

Referee report on the doctoral thesis

Generating Methods in GR and Properties of the Resulting Solutions

by Mgr. Jakub Hruška

There are different approaches how to generate solutions of the Einstein(-Maxwell) equations from which conformal transformation technique is the one employed in the present doctoral thesis.

The first introductory part of the thesis is an overview of various topics employed in the second part of the thesis, namely generating techniques in GR and their applications, conformal transformation and its properties, spinors and Newman-Penrose formalism, equivalence problem approached either using Cartan scalars or curvature invariants and Maxwell fields.

In the second part of the thesis new results are presented using the generating method based on conformal transformation exploiting conformal invariance of the Maxwell field. First, a suitable null (electrovacuum) Einstein-Maxwell spacetime (conformally flat plane wave) is used as a seed metric. The conformally transformed solution is required to satisfy Einstein-Maxwell equations as well, leading to another plane wave of the McLenaghan type. It is checked that the seed metric and the resulting solution are not equivalent and therefore, to the author's knowledge, it is a first case of non-equivalent conformally related non-vacuum spacetimes.

For null Einstein-Maxwell fields, integrability condition for the main equation for the conformal factor is then used to systematically search for all allowable seed metrics for the considered generating method and it is proven that the seed metric can only be a pp-wave of the McLenaghan type.

The generating method is further generalized to allow for resulting Maxwell field independent on the seed Maxwell field. Pairs of null E-M fields connected by conformal transformation with two generally different source-free null electromagnetic fields propagating in algebraically special spacetimes and automatically aligned with PND of the Weyl tensor are searched for. Class of non-vacuum, conformally connected Petrov type III and N Kundt spacetimes with null Maxwell fields is recovered for null conformal transformations. For non-null conformal transformations, only existence of isometric conformal pairs with null EM field is achieved due to the complexity of the task.

Finally possible future research directions are briefly discussed.

The thesis is lucidly and pedagogically written and well organized. Most of the new results are contained in two research papers – one already published in *Classical and Quantum Gravity*, which by itself indicates high quality of the work. Technical quality of the thesis is good, there are only some typos, the only ones worth mentioning are a missing minus sign in eq. (1.2) in front of $2y$ and M_{RN} instead of M_{RN} on page 44.

Jakub Hruška in his thesis, which satisfies the required conditions, demonstrated ability to perform independently creative research in theoretical physics and I recommend his thesis to be accepted as a doctoral thesis at the Faculty of Mathematics and Physics of the Charles University.

I would suggest the following question:

It is well known that in higher dimensions the energy-momentum tensor of electromagnetic field has a non-vanishing trace and the traceless property seems to be important ingredient in your approach. Could this generating method be still used in higher dimensions?

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