In this work we describe transformations of the 3-dimensional and the 4-dimensional Euclidean space. First we show how one can elegantly describe reflections and rotations in these dimensions using quaternions and we prove 2 structural theorems concerning the connection between the group of unit quaternions and the special orthogonal groups SO(3) and SO(4). Next we recall a part of the conformal mapping theory, which we use later in the description of the Möbius transformations. We define the Möbius transformations in dimension 4 as compositions of an even number of spherical inversions and reflections. We show that one can describe them also in dimension 4 as linear fractional transformations in an analogous way as in dimension 2, if we use quaternions instead of complex numbers. We then outline a classification of Möbius transformations into elliptic, loxodromic and parabolic classes and in dimension 4, we describe what each class looks like.