive reasoning see [**74**].

succesful can this theory possibly be in evaluating properly all these different kinds of reasoning. In the sections 3.3–3.5 I will merely attempt to show some alternative approaches to plausible dialectical reasoning that will be no less scientific, but hopefuly greatly more intuitive.

As already noted, rationalistic philosophy, following Descartes and Pascal, was overly sceptical to the role of deductive reasoning.

Kant's famous remark that Aristotle's logic is of permanent value has to be understood in the way Coffa ([**15**], p. 680) explains it: Not that syllogistic already contains all desirable canons of all judgements, quite the contrary.

Kant believed that Aristotle's semantics already provides sufficient ground for all analytical (apagogical or linguistic) judgements, but could not be extended to cover more interesting cases of synthetic apriori judgements. In this light Kant's praise ironically reveals itself as a harsh critique—nothing more needs to, and in fact can be, added to Aristotle's logic and because it clearly does not suffice to provide foundations of mathematics, there is no prospect it will ever be. Synthetic apriori judgements of mathematics are mostly grounded on epagogical reasoning and therefore justified by pure intuition, according to Kant.[31].

Leibniz's philosophy of mathematics was quite exceptional as it went in the opposite direction to this dominant trend of rationalistic philosophy. Leibniz had recognized an intimate connection between calculation and deductive reasoning involving symbolic transformations and thus he greatly anticipated the development of modern logic and became recognized as a precursor of Boolean and Fregean tradition of modern logic alike[32].

According to Leibniz, all reasoning is connected to some signs or characters which can be used not only to stand for things themselves, but also for ideas of things. Leibniz believed that the languages of arithmetic and algebra partially possess this quality, but that it can be extended to the whole of human thought. Moreover, this whole of human thought could be reduced to a list of simple thoughts[33].

Natural languages, however, are not suitable for this task, as their concepts are ambiguous and bear only arbitrary and nonsystematic relation to the ideas they express ([**44**], p. 7).

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[31]Brief summary of this development and some explanatory comments can be found in the first chapter of [**31**]. An impact of the ideal of CMS on philosophy of Kant, Bolzano and Frege, particularly on their notion of synthetic and analytic, is described in [**17**].

[32]For Leibniz's appraisal of medieval Aristotelian logic see [**57**].

[33]Items 1) and 2) of CMS

In an ideal language, each simple thought would be represented by a unique symbol. Leibniz called this language by several names, such as *lingua generalis*, *lingua universalis*, *lingua rationalis*, or *lingua philosophica*, but later it became known, a little bit confusingly, as *lingua characterica* ([**44**], p. 5).

I will stick to this more recent, though inaccurate terminology. Leibniz himself used the term *ars characteristica* for the method of discovering such language in particular fields of human knowledge. The term *characteristica* or *characteristica universalis* was used to denote a general theory of this method.

It is an inherent feature of lingua characterica that its expressions bear a non-arbitrary relationship to concepts they express. In Leibniz's theory of concepts all complex concepts are compound from atomic ones and all expressions of lingua characterica should be compound from atomic signs corresponding to those concepts by operations which are analogous to concept-forming operations ([**33**], p.286).

Once a representation of some problem in characteristic language has been found, it could be solved by the *calculus ratiocinator*. Calculation with this calculus already did not require any insight, just mechanical application of symbolic transformations ([**33**], p. 286).

It was therefore characteristica which, according to Leibniz, stood for a general science of human reasoning and it therefore served the same purpose as Aristotle's organon. A goal of a scientist was just to find the proper representation of a certain particular problem and then, once this was done, the problem could have been easily solved by a machine ([**44**], pp. 7–8).

Aristotle's original ideal of deductive logic as organon, transformed by Leibniz into the ideal of lingua characterica and calculus ratiocinator, was also adopted by Frege[34].

Peckhaus remarks that while Frege was aware of the utopian character of this project, he suggested that it could be realized at least in some parts ([**44**], p. 8). His primary concern was lingua characterica for mathematics (ibid. 10).

However, it seems, Frege himself wasn't always quite clear-cut and consistent in the questions of purpose of his logic. Despite the above mentioned statement, he is also known for his remark that logic is like a tool designed for certain scientific purposes that shouldn't be criticized for being unfit for others.

[34]Although for Frege, unlike for Leibniz, logic was not only a method whose auxillary propositions are actually void of any meaning, but theoretical discipline in its own right, whose theorems, describing specific domain of entities (logical relations, inference, proofs...), can properly be called truths of logic (see [**17**],[**16**]).

His formal language was clearly intended to serve as a foundation of arithmetic, in a radical sense that all arithmetic could be reduced to it. This purely epistomological programme of logicism was aimed against Kant's doctrine that arithmetical truths are synthetic a priori as logic is taken to be analytic by definition[35].

Whether Frege's project, that is to provide lingua characterica for mathematics only, succeeded or failed is of little interest to us. While this is a very exciting and interesting problem, it is also very complicated and it has been dealt with competently and with great detail in books [**31, 32**]. It is mainly the later reception and interpretation of modern logic which influenced plenty of current accounts of logic.

The importance of modern logic, stemming from work of Boole and Frege, was recognized by philosophers of Vienna Circle, also referred to as *logical empiricists*, who thus became heirs of the classical model of science. Logical empircists developed their own variant of the classical model of science.

First, they replaced Aristotle's syllogistic in its role of the fundamental method of demonstrative reasoning by modern logic (therefore 'logical').

Second, their system S should also include sentences describing results by empirical observation. Such sentences which certainly were not fundamental could hardly ever be considered necessary, and could even be true, without being known to be so (therefore 'empiricists' or 'postitivists').

Logical empricists also recognized limitations of deductive reasoning and hoped to supplement it with some kind of inductive logic, or with probability theory, but this theory was also intended to serve exclusively as a theoretical description of purely scientific reasoning.

While the original manifesto of Vienna Circle did proclaim the ambitious project to delimit all rational discourse, later it was abandoned for a more modest and fruitful plan to develop a reasonable methodology of science[36].

Textbooks of logic, written within this tradition, are therefore often modest in proclaiming what they hope to achieve. Quine's *Method of*

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[35]Detailed description of semantic tradition that was antagonistic to Kant's philosophy of mathematics is described in detail in [**15**] and within a larger context also in the relevant chapter of [**14**]. Frege's philosophy of logic is summarized in [**31**]. You can find an account of the mentioned ambivalence in Frege's concept of logic on page 33 of this book. Frege's project and its relation to the Aristotelian project which author considers to be incomplete, is summarized in [**72**] pp. 139–142.

[36]Insightful description of philosophy of logical empiricism is summarized in [**14**].

Logic which was very influential mainly in anglophonic countries, is a prototypical representative of this approach. It was mainly because of the more ambitious textbooks like Copi's *Introduction to Logic* that these methods were applied outside their intended scope of a scientific model, i.e. for description and evaluation of dialectical, practical argumentation.

It is therefore demonstration which is a subject of modern formal logic, at least in its positivistic version.

The logical analysis of natural language argumentation which I have criticized in the section 1.3 is a result of somehow unfortunate blending original Aristotle's methodology with Quine's account of logic. Because Quine is a heir of logical empiricism, the result was a quasi-dialectical theory which was actually somehow weakened theory of demonstrations[37].

In the section 2.2 I will elaborate more on this positivistic account of logic, in order to explore its fundaments criticized by informal logicians. I will illustrate the importance of the notion of logical form for this account and hence explain the notion 'formal logic' properly. It is beyond the scope of this thesis to explore other philosophies of logic in detail.

[37]See [**72**].

CHAPTER 2

Logic and Form

2.1. Contemporary approaches to logic

I would like to draft some important terminological distnictions in this section which will be clarified properly later. It is my goal here to simply draw attention of the reader to these distinctions and briefly sketch their motivation and historical background.

Unlike the *traditional logic* of Aristotle, Leibniz's, Boole's, and Frege's logics were described using a lot of symbolism. While Leibniz and Boole used algebraic operators and constants to express thoughts about judgements, Frege and Peano invented their own symbolism which was novel in many aspects. *Modern logic* which follows mainly work of these founders, is therefore sometimes called *symbolic logic*.

We should not identify *symbolic logic* reasoning with *formal logic*. A logic is formal if it gives an account of valid inferences by an instrument of *logical formalisms*, that is systems of valid argument forms. I should probably explain more every single term of this definition, but this is what I will actually need to do later in section 2.3, so I kindly ask for readers patience.

I will remark here only that Aristotle's syllogistic is also formal, although it is mostly not symbolic, while some modern accounts of arguments which I will briefly mention in section 3.3, are symbolic but not formal.

The term *mathematical logic* is often used nowadays to coin a certain bunch of mathematical disciplines. Mathematical logic is often subdivided into *proof theory*, *model theory*, *set theory* and *recursion theory* (also called *theory of algorithms*) and often also *computational complexity*. These highly mathematical disciplines originated from works of modern logicians, partially motivated by the aim to provide foundations of mathematics, partially by studying mathematical properties of logical formalisms.

To a certain degree mathematical logic would better be called mathematics of logic, as it is an external description of certain properties of logical formalisms. This would be debatable terminology, though, as it is still not clear whether mathematic is applied logic, or whether logic is a branch of mathematics, much like algebra, calculus, combinatorics, planar geometry, or graph theory. Actually, whether mathematical logic is a discipline of mathematics or rather description of mathematical methods we need not to decide.

While mathematical logic is usually carried out by mathematicians, who often may not be interested neither in philosophical foundations and origins of their discipline, nor in the application of their results in fields of philosophy or linguistics, literature on philosophy of mathematical logic is a discipline in its own right[1].

Mathematical logic is often contrasted with *philosophical logic*, a term which is notoriously ambiguous and seems to encompass all diverse themes traditionally associated to logic, but not being described by mathematical logic[2].

We may be inclined to think that philosophical logic would mainly be concerned with dialectical reasoning, as demonstrative reasoning has already been described by mathematical logicians. This, however, is often not the case, although if there is a topic that would befit ours delimitation of philosophical logic, it is this one.

Philosophical logicians are the ones who are usually interested in natural language reasoning and argumentation, but the dominance of mathematical logic is so strong that even philosophical logicians usually approach this task with formal methods and often look at the natural language through the optics of logical formalisms.

I do not wish to imply such an approach is unjustified or unfruitful. I just want to stress the fact that it studies only limited function of natural language, that is demonstrations. I personally believe that demonstrations are perhaps not that important for argumentation.

The methods of formal logic were developed by mathematicians for mathematicians and their application to natural language is limited, as I will discuss in section 3.1.

The original logical formalisms were designed in order to describe mathematical reasoning indeed. Therefore most of these logical formalisms were unsuited for the desrciption of natural language which forced philosophical logicians to explore their expressive limits and design new ones.

These new formalisms were supposed to refine the crude definitions and methods of the old ones (intensional logics), to avoid or explain paradoxes in natural language (relevant, multivalued, paraconsistent logics), or to provide a tool for analysis of a specific natural language

[1]A laborous endeavor of unveiling intricate relation of modern logic to mathematics was undertaken for example by Kolman [**32**].

[2]It is sometimes possible to encounter an explanation of the difference between philosophical and mathematical logic as a distinction between the study of interpreted and un-interpreted formalisms [**24**].

This is, however, not quite meaningful, as it suggests as if there could be some uninterpreted formalisms. The sentences of formalisms used in mathematics might not have a direct translation into any natural language, but have their own, that is mathematical, interpretation, often described by means of model theory or proof theory. The language of mathematics is also a language after all.

discourse, not particularly relevant to mathematics (alethic, epistemic, temporal, deontic, doxastic logics...).

On the vague border between mathematical and philosophical logic lies the purely mathematical study of such non-classical logical formalisms[3]. That is in fact what one encounters, when he opens *Handbook of Philosophical Logic*.

Often there is no difference in methods and problems in handbooks of mathematical and philosophical logic, it is just the subject that is slightly different in each of the cases.

To identify study of non-classical logical formalisms with philosophical logic would be quite narrow and also historically inaccurate. While it is a historical fact that many nonclassical logical formalisms originally arise due to the activity of philosophical logicians in order to explain some of the natural language arguments, including philosophically interesting concepts like 'necessity', 'time', 'knowledge', 'belief', 'obligation' etc. (explicated in modal logics), origins of some other non-classical logical formalisms were motivated by mathematical interests (for example intuitionistic logic), or interests of computer science and artificial intelligence (substructural logics, dynamic logics and and certain modal logics as well).

Moreover, those logical formalisms which are now considered as an object of research of mathematical logic were originally described in philosophical terminology motivated by aim to model and clarify concepts such as 'meaning', 'truth', 'consequence', or 'necessity', as we have observed this in Frege's case.

It was only later that these concepts were replaced with new ones like 'reference', 'model', or 'entailment'. Also many nonclassical logic have gone through the same development and are now extensively and fruitfully studied with mathematical methods.

While mathematical logic delimits one side of philosophical logic, *philosophy of logic* delimits the other one.

Much like a mathematical logician, a philosopher of logic studies logical formalisms (classical or non-classical) however, his viewpoint is different.

He is interested in the philosophical foundations of logic, the philosophical interpretation of its results and questions concerning their relevance for philosophy in general—epistemology or ontology in most cases. This often reduces to studying philosophical aspects of formal

[3]It is usually two valued first-order logic which is automatically considered to be 'classical logic' nowadays. Emergence of first order logic is described in [**37**] and some of the important foundational questions on nature of the quantification are further described in [**23**].

logic and inquiry into foundational notions used to motivate basic concepts of logical formalisms.

These concepts often include 'argument', 'valid argument', 'reasoning', 'demonstration', 'sentence', 'truth', 'meaning', 'reality'[4].

While in the past this endeavor often crossed borders with ontology and epistemology, for philosophers of logic who belong to the semantic tradtion of analytical philosophy, these enquiry often interlaps with linguistic and semantical investigations.

However, the influence of modern formal logic which is unlike its ancient predecessor connected to mathematics often dominates even such philosophical endeavors. Philosophers of logic often study logical formalisms which have been derived from Frege and therefore bear a strong imprint of lingua characteristica of mathematics, despite their various mutations and deviations even those which are so unsuited for mathematical practice, such as modal or intensional logics.

The concepts mentioned above are relevant for the description of demonstrative reasoning exclusively. Even the concepts of argument and validity are often defined in such a way so that they were be applicable only for demonstrative reasoning, as I will show in sections 2.2–2.6.

This should not be interpreted that there is no distinction beween mathematical logic and philosophy of logic.

Most mathematical logicians need not to be concered with those concepts. In fact, one rarely encounters them in most textbooks of mathematical logic. They have been all substituted by technical and artificial notions of 'formula', 'satisfaction', 'reference', 'entailment', 'model', 'proof'.

Still, it would be mistaken simply to indentify those concepts with the intuitive concepts they were meant to capture. The latter can be only viewed as the explication of the former, as is explained in [**46**]. Or alternatively again they can be understood as relatively self-contained concepts without a need of further explication beyond the model-theoretical 'semantics' provided for them.

Tarski's famous article on truth [**63**] can also be interpreted in this light. Its aim was not to provide yet another definition of truth, but to delimit just what attributes of this notion are relevant for logical inquiry and therefore what minimal criteria should this concept satisfy

[4]Frege's definition of logic as the scientific discipline studying *general laws of truth*; his exact formulation is the following: *'Der logik kommt es zu die Gesetze des Wahrseins zu erkennen'* p. 30 of [**22**].

in order to serve its intended role in what later became known as model theory. I will explain this claim in greater detail on the page 55.

Only on the background of all those approaches to logic, we can arrive at a meaningful and correct idea of *informal logic*. This term is often misunderstood and informal logic is often identified either with philosophical logic or more broadly philosophical analysis of certain concepts using logical formalisms, as is suggested for example by G. Ryle [**56**][5].

It is also tempting to contrast formal with informal logic, on the basis that formal logic often employs 'formal' mathematical methods, while informal logic employs more or less merely a heuristic, if not only literary devices.

The informal logic is quite a recent discipline which arise mostly from pedagogical concerns, the fundaments of its respective theory have not yet been fully laid out and its current lack in utilization of rigid methods is, I believe, also a mere consequence of historical circumstances rather than a systematic feature.

Nor we can say that formal logic is primarily focused on the description of formal languages and is tied closely to mathematics, while informal logic is concerned with arguments in natural languages. The tight relation between formal logic and mathematics can not be taken to be a universal characterization of formal logic, as it pertains mostly to modern accounts of formal logic.

Aristotle's logic, was also formal to a great degree, but it didn't play any important role in mathematics. On the other hand the application of logical formalisms (classical or non-classical) to natural language analysis is a common practice in philosophical logic.

It is also common in many textbooks on logic to draw a line between formal and informal reasoning. Informal reasoning is the corresponding manipulation with statements of natural language, according to the rules and principles explained in natural language, while formal reasoning is usually understood as the manipulation with statements of certain artificial (formal) language using apparatus of a certain logical calculus, usually designed to model the informal counterpart of this reasoning (such as natural deduction calculus).

Informal logic has nothing to do with such kind of 'informal reasoning' which is actually based on considerations of logical forms of natural language sentences in question, although carried out directly without translating them first into specifically designed formal languages. Formal logic, I repeat, is not exactly the same thing as symbolic logic.

[5]See also ([**27**], p.95)

A more important feature is that formal logic is the study of logical formalisms which aim at description of logical forms of arguments. Logical formalisms can be studied from purely mathematical point of view within so called mathematical logic, but their applications and philosophical motivations are studied by philosophical logicians, who are also concerned with formal logic, although they do not resort to mathematical language and methods so much.

On the other hand an informal logic is concerned with the context of arguments and their interaction. This can be carried out in highly symbolical and abstract manner too.

From my point of view, it does not matter, to what degree mathematical methods and symbolic languages are used to describe subjects of formal and informal logic and to what degree literary explanation is used. It does not matter neither to what languages are such methods typically applied.

All that matters for the distinction is what aspects of a given language are being studied. Language of autoepistemic logic, for example, can be defined recursively and is therefore formal. However, the valid inferences of this logic are not closed under substitution of extra-logical terms and therefore there is no description of forms of those inferences.

Whether we formulate our laws for statements of natural language, or for some more or less symbolic language (for example language of arithmetic, or set theory), and whether we formulate these laws themselves using natural language, symbolic language (usually set-theoretical or algebraic language), or even using graphical representation (truth-tables), are two different issues.

We are not doing formal logic just because we are operating with symbolic languages either at the level of our object-language or at the level of meta-language (usually both). Should we wish, we could devise a formal logic for a fragment of any natural language, but we would have to do away with certain difficulties which do not appear in artificially constructed languages and which would be of little relevance to our project.

