Report on the Ph.D.-Thesis

Study of exact spacetimes

by Robert Švarc

The general field of this thesis is the study of exact (radiative) space-times in general relativity and it consists of two parts. The first part (chapter 2) deals with geodesics in impulsive spherical gravitational waves, while the second part (chapters 3–6) is concerned with describing physically relevant properties of higher-dimensional space-times using geodesic deviation.

Following a short but nicely written historical introduction (chapter 1), the first part of the thesis (chapter 2) consists of the paper "Refraction of geodesics by impulsive spherical gravitational waves in constant-curvature space-times with a cosmological constant" co-authored by R. Švarc with his supervisor and already published in Physical Review **D** (see [6]). This work deals with the problem of solving the geodesic equation for expanding impulsive waves propagating in a constant curvature background. This problem was first tackled for the special case of a flat background in [5], which mainly focused on a special class of geometrically privileged observers. Assuming that the geodesics are of $\mathcal{C}^1$-regularity across the shock surface the work under review succeeds in significantly generalizing its predecessor by deriving refraction formulas not only for *general* geodesics but also in the case of a *non-vanishing cosmological constant.* Moreover, the authors analyze in detail the particle motion in the important special case where the wave is generated by a snapped cosmic string.

The second part of the present thesis starts out with the paper "Interpreting space-times of any dimension using geodesic deviation" (chapter 3) again co-authored by R. Švarc with his supervisor and also already published in Physical Review **D** (see [7]). Here the authors present a systematic analysis of the motion of freely falling test particles in space-times of any dimension $D \geq 4$, based on a projection of the equation of geodesic deviation onto suitably chosen frames. In this way they generalize to an arbitrary number of dimensions (a variant of) the well-known Newman-Penrose formalism (see e.g. [3]). Using a decomposition of the Riemannian curvature tensor into the Weyl and Ricci tensors and the curvature scalar they isolate the specific effects of the gravitational field on a freely falling observer: transverse and longitudinal components, Newton-Coulomb-type effects, and isotropic influences of the cosmological background. By this the authors construct a framework which allows for a convenient analysis of physically relevant effects of higher dimensional gravity. As a first application pp-waves are treated where an interesting effect can now be easily detected: while non-trivial vacuum pp-waves in $D = 4$ induce a motion in both transverse directions (with amplitudes $\mathcal{A}_2 = -\mathcal{A}_3$), in higher dimensions the amplitudes are coupled by the constraint $\sum_{i=2}^{D-1} \mathcal{A}_i = 0$. In principle this effect of a higher-dimensional pp-wave is observable as a violation of the standard TT-property. Finally, it has to be explicitly pointed out that the (historical) overview given in this article is very informative and that the bibliography, which lists 141(!) items, is impressive.

The following chapter 4 specializes the above framework to the case of space-times possessing a *non-twisting null congruence of geodesics.* All the frame decompositions are carefully derived and explicitly given. At this point we remark that in the present construction the frame is not necessarily parallely propagated along the observer's world line, which allows for a more sensible adaption of the frame to the symmetries of the space-time. Chapter 4 ends with the useful observation that the frame is already parallely propagated if only the $D-2$ spatial components as well as the u-component (tangent to the observer's world line) of the frame vectors are parallely propagated.

In chapter 5 the author applies the construction to the class of Kundt space-times. These geometries, where the non-twisting geodesic null congruence is also assumed to be *non-expanding and shear-free*, have recently been investigated in detail (in $D \geq 4$) in [8]. This analysis is complemented by the present chapter, where again all the relevant frame decompositions are presented in detail. Also the author discusses important special cases belonging to the class of VSI-space-times, in particular, he includes a much more detailed discussion of the properties of higher-dimensional pp-waves already mentioned above.

The final chapter 6 is devoted to the class of (again possibly higher-dimensional) Robinson-Trautman space-times, which, by definition possess a non-twisting, *shear-free but expanding* geodesic null congruence. This family of solutions has been investigated in [4, 2], where it has been shown that its structure is not as rich as in $D = 4$, i.e., their algebraic type is D or more special, while in $D = 4$ the algebraic type is II or more special. Again the present chapter is an interesting complement to the works cited above: it turns out that suitable observers in higher-dimensional Robinson-Trautman space-times will only measure Newton-like tidal forces (which are typical type D-effects) and *no transverse and longitudinal* effects, which are typical type-II effects and which are present in $D = 4$.

Let me now come to my assessment of the present thesis. Both parts contain new and interesting results. The explicit junction conditions for geodesics in expanding impulsive waves, derived in chapter 2, may well serve as an essential building block for a mathematically rigorous description of these geometries along the lines of [1], which only deals with the non-expanding case in a flat background. On the other hand the formalism developed for interpreting higher-dimensional space-times in chapter 3 has very high potential for future application in the study of exact solutions. This already becomes apparent in chapters 4–6, which contain such applications that complement previous investigations of higher-dimensional space-times of the Kundt ([8]) and Robinson-Trautman ([4, 2]) classes. It serves very well to extract and physically understand effects that distinguish higher dimensional space-times from 4-dimensional ones. Here the "(non-)vanishing trace"-effect in pp-waves is certainly a highlight: in principle it would allow to experimentally observe the existence and influence of higher dimensions.

Although essential parts of this thesis have been published by the candidate together with his advisor the overall form and content of this work clearly shows the candidate's ability to pursue original research and to creatively contribute new and interesting results to his field. Summing up, the present thesis is of very high quality and I am convinced that the candidate well deserves to be awarded a Ph.D. degree in (Theoretical) Physics.

Ao. Univ. Prof. Mag. Dr. Roland Steinbauer
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References

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