If the previous account has been confusing for the reader, it is probably because the explanation of the key concepts used in the previous paragraphs is what sections 2.2–2.5 aim at. So far I plead for the reader's patience, I chose to introduce various accounts of logic very briefly here, before explaining them in greater detail, so that a possible mismatch with informed reader's and mine understanding of some of the concepts did not obscure his understanding of the expositions to come on page 49.

Let us proceed by explaining origins of positivistic doctrine of logic and explore the fundaments of the previously mentioned approaches to logic.

2.2. Logical form of arguments

In following paragraphs I will briefly retrace some well-known facts from history of philosophy, in order to setup the stage for discussion of relation of the above mentioned concepts to concepts of substantial and analytic argument, mentioned in section 1.3.

As we have observed in section 1.6 the notion of necessary truth played an important role in the history of western philosophy of science.

The distinction between necessary and contingent truths might be called *ontological*, as it has traditionally concerned metaphysicians. It should not be confused with *epistemological* distinction between *a priori truths*, truths known to us independently of our experience, and *a posteriori truths*, that is truths known to us due to our experience.

Neither we should mix any of these with the third traditional distinction, *semantical* distinction of *analytic* truths (*formal* or *generally valid*), that is truths due to the meaning, and *synthetic* truths (also called *material*), the matters of fact, so to say.

Necessarily true sentences can be defined in Leibnizian terminology as sentences true in each possible world. We can define deductively valid arguments in the same manner. An argument is deductively valid if and only if in each possible world where all of its premises are true, so is its claim. In other words, there is no such possible world, where all premises of the deductively valid argument would be true, yet its claim was false.

Necessarily false sentences are false in all possible worlds. Sentences which are neither necessarily true nor necessarily false are contingent. They are true in some possible worlds, false in the others. A sentence is true or false if and only if it is true or false in the actual world.

Coffa ([**14**], p. 33) contrasts this Leibnizian approach with the one of Bolzano. Bolzano gives an account of generally valid sentences which are sentences true due to their form.

In Bolzano's spirit we can define valid arguments as arguments, that for all their forms which have true premises their claim will be true as well[6].

[6]Bolzano's is considerably more complicated and refined, but it is this latter elaboration of his approach we want to expose.

I do not pretend to give historically exact account of origins of those concept and do not stick to the original terminology in which they were formulated, nor to the

Bolzano's explication of Leibniz's definitions, as Coffa explains, was motivated by his concerns for philosophy of mathematics.

Since Kant, necessary truths were usually identified with a priori truths. According to the famous argument of Hume, also adopted by Kant later, we cannot establish necessary truths purely on the basis of empirical observations. Therefore they must be a priori, known prior to and independently of any possible experience.

The second distinction between analytic and synthetic truths is already due to Leibniz. An example of an analytic truth is expressed in the sentence 'All bachelors are men.' To realize this is a true sentence, we only need to analyze the idea 'bachelor' as 'unmarried man'. We obtain 'All unmarried men are men.'

The subject of this sentence is obviously included in its predicate. It is therefore a priori known to us. We do not need to collect any evidence that would confirm this statement—that is to check for each individual bachelor, whether he is married or not. That would clearly be pointless and absurd.

For Kant all analytic statements are a priori, but there exist synthetic a priori sentences too. Kant's well-known example of synthetic a priori statement is the statement '7+5=12'. To verify this mathematical statement we do not need any empirical evidence. Yet we cannot say that the subject of this sentence is contained in the predicate. So it is synthetic.

A priori truths of mathematics are therefore neither based on our experiences, nor on meanings of concepts and must be a function of our pure intuitions of space and time, that are universal and unique categories in which any experience we can possibly have is given to us.

However, for Bolzano, who disagreed with Kant's philosophy of mathematics and his concept of pure intuition, the above arithmetical statement is also analytical. His notion of analytic sentence was that of one that is either generally valid or generally contravalid, both of these properties result from the meaning of the terms used in the sentence.

The understanding of analyticity is the one that prevailed in what later became to be known as analytic philosophy. It is therefore Coffa's merit that he recognized these nuances, when he called this tradition originating in Bolzano and leading to contemporary analytic philosophy as semantic tradition. The philosophy of logical empiricism which I have discussed in section 1.6 stands in this tradition.

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philosophical background of their authors, as it is their positivistic interpretation which interests me.

According to common interpretation in the semantical tradition, the notion of analytical both in the work of Leibniz and Kant was largely determined by Aristotle's theory of deductions. Bolzano's critique overcame this adherence to semantics of Aristotle's work which was not broad enough to give an adequate account of all analycal judgements.

For philosophers of the semantic tradition any necessities and therefore a priori truths must be of semantical nature. We cannot reach any objective certain knowledge from introspecting our subjective cognitive activities while postulating their universality.

The language seems to be the only domain which is accessible to our scrutiny, inter-subjective, and which yields necessary truth. Necessities of other than semantic kind would probably be far beyond the reach of our cognition.

I will not present arguments for or against this particular philosophical tradition, this account is only meant to introduce the reader to the fundamental postulates and concepts of this tradition. Bolzano's account of necessary truths and of deduction in terms of grammatical terms follows now. This account is from [**61**].

Given a sentence we obtain its *sentential form* if we replace some of its concepts with variable signs[7]. Clearly we can obtain the original sentence from its form by substituting the original concept for the variable which replaced it.

The process of moving from sentence towards it form is called *abstraction*, while the reverse process of obtaining sentences from their forms can be called *instantiation* as in [**59**], because we move from a sentential form to one of its instances.

Further we can define *substitution*. By substituting some string of symbols t by another string of symbols t' in a sentence s, we will simply replacef all occurences of t in s by t'. This is clearly what we would obtain by abstracting away from t and then instantiating the variable replacing t with a string t'. This procedure is called *reinstantiation* in [**59**].

[7]To avoid confusion with variables ranging over objects of a domain of discourse which are part of the language predicate logic, sentential variables are called *parameters* in [**59**].

Because I am not going to expose the language of predicate logic in this thesis, I need not to be that strict about terminology and therefore I will use term 'variable' for all kinds of symbols which serve as a placeholder for a variety of strings of signs.

Also, Bolzano's account is given in terms of ideas, but under certain philosophical assumptions, I need not to explain it in detail here, we may limit our attention to concepts which express those ideas.

Usually we do not want to abstract from, instantiate, or reinstantiate any strings of symbols. We will only want to apply these procudures to a *concept* or *name*.

What is a name? Some meaningful string of symbols which denotes (or refers to) some object, be it a sentence, thing, relation, function, set, truth value etc. What is meaning and denotation? That is a semantical question. Logicians will usually work with some artificial languages, where class of names and sentences is unambigously identified and their referents fixed in a unique way.

We can illuminate the important notion of logical form using only intuitive concepts of sentence, word, meaning and reference, there is no need to go into semantical details here.

A substitution is called admissible if and only if the resulting sentence is meaningful, provided the former sentence was. Necessary but not sufficient condition is that t and t' are of the same grammatical category. We cannot substitute 'but' for 'snow' in sentence 'Snow is white.' because the result 'but is white.' is not a grammatically correct sentence.

However, we cannot substitute 'mortgage' for 'snow' neither. The resulting sentence would be grammatically correct, but meaningless, as it makes no sense to apply the predicate 'white' to the subject 'mortgage', as far we are not concerned with metaphorical or poetic use of language. The substitution is admissible, roughly speaking, if we replace a name by another name which denotes objects of the same kind.

To determine whether a substitution is admissible is therefore not only a task of a grammarian and lexicographer, but also of a semanticist. Again, we are not concerned with semantics, we may leave these notions further unexplained. I give their account only to show later on page 49, that some fundamental logical concepts have been inspired by those concepts.

Now lets move to the Bolzano's account of general (universal) validity.

Some sentence s is a variant of the sentence p relatively to a certain set of names T , if s is a result of substituting arbitrary names of appropriate grammatical category for all names from T in p. A sentence is *generally valid* relatively to the set of terms, if all of its variants relatively to that set of terms are true. It is *generally contravalid* relatively to the set of terms, if all of its variants relatively to that set of terms are false. A sentence is analytic due to Bolzano, if it is either generally valid or generally invalid.

Consider the sentence ‘If snow is white, then snow is white.’ This sentence is generally valid relatively to a name of the meaningful sentence ‘Snow is white’. Whenever we substitute another meaningful sentence for ‘Snow is white’ (such as ‘Snow is black’ or ‘Snow is white and snow is not white’, or ‘Gorillas like bannanas’) into the original sentence we obtain a true sentence.

How do we get rid of the relativity to a set of terms in our definition? Bolzano calls a sentece *logically analytic* if it is analytic relatively to a set of all but *logical names*. Accordingly we can define notions of *logically valid* and *logically contravalid* sentences. Just what names are logical is a matter of great controversy because such definition would delimit boundaries of logic once for good. This particular problem will be tackled few sections later.

Bolzano himself does not give any clear answer to such question. Coffa ([**14**] p 34.) paraphrases Bolzano’s words: ‘The whole domain of concepts belonging to logic is not circumscribed to the extent that controversies could not arise at times.’

A notion of *deducibility* is defined in approximately in the following way. A set of sentences A is deducible from set of sentences B, with respect to some set of names T, if and only if the union of sets A and B is compatible and if in each variant of the union of A and B with respect to T holds, whenever all sentences from A are true so are all sentences from B. We obtain a logical concept of *logical deducibility* if T is a set of logical concepts only[8].

We can use this terminology to define our own concepts of valid arguments. An argument is *deductively valid* due to Bolzano if its claim is deducible from its premises and it is *logically valid* if its calim is logically deducible from its premises.

Despite the fact that Bolzano’s account makes a distinction between logically valid arguments and deductively valid arguments in general, his deductively valid arguments include some factually valid arguments as well. Truly, deductively valid arguments can be either logically valid, or anallytically valid or even factually valid.

[8] A set of sentences is compatible if there exists at least one variant of sentences from this set which makes all of them true. Modern accounts of deduction, however, usually drop this requirement of compatibility so I do not need to explain it in detail.

Consider the following three arguments:

A6:

Premise: Leipzig is to the north of Dresden.

THEREFORE

Conclusion: It is not the case that Leipzig is not to the north of Dresden.

A7:

Premise: Leipzig is to the north of Dresden.

THEREFORE

Conclusion: Dresden is to the south of Leipzig.

A8:

Premise: Leipzig is to the north of Dresden.

THEREFORE

Conclusion: In winter, the days are shorter in Leipzing than in Dresden.

Now replace in each of thes arguments names 'Dresden' and 'Leibniz' with variables x and y ranging over settlements. No matter what you subtitute for these variables, the resulting arguments will be still valid. The first of them due to the meaning of logical construction of negation, second due to the meaning of the terms 'north' and 'south', the third will be valid due to some general laws of astronomy.

It should be already somehow clear, what do these two accounts, Leibnizian and Bolzanian, have in common. To paraphrase Coffa's ([**14**], p. 33) interpretation: Leibniz takes a firm sentence, fixes its meaning, lets the world vary within a certain range of possibilities, and observes how it affects the truth of this given sentence. Bolzano, on the other hand, takes the same sentence, lets its meaning vary within a certain range of possibilities, and observes how it affects the truth of the sentence in the actual world.

Of course, to what degree is the Leibnizian definition of necessary truth truly equivalent to Bolzanian definition of generally valid sentence depends on how we interpret the key notions of both definitions—that is the notion of *possible world*, the notion of *sentential form*, and the notion of *truth*. The case of argument **A8** illustrates an objection against Bolzano's account that general validity does not imply necessity. The sentence 'All dogs which do have a heart also do have kidneys.' is generally valid relative to the idea 'dog', but it is not obvious why it would be necessary that anything with a heart also has kidneys.

Nonetheless, I will argue, all necessary truths are generally valid and consequently all deductively valid arguments in Leibnizian sense are deductively valid in Bolzanian sense. This inclusion undoubtedly holds. Therefore cogent arguments which are not deductively valid in the Leibnizian sense, as I have already shown, cannot be valid due to their form, logical or other. This is what I am aiming to establish and what I will hopefully fully demonstrate in section 2.6.

I will begin with modern accounts of logical truth and of logical necessity. Because they are generally valid, logical truths can be studied as instances of certain logical forms. Formal logic, therefore, will be a discipline studying such forms. I hope to establish that formal logic is a discipline studying logical forms and therefore logically valid (therefore) deductive arguments which in turn are used in demonstrations and are subject to demonstration.

2.3. Logical formalisms and formal logic

Quine [**52**] distinguishes four possible accounts of logical truth—in terms of *substitution*, *structure*, *models*, and *proofs*. However, he also claims that all these accounts of logical truth rest on two principal accounts: that of *grammar* and of *truth*.

He clearly believes all these accounts elaborate on Bolzano's definitions and can be provided using linguistical terminology only. He is very cautious not to mention the 'ontological' concept of possible world or any other equivalent to this concept in his treatise. To give such accounts for natural language we would need to go deep into ontology or semantics.

If we limit our attention to mathematical demonstrations only, it is the orderly and well-defined language of mathematics, that we will need to describe. It is not only much easier to provide theory of validity and demonstrations for mathematical language, but also much more desired because demonstration is a crucial for mathematical practice.

This fruitful marriage of logic and mathematics inspires formal logicians even today, when they often prefer to study artificial languages defined in such a mathematical manner.

One of advantages of using formal languages is that there is no place for ambiguity, indeterminacy and contextual dependence of meanings in such a language. The lexicon of the language may be desgined to be unequivocal and extensional—independent of the context of its use.

The grammar of such language will typically be defined in an exhausting, alghorithmical and totally unambiguous way. Consequently each item of such formal language will denote a unique object which will fall into one of previously specified categories.

The second advantage of using formal languages is the overcoming of the problem of determining the logical form. While designing such an artificial language, we may arbitrarily decide which names of this logic will be counted as logical and hence what logical forms will each sentence of this language have. Therefore notions of logical form and substitution will have unproblematic meanings in language of this kind.

Let me now, for illustration, introduce perhaps the simplest of such artificial languages—the language of *sentential* logic and illustrate its characterization of the fundamental notions mentioned above.

The set of basic names of language of the sentential logic (sometimes also called propositional,truth-functional, Boolean, or zero-order) consists of infinite number of sentential atoms $\{s_1, s_2, \ldots\}$ and a set of sentential operators (also called connectives) $\{\neg, \wedge, \vee, \rightarrow\}$.

The sentential atoms will be considered as *extra-logical names*, while the sentential operators will be considered as *logical constants*[9]. We define sentences of such a language recursively:

(1) Each sentential atom is a sentence.
(2) Whenever ϕ is a sentence, also $\neg\phi$ is a sentence.
(3) Whenever ϕ and ψ are sentences, also $\phi \wedge \psi$, $\phi \vee \psi$ and $\phi \rightarrow \psi$ are sentences.
(4) Only such expressions are sentences which have been created in the above manner.

Greek letters ϕ and ψ therefore stand for variables (parameters) ranging over sentences of the language.

These symbols do not belong to the language of sentential logic, but to a certain meta-language used for the description of sentential language. They are mere dummies, in Quine's terminology ([**52**], p. 50).

Now that we have defined grammar of sentential logic, it remains to define some truth criteria for strings of this language so that we would be justified to call this set 'language' in the first place.

A notion of truth for formal languages is usually described by means of so-called interpretations. One way to define interpretations for a particular language is to identify them with sets of sentences of a given language obeying certain principles. Let me give an example of *classical* interpretation of language of sentential logic.

A classical interpretation of sentential language **S** is any set of sentences of sentential logic, satisfying the following *logical laws*:

[9]See chapter 2 of [**24**] for more details on logical constants.

(1) *Law of bivalence* or truth principle: For each sentence ϕ either $\phi \in \mathbf{S}$ or $\phi \notin \mathbf{S}$. It is never the case that both $\phi \in \mathbf{S}$ and $\phi \notin \mathbf{S}$[10].
(2) *Classical negation*: For each sentence ϕ either $\phi \in \mathbf{S}$ or $\neg\phi \in \mathbf{S}$. It is never the case that both $\phi \in \mathbf{S}$ and $\neg\phi \in \mathbf{S}$.
(3) *Classical conjunction* : For each sentences ϕ and ψ: Whenever $\phi \wedge \psi \in \mathbf{S}$ also $\phi \in \mathbf{S}$ and $\psi \in \mathbf{S}$. Whenever $\phi \in \mathbf{S}$ and $\psi \in \mathbf{S}$ also $\phi \wedge \psi \in \mathbf{S}$.
(4) *Classical disjunction*: For each sentences ϕ and ψ: Whenever $\phi \vee \psi \in \mathbf{S}$ either $\phi \in \mathbf{S}$ or $\psi \in \mathbf{S}$. Whenever $\phi \in \mathbf{S}$ or $\psi \in \mathbf{S}$ also $\phi \vee \psi \in \mathbf{S}$.
(5) *Classical conditional*: For each sentences ϕ and ψ: Whenever $\phi \to \psi \in \mathbf{S}$ also $\neg\phi \vee \psi \in \mathbf{S}$. Whenever $\neg\phi \vee \psi \in \mathbf{S}$ also $\phi \to \psi \in \mathbf{S}$.

Alternatively we may identify classical interpretation of a sentential logic with a truth-assignment function, that is a total function from the set of all the sentences of the sentenital logic into the set of truth-values {TRUE, FALSE}.

Clearly, the requirements that this assignment is a function and that it is total ensure that interpretation does not assign each sentence exactly one truth value. This alone would guarantee the law of bivalence.

In addition, such function would have to fulfill additional properties for evaluation of compound sentences, corresponding to conditions (2)-(5) in the previous definition. Those requirements can be expressed using the notoriously known *truth-tables*[11]:

ϕ	$\neg\phi$
TRUE	FALSE
FALSE	TRUE

ϕ	ψ	$\phi \to \psi$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	TRUE
FALSE	FALSE	TRUE

[10]This is an immediate fact whenever we decide to model possible worlds with sets of some classical (i.e. not fuzzy) set theory which is why this law is ignored in some accounts and incorporated directly into the following law.

[11]We could also simply define states of affairs as sets of *atomic* sentences, and truth-assignments as total functions from the set of *atomic* sentences to the set of truth-values, and then define truth of a complex sentence using truth-tables.

This is only a technical detail, though.

ϕ	ψ	$\phi \wedge \psi$	ϕ	ψ	$\phi \vee \psi$
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	FALSE	TRUE
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

We may now use the technical notion of interpretation to define logically true sentences of classical sentential logic (CSL).

A sentence of sentential language ϕ is a logical truth of CSL (also called *classical tautology*) ($\vdash_{CSL} \phi$) if and only if it is a member of all classical interpretations of sentential lanugage, or alternatively if and only if it is assigned the value TRUE by all classical truth-assignments for sentential language.

A sentence ψ is a *CSL consequence* of the set of sentences $\phi_1 \ldots \phi_n$ ($\phi_1 \ldots \phi_n \vdash_{CSL} \psi$) if and only if each classical truth-assignment for sentential language which assigns value TRUE to all sentences $\phi_1 \ldots \phi_n$ also assigns the truth-value TRUE to the sentence ψ.

The notion of CSL consequence is therefore our approximation of the notion of logical deducibility and the notion of classical tautology is our approximation of the notion of logical truth.

We can illustrate the relation between logical truths and logically valid arguments on the case of a relation between classical tautologies and CSL consequences.

The following theorems also hold for CSL:

(1) $\vdash_{CSL} \phi$ if and only if $\emptyset \vdash_{CSL} \phi$
(2) $\phi_1 \ldots \phi_n \vdash_{CSL} \phi$ if and only if $\vdash_{CSL} (\phi_1 \wedge (\ldots \wedge (\phi_n) \ldots) \rightarrow \phi$.

In our CSL approximation, each logically valid argument corresponds to some logical truth in the form of a conditional statement and each logical truth corresponds to a conclusion of certain logically valid argument which has no premises.

In sufficiently rich languages (formal or natural, containing something like classical conditional), we may therefore treat the problem of determining logical truths interchangeable with the problem of determining logically valid arguments.

Further we say, a set of sentences Ψ is CSL *consistent* if and only if there exists a truth-assignment which assigns all sentences from Ψ the truth-value TRUE. The following conditions are equivalent.

(1) Ψ is consistent.
(2) There is no such sentence ψ that both $\Psi \vdash \psi$ and $\Psi \vdash \neg\psi$.

Defining logical truth in terms of interpretation or *models* in Quine's classification, is nowadays often called *logical semantics*, as opposed to definitions in terms of *proofs*, often called *logical syntax*.

If we pair a formal language with one such account, or procedure for determining logically valid arguments of that language (or more procedures yielding same results), we obtain what I call *logical formalism* and what is more often called *logical system*.

A logical formalism will simply be a pair of a formal language and some criteria defining valid arguments of this language be it a model semantic, or some syntactical procedure[12].

We can also finally delimit *formal logic*, as the discipline concerned with the study of logical formalisms. Different logical formalisms can be understood as different artificial languages, as I will argue later on page 2.4.

So called formal language, without any interpretation, is not a genuine language at all. Truly an account of grammar without any account of truth does not constitute anything that would be worth the name of language.

Of course, there remain accounts of logical truth in terms of structure and in terms of substitution. Those are, however, usually unproblematically equivalent to that in terms of models for most known logical formalisms.

This fact is quite important for my argument, as it demonstrates equivalence of Leibnizian account (interpretations as possible worlds) and Bolzanian account (in terms of substitutions and sentential forms) for such languages. I will, nonetheless, do not demontrate it here and redirect reader for technical details to [**45**].

The relation of the account in terms of models and in terms of proofs is often not that unproblematic. It is a subject matter of completeness theorems which are considered often a backbone result of mathematical logic. The completeness theorem do not hold for all logical formalisms though. There may be such accounts of logical truth in terms of models such that there is no equivalent account in terms of proofs.

This may seem as a serious objection to such logical formalisms, as the notion of logical truth is intended to clarify the notion of logically valid arguments and logically valid should be demonstrable by logical

[12]Note that the exisence of criteria for determination of validity of arguments does not necessarily imply the existence of a definite alghoritm, that could decide the validity in finite time for such a scheme and was guaranteed to terminate on each input.

Many important logical formalisms are undecidable, that is, there is no such algorithm for determining logical truths of those formalisms.

means. Are there therefore some logically valid arguments that cannot be demonstrated to be so?

Well that depends on what we consider to be an acceptable proof. The incompleteness result merely shows that a certain concept of proof is perhaps too narrow[13].

The concept of proof is not yet quite well understood and circumscribed in any definite manner, nor is that of model or structure of sentences. It is a goal of various accounts of logical truth to help to illustrate just what these notions are.

There exists plurality of approximations of logical truth and demonstration. Just before explaining why this plurality poses no real problem in the section 2.4, let me show under which philosophical assumption we may conclude, that logical truths of CSL, are indeed the only necessarily true sentences.

2.4. Form, necessity, meaning and truth

In section 2.3, I have introduced the logical formalism of classical sentential logic by means of classical truth-tables, just to tell the reader to see other accounts for himself and check their equivalence on his own. So what was the deal with that?

Our primary interest is not classical sentential logic as such, nor notions of CSL validity consequence. It is their function as approximations of notions of logical truth and logically valid argument that interests us.

Even such an easy tool as is sentential logic helped us to illuminate some fundamental properties we would expect these latter notions should have. That is the relation of notion of logical truth to that of logically valid argument.

Classical truth-tables are of particular interest because their origins can be traced to Wittgenstein, who first came up with interpretations for sentences of sentential language in his Tractatus [**70**].

Wittgenstein's alternative to the classical notion of possible world and precursor for more recent notion of interpretation is a notion 'state of affairs'. A state of affairs, for Wittgenstein, is a configuration of facts that are the case.

Each meaningful sentence of a certain language corresponds to some fact. A sentence is true in a state of affairs whenever a corresponding fact is the case in that state of affairs, otherwise it is false. Some of the sentences are atomic and the other ones are complex. The atomic

[13]This is one of the main arguments of [**32**].

sentences correspond to atomic facts and all combinations of atomic facts are possible.

Consequently all distributions of truth-values over atomic sentences are possible. Whether a complex sentence is true depends in a determinate and unique way on the truth-value of the atomic sentences from which it is constructed. Therefore each composed sentence will also have exactly one of the two truth-values.

It may seem at the first sight that Wittgenstein offers just a rephrasing of the old ontological justification of the law of bivalence. A closer look, however, reveals a different perspective. For Wittgenstein, laws of logic are the only laws we can reasonably say that they hold in all states of affairs. He writes:

> It used to be said that God could create everything, except what was contrary to laws of logic. The truth is, we could not *say* of an 'unlogical' world how it would look. ([**70**] 3.031).

The logical laws are therefore not the most fundamental laws of existence even God could not violate—they are the most fundamental laws of all reasonable descriptions of such reality.

The boundary between possible and impossible does not lie within reality itself and even if it did, we could not apprehend it, but within our own language which we use to describe it.

This also justifies why we do not need to take into an account any other laws governing all possible states of affairs. Logicians are simply not concerned with what actually could be or could have been the case, they are concerned only with what we could actually describe or think as a case.

> The thought contains the possibility of the state of affairs it thinks. What is thinkable is also possible. ([**70**] 3.02).

And what is thinkable? Only that is logical which conforms to laws of logic, that is the laws governing the most central ways we use our language.

The law of bivalence, for example, is to be understood as the law governing our usage of notions of truth and falsity and not as a law describing configurations of facts which actually might sometime come to be the case. This is why, according to Wittgenstein, logic is not a proper science with its own subject.

Facts may be seen as correlates of true sentences we project into reality. In Frege's words: 'Fact is a true sentence.' ([**22**], p. 74).

The remaining laws concerning meaning of the sentential connectives are of the same kind as the truth-principle ([**70**], s. 6). They are not to be justified by ontological considerations about common feats of all possible worlds.

They delimit what laws any situation or state of the world must satisfy so that we could consider it to be possible (that is intelligible) which in turn is logically describable, according to Wittgenstein.

Laws of logic are not to be discovered in reality or thought, instead they are prescribed to it. We are looking at the reality through the looking glass of our language which already obeys those laws and only through language we are capable of describing the reality as the case.

Note also, that we do not need any strict correspondence theory of truth. To say that truth sentences describe facts is a truism because this is how facts have been constituted, yet what actually is a matter of fact and what not and how can we decide this issue does not matter for the semantical account of logical truth at all.

Modern formal logic has been fully liberated from foundational questions about nature of truth thanks to the work of Alfred Tarski[14]. Tarski provided something what could be called a theory of theories of truth.

In his famous article [**63**] he summarized a criteria he believed all theories of truth should fulfill. Therefore what truth actually is does not bear any impact on the delimitation of logical truth, as long as the concept of truth obeys the laws of logic we already had before making any attempt to define the truth.

Now one may dispute about various items on Tarski's list, but it is the general approach of Tarski I wish to emphasize. It is our notion of truth which is determined by our notion of logical laws, not vice versa.

It is our understanding of nature of logical laws, that is as a certain specific class of generally valid sentences which are fundamental for our understanding, what constitutes a possible state of affairs and what does not, and not vice versa.

This does not mean Tarski delimited the notion of truth once and for all. He only delimited necessary criteria that each account of truth should satisfy, without adding they are also sufficient.

For example Wittgenstein's delimitation of truths which trivially satisfies Tarski's criteria, is obviously quite narrow. After all a notion of CSL tautology was only a partial approximation of the notion of

[14]For various accounts of truth and Tarski's special contribution to this topic see [**29**].

logical truth and beside that also analytical truths are necessary after all.

However, the most important impact of Tarski's theory and semantical approach in general is that we de not need to make logic dependent on notion of truth which in turn is dependent on our notion of reality.

Suppose we adhere to naive realism and correspondence theory of truth. According to this particular ontology and theory of truth a sentence is true if and only if it corresponds to reality which could be alternatively rephrased that the fact it expresses is the case.

Suppose further that we believe that each fact either can be the case or not be the case, but not both. Consequently each sentence will either be true or false, but not both.

Have we just hereby justified the law of bivalence? And what about logical systems that deliberately break it? Are they blatantly false, or just pure meaningless calculi only pretending to be systems of the 'true logic'?

Probably we could defend the above position, only if we could provide reasonable and justified answers to questions such as: What is reality? What facts does it consist of? How do we know each fact must be the case or not be the case and not both? How do we recognize which sentence corresponds to which fact? Are there some negative or disjunctive facts corresponding to negations or disjunctions of sentences respectively?

This approach will become even more problematic if we are faced with the second philosophical problem of possible worlds. What combinations of facts are possible?

Does a concept of classical sentential interpretation do justice to the notion of the possible world? Have we included all possible worlds and included all impossible ones by listing logical laws?

Some might argue, that as there is a multitude of various, even competing, lists of logical laws (for example laws of intuitionistic logic), we might have just picked the wrong list. Also a question whether some logic or another has to have the final and decisive word on what is possible and impossible can be raised.

Now we can clearly see the merits of semantic interpretation of logical laws. While a fact of multitude of different accounts of logical truth would have to be interpreted as a fact of multitude of various accounts of basic features of reality or thought by ontological and psychological philosophies of logic respectively.

This is a great embarassment and naturally leads to questions which of these accounts is the correct one, the very same fact merely implies that there is a multitude of different languages, if we interpret logical laws semantically.

But what about the case that we have a competing account of valid forms for a single language? Do we not have a conflicting account of semantics of this language? Well, this is a problem only if we presuppose that the logical terms of the formal language derive their meaning from some other language which they are meant to represent.

For example the law of classical conjunction must be clearly valid, as this is how connective 'and' behaves in English and we do intend our connective *and* to model its meaning. Or does it? And do we?

Now it is when we must realize that the semantics of formal languages are not meant to be the semantics of expressions of natural language which we might want to translate into language of this formalism.

Different logical formalisms represent different languages. They are clearly different from natural language, however, some fragments of it can be translated into language of these formalisms.

They are, nonetheless, also distincively different one from another, even though they have exactly the same set of symbols and even sentences, but diverge in an account of logical truth for such language[15].

In such a case these formal languages are identical only in a way of symbolic representation, but a sentence of one logical formalism, although it is an identical string of symbols as the sentence of the second logical formalism, has a different meaning, due to the fact that other set of logical laws applies to it.

For example, a sentence $p \vee \neg p$ has a different meaning in system of classical and intuitionistic logic. However, there is an uniform translation of the language of classical sentential logic into the the language of sentential logic, such that it preserves a notion of logical truth.

On a surface level it is a mapping from a set of strings composed from a given set of elementary symbols by some means of given grammatical rules into itself[16].

But in fact it is a genuine translation of one language into a distinct language, such that it preserves all the properties of the sentences of the original language we are interested in, that is logical truth[17].

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[15]This insight is clearly expressed by Quine in his discussion of so-called 'deviant logics' ([**52**], chap. 6).

[16]It is not surjective mapping, so, in a way, language of intuitionistic sentential logic is richer than that of the classical sentential logic.

[17]Details of this translation and further explanatory commentaries are to be found in [**45**].

It is therefore the general approach of philosophers of the semantic tradition which I wished to illustrate on Wittgenstein's example. It is our understanding of logic and of form which determines our understanding of possibility, necessity, and impossibility, and not the other way around.

This is a strong thesis that many philosophers would disagree with. There might be some necessary truths which are not analytic and vice versa. Still various different accounts of logical laws will be acceptable.

These laws of logic may be considered to be laws governing the *meaning* of logical concepts involved as opposed to some more traditional interpretation of logical laws as general metaphysical laws governing the innermost structure of reality, or of our cognition, reason, or thought.

Nonetheless, the controversy about the nature of meaning is stil vivid, until recent days. There are many competing philosophical doctrines of logic. For example, while *platonism* and *pragmatism* offer different accounts of grounds for meanings of logical terms, account of laws of logic as (either normative or empirical) laws of thought is still alive in a philosophical doctrine of *psychologism*[18].

What are meanings and how do we grasp them, that is yet another level of debate which has no particular relevance for understanding what is and what is not logical truth of a given language.

Nonetheless, even for a semanticist, the plurality of logical formalisms does not imply plurality of accounts of logical truth for one particular language.

Logical formalisms are languages on their own. If we are describing logical truths of one particular language, say English, we are bound to obey its semantics which cannot just be postulated as if we were designing an artificial language of logical formalism on our own.

Logical formalisms can be used as a tool for analyzing English, but the question of correct translation between English and a particular logical formalism emerges.

To sum this discussion up. For my current account it would be contraproductive to delve into ontology, epistemology, or psychology, and semantics too much.

I am not fully aware what arguments do these philosophies for or against such interpretation of multitude of logical formalisms as multitude of different languages, that are meant to be used as instruments.

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[18] A brief overview of these competing accounts is to be found in ([**59**], sec. 1.2–1.3.).

All that I needed to establish is that all logical formalisms aim to provide a description of logical truths of a given formal language which can be used to analyze natural language, either properly or improperly, depending on the semantics of natural language.

Just how to apply those tools for practical analysis of natural language is yet a different issue which will be briefly mentioned in sections 3.1–3.2.

For now let us stay on a theoretical level of debate and delimit the notion of logical formalism a little bit more.

2.5. Limits of logical formalisms

Even though we have explained the multitude of logical formalisms as a rather innocent multitude of different languages, we still have to address the other side of the problem and determine where exactly boundaries of logical formalisms lie and whether there is something common to all those formalisms after all.

This is also a task of the authors of [**6**].

In this section I will finally illuminate some of the concepts used in the section 2.1, such as formal vs. symbolic language and central notion of logical formalism.

The notion of logical formalism remained quite vague in the preceding sections because no precise specification of the admissible methods for determining the set of logically valid arguments has been given.

It was vague intentionally because the domain of logical formalisms should not be limited to include only those using some already established criteria for validity. The family of logical formalism must always remain capable of accomodating new systems which continue to emerge even those with quite unprecedented methods to recognize valid arguments.

Therefore the list of four possible accounts of logical truths suggested by Quine for the case of classical predicate logic should by no means be taken dogmatically as a standard for delimitation of logical formalisms. However, there will have to be at least some standards.

Often we can encounter a general definition of logical system in set-theoretical terms. We are told that a logical system is simply an ordered pair $\langle L, \vdash \rangle$, where L stands for a language, here interpreted as an arbitrary set of sentences, and $\vdash$ stands for the relation of *consequence* (or *entailment*) $\subseteq 2^L \times L$.

An argument is a tuple $\langle A_1 \dots A_n, B \rangle$, where $A_1, \dots, A_n, B \in L$ are premises and claim respectively. This argument is logically valid according to such system, if and only if, $A_1, \dots, A_n, B \in \vdash$, usually abbreviated as $A_1, \ldots, A_n \vdash B$—that is if the claim is consequence of the premises (it is entalied by the premises, it logically follows from them etc.).

This might seem as an inadequate definition as it does not offer us any decision algorithm that would recognize logical truths of the given language. However, the main problem of this definition, I believe, is even more fundamental. It is too broad to be useful.

Sets such as $\langle \{\clubsuit, \heartsuit, \triangle, \diamondsuit\}, \{\langle \{\clubsuit, \heartsuit\}, \triangle\rangle, \langle \{\triangle, \heartsuit, \clubsuit\}, \clubsuit\rangle\}\rangle$ can be called 'logical formalism' only by the most extreme inclusivists, as no considerations of form play any role in its constitution. The relation of consequence must have a certain property which Beall and Restall [**6**] call *formality*.

However, the notion of formality of consequence relation becomes intelligible only, if we are capable of determining at least some form for all sentences of the language for which the relation is defined. The set of sentences must already have some formal structure.

These two requirements deserve to be pointed out.

(1) The language of logical formalism is formal.
(2) The relation of consequence of logical formalism is formal.

Ad 1) By formal language I do not mean any language which is often referred to as symbolic language—that is a language using other symbols than those of standard alphabets, such as symbols for sentential connectives $\neg, \vee, \wedge$ etc.

The 'language' used in the above example also uses 'exotic' symbols, yet is not formal in the sense that I have in mind. This is quite a trivial observation, but surprisingly enough the confusion of formal language with those containing 'formal' symbols in its lexicon is quite widespread.

It makes no sense to distinguish formal reasoning from informal purely on the basis that the one is carried within a language containing special symbols of sentential logic, while the second is formulated in English with phrases such as 'and' or 'or', if the latter follows the same pattern as the former.

A formal language is a language whose sentences are built up recursively from a set of primitive terms of different gramatical categories using a set of unambiguous production rules. Due to the way the language is constructed, it usually contains an infinite number of sentences and, more importantly, such concepts, like 'logical form', 'valuation', and 'substitution' can be meaningfuly and unambigously defined for it.

Optimally finding the logical form of a sentence or substituting for terms within a sentence of formal language should be achievable by purely mechanical procedures.

This delimitation of formal languages is quite open-ended as it is not specified what grammatical categories of terms there have to be, or how do the rules for sentence forming have to look like.

This definition encompasses languages of all currently recognized logical formalisms, yet excludes the previous language, as no account of logical forms of its 'sentences' has been provided along with the 'definition' of the sentences of this language.

Ad 2) It should hold, that whenever some argument is listed as valid within the formalism, there should exist some logical form of it, a notion meaningful due to the requirement 1), so that also other arguments of the same form were logically valid.

In more technical jargon: The relation of consequence should be closed under substitution[19].

However, even previous specification encompasses formalisms which we would be hesitant to call logical. Consider for example a following system, I will referr to it as **N**[20].

We can define the language of **N** recursively using a constant symbol **0**, an unary function symbol $\int$, and a binary relation symbol $\leq$:

(1) **0** is a name.
(2) Whenever **n** is a name, so is $\int \mathbf{n}$.
(3) Only such expressions are names which are described in points 1)–2).
(4) Whenever **n** and **m** are names, an expression $\mathbf{n} \leq \mathbf{m}$ is a sentence.
(5) Nothing else is a sentence than that what is described in 4).

Clearly this is an account of a formal language satisfying condition 1) of the definition from the beginning of this section, while we can arrive at a from of a sentence, if we replace some name in it by variable **m** ranging over numerals.

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[19] This is not a feature satisfied by a family of so-called non-monotonic logics (characterized usually by lack of other properties of consequence relation), as is proven in [**35**].

For those reasons I am hesitant to call such systems as logical formalisms. Their specific role in description of dialectic reasoning will therefore be highlighted in the section 3.3.

[20] Lorenzen [**34**] introduces a similar calculus **K**, but I would like to avoid using the somehow philosophically problematic relation of equality in a calculus which I introduce with the sole purpose to demonstrate some general features of formal systems.

Therefore sentence $\int\int\int \mathbf{0} \leq \int\int\int\int\int \mathbf{0}$ does have following forms:

$$\int\int\int \mathbf{m} \leq \int\int\int\int\int \mathbf{m}$$
$$\int\int \mathbf{m} \leq \int\int\int\int \mathbf{m}$$
$$\int \mathbf{m} \leq \int\int\int \mathbf{m}$$
$$\mathbf{m} \leq \int\int \mathbf{m}$$

Obviously only the first form on the list would be counted as a logical form of this sentence, should we treat **0** as the only extra-logical term of our language. Speaking about logical forms in this case is quite superflous and it will be the more general forms that will interest us.

It is due to the fact that we can give an account of valid logical forms by enumerating recursively all valid forms of sentences of which logical forms are just a specific case:

(1) Any sentence of a form $\mathbf{m} \leq \mathbf{m}$ is true.
(2) Whenever a sentence of the form $\mathbf{m} \leq \mathbf{n}$ is true, so is a sentence of the form $\mathbf{m} \leq \int \mathbf{n}$.
(3) No other sentences are true, but those of the forms described in points (1) and (2)

This would be a proof procedure for determining true sentences of our language.

We immediately see that whenever a sentence is true, so will be all the sentences of the same form. So we have indeed constructed account of truth that satisfies condition 2) from the definition of logical formalisms.

We can recursively check that the sentence from our example is true according to this definition. The point 1) in the previous definition tells us that a sentence of the form $\mathbf{m} \leq \mathbf{m}$ is true, we apply rule 2) to obtain that $\mathbf{m} \leq \int \mathbf{m}$ is true and apply it once more to obtain that also $\mathbf{m} \leq \int\int \mathbf{m}$ true. But that already is the form of the sentence in question. We can now instantiate **m** with $\int\int\int \mathbf{0}$ to obtain $\int\int\int \mathbf{0} \leq \int\int\int\int\int \mathbf{0}$ as desired.

The previous calculus can clearly be used to describe a fragment of arithmetical truths—in particular true inequalities among numerals. Despite Frege's programme of reducing arithmetic to logic, thruths of arithmetic are considered to be of slightly different kind than truths of logic.

But that clearly does not matter, as I could have suggested a completely different calculus instead. The formalism **N** could still be 'interpreted' as a regimented language of some statements comparing natural numbers.

There is no reason why we could not design a formal language without any intended interpretation in mind and add some procedure determining what is a valid form of this language.

In fact, arithmetical and logical calculi are just specific examples general calculi which can be constructed without any prior semantic concerns. Lorenzen [**34**] describes general features of such calculi and calls inquiry about their general properties *protologic*.

But when does a calculus cease to be a mere generic calculus and becomes a logical calculus? Are we not in the end forced to use semantic categories and resort to saying that a logical calculus is the one that captures meanings and laws of logical concepts, while, for an example, arithmetical calculus is the one that describes laws of arithmetic?

The Leibnizian distinction between a mere meaningless calculus and calculus rationicator of lingua characteristica seems to be relevant at this point of discussion.

We may have arrived at the language of sentential logic by the process of regimentation and abstraction of natural language (these processes will be described later on page 80).

But at the beginning of this process, we already had at hand a set of concepts we decided to consider to be logical constants. We could just as easily have another one.

Later we have postulated the classical logical laws thus arriving at logical formalisms of classical sentential logic. But we could have chosen laws of intuitionistic logic instead and then just provide a different translation of the sentences of this logic to natural language, perhaps through translating them to classical logic first, as mentioned above on the page 57.

The delimitation of logical formalisms must be seeked for within the structural syntactical properties of the calculus itself, rather than by invoking semantics of concepts our calculus is supposedly designed to describe in some meta-language[21].

I will not present details of Lorenzen's solution, but the main idea of his approach can be summarized in a following way: Logical concepts are such concepts that it is convenient to introduce to all calculi as they will allow us to make some protological features of all the calculi explicit.

It is for example useful to be able to derive in every calculus the word $A \to B$ whenever a rule allowing us to derive B from A would be

[21]Tarskian semantical delimitation of logical concepts ([**64**]) is of this kind. The invariance of logical concepts with respect to certain transformations is nothing if not trivial corollary of the (meta-language) definition of these transformations.

admissible, that is by adding a rule that B can be generated from A to the original set of rules, no more additional words can be generated (for example [**34**], p. 68).

For example in $\mathbf{N}$ we would have to be able to derive $\int \mathbf{m} \leq \int \mathbf{n} \rightarrow \mathbf{m} \leq \mathbf{n}$.

This is a purely pragmatic criterion and as such it is quite open ended. It leaves us to decide what principles are convenient for us to work with.

This claim that logical concepts are usually designated with some practical purpose in mind is illuminated by actual historical developments in modern logic.

The class of logical concepts clearly evolves as new logical formalisms are proposed to articulate and solve some problems that were not addressed adequately in previous formalisms.

Those problems might come from a reservoir of classical antimonies and paradoxes, or be motivated by epistemic or metaphysical concerns, or be aimed directly at some practical application in mathematics or computer science.

Often the only motivation is just a refinement of old concepts which do not work as intended or do not work in as many cases as would be desirable.

Besides their utility for description of problems in related fields, also intrinsic features and properties of logical formalisms determine their practical value. Only such formalisms are accepted by the community of logicians, and become recognized and well-established, that are describable in a feasible way.

It is desirable that some classical methods to describe and study logical formalisms can be applied to them (model theoretic semantics, sequent calculi, Kripke semantics), that they have some of the properties recognized as important (inference relation is reflexive, recursive axiomatization etc.) and that some results proved for other formalisms hold for these as well (compactness, completeness).

While we can strive for some more universal set of properties which logical formalisms should always posses, every such account should be able to explain the whole multitude of logical formalisms there are.

The only necessary requirements I could identify, have been already mentioned in this section. Nonetheless, I do not dare to claim they are sufficient and that more refined adequate criteria could not be found. Still, I believe we cannot delimit a class of logical concepts in a unique absolute way.

2.6. Limits of formal logic as such

The restrictions of formality, we imposed on logical formalism, enable us to conclude that each logically valid argument according to this formalism is generally valid with respect to the set of concepts, designated as logical in this formalism.

The notion of interpretation and laws of truth must not always be straightforwardly evident for sentences of many of logical formalisms, such as substructural, dynamic, erotetic or deontic logics for example.

There are, nonetheless, different methods to delimit class of logically valid arguments, we can delimit them in terms of proof, for example (again that does not imply we will be able to carry out this procedure in each case and actually prove every logically valid argument of our formalism).

Because those arguments will be valid due to their logical form, they will also always be valid deductively, under the standard explication of Bolzano.

Including more arguments, we would not construct a language for logical analysis, but perhaps for the analysis of material truths of a certain field which would be, however, merely analytical as far as we agree with the positivistic interpretation of necessity.

Omitting some logically true sentences from our account of necessary truths, on the other hand, would be an unprecedented and hardly explicable philosophical blunder.

Formal logic, if its concern is approximation of logical and hence formal validity, is limited to recognition of deductively valid arguments indeed. But does the inverse inclusion also hold? Are all deductively valid arguments logically valid?

Remember that we have been told that only such arguments are deductively valid, that the world where the argument would not establish its conclusion is impossible. According to Wittgenstein we should understand this impossibility merely as logical impossibility. And only such worlds would be illogical, which means it could not be properly talked about in a language that has a certain logical structure.

That practically amounts to saying that logic can correctly determine what arguments are deductively valid because logic can determine which states of affairs violate its laws and there are no other laws. Logically impossible worlds are clearly impossible, but again—does the opposite inclusion also hold?

The non-trivial direction of the claim, that all impossible worlds are already logically impossible is equivalent to the claim that all deductively valid arguments are logically valid and therefore they can be recognized by logical methods. This is much more problematic claim.

We may want stronger concepts of impossibility for various reasons and not all of them have to be a result of some realistic plead for metaphysical laws.

Not all impossibilities, clearly, are of this logical kind, but that does not necessarily mean they are epistemological or metaphysical. Clearly analytically valid arguments which are valid due to the meaning of some other than just logical terms will be deductively valid too.

But are there any other than logical concepts? Why do we need to designate a special category of logical concepts after all? The definition of logical concepts troubled Bolzano because he wished to delimit a logical form.

Now that we have various accounts of logical forms in the languages of logical formalisms, we have also various accounts of logical concepts for those languages.

This leads to a question: Are intended English correlates of logical concepts in such formal languages also logical concepts of English?

Beall and Restall aim to provide a general criteria any logical account of arguments should satisfy from which an answer on the question 'What are logical concepts?' could be deduced.

Let us now asses a slightly modified list of argument schemes, adopted from their book ([**6**], p.20):

AS1: **A**. If **A** then **B**. Therefore **B**.

AS2: x is a sulphuric acid. If x is sulphuric acid, then x is acid. Therefore x is acid.

AS3: x is y. y is z. Therefore y is z.

AS4: x is y. x is A. Therefore y is A.

AS5: $x \leq y$. $y \leq z$. Therefore $x \leq z$.

AS6: Ph of $x \leq 5$. Therefore Ph of $x \leq 7$.

AS7: Ph of $x \leq 5$. Therefore x is an acid.

AS8: H_2SO_4 is A. Therefore Sulphuric acid is A.

AS9: x is sulfuric acid. Therefore x is acid.

AS10: x is an acid. Therefore x has mass.

AS11: x is an acid. Therefore x is a substance.

AS12: x is an acid. Therefore x reacts with a base.

Clearly small italic variables **x**, **y**, **z** should be instantiated with names of objects, capital italic variables **A** and **B** with predicates (properties, verbs...), **R** with binary relation and bold capital variables **A** and **B** with meaningful declarative sentences.

I believe that to address the question of boundary between various kinds of forms appropriately, we need to recall why do we want to draw any boundaries in the first place.

The category of logical concepts is of pragmatic importance because there exists a bunch of relatively related methods and algorithms for determining generally valid statements relative to those concepts. Those methods have been developed continuosly within the tradition of formal logic.

However, there also exist concepts, whose meaning could not yet be captured faithfuly by known methods of logical formalisms. Consequently, although some arguments are analytically valid with respect to a certian set of terms, we cannot give a comprehensive and exhaustive list of all valid arguments with respect to these set of terms or any algorithm to generate such.

This is a well-known result of a series of theorems showing in different versions that such a complete and decidable account is impossible even when we limit our scope to natural numbers. However, these are rather complicated results and theirs history is long and intricate so I cannot explain them here but merely refer to [**32**].

Moreover, remember that analytically valid arguments could be reduced to logically valid arguments. Logically valid arguments should be, in some sense, already irreducible.

Logically valid arguments, therefore, would be such arguments that it makes no sense to explicate them any further by reducing them to more elementary arguments by adding an additional premise.

It would not make any sense to extend argument form **AS1** by adding its internalization

*****: If **A** and ‘If **A** then **B**’, then **B**.

among its premises. In this additional premise we merely explicate the well-known rule of *modus ponens*. It is this rules which justifies the inference from premises of **AS1**. ***** postulates this principle as a logical law, that is as a special kind of truth—necessary truth of logic. But to apply this law for justification of inference of **AS1**, we would already need some ‘meta’-modus ponens.

So the argument of a form **AS1** is in a certain sense elementary, as it cannot be logically justified without reference to itself, unlike **AS5** which can be reduced to **AS1**.

The case of **AS1** is quite clear-cut, but a general definition of meaningful reduction of an argument is hard to come by, to provide universal criteria for logical concepts.

Again, we are left only with methodical considerations as to which concepts should be picked as fundamental in order that we could explain meaning of the remaining ones using those with relative efficiency. But there evidently is more than one way how to do so.

While we could possibly aim at finding some basic logical operations, such as predication, equallity, implication, negation, existence, abstraction etc. from which others could be defined we possibly could not arrive at a unique set of logical constants in this way neither, as it is not clear what constitutes the totality of arguments we want to explain. Should we, for example, include modalities?

Also, sometimes even a weaker additional premise, than the internalization of the original argument, is required to explicate the argument, therefore it is quite difficult to tell, whether some non-circular explication of certain logical terms could not be given after all. To find a proper enthymeme can be a particularly tricky matter indeed.

While the distinction between logical truths and lexical truths is important mostly from a methodical perspective, the distinction between analytic and factual truths is identified with that between necessary a priori truths and contingent a posteriori truths within semantic tradition.

So, we may ask which of the argument schemes **AS6**–**AS12** is factually valid? They are all deductively valid according to Bolzano: that is whatever object we substitute for x we obtain a valid argument.

Accordingly their internalizations should be expressions of general empirical facts. But how can we be sure that all of the potential infinite instances of these internalizations are true?

This can clearly be a problem, if there is an infinite number of true instances of the premises of such an argument. In case there can be only a finite number of such instances, such as with the argument scheme:

AS13: x is a whale. Therefore x is a mammal.

But is a statement that all whales are mammals a general fact which can be falsified by observation or rather a definition of a whale? What if new creatures were discovered in the uncharted oceans depths which would look exactly like whales but were blind, hatched from eggs and had fins instead of lungs.

Would the internalization of **AS13** be falsified by this discovery? Or would it mean those creatures are not whales, but rather some close relatives of them, a new species which we may actually call 'deep-whales' or something alike? Or have we falsified the claim that all mammals breathe air?

Remember that the discovery of platypus, a mammal who actually hatches from eggs, was also hard to believe.

Actually the very claim that all whales are mammals is doubtful, as the anatomy, life environment, and habits of whale, resembles more that of a fish, than that of most mammals.

In his famous essay [**50**], Quine argues that a boundary between statements of meaning and statements of fact is volatile. Any surprising encounter of the above kind would not falsify one particular claim, but would affect the whole body of our knowledge and the internal implicit relations between members of this set.

Actually in the above case all three solutions would be accessible, the first one might be the most plausible though, as it would cause 'minimal mutilation' ([**52**], p. 7). For this reason not even truths of logic are immune to revision (ibid. p. 100.).

Truths therefore cannot be clearly divided into necessary (analytical and logical) and contingent (empirical) ones—they merely enjoy a certain degree of certainty which they derive from their centrality in the web of knowledge.

As the web of knowledge perpetually evolves through the time, what might have been a necessary truth yesterday, might be a falsifiable statement of a fact and falsified tommorow.

To sum up previous discussions—in a semantic tradition a notion of necessary truth is explained as that of analytical truth. However, this concept is very problematic.

We are confronted with the following controversy—Bolzano's delimitation of necessary truths as generally valid statements was too broad. It included also generally valid truths.

Meanwhile different logical formalisms enable us to constitute different accounts of logical truths of formal languages which we may or may not identify with certain logical truths of some natural language, according to our semantic interpretation. They provide us only with partial approximation of the notion of logically valid and hence necessarily valid truths.

In this way we may arrive at various tools that we can use to describe a segment of logical truths of natural language. Nonetheless none of these accounts will probably ever describe all logically valid truths, not to mention necessary truths.

The set of analytical truths, that is necessary truths of meaning, will probably never be completely exhausted by logical truths and the complement of those two sets will probably be nonempty.

This thesis is not amenable to any argument or demonstration, as the notion of logical formalisms was not delimited with a definite clarity. It could be possibly understood, with a great licence of course, as a certain anti-thesis to Church's famous thesis.

Church's thesis approximately says we will never be able to come up with a notion of algorithm, that would not be equivalent to one of the already existing accounts.

The anti-thesis I have in mind can be put in the following manner—we will probably never be able to come up with any logical formalism, that is a formal or semi-formal account of logical truths, that would describe all truths of meaning.

From the other side, the boundary between analytical and factual truth is also amenable to change. Therefore the concept of necessary truth and of deductively valid argument remains open to various revisions. Just whether all analytical truths are also necessary and a priori and vice versa, is yet another issue.

Hardly anyone, though, would be willing to deny that logical truths are indeed necessary and consequently any analysis of natural language by means of logical formalisms will reveal only deductively valid arguments of natural language.

We still can analyze plenty of analytically and factually valid arguments by means of logical formalisms using our non-trivial linguistic or factual knowledge, still we can never describe any substantial argument properly. So what is the conclusion so far?

I have presented this significantly superficial overview of a complex matrix of concepts, such as logical form, necessity, analycity, proof etc. just to point out this two important facts:

1) Designing logical formalisms is a task intricately bound with the idea of classical model of science and so are epistemological and ontological concepts in which role of logic in science is explained [**17**].

2) Substantially valid arguments which contain merely presumptive premises cannot be reduced to deductively valid arguments.

That means no logical formalism can be designed which would help us to recognize them.

CHAPTER 3

Argument and Form

3.1. Argument and inference in natural language

So far I have aimed at providing some negative results about the scope of formal logic. It should be evident by now that in our everyday reasoning we mostly employ arguments which cannot be properly evaluated by methods of formal logic, just because they are substantial, therefore not deductive and hence not formal.

I have therefore explained why I think it is contraproductive to do baby logic. Not only baby logic omits the most important parts of formal logic, but justification for such an approach is based on false presumptions about utility of formal logic for understanding of role of argumentation in commonday reasoning.

This claim, however, should not be interpreted in the sense that formal logic cannot help us to understand anything about reasoning natural language. Formal logic, once again, is a science about logical validity and demonstrations.

At this point the above mentioned distinction of formal languages and symbolic languages becomes all important. There is no need to presume that methods of formal logic cannot be applied for studying natural language because it is not 'formal' as some informal logicians may claim.

Any language will be formal as long as the notions of logical truth and substitution for this language will be so defined, as to satisfy Bolzano's insights that each logicallz true sentence has some form, such that each sentence of this form is also logically true.

The problem, of course, is just how to define admissible substitutions and notion of logical truth for natural languages. It is, however, problem merely technical and semantical and not principal. That is why application of methods of formal logic on studying natural languages can bear interesting insights.

Remember that the subject of demonstration was to show that whenever some premises are true, so is a certain claim is necessarily also true, by providing chain of immediately obvious inferences that 'take us' from the premises to the claim.

We cannot achieve this certainty with a chain of substantial arguments. So it is indeed deductions, or inferences, that we are interested in.

Philosophical logicians, who often use formal logic in this way, however, should be aware that what are they studying indeed is *inference* in natural language, rather than argumentation in general. This terminological distinction which I mentioned only briefly on the page 2 of the thesis may save us a lot of confusion.

We may simply reserve a term ‘inference’ for deduction (syllogismos) and term ‘argument’ for each sequence of somehow structured sentences, that seeks to establish some particular goal during a certain specific kind of discussion[1]

At this point someone it might seem that I have spent mos of my thesis just opting for using term ‘inference’ rather than ‘argument’ for basic subject of formal logic. This distinction, however, when it is clearly recognized and articulated, enables us to realize that there are important and interesting problems concerning relation of these concepts.

What uses of arguments are there, in which contexts and situations we are entitled to employ some kinds of arguments and how, and specifically in which contexts and how should we use demonstrations and inferences.

Moreover, even logicians who acknowledge difference between arguments and inference are often inclined to think that the latter are somehow reducible to the former. I have already discussed the classical approach of treating genuine arguments as enthymemes, that is valid arguments with missing premises, in section 1.2.

I have shown, that this is principally impossible, unless we integrate merely plausible or probable sentences into our semantics. This is actually what eventually logical positivists had to do.

Or we may generalize the notion of inference to the degree, that it will lose some of its fundamental properties, such as monotonicity and formality[2].

In such case we have identified inference and argument once again, as all that we require from valid inferences now would be that provided that their premises are acceptable somehow entitle us to make their claim.

Under such definition it will perhaps be possible to treat arguments as material (informal, factual) inferences of certain kind, but we still have to come up with such a notion of truth for the language in question, so that we would be able to refine arguments such as **A4** to either prove as invalid or valid, with true claim.

[1]This is again very broad and vague definition but it is so because there are yet very few comprehensive and exhaustive theories of arguments. The argumentation theory is rather young scientific discipline and did not achieve proper delimitation of its subject yet.

[2]I will discuss the importance of these properties for theory of demonstration on page 92.

To what extent are other functions of language reducible, explainable or derived from this function, as inferentialists believe, is debatable and I do not intend to participate in this discussion.

What occurs to me as suspicous, though is, the very idea that the possibility of such reduction poses an important problem. It seems to me, that it is the feat shared by all reductionism be it physicalistic, psychologistic or phenomenalistic reduction.

They are often motivated by ambitions of the researchers of the respective field to apply their methods to study different field, whose laws, as the reductionist firmly believes, can be explained by laws of the field to which they are to be reduced.

I cannot imagine how it would be possible to argue against principal impossibility of such reduction. It seems quite obvious to me, nonetheless, that while possible in theory, in practice such reduction would be immensely complex and therefore would probably not make the reduced field any more intelligible. At this point we just need to recall Toulmin's critique of epistemological costs of reductionisms.

The reductionistic agenda is often driven by 'ideological' reason, rather than by pragmatic scientific concern for more intelligible explanation of given phenomena.

Instead of arguing against principal impossibility of reducing all good arguments to inferences of some kind, I will offer more intuitive treatment of arguments in sections 3.3–3.5.

Let us now deal with the opposite extreme, that is the opinion that formal logic has nothing to tell us about reasoning in natural language at all.

Before addressing this critique review in the following section, in the rest of this section I will introduce some basic methods of logical analysis of natural language.

The process of arriving at a certain logical form of a sentence via formal languages to which I will refer to as *formalization*, has two phases described in [**59**], called *regimentation* and *abstraction*. When we do perform these acts, we already do it with a certain logical form in mind, preferably one that is described in one of the available logical formalisms.

During regimentation, all features of the sentence which are not relevant from the point of view of its role within the argumentation, for example its stylistic features, are omitted. Let us take the following list of examples:

S1: If Barney loved Lassie, Lassie would not love him.
S2: Lassie would not love Barney, had he loved her.
S3: If it was cloudy, the sun would not shine.

S4: When there are clouds, the sun does not shine.

With a certain idealization we may see, that sentences S1)-S2) can be substituted one for another within an argument, without a change in any of its attributes. The differences between them are merely pragmatical and styllistical, but we abstract from them.

By regimenting these sentenes we can arrive at their translations into the language of sentential logic. We will replace the primitive sentences with (extra-logical) sentential atoms and sentence connectives with truth-functional operators. The resulting regimentation will yield a sentence of the language of sentential logic, called *sentential formula*:

SF1: $s_1 \to \neg s_2$ for S1) and S2)

SF2: $s_3 \to \neg s_4$ for S3) and S4)

Where extralogical terms s_1, s_2, s_3, s_4 stand for 'Barney loves Lassie.', Lassie loves Barney.', 'It is cloudy.', and 'Sun is shining.' respectively.

The second part of formalization of natural language sentences is abstraction. Following Bolzanian approach, the actual content of the extra-logical terms is unimportant. Formulas SF1)-SF2) share the same form. Such form is obtained by abstracting from the meaning of extra-logical terms and is following:

F1: $\phi \to \neg\psi$

Expression **F1** therefore isn't, strictly speaking, itself a sentence of the formal language of sentential logic and is meaningless. It is a schema from which such expression can be generated.

Let us notice that it seems that even within a given single language we can identify a multitude of forms of sentences.

In our example the logical form **LF1** is the most specific form of formulas **SF1**, **SF2**. Each of it would be obtained by substituting a certain sentential atom for sentential variable, specifically A for ϕ and B for ψ to obtain **SF1** and C for ϕ and D for ψ to obtain **SF2**. However, these formulas could also be obtained from the following forms:

F2: $\phi \to \psi$

F3: ϕ

We would have to substitute s_1 for ϕ and $\neg s_2$ for ψ into **F2** to obtain **SF1** and s_3 for ϕ and $\neg s_4$ for ψ into the same form to obtain **SF2**. We would have to subsitute **SF1** and **SF2** themselves for ϕ into **F3** to obtain **SF1** and **SF2** respectively. The form **F3** is of course the most general and it is the form of all formulas. Each formula can be obtained from it when we subtitute it for ϕ.

This multitude arises due to the fact that not only sentential atoms can be substituted for such sentential variables, but sentences in general, including complex sentences. Therefore **F1** could be instantiated with **SF1** for ϕ and **SF2** for ψ. The resulting sentence would be:

SF3: $(s_1 \to \neg s_2) \to \neg(s_3 \to \neg s_4)$

This sentence also has a form **F1**, but also a more specific form:

F4: $(\phi \to \neg\psi) \to \neg(\xi \to \neg\theta)$

Properties of sentences of natural language are studied by mapping these into sentences of such formal language and by studying the corresponding properties of the resulting translastions. Formal languages therefore fulfill a role of partial lingua characterica. By choosing a certain formal language to translate our sentence into, we already chose which names of the original sentence will be mapped onto names of the artificial language that we have decided to designate as logical names.

As a result we may arrive at a logical form of this translation by mechanical replacement of all other names by variables of appropriate categories. This will correspond to logical form of the natural language sentence, relative to the given formal language, of course[3].

3.2. Problems of formalization

This analysis of natural language arguments, however, is often not so straightforward. It is neither obvious which formal language to choose for such analysis and how to translate the arguments of natural language into arguments of this formal language[4].

A formal logician may merely give some general principles that should guide our selection of appropriate logical formalisms, such as principles of *transparency* and *efficiency* ([**59**]).

The latter advices us to choose such formalization, that will 'do justice' to the original argument, as it will give adequate explanation of meaning of terms on which the original argument rests.

The former principle advices us to choose such formal language, that the translation will be quite straightforward and the resemblance of form of the original argument and the formal argument will be transparent.

[3]See section 3 of [**73**] for detailed description of the technique of arriving at sentential forms using the formalism of sentential logic.

[4]A brief account of most common approaches to logical analysis is summarized in [**60**].

Common philosophical error of identyfing meaning of natural language terms with interpretation of their standard translations into formal languages is exposed in [**46**].

Evaluating natural language arguments by means of logical formalisms, however, is even more complicated, as it presupposes that we have already been able to identify the natural language argument in the first place and reconstruct it properly. It is therefore common to distinguish stage of *identification* of arguments and stage of *reconstruction* of arguments.

These two are preliminary to the already mentioned *evaluation* of arguments which is often carried out by their formalization and subsequent checking for logical validity by some of the many methods available for logical formalisms (see [**19**]).

The process of argument identification consists of the identification of sentences which constitute the premises and the claim of an argument. This procedure therefore already requires some non-trivial linguistic insight as the premises and claim of a single argument may often be disconnected in the text and may contain various anaphors, metaphors, etc.

During argument reconstruction, we often have to go even beyond the original text and add premises which are not explicitly mentioned in the text, but which the author evidently assumes as true, commonly believed or self-evident.

A detailed understanding of the intended purpose and audience of the text, its historical and social context, personality and philosophy of the author, and literary genre is often essential for such reconstruction. It is not only crucial to find the missing premises, but also to identify their epistemological status. Are they considered to be true or merely plausible?

Again, there is very little a logician can tell about these two processes. He may give us some general principles guiding our identification and reconstruction of arguments, such as the principle of charity [**19, 21**] and list of the most common errors and impedements to good reasoning [**19, 59**].

Once we succesfuly and correctly perform all these tasks, we can finally employ logical methods to see what follows from the text.

Obviously, we need more than to understand formal logic in order to understand demonstrations in natural language, we need to understand natural language as well. That should, however, not be a great impedement for competent speaker of the language and excuse why it makes no sense to perform logical analysis.

After all, competent science in every respectable field must posses at least elementary understanding of language. That is why critical thinking skills are not associated with logic anymore than with any other rational inquiry.

While knowledge of formal logic is evidently not sufficient for understanding demonstration of natural language, informal logicians often claim that it is also not necessary. According to some using language of logical formalism for analysis of logical language is not only useless detour, but also one where most of the dangerous pitfalls occur.

Any formalization of natural language arguments must fulfill what Woods ([**73**], p.69) calls *backwards reflection property* of logical properties.

That is—whenever a formalization of a certain sentence, set of sentences, or an argument does have a certain logical property (such as *logical truth*, *consistency*, *validity* or *invalidity*, just what is a logical property needs not to be circuscribed prior to this discussion) according to logical formalism, so should the original sentence, set of sentences, or argument in natural language. This is, after all, goal of any logical formalization.

To achieve this, our formalization must follow a certain principles, called *disambiguation rule* (ibid., p.70) and *logical inertia rule* (ibid., p. 72). To illustrate the importance of these two rules consider following three examples given by Woods:

A9:

Premise 1: If Sarah has been awarded the first University degree, then Sarah is a bachelor.

Premise 2: If Sarah is a bachelor, then Sarah is an unmarried man.

THEREFORE

Conclusion: If Sarah has been awarded the first University degree, then Sarah is an unmarried man.

A10:

Premise: This shirt is red.

THEREFORE

Conclusion: This shirt is colored.

A11:

Premise: This figure is a triangle.

THEREFORE

Conclusion: This figure is a circle.

Argument **A9** has a following logical form in sentential logic.

AF1:

Premise 1: $s_1 \rightarrow s_2$

Premise 2: $s_2 \rightarrow s_3$

THEREFORE

Conclusion: $s_1 \rightarrow s_3$

This is a valid argument due to sentential logic. The original argument **AF1**, however, obviously is not valid. Our formalization therefore does not have the backward reflection property of validity.

This is because we have violated the disambiguation rule, by equivocating the ambiguous English term ‘bachelor’ in premises 1 and 2 by replacing it with a single sentential atom s_1. Would we formalize the original argument properly, we would have obtained:

AF2:
Premise 1: $s_1 \to s_2$
Premise 2: $s_4 \to s_3$
THEREFORE
Conclusion: $s_1 \to s_3$

Arguments **A10** and **A11** share the same sentential form:

AF3:
Premise 1: s_1
THEREFORE
Conclusion: s_2

This rather trivial argument form is invalid. But the original argument **A10** was valid. Also its premise and conclusion form a consistent set. But premise and conclusion of **A11** are inconsistent. What has gone wrong this time?

We have transgressed the logical inertia rule which commands us that no two sentences of natural language, we are formalizing as sentential atoms, must imply one another, or be inconsistent one with another[5].

The necessity of following these two rules in any formalization results in what Woods calls the *bootstrapping problem*.

In order to follow the logical inertia rule, we must already recognize logical implications and inconsistencies between simple sentences of natural language in a systematic way. However, this is what we hoped to achieve by methods of formal logic in the first place.

Similarly, to enforce the disambiguation rule, we must already be aware of ambiguities and synonymy of terms in natural language. But any principled theory of synonymies, as Quine ([**51**] section 5, pp. 60–64, also [**50**], section 3. pp.27–32) points out, is equivalent to the theory of analycity.

[5]It is therefore unclear, what are elementary sentences of Wittgenstein’s picture theory from his Tractatus, as we can hardly find such basis of logically inert sentences, with no logical relations among them, in any natural languages from which all other complex sentences could be combined.

We have lost the backward reflection property for logical properties of invalidity and inconsistency in our formalization because the language of sentential logic is too crude to reflect the logical form of the original structure and hence theirs logical relations.

Yet to provide such formal language, we would need it to reflect on meanings of terms, such as 'red', 'colored', 'triangle', and 'circle' and preserve all analytic validities. This is unrealistic expectation, as the section 2.6 suggested.

All these difficulties have been understood as qualifications against employing formal logic in any kind of argument evaluation. Once we can properly identify, reconstruct and formalize sn argument, the proponent of informal logic claims, all that remains is to perform the routine and trivial task of applying some of the appropriate logical methods for determining the validity of the formal counterpart of the original argument.

However, all the substantial and important work has already been done before the machinery of formal logic could even be employed. Wouldn't it hence be easier to get rid of this unnecessary fetour and to concentrate on what is substantial for evaluating the original argument, rather than thinking hard how to transform it into something that a formal logic can handle? Shouldn't we concentrate on the evaluation of *arguments on the hoof*, as Woods calls them ([**73**], p. 66)?

Some logicians, most prominently Quine (see [**52**]), would simply accept those objections, confirming logic is not the theory of natural language arguments and theory of natural language arguments is not logic. Others usually address these objections in two different ways.

First, it is not quite true that the step of evaluating validity of the resulting formal argument is always so trivial and contribution of formal logic so insubstantial. In some, admittedly rare, cases the logical structure of the original argument can be intricately complex and without elaborate methods of formal logic we have no chance to grasp it adequately. In other words, a proof is required to make the validity of such argument transparent.

Nonetheless, such complex arguments are often really hard to come by outside mathematical or philosophical works which often presuppose the readers will to employ logic from the very beginning and prove many conclusions for himself.

Formal logic is indeed important for the understanding of proofs, however, in practice validity of very few arguments rests on proof of some kind, so the objection against employing formal logic holds in most cases of argument evaluation.

Second, formal logic is not merely a practical tool for argument evaluation, but aims to provide fundaments for a general theory of argument. For these reasons not only particular arguments are studied in formal logic, but whole classes of arguments which share the same logical form. Only because of this general approach may formal logic seriously be considered to be a scientific theory of its own kind.

Informal logic has not yet been able to provide any comparably general and efficient approach to arguments and so formal logic is simply the best tool we have. In most cases it may be our best bet simply to employ our common sense and linguistic insight, but by pleading for it, we have not yet presented a scientific alternative.

According to the authors of [**59**] a (formal) logician, is like a cartographer, who draws a simplified version of the landscape. In order to be able to do so, he already has to have prior detailed knowledge of the landscape (that is natural language). Obviously maps cannot be the most important, or even sole and unique, source of our knowledge about landsacape.

Neither can any map cover all notable aspects of the landscape and include all details. It is actually the goal of maps to simplify the landscape after all and this goal is achieved by highlighting certain of its aspects for a given particular practical purpose (touristic maps, highway maps etc.).

A visible landmark can be left out from the map on purpose, as well as 'invisible' and 'imaginary' aspects of the landscape can appear on the map, for example levels of polution.

This is, admittedly, also true, nonetheless the above mentioned pracitcal limitations of applying formal logic for argument evaluation aside, it still is the case that only a menial fragment of all valid arguments can be explained in logical terminology.

It is therefore perfectly justifiable to use particular logical formalism for the analysis of a certain specific fragment of natural language, as long as we are aware that we are dealing with this particular fragment.

Moreover, and this is scarcely recognized by advocates of the usefulness of formal logic for argument evaluation, by using any kind of logical formalism, in the sense explained in section 2.4, we limit our attention to proofs or demonstrations of one kind or another, rather than on the rational argumentation in general.

So is formal logic useful for argument evaluation or not?

Of the three critiques of formal logic mentioned in [**28**], that is pedagogical, empirical (psychological) and internal, I believe only the internal one should be considered and even so only some parts of it[6].

It is not the very idea of using formal languages for argument analysis that is flawed. After all, any truly scientific treatment of argumentation will have to rely on certain generalizations and therefore idealizations and abstractions.

Nonetheless, the somehow fuzzy limitations imposed on all members of family of all existing and prospective logical formalisms which have their justification in mathematical origins of modern formal logic and which I have explored briefly in section 1.6, result in a somehow narrowed scope of arguments, to which methods of formal logic can be applied. That is demonstrative arguments.

The extension of methods of formal logic for the description of plausible arguments is possible nonetheless, as I will attempt to demonstrate in the sections 3.3–3.5, although some cherished properties and concepts of formal logic, essential for its application in mathematics, will necessarily have to be put aside in such process.

This application of formal logic for general argument evaluation will follow Aristotle's original ideas and intuitions surprisingly, proving that we still have much to learn from this ancient giant.

3.3. Informality of defeasible arguments

Aristotle's classification of deductions depended purely on the nature of their premises. Either they were necessarily true and known to be true, or they were merely plausible.

The shift in paradigm of science put Aristotle's logic into a slightly different use. Deductions, whose premises were merely true (either necessarily or merely contingently), were of sufficient importance for the positivistic model of science. Sound arguments are therefore required to have only true premises[7].

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[6]Pedagogical critique has been already mentioned somewhat in the preface.

Psychological critique, dwarfing formal logic for, actual or fictional, misrepresentation of factual ways in which people actually think or argue, is relevant only on the unlikely premise, that formal logic aims to describe human thought.

Why this psychologistic doctrine of logic is somehow misguided is briefly yet sufficiently explained for example in the preface to [**45**].

[7]Also authors, who aim to provide accounts of dialectical uses of logic often simply identify rational arguments with sound ones.

This strict identification, often combined with naive realism and some version of correspondence truth theory or another, may lead them to worry that some soundness of arguments can never be achieved and recognized, as there are truths

The notion of plausibility did not draw that much attention of philosophers, as it seems[8].

It is therefore quite peculiar that this notion regained its importance quite recently within the field of Artificial Intelligence in connection with practical attempts to simulate more numerous cases of commonsense reasoning using computer models[9].

This development initiated from within a community of theoretical computer science researchers, who aimed to defend the claim that logic is actually a useful tool for description of common-sense reasoning which often included reasoning with vague, uncertain, contradictory, and incomplete information.

Such kinds of reasoning were usually described using probabilistic and stochastic models. Even nowadays Bayesain networks, to mention just one example, are used extensively in computer science and constitute a potent alternative to logic-based approaches to uncertain reasoning.

The oldest approach to knowledge representation in computer science was based exclusively on logic. Sets of sentences or schemes of first-order logic were used to represent knowledge and some classical inference mechanism was used to deduce their consequences and answer potential queries into such database.

The need to represent all the 'imperfect' knowledge and defeasible reasoning was demanded by practical applications. Logical programming might be considered to be the most important of them.

Indeed, it was necessary to provide a logical model of, what Toulmin would calls substantial, but what is now more commonly called *defeasible reasoning*[10].

Such theories of defeasible reasoning were therefore originally built mostly as extensions or modifications of classical first-order logic. I will

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which we do not know or even cannot know. This may lead some of them to embrace fallibilism[**19**] in the end.

For Aristotle this problem did not originate at all because both episteme and endoxa are, by definition, required to be knowable and in fact known to all reputable men among which a case of argumentation may arise.

[8] Walton ([**66**], pp.16–28) mentions bright exception of John Locke, to whom we owe fundaments of modern theory of fallacies.

[9] See for example the extensive *Handbook of Logic in Artificial Intelligence*, particularly vol. 3 dedicated to non-monotonic reasoning.

[10] See [**49**] for general theory of defeasible reasoning. It would be quite an interesting research topic to investigate philosophical motivations, explicit, implicit or alleged, of some of the pioneering articles, such as [**54**] or [**36**], of course, but that would require yet another dissertation.

follow this line by introducing Reiter's default logic which is a system extending classical first-order logic.

I will follow with introduction of Poole's theory of explanation which does not extend classical logic, but 'merely puts this logic to another use'. This account will be ended by a completely abstract account of argumentation—mainly due to the landmark paper of P. M. Dung [**18**][11].

The contemporary boom of argumentation-theoretic approaches to classical areas of interest in AI, such as multi-agent systems, planning, problem solving, knowledge representation particularly in respect to incomplete, inconsistent, vague or probable information, has partly liberated itself from its logical origins.

Still a fruitfull interaction between argumentation theory and formal logic continues in this area of artificial intelligence. For example in Besnard's and Hunter's model of deductive argumentation [**8**].

But let us start at the beginning.

The beginnings of logical theory of defeasible argumentation can be traced to multiple authors and systems, but perhaps the most transparent, comprehensive and cited take on 'logic' of defeasible argumentation was given in Reiter's milestone article [**54**] in which he introduced his notion of *default*.

In general a default is a sort of conditional argument of the form $\frac{p_1 \dots p_m : q_1 \dots q_n}{r}$, where $_1 \dots p_m$ are *prerequisites*, $q_1 \dots q_n$ *justification* and r *consequent*. This rules entitles us to conclude the consequent anytime we are entitled to say we know that all the prerequisites are true and we have no information to the contrary of any of the justification.

A default is therefore intended to represent certain defeasible argument or inference because the consequent might be withdrawn 'later', when new information to the contrary of some of the justifications becomes available to us.

The key concept of this definition which makes it special is that of 'lack of information to the contrary' and what does it mean that it is not available at a certain point, but becomes known 'later'. These are crucial notions indeed which I will explain soon.

Originally, Reiter's theory of defaults was designed as an extension of classical first-order logic. In order to maintain simplicity, I will introduce its modification for classical sentential logic, introduced in section 2.3.

[11]A very good and comperhensive overview of argumentation models in Artificial intelligence is available in [**7**].

While motivations of Reiter's original definitions were concerns for application of his theory in computer science, in order to present its main ideas and merits I do not need to bother with many technical details. Let us from now on assume that we are working with a sentential language.

An example of a default would be:

$\frac{\text{FLIES(Oscar):FLIES(Oscar)}}{\text{FLIES(Oscar)}}$

The conditions of consistency can be further specified by adding material inference, such as

$\frac{\text{OSTRICH(Oscar)}}{\neg\text{FLY(Oscar)}}, \frac{\text{PENGUIN(Oscar)}}{\neg\text{FLY(Oscar)}}.$

Such defaults with no justifications are called *residues*.

For now let's compare this notion of default rule to Aristotle's notion dialectical argument and Toulmin's model of argument.

A default is slightly closer to Toulmin's model of argument rather than 'classical argument' consisting only of premises and conclusion. In his terminology prerequisites correspond to data, consequent to conclusion, while contraries of justifications would represent rebuttals of the argument.

The qualification, warrant, and iits backing of argument are not thematized in Reiter's system, but they can vary depending on the theory we are trying to build.

In our example a warrant for the default in question would be a statement that birds usually do not fly, why its backing would be either some generalization of observations, or ornitological knowledge.

In the Aristotelian model, the prerequisites of the default are required to be episteme, while the qualifications can be understood as additional premises which are merely endoxa.

Nonetheless, contrary to Aristotle's theory, the default itself does not necessarily have to be a deduction, its conclusion need not always logically follow from the prerequisites and justifications of the default taken together. Actually, defaults were intended to be additions to a deductive system, in particular that of logical formalism of predicate logic, as mentioned above.

The set of some default rules D (including residues), together with set of sentences F, representing facts, give together a *default theory* T.

The concept of 'lack of information to the contrary' is interpreted as the consistence (in the sense given by the underlying logic, in our case CSL) with a certain theory E. However, this theory E is not simply the original theory F, but rather includes everything that can be inferred from it by means of rules of the underlying logic, most often classical predicate logic, combined with the default rules D.

This requirement is what results in additional complexity of default reasoning. In order to decide, whether a given default in question can be used, we already have to know what would the final theory we are currently constructing look like.

Therefore technically a concept of applicable default must be explained by means of quasi-inductive or fixed point definitions. Here are examples of such definition.

We say a default $\frac{p_1 \ldots p_m : q_1 \ldots q_n}{r}$ is *applicable to a set of sentences* E if and only if $p_1 \in$E$\ldots p_m \in E$ and $q_1 \ldots q_n$ are consistent (in a sense of CSL) with E.

A set E is an *extension* of a default theory $T = \langle\ D\ ,\ F\ \rangle$ if and only if it is the least such set, that

(1) $F \subset E$
(2) E is deductively closed (contains all CSL consequences of itself).
(3) Resolutions of all defaults applicable to E are already members of E.

Alternatively if Λ_T is a function which assigns to a set of sentences S the least set of sentences which contains F, is deductively closed, and contains resolutions of all defaults from D applicable to S, then S will be an extension if and only if $\Lambda_T(S) = S$.

An extension of a given theory is therefore supposed to represent the complete yet minimal 'worldview' that can be based on the given default theory[12]

[12]It might be tempting to interpret facts as residues with empty prerequisites and for sake of completeness also introduce defaults with empty prerequisites, representing hypotheses. However, the semantics require that each of the extensions contains all the facts which are known prior to any application of defaults or even residues.

Extensions can also be constructed in an algorithmic way (for details see [**1**] Theorem 4.4. and [**35**] Theorem 4.12.), where a different order of application of defaults yields different extensions on the output.

Therefore it is not actually correct to represent facts by residues as I did in the previous example. By applying first the default rule on Tweety, the penguin, we would conclude that he flies and with application of the residue we would obtain that he does not fly and so we would end up with a contradictory theory.

Similarly the built-in requirement that each extension has to be deductively closed is ensured in the construction by making a deductive closure of set obtained at each stage, before applying any other defaults to it.

This priority of logical inference prohibits us to represent logical laws in the form of residues which can be applied in a random order. Such abstraction from logical mechanisms will become more transparent and straightforward with the simpler system of [**47**], that I will discuss in section 3.4.

Numerous variants, extensions, and modifications of Reiter's system, as well as some algorithms for constructing extensions and their complexity must not interest us right now[13].

It is the role of logic in this theory of argument, I would like to examine.

Note that default theory arises when a certain logical formalism is extended by a set of default rules which are intended to capture arguments, that do not establish their conclusions with necessity.

So at the foundation of default theories lies a certain system of deductively valid inferences, but the most intriguing and confusing fact is that we can define another system of valid inferences (although provably much more complex) on basis of such a theory.

How could this be achieved? Consider for example a following infamous example of a default theory, known as the 'Nixon diamond':

$F = \{\text{REPUBLICAN(Nixon)}, \text{QUAKER(Nixon)}\}$,

$D = \{\frac{\text{REPUBLICAN(Nixon):DOVE(Nixon)}}{\text{HAWK(Nixon)}}, \frac{\text{QUAKER(Nixon):HAWK(Nixon)}}{\text{DOVE(Nixon)}}\}$.

We know for certain that president Nixon is both a republican by political affiliation and quaker by religion. We also know by default that typical quakers are doves, while typical republicans are hawks, but there might be exceptions of course.

Now we may construct two extensions of this theory. One which includes all CSL consequences of

$\{\text{REPUBLICAN(Nixon)}, \text{QUAKER(Nixon)}, \text{DOVE(Nixon)}\}$

and one that includes all CSL consequences of

$\{\text{REPUBLICAN(Nixon)}, \text{QUAKER(Nixon)}, \text{HAWK(Nixon)}\}$.

The given default theory therefore does not allow us to deduce with certainty neither that Nixon is hawk, neither that he is a dove, as we do not know whether he is rather a prototypical republican and exceptional quaker or prototypical quaker and exceptional republican.

But there is another point of view. It is consistent to assume both that Nixon is a dove as well as that Nixon is a republican, as well as we do not assume both these at the same time. We may therefore define a credulous consequence and sceptical consequence of a default theory T in a following way:

A sentence s is a *credulous consequence* of T if and only if s is a member of some extension of T. It is a *sceptical consequence* of T if and only if s is a member of any extension of T.

We can see now that neither HAWK(Nixon), nor DOVE(Nixon) are sceptical consequences of T.

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[13] Overview of Reiter's theory and its later development can be studied from [**1**] chap. and [**62**] chao.

On the other hand, both HAWK(Nixon) and DOVE(Nixon) are credulous consequences of T.

If we now fix a certain set of default rules of a given language, we immediately obtain two relations of 'consequence' for $\vdash_D^s$ and $\vdash_D^c$.

Has Reiter therefore described an infinite number of ways how to extend some logic by adding default rules and transforming it into new logic? We have to be careful with such conclusions.

If we simply take a logical formalism to be an arbitrary language (here identical with the language of the original underlying logic) with any relation of consequence, then yes. However, I had pleaded in section 2.6 that we should resist such an approach.

The relation defined in the previous way may, and often also will lack certain important properties, that characterize intuitive notion of demonstrability.

The most often mentioned feature of such relations is their *non-monotonicity.* A consequence relation $\vdash$ is said to be *monotonic* if and only if for each sentence s and sets of sentences Γ and Δ, such that $\Gamma \subseteq \Delta$ and it holds: whenever $\Gamma \vdash s$ then it is also true that $\Delta \vdash s$. Otherwise it is *non-monotonic.* Intuitively what we can demonstrate from a given set of premises, we should also be able to demonstrate from an even larger set of premises.

Non-monotonic relations are therefore suspicoius candidates for relation of inference and demonstrability.

Now take for example the following simplification of an infamous theory given at the introduction of this chapter:

$F = \{\text{BIRD(Oscar)}\}$

$F' = \{\text{BIRD(Oscar)}, \text{OSTRICH(Oscar)}\}$

$D = \{\frac{\text{BIRD(Oscar)}:\neg\text{OSTRICH(Oscar)}}{\text{FLIES(Oscar)}}$

Clearly both $F \vdash_D^s$ FLIES(Oscar) and $F \vdash_D^c$ FLIES(Oscar), but neither $F' \vdash_D^s$ FLIES(Oscar), nor $F' \vdash_D^c$ FLIES(Oscar).

Neither of those two relations is monotonic.

There exists a myriad of so called non-monotonic logics describing decision procedures for such non-monotonic relations in one way or another.

Makinson [**35**] classifies most of them as modifications of standard monotonic logics by means of three related, yet slightly different approaches. Either by adding pivotal and default assumptions (Poole's theorist [**47**] discussed in section 3.4 is of this kind), by limiting set of possible valuations (preferential models and circumscription), or by adding some rules (default logics or non-monotonic modal logics and autoepistemic logics).

All these approaches, however, share the very same idea, that is allowing defeasible arguments which may later be defeated and rebutted.

However, more importantly, such relations will cease to be formal, even when we obtain them by extending some logical formalism.

Actually, defaults, when added to a certain logical formalism, always allow us to infer at least as much as we could infer in the original formalism. By adding any defaults to CSL we will always obtain consequence relations which are *supraclassical*.

Makinson proves (see [**35**] Theorem 1.1) that there can be no supraclassical relation for the sentential language, that would be closed under substitution, except for the classical relation itself and the total relation[14].

He pleads that reader of his textbook first suspended his habit of looking at valid arguments as instances of certain valid argument forms, least he will not be able to make any sense of the following text.

This comes as no surprise to me. While leaving territory of necessary and hence formal truths, we have also abandoned grounds of demonstrations and necessary consequences.

We must therefore resist the temptation to look at non-monotonic logics simply as on other members of family of non-classical logical formalisms.

They are not formalisms in general, in the sense of page 60, although their language is often symbolic and their key concepts are defined in mathematically rigorous ways.

They also do not have anything naturally to do with logic, unless we simply deliberately decide to build in some already established logical formalism into their foundations, or transform them into yet another 'logics' by introducing notions of sceptical and credulous consequences and define them using notion of extension.

That neither of those decisions is in fact crucial for non-monotonic logics to have fruitful applications, but that relating theory of defeasible arguments to that of classical logics is rather a result of certain respect for tradition and justifying relevance of logic for understanding argumentation and commonsense reasoning, is what the section 3.4 will demonstrate.

That does not mean, however, that such accounts of non-monotonic inference relation cannot be principally studied by logical methods.

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[14]Adding defaults to some weaker logical systems, for example intuitionistic or substructural logic would be an interesting area of research, but extremely complex with no intuitive interpretation or philosophical justification and of questionable relevance.

There exist accounts of such non-monotonic relations in terms borrowed from proof or model theory[15], however, it is very difficult to interpret such 'proofs' as demonstrations, and such models and interpretations in clasical terms, such as 'true' or 'denotation'.

So instead of rebutting such approach, I will present an alternative approach to defeasible reasoning which is hopefuly far more intuitive[16].

3.4. The uses of arguments in science

Another, even more transparent, approach to defeasible reasoning was provided in Poole's article [**47**].

Unlike Reiter, Poole believes we do not need to enhance classical first-order logic, or provide new semantics for it. Instead of changing the logic, we need to change the way we use it.

To paraphrase words from the abstract of this article: Rather than expecting reasoning to be just deduction (in any logic) from our own knowledge, logic should be used rather as a tool for theory formation.

In his article, he shows how we can use classical first-order logic to make an *explanation* of certain observations and on the basis of known facts and possible hypotheses.

He has also shown relations of his system to Reiter's theory of defaults[17]. In later article [**48**] Poole appplied his theory for other practical examples of theory formation, that is *prediction*.

The version I will present here is based on account of Poole's system given by [**1**].

A *theory* T is an ordered pair of sets of sentences: *facts* F and *hypotheses* (also called defaults and abbreviated D). A *scenario* of T is a consistent subset of $F \cup H$. An *extension* of theory the T is a set of consequences of some maximal (w.r.t. set inclusion) scenario of T. A sentence p is *explainable* from T if and only if there exists a scenario of T such that p is deducible from such scenario.

A set of observations O is explainable from T if and only if there exists an explanation of each its member[18].

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[15]For a guide into some proof theory for default logics [**9**]. An overview of model theory for default logics is described in section 4.1.3. of [**62**].

[16]Some arguments for such approach are also to be found in [**49**].

[17]See [**47**] Theorem 4. This proof is also elaborted in and set into larger context within [**1**] (Theorem 17.1.).

[18]Poole also introduces a variant of his framework adding a set C of *constraints*. Constraints are additional limitations of scenarios, in the sense that any scenario must be consistent with those constraints, but are themselves not included in the scenario and can therefore not be used in deductions, used for explanation of observations, or forming of extensions.

A straightforward theorem proved in ([**47**] Theorem 3) establishes that p is explainable from T if and only if p is in some extension of T.

To accomodate this model for explanation, Poole enlarges it with a set of *conjectures* K.

Given a theory $T = \langle\ F,\ H,\ K\ \rangle$ and set of observations O a sentence p will be *predictable* from T relative to an explanation E of O from T (a subset of $F \cup H \cup K$, such that $F \cup H \cup K$ is consistent), if and only if p is included in all extensions of theory $T' = \langle\ E,\ H\ \rangle$[19].

Poole needs to differentiate between hypotheses and conjectures because his underlying logic is that of classical first-order logic which is used to reason about properties of individuals.

Hypotheses therefore model assumptions we can make for typical cases, such as people normally do not have tumors, while conjectures are used for explanation only and therefore model atypical cases, such as that a person does have a brain tumor which we may wish to assume only in cases we wish to explain his unusual symptoms.

Explanations and predictions are special cases of abductive reasoning. That is reasoning in which we seek set of general hypotheses that would allow us, together with already known facts, demonstrate all the facts we have observed[20].

Observe that in all applications we did use logical methods just for two purposes—first to establish that a certain set of hypotheses is *inconsistent* with known facts and second, to draw all *conclusions* of a certain scenario to form an extension.

We may already start to feel that the application of logic in this model corresponds to Aristotle's idea of dialectical reasoning as deductive reasoning from plausible statements towards contradiction.

[19]It is important to remember that this notion of predictability is relativized w.r.t. to some explanation.

A notion of absolute predictability could be probably proposed, as predictability w.r.t. all posible explanations, however, it is not my goal to suggest such modifications of the already established theory.

[20]There are numerous variants of this framework used to model both abductive and inductive reasoning (see [**62**]). They are mainly devised for practical purposes in computer science, but can be interpreted as idealized models of ideal scientific methodology. According to CMS, however, there was no need for any other methods, but demonstrations. Scepticism of founders of modern science towards formal logic might have been particularly motivated by (justified) opposition towards such doctrine of scientific methodology. The utility of formal logic for scientific disputation and demonstration is, however, undoubtful, as already Leibniz observed. That does not mean formal logic is or should be also the method of science itself.

This observation allows us to abstract from the logic entirely and simply take the notions of demonstration and inconsistence as primitive. This is the idea behind *assumption based frameworks* (ABFs) introduced in [**11**].

Assumption based frameworks are built upon *deductive systems* which are nothing but a pair of (countable and formal) language L and a set R of inference rules of a form $\frac{p_1 \dots p_n}{c}$.

In particular axioms of any (recursive) logical formalism can be represented as inference rules for $n = 0$ and rules of inference for such logical formalism may be represented in a such way, even though we will typically have to represent countable many instances of the inference schemes and axiom schemes.

Any subset of L is called a *theory.* A *deduction*[21] from a theory T is a sequence $r_1 \dots r_m$, such that for all $i = 1, \dots m$:

- $r_i \in T$
- there exists $\frac{p_1 \dots p_n}{r_i} \in R$, such that $p_1 \dots p_n \in \{r_1 \dots r_{i-1}\}$

A sentence r of L is *demonstrable* from sentences $p_1 \dots p_n$ of L ($p_1 \dots p_n \vdash r$) if and only if there exists a demonstration of r from $p_1 \dots p_n$.

This relation of demonstrability is a decent one and has all properties it should have—it is monotonic, compact, reflexive and transitive[22].

The languge L might correspond to a language of some logical formalism and the relation of demonstrability might as well be described by the underlying logical formalism, but it necessarily does not have to be so—we may select the language L and set of rules R arbitrarily, should we desire to do so.

Neither do we require that the language deductive system itself was capable of expressing negation or even inconsistency. Instead, it is yet another primitive component of the ABFs.

Given a deductive system $\langle\ L,\ R\ \rangle$ an ABF w.r.t. this deductive system is a tuple $\langle\ T,\ Ab,\ \overline{x}\ \rangle$, where

- $T, Ab \subseteq L$ and $Ab \neq \emptyset$
- $\overline{x}$ is a mapping from Ab to L, where $\overline{p}$ denotes the *contrary* of p.

Note again, that no specific properties of the contrary function are presupposed in this version of the framework.

[21] I am more inclined to use the term 'deduction' for one step derivations only and call larger sequences properly 'demonstrations', but let's keep to the original terminology.

[22] See [**35**] for explanation of those concepts.

By generalizing Poole's account, authors of [**11**] arrive at so-called *naive semantics*. They replace Poole's requirement of maximal consistency of extensions by a requirement of being maximally conflict-free.

This notion is defined in a following way—given a deductive system and associated assumption based framework $\langle\ T,\ Ab,\ \overline{x}\ \langle$, we say a set $H \subseteq Ab$ attacks an assumption $h \in H$ if and only if $T \cup H \vdash \overline{h}$. Further we say that H attacks another set of assumptions $H' \subseteq Ab$ if and only if H attacks some assumption from H'. Finally we say H is *conflict-free* if and only if for no $h \in Ab$, $T \cup H \vdash h$ while also $T \cup H \vdash \overline{h}$.

Clearly each set of assumptions that is conflict-free does not attack itself. The converse, however, does not hold in general. It is a special property of *closed* sets of assumptions, that is sets H such that $H = \{h \in Ab | T \cup H \vdash h\}$.

A maximal conflict-free set is called a *naive extension*. Due to their maximality naive extensions are closed and therefore do not attack themselves. Naive extensions always exist, but do not have to be unique.

Now we can associate each theory $\langle\ F,\ H\ \rangle$ of Poole, with a corresponding ABF $\langle\ T,\ Ab,\ \overline{x}\ \rangle$, where $T = F$ and $Ab = H$ and function of contrariness will associate each sentence with its negation.

The underlying deductive system of the ABF will correspond to an underlying logic of Poole's theory[23].

Now by Theorem 3.12. of [**11**], an extension of Poole's system is exactly the same as a maximally conflict-free set of the corresponding ABF.

This does not seem to be a very deep result. However, that is because Poole's theory greatly motivated basic concepts of ABFs. The technique of ABFs can be used to provide semantics to many more systems of non-monotonic inference, including Reiter's theory of defaults (Theorem 3.16.), logic programming (Theorem 3.13.), autoepistemic logics (Theorem 3.18.) and non-monotonic modal logics (Theorem 3.19.).

That is where this tool really starts to be useful and efficient, as it provides natural Poole-style interpretations of otherwise complex accounts of extensions, by associating the original theories with corresponding ABFs.

[23]The distinction between facts and constraints does not have to concern us for now because we are interested only in construction of extensions, where constrains play the very same role as facts.

It is only when we seek to construct all consequences of a given extension that we must distinguish between facts and constrains.

These more complex theories also require more sophisticated semantics which, however, remain very intuitive and which I will present in the section 3.5.

Poole's framework is so simple, that its associated ABF has a certain specific property called *normality*. In normal ABFs, each naive extension is also *stable extension* and therefore also a *preferred extension* (Theorems 4.8.).

Characterizations of those extensions do not matter that much, what is important that there exist natural dialectical proof-procedures for preferred and stable semantics [**13**].

The above mentioned results allows us to apply Prakken's and Vreeswijk's argumentation games for preferred semantics to decide, whether a certain set of sentences is a subset of all extensions, or of some extension (or none), and consequently whether an argument with premises from this set can reasonably be used to predict or explain its conclusion.

As I will carry this out in the section 3.5, I will connect these models of scientific reasoning which incorporate logical methods, but are not reducible to them, with the notion of dialectical argument.

3.5. From arguments to dialog

Before presenting the rules for dialogical games, one more technicallity has to be clarified for someone, who would like to study technical details of the dialogical method and its adequacy from cited literature.

These semantics were designed for a more general abstract argumentation framework[**18**].

In this framework the internal structure of arguments is abstracted away. The notions of deducibility and incosistency which were used to define the relation of attack between two sets, are no more primary notions. It is the notion of attacks between arguments.

Dung's abstract argumentation framework could not be more abstract indeed, it consists of an arbitrary set of arguments and an arbitrary binary relation of attack on this set of arguments. Extensions are defined as sets of arguments having certain properties, analogous to those of extensions of ABFs.

It would be hasty to simply identify arguments of ABFs with arguments in the abstract argumentation framework (AAF) because in ABF a relation of attack is not defined for two single arguments, but rather relates a set of argument to a single argument.

However, this relation is used to define another relation of defeating between two sets. Therefore sets of arguments of ABF is what we need to identify with arguments of AAFs. Semantic properties will therefore be defined for sets of sets of arguments of ABFs within AAFs.

Nonetheless, it can be verified that whenever a set A of arguments does have some of the properties studied in ABF (for example it is a stable extension) its powerset 2^A will have the corresponding property defined for AAFs.

In the other direction, whenever a set of sets of arguments B is, say, a preferred extension in sense of AAF its union $\bigcup B$, will be the preferred extension of ABF.

Dialectical games have to be interpreted in this light. There are two such games, *sceptical* and *credulous*. The first one determines, whether a given argument is in some preferred extensions of AAF, the other one determines whether it is in all preferred extensions AAF.

Consequently we can apply it to a set of assumptions and determine whether a given set is a subset of some preferred extension of ABF (by a credulous game), or all of them (by a sceptical game).

In the end we may use some method of formal logic, Lorenzen's games for example—should we wish to remain in a playful setup [**34**], whether a given conclusion can be demonstrated from such set.

The rules of credulous games are the following:

- Proponent (P) and opponent (O) take turns. P begins by positing a set of hypotheses he wants to defend.
- Each move of O is an attack on some of (not necessarily preceeding) previous move of P.
- Each move of P (except the first one) is an attack on the directly preceeding move of O.
- O is not allowed to repeat its own moves, but is allowed to repeat P's moves.
- P is not allowed to repeat O's moves, but is allowed to repeat its own moves.

A credulous game is won by the proponent if and only if the opponent cannot move anymore. It is won by an opponent, if the proponent cannot move anymore, or if the opponent manages to repeat one of proponent's moves.

An argument is in some preferred extension if and only if the proponent can win a credulous game (Theorem 3 in [**13**]) .

In frameworks, where each preferred extension is also stable, such as the ones resulting by abstraction from normal frameworks, this game can be used to determine membership in all preferred extensions. In such a case an argument is in all preferred (stable) extensions if and only if none of its defeaters is in any of them.

That means that an opponent has a winning strategy for each of these arguments. But that just means that the proponent has a winning strategy in the original credulous game (Theorem 5 in [**13**]) .

Let me demonstrate this dialectical method on a pratical example. Suppose the following theory is given:

$L = \{p, q, r, s, \neg p, \neg q, \neg r\}$
$R = \{\frac{p,s}{\neg r}, \frac{q,r}{\neg p}, \frac{\neg p}{\neg r}\}$
$F = \{s\}$
$H = \{p, q, r\}$

Our question is, can we explain $\neg p$ in such framework? Or can we predict it (suppose no conjectures are specified, so we only need to check that $\neg p$ is in all extensions of this framework)?

We construct a corresponding ABF, simply taking $\overline{p} = \neg p$, $\overline{q} = \neg q$, $\overline{r} = \neg r$, and $\overline{\neg p} = p$, $\overline{\neg q} = q$, $\overline{\neg r} = r$, although we could have chosen any other assignment. We do not need to define contraries of facts.

Now this is a winning debate for a propponent, that proves that $\neg p$ is in fact explainable from this framework:

(1) P: I assume q, r and and conclude $\neg p$. (There is no other set of assumptions from which P could conclude $\neg p$).
(2) O: I assume $\neg p$ and and conclude $\neg r$. (O attacks second assumption of P's first move, using its conclusion).
(3) P: I assume p and conclude p. (P is allowed, for sake of argument, assume the contrary of his original thesis).

The only possible counterattack on this move would be to assume $\neg p$ again, but O is not allowed to repeat his moves and therefore loses. Indeed, $\{q, r\}$ is an extension of given framework and we can demonstrate p from it.

However, P does not have a winning strategy in this game. The following dialogue proves that O can actually win the argument by using better strategy:

(1) P: I assume q, r and and conclude $\neg p$.
(2) O: I assume p and and conclude $\neg q$. (O attacks first assumption of P's first move).
(3) P: I assume q, r and and conclude $\neg p$. (P is allowed to repeat his first move).
(4) O: I assume $\neg p$ and and conclude $\neg r$. (O attacks the previous move of P, as there is no way he could attack his first move, without repeating himself).

The only possible move for P would be to assume p, but this he is not allowed to do because this is what his opponent alrady did in step 2), although in order to draw different conclusion. P wins, the set

$\{p\}$ is an extension of the given framework, such that $\{q, r\}$ is not its subset and $\neg p$ therefore cannot be concluded from this set.

What we are actually checking by this dialogue games is wheter set of assumptions $\{q, r\}$ is in all some/all extensions of the corresponding AAF which is equivalent to checking that this set is a subsef of some/all extensions in the sense of ABF.

Because this set is the only one we can use to deduce the desired conclusion, this is equivalent to checking whether the desired conclusion is in some/all extensions.

If P was able to base his argument on different set of assumptions, such that he would have a winning strategy in a game starting with this argument, he would be able to predict the desired conclusion after all, no matter that his original selection of hypotheses was quite unfortunate. That is—his first move is untrivial because he has to select the correct set of assumptions.

Now many other applications of arguments in reasoning could be possibly described in a manner of rules of dialogue. This trend is also obvious in informal logic [**66, 68**].

Note, however, that I deliberately chose an example which has nothing to do with formal logic. The symbol of negation was used just for notational convenience and the definition of contrariness was so familiar in order to make the example more intelligible, but that was my design choice.

There is no reason not to incorporate logic into the model by choosing L and R accordingly.

Before proceeding to conclusion of all of my thesis, let me summarize the previous few sections.

Their goal was to show that formal logic can be used for analysis of natural language (although its import will often not be that substantial) and eventually for analysis of defeasible scientific argumentation after all.

Nonetheless I argued that none of these fields is its primary domain. Thoughtful application of formal logic in these subjects can be fruitful, but other methods are more fundamental to understand written argument in natural language, or adequacy of a certain way of argumentation wihin a specific kind of dialgoue.

The important moral is that formal logic is the science of demonstrations. However, there are many more other uses of arguments and unlike demonstrations which due to their traditional role in CMS are supposed to proceed from episteme have to deal with mere endoxa.

For merely plausible statements, their acceptability simply cannot be identified with that of impossibility of theirs refutation, but with absence of such final refutation in some kind of dialogue.

Reasoning operating with such plausible statements, therefore cannot abstract from particular content of those statements and the setup of other plausible statements, as well as the particular kind of dialogue in which arguments are used. It is therefore impossible to try to abstract from these conditions and arrive at a valid form of such arguments.

Nonetheless the notion of demonstration and inconsistency remain formal and are often preliminary for deciding which move in a particular argument game is admissible and which not.

This is why a good theory of demonstration is necessary, although not sufficient, preliminary for any good theory of argument.

Aristotle would surely be happy about such an implementation of his ideas.

CHAPTER 4

Conclusion

In the preface to this thesis I promised severe criticism of baby logic served as theory of argumentation. This has been already achieved by Toulmin decades ago and I have simply rearticulated this critique, its consequences, and traditional responses in section 1.3.

Nonetheless, and this I hoped to explain with my own words, Toulmin's criticism was mainly misunderstood or simply ignored by many philosophers of logic.

This is mainly because his distinction between substantial and analytical arguments might be inappropriately identified with the classical distinction of analytic and synthetic arguments.

Analytic arguments, we are told, establish their conclusions with necessity because of meaning of terms, while synthetic (factual, material), also because of the information they convey and the state of the external world.

The analytic-synthetic distinction is originally applied to truths. But some statements, although reasonable, simply cannot be taken to be true, unless we embrace some very inclusivist truth theory.

So it makes no sense to ask, whether such statements are synthetic or analytic, as this distinction was primarily intended to be applied to statements of different epistemic status, that is on truths or statements of facts.

Consequently arguments based on such assumptions can be neither logically, analytically, nor even factually valid because there simply is no fact underlying them, not in the conventional sense of the word at least.

I hoped to explain the origins of those distinctions and traced them back to Aristotle. Recent Aristotelian scholars I quote identified two different traditions in logic, one in which logic was used as a tool of science, one in which it had to explain argumentation.

It is the first use of logic which prevailed and which pretty much defined the terminology and categories in which we nowadays mostly think about logic, as well as the foundational problems that are associated with it.

I followed the development of ideal of CMS until the beginning of 20th century in order to explain that logical positivists were its modern heirs. So if we are not willing to accept their quest to found all human knowledge and reasoning on the model of science they promoted, we should not apply the categories of synthetic and analytic on those kinds of reasoning that are not scientific in nature and we should respect different uses of arguments which they may have in law, art, or religion.

However, Toulmin is interested in different aspects of arguments because his account was intended to explain their dialectical use, not their use in scientific demonstrations.

His concern is whether we merely repeat in the conclusion of an argument, what we already presuppose in (explicit or implicit) premises, something that could be verified directly if needed, or whether we do conclude something substantial—something that would be otherwise unjustifiable, if we did not back it up with an argument of a similar kind.

When relating formal logic to argumentation it is necessary to draw a clear border between substantial and analytic arguments and this is what for example authors of [**59**] neglect and consequently it is not quite clear from their account how do terms like argument, inference, or reasoning relate one to each other.

They also include chapter on Toulmin, but they underestimate consequences of his critique and thus fail to address it in a manner it deserves.

Nonetheless [**59**] is a textbook philosophical logic and not argumentation, so this does not have any serious consequence or impact on the subject of the textbook later on. It just creates a mismatch between what is promised to the reader in the preface to the book and what the book actually achieves.

If this book should be inserted between first chapter of each textbook of mathematical logic, as authors proclaim, I would suggest inserting yet another book before the first chapter of this one. This is, however, fate of most scientific treatises—it is not possible to question ones assumptions infinitely and one simply has to take certain thinks for granted for the start.

Feldman [**19**] recognizes difficulties with classical identification of reasonable arguments with logically sound arguments. Despite that, he bends the classical definitions of soundess until he is capable to present us with a strange subspecies of good arguments we can practically recognize by using formal logic.

The merits of such approach are, nonetheless, incomparable to the problematic philosophical assumptions he must make, so his account is the prototypical example of an approach that informal logicians would criticize.

The Quinean tradition stands is much more immune to this critique because it is much less ambitious in its goals.

I have also explained why formal logic is essentialy unfit for a role of general theory of arguments.

It is not a mere coincidence or shortage of detail we could fix by improving our logics, unless we do not wish to give up on methods which are fundamental to all such logical formalisms. I argued that a distinctive and crucial feature that makes an arbitrary relation a logical relation is that of formality.

I have shown that the notion of formality is essentially connected to the notion of necessary validity and that this is in turn essential for notion of demonstration, the notion that logic was primarily intended to explain. This claim was derived from philosophies of Bolzano and Wittgenstein and demonstrated on the embodiement of their ideas in formalism of classical sentential logic.

Later (page 60), I have argued, that due to the great plurality of logical formalisms, they are all based on the same insights.

I often had to touch very complex and delicate topics very superficially. However, my goal was not to wage arguments, concepts and interpretations from all numerous philosophies of logic.

I had to proceed very selectively, hoping that most of the subtleties, distinctions, and issues debated in philosophy of logic are not that relevant for the final outcome—that the notion of logical form is central to all logical accounts of truth and that is why formal logic is inherently unfit for description of substantial and therefore informal reasoning.

The confusing notion of informal reasoning was also somehow clarified for this account.

This rather superficial overview, I believe, is indispensable, despite its many open ends because it provides the essential bridge between the first and the third chapter. It points out what is formal logic really good for and so sets grounds for the exploration of its possible utility for studying argumentation after all.

Finally I have also briefly illustrated two possible areas of application of formal logic outside the traditionally associated field of mathematics, that is as a tool for language analysis and an underlying theory for various models of reasoning and knowledge representation in artificial intelligence.

The actuall contribution of logic to these fields had to be delimited in order to deflect various objections aimed against logic. The most fundamental objection to formal logic that it is unable to give appropriate account of substantial reasoning was also adressed.

I also presented the development of logical theories of defeasible reasoning in the light of the previous account.

First, the role of plausible statements was demonstrated on the case of Reiter's default logic and than the defaults were removed in favor of explicit distinction of facts and hypotheses. Later, new uses of logic suggested by Poole were briefly examined. This exposition was concluded dialogical account of admissibility.

This exposition was also very brief, but it merely served to illustrate the importance of the pre-technical concepts of plausibility and refutation and point out some promising areas of logical research. I did not present all the definitions, theorems and proofs because it would detract from the flow of the main argument, but all these can be found in cited literature.

The main contribution of my work, as I see it, is that of bringing together research of informal logicians, philosophers and historians of logic and science, and researchers in computer science to explore arguments for the central thesis of this paper and its implications.

This thesis can be summarized in the following words: "In order to arrive at a more appropriate theory of argument, it is fruitless trying to fix, alter, modify, or enhance current systems of formal logic, because they were devised for different purpose. That does not mean, however, that formal logic cannot help us to understand argumentation better. We must merely employ it within a larger theory, studying argument exchange and interaction".

This may seem as a little unimpressive and unambitious result compared to the overwhelming list of questions that remained unanswered:

- Is the probabilistic account of defeasible arguments justifiable and if so, what is its relation to accounts presented in this work?
- Is there no interpretation of various accounts of non-monotonic inference relation as formal logics after all and are there no possible appplications of those systems in mathematics?
- Would it be reasonable to extend some non-classical logical formalisms by adding defeasible rules?
- Do philosophical logicians actually study relevant properties of natural languages, when concentrating primarily on demonstrations, or is the ability to back our claims by demonstrations in some sense primary to other functions of language, such as explanations or predictions?
- How much is an account of formal logic as a general theory of demonstration dependent on a particular philosophy of logic which I have preferred?
- Is there actually some historical connection between Aristotle's ideas, Toulmin's model of argument and defeasible logics, or was it constituted only additionally?

- To what extent is an informal logic movement influenced by the parallel development in philosophy of logic and in AI and how much did it actually contribute to it[1]?

These are only few of the questions that remain unanswered in my thesis. Perhaps concentrating only on one of those would produce a more definite outcome, but for the cost that probably nobody, including me, would understand the relevance of the answer in the proper context.

[1]So that the list of my omissions was complete: Neither have I given technical details of abstract-argumentation approaches, nor explained theirs motivation or possible variants.

Relation of dialectical semantics for argumentation to dialogical logics, or various known dialogical semantics for formal logics also remain extremely unclear.

Bibliography

[1] ANTONIOU, G. *Nonmonotonic Reasoning*. Cambridge (Mass.): MIT Press. 1996. p. 285. ISBN 0–262-01157–3.

[2] ARISTOTLE. *The Complete Works of Aristotle*. Vol. 1. transl. JENKINSON, A.J. ed. BARNES, J. Princeton: Princeton University Press. 1995. pp. 1–1250. 6th ed. ISBN 0-691-01650-X. Chap. 3, Prior Analytics, pp. 39–113.

[3] ARISTOTLE. *The Complete Works of Aristotle*. Vol. 2. transl. RHYS ROBERTS, W. ed. BARNES, J. Princeton (NJ): Princeton University Press. 1995. pp. 1251–2465. 6th ed. ISBN 0-691-01651-8. Chap. 15, Rhetoric, pp. 2152–2269.

[4] ARISTOTLE. *The Complete Works of Aristotle*. Vol. 1. transl. PICKARD, W. A. ed. BARNES, J. Princeton (NJ): Princeton University Press. 1995. pp. 1–1250. 6th ed. ISBN 0-691-01650-X. Chap. 6, Sophistical Refutations, pp. 278–314.

[5] ARISTOTLE. *The Complete Works of Aristotle*. Vol. 1. transl. PICKARD, W. A. ed. BARNES, J. Princeton (NJ): Princeton University Press. 1995. pp. 1–1250. 6th ed. ISBN 0-691-01650-X. Chap. 6, Topics, pp. 167–277.

[6] BEAL, J. C. - RESTALL, G. *Logical Pluralism*. New York: Oxford University Press. 2006. p. 143. ISBN 0–19–928841-0 (Pbk.).

[7] BENCH-CAPON, T. J. M. - DUNNE, P. E. Argumentation in Artificial Intelligence [online]. *Artificial Intelligence* January 2007. Vol. 171. No. 1. pp. 619–641. ISSN 004–3702.

[8] BESNARD, P. - HUNTER, A. *Elements of Argumentation*. Cambridge (UK): MIT Press. 2008. p. 298. ISBN 978-0–262-02643-7.

[9] BONATTI. P. A. - OLIVETTI, N. Sequent calculi for propositional nonmonotonic logics [online]. *ACM Transactions on Computational Logic (TOCL)*. April 2002. Vol. 3. No. 2. pp.226–278. ISSN 1592–3785. EISSN 1557-945X.

[10] BOCHENSKI, J. M. *Formale Logik*. Freiburg: Alber. 2002. p. 645. ISBN 3–495–48071–4.

[11] BONDARENKO, A. et al. An abstract, argumentation-theoretic approach to default reasoning [online]. *Artificial Intelligence*. December 1997. Vol. 93. pp. 63–101. ISSN 004–3702.

[12] BONDY, P. Truth and Argument Evaluation [online].*Informal Logic* 2010. Vol. 30, No.2. pp.142–158. ISSN 0824–2577.

[13] CAMINADA, M. - WU, Y. An Argument Game for Stable Semantics [online]. *Logic Journal of the IGPL*. 2009. Vol. 17. No. 1. pp. 77–90. ISSN 1367-0751. EISSN 1368-9894.

[14] COFFA, J. A. *Semantic Tradition from Kant to Carnap: To the Vienna Station*. ed. WESSELS, L. New York (NY): Cambridge University Press. 1991. p. 445. ISBN 0-521–44707-0 (Pbk.).

[15] COFFA, J. A. Kant, Bolzano and the Emergence of Logicism [online]. *The Journal of Philosophy*. November 1982. Vol. 79. No.11. pp.679–689. ISSN 022–362X.

[16] DE JONG, W. R. - BETTI A. The Classical Model of Science: a millenia-old model of scientific rationality [online] .*Synthese*. 2008. Vol. 174. No. 2. pp.185–203. ISSN 0039-7857. EISSN 1573-0694.

[17] DE JONG, W. R. The analytic-synthetic distinction and the classical model of science: Kant, Bolzano and Frege [online].*Synthese*. 2008. Vol. 174. No. 2. pp.237–261. ISSN 0039-7857. EISSN 1573-0694.

[18] DUNG, P. M. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*. September 1995. Vol. 77. Issue 2. pp. 321–357. ISSN 004–3702.

[19] FELDMAN, R. *Reason and Argument*. 2nd ed. Upper Saddle River (NJ): Prentice Hall. 1999. p. 462. ISBN 0–13-624602-8 (Pbk.).

[20] FISHER, A. *Logic of Real Arguments*. 2nd ed. Cambridge (UK): Cambridge University Press. 2004. p. 224. ISBN 0-521-65481-5 (Pbk.).

[21] FISHER, A. *Critical thinking: an introduction*. Cambridge (UK): Cambridge University Press. 2006. p. 249. ISBN 0-521-00984-7. (Pbk.).

[22] FREGE, G. *Logische Untersuchungen*. ed. PATZIG. G. *Göttingen*: Vandenhoeck & Ruprecht. 1966. Chap. 3, Der Gedanke, p. 141.

[23] GOLDFARB, W. Logic in the Twenties: The nature of Quantifier [online]. *Journal of Symbolic Logic*. September 1979. Vol. 44 No. 3. pp. 351–368. ISSN 0022–4812.

[24] HAACK, S. *Philosophy of Logics*. New York (NY): Cambridge University Press. 1978. p. 276. ISBN 0-521–29329–4 (Pbk.).

[25] VAN HEIJENOORT, J. Logic as calculus and logic as language [online]. *Synthese*. 1967. Vol. 17. No. 1. pp. 213–264. ISSN 0039-7857. EISSN 1573-0694.

[26] JAURIS, M. Výuka elementární logiky (Sylogistika, booleovská logika, nebo něco jiného). *Miscellanea Logica*. 1998. Vol. 1. pp. 10–35.

[27] JOHNSON R. J. - BLAIR, J. A. Informal Logic: An Overview [online]. *Informal Logic*. 2000. Vol.20. No.2. pp.93–107. ISSN 0824–2577.

[28] JOHNSON R. J. - BLAIR, J. A. *Handbook of the Logic of Argument and Inference*. eds. GABBAY, D. M. et al. Amsterdam: Elsevier. 2002. Vol. 1. p. 498. 1st ed. ISBN 0–444-50650-0. ISSN 1570–2464. Chap. 8, Informal logic and the Reconfiguration of Logic, pp. 339–396.

[29] KIRKHAM, R. L. *Theories of Truth: a critical introduction*. Cambridge (Mass.): MIT Press. 1992. p 401. ISBN 0–262-61108–2.

[30] KNEALE W. - KNEALE, M. *The Development of Logic*. Oxford: Clarendon Press. 1984. p. 783. ISBN 0–19-824773-7 (Pbk.).

[31] KOLMAN, V. *Logika Gottloba Frega*. Praha: Filosofia. 2002. p. 295. ISBN 80-7007–164-8.

[32] KOLMAN, V. *Filosofie čísla: základy logiky a aritmetiky v zrcadle analytické filosofie*. Praha: Filosofia. 2008. p. 670. ISBN 978-80-7007–279-0.

[33] KORTE, T. Frege's Begriffsschrift as a lingua characteristica.*Synthese*. 2010. Vol. 174. No. 2. pp.283–294.

[34] LORENZEN, P. *Constructive Philosophy*. tansl. PAVLOVIC, K. R. Amherst: University of Massachusetts Press. 1987. p. 291. ISBN 0-87023-564-8.

[35] MAKINSON, D. *Bridges from Classical to Nonmonotonic Logic*. London: King's College Publications. 2005. p. 216. ISBN 1-904987-00–1 (Pbk.).

[36] MCDERMOTT, D. - DOYLE, J. Non-monotonic Logic I *Artificial Intelligence*. April 1980. Vol. 13. No. 1. pp. 41–72. ISSN 004–3702.

[37] MOORE, G. H. *The Emergence of First-Order logic* [online]. 11.11.2011. URL: ⟨http://www.mcps.umn.edu/philosophy/11_4Moore.pdf⟩.

[38] NEVRKLA, S. Logic and Argumentation. *Miscellanea Logica viii: Foundations of logic*. 2010. Acta Universitatis Carolinae Philosophica et Historica. Vol. 2 –2007. pp. 41–70. ISSN 0567-8293

[39] NEVRKLA, S. K čemu je filosofii logika a logice filosofie. *Filozofia*. 2010. Vol. 65. No. 4. pp. 383–387. ISSN 0046–385 X

[40] NEVRKLA, S. Neformální logika: contradictio in adiecto? *Organon*. 2012. Vol. 7. No. 1. pp. 69–80. ISBN 978-80–261-0035-5.

[41] NEVRKLA, S. Is Formal Logic a Science About Rational Arguments? *Organon F*. 2011. Vol. 18. No. 4. pp. 499–511. ISSN 1335-0668.

[42] NEVRKLA, S. *The Logica Yearbook 2011*. eds. PUNČOCHÁŘ, V., CHVALOVSKÝ K. London: College Publications. To appear in 2012. Sceptical and Credulous Approach to Deductive Argumentation.

[43] NEVRKLA, S. Argument a odvození. *Organon F*. Submitted for review. 2012. ISSN 1335-0668.

[44] PECKHAUS, V. Calculus rationicator versus characteristica universalis? The two traditions in logic, revisited [online]. *History and Philosophy of Logic*. 2004. Vol. 25. No.1. pp.3–14. ISSN 0144-5340. EISSN 1464-5149.

[45] PEREGRIN, J. *Logika a logiky*. Praha: Academia. 2004. p. 205. ISBN 80–200–1187-0

[46] PEREGRIN, J. Two Concepts of Validity and Completeness. *Miscellanea Logica viii: Foundations of logic*. 2010. Acta Universitatis Carolinae Philosophica et Historica. Vol. 2 –2007. pp. 23–39. ISSN 0567-8293

[47] POOLE, D. A Logical Framework for Default Reasoning [online]. *Artificial Intelligence*. December 1988. Vol. 36. No. 1. pp. 27–47. ISSN 004–3702.

[48] POOLE, D. Explanation and Prediction: an architecture for default and abductive reasoning [online]. *Computational Intelligence*. May 1981. Vol. 5. No.2. pp. 97–110. ISSN 0824-7935.

[49] PRAKKEN, H. - VREESWIJK, G. *Handbook of Philosophical logic*. eds. GABBAY, D. M. - GUENTHER, F. Dodrecht: Kluwer Academic Publication. 2002. Vol.4. p. 431. 2nd ed. ISBN 1–4020-0139-8. Chap. 3, Logics for Defeasible Argumentation, pp. 219–318.

[50] QUINE, W. V. O. *From a Logical Point of View*. 2nd ed. 2003. Cambridge (Mass.): Harvard University Press. p. 184. ISBN 0-674–32351–3 (Pbk.). Chap. 2, Two dogmas of empiricism, pp. 20–46.

[51] QUINE, W. V. O. *From a Logical Point of View*. 2nd ed. 2003. Cambridge (Mass.): Harvard University Press. p. 184. ISBN 0-674–32351–3 (Pbk.). Chap. 3, The problem of meaning in linugistics, pp. 47–64.

[52] QUINE, W. V. O. *Philosophy of Logic*. 2nd ed. 1986. Cambridge (Mass.): Harvard University Press. p. 109. ISBN 0-674-66563 (Pbk.)

[53] RÁLIŠ, J. *Věda, principy a dialektika v Aristotelově díle*. Olomouc. 26.6. 2010. p. 73. Master thesis at the Department of Philosophy, Philosophical Faculty of Palacký University in Olomouc. Supervisor Mgr. Pavel Hobza, PhD.

[54] REITER, R. A Logic for Default Reasoning [online].*Artificial Intelligence*. 1980. Vol.13. No. 1. pp. 81–132. ISSN 004–3702.

[55] RENON, L. V. Aristotle's Endoxa and Plausible Argumentation [online]. *Argumentation*. 1988. Vol. 12. No. 1. pp. 95–113. ISSN 0920–427X. EISSN 1572-8374.

[56] RYLE, G. *Dilemmas*. Cambridge (UK): Cambridge University Press. 1954. p. 129. Chap. 8, Formal Logic and Informal Logic, p. 111–129

[57] SOUSEDÍK, S. Technika filosofické disputace v 17. století.*Filosofický časopis*. 1967. Vol.15. No. 1. pp. 132–152 (1967)

[58] SELLARS, W. Inference and meaning. *Mind 62*. July 1953. Vol.62. No. 247. pp. 313–338. ISSN 1460–2113. EISSN 0026–4423.

[59] SVOBODA, V. - PEREGRIN, J. *Od jazyka k logice: Filozofický úvod do moderní logiky*. Praha: Academia. 2009. p. 428. ISBN 978-80–200–1740-6.

[60] SVOBODA, V. et al. *Logika a přirozený jazyk*. Praha: Filosofia. 2010. p. 299. ISBN 978-80-7007–325–4.

[61] ŠEBESTÍK, J. Bolzano's Logic [online]. *The Stanford Encyclopedia of Philosophy*. URL: ⟨http://plato.stanford.edu/archives/win2011/entries/bolzano-logic/⟩. 11.11.2011.

[62] ŠEFRÁNEK, J. *Inteligencia ako výpočet*. Bratislava: IRIS. 2000. p. 427. ISBN 80-88778-96–4.

[63] TARSKI, A. The Semantic Conception of Truth [online]. *Philosophy and Phenomenological Research*. 1944. Vol. 4. No. 1. p. 341–375.

[64] TARSKI, A. What are logical notions? [online]. *History and Philosophy of Logic*. ed. CORCORAN, J. August 1986. Vol. 7. No.1. pp.143–154. ISSN 0144-5340. EISSN 1464-5149.

[65] TOULMIN, S. *The Uses of Argument*. New York (NY): Cambridge University Press. 2005. p. 247. ISBN 0-521-53483-6 (Pbk.).

[66] WALTON, D. *The New Dialectic: Conversational Contexts of Argument*.Toronto: University of Toronto Press. 1998. p. 304. ISBN 0-8020–4143–4.

[67] WALTON, D. *Abductive Reasoning*. Tuscaloosa (Alab.): University of Alabama Press. 2004. p. 303. ISBN 0-8173–1441-5.

[68] WALTON, D. *Informal Logic: a pragmatic approach*. Cambridge (Mass.): Cambridge Univerity Press. 2008. p. 347. ISBN 978-0-521-71380–1 (Pbk.)

[69] WILLIAMSON, J. *Handbook of the Logic of Argument and the Inference*. eds. GABBAY, D. M. et al. Amsterdam: Elsevier. 2002. Vol. 1. p. 498. 1st ed. ISBN 0–444-50650-0. ISSN 1570–2464. Chap. 9, Probability Logic, p. 397–424.

[70] WITTGENSTEIN, L. *Tractatus Logico-Philosophicus*. intro. RUSSELL, B. 1922. London: Paul Kegan International. p. 189.

[71] WOODS, J. *Handbook of the History of Logic: Greek, Indian and Arabic Logic*. eds. GABBAY, D. M. - WOODS, J. Amsterdam: Elsevier. 2004. Vol. 1. p. 618. 1st ed. ISBN 0–444-50466–4. series ISBN 0–444-51596-8. Chap. 2, Aristotle's Early Logic, p. 27–100.

[72] WOODS, J. How Philosophical is Informal Logic? [online] *Informal Logic Vol. 20, No.2 (2000)*. 2000. Vol. 20. No. 2. pp.139–167. ISSN 0824–2577.

[73] WOODS, J. *Handbook of the Logic of Argument and the Inference*. eds. GABBAY, D. M. et al. Amsterdam: Elsevier. 2002. Vol. 1. p. 498. 1st ed. ISBN 0–444-50650-0. ISSN 1570–2464. Chap. 2, Standard Logics as Theories of Argument and Inference: Deduction, pp. 41–104.

[74] WOODS, J. *Handbook of the Logic of Argument and the Inference*. eds. GABBAY, D. M. et al. Amsterdam: Elsevier. 2002. Vol. 1. p. 498. 1st ed. ISBN 0–444–50650-0. ISSN 1570–2464. Chap. 3, Standard Logics as Theories of Argument and Inference: Induction, pp. 105–170.

[75] ZASTÁVKA, Z. Je neformální logika logikou? *Miscellanea Logica*. 1998. Vol. 1. p.79–84.