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Bachelor Thesis

Global Games and its Applications in Economics : Creditor Coordination Puzzle

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Abstract
This thesis focuses on the creditor coordination problem. The creditor coordination problem is a problem that arises if there are multiple creditors, which leads to herd behavior of the creditor and self-fulfilling prophecies. Theoretical framework I use for solving this is global games theory, which is a part of non-cooperative game theory. The question I ask is if there is any explanation for behavior observed in the German banking system, which contradicts previous results in global games literature. Namely the fact that the size of effect of the large creditor becomes detrimental for rolling over the debt, if size of the large creditor exceeds some optimal value. I found that any of the results do not give conclusive answer to this problem. On top of that, the result I obtained from my model suggests that the size effect of the large creditor should be even more positive. Thus I conclude that this behavior can be attributed to quasi-rents that the large creditor usurps, as was shown in Hubert and Schaefer [2000], as any supposable setup does not explain this.
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Abstrakt
Tato práce se zabývá problémem koordinace věřitelů. Problém koordinace věřitelů je problém který vyvstává v případě, že je věřitelů více, což vede ke stádovému chování a sebenaplňujícím proroctvím. Teoretický aparát, který k řešení tohoto používám je teorie globálních her, jenž je částí nekooperativní teorie her. Otázka, kterou se snažím zoodpovědět je zdali lze v ní nalézt vysvětlení pro chování pozorované v německém bankovním sektoru, jenž je protichůdné k dosavadním výsledkům v literatuře aplikující přístup globálních her. Konkrétně fakt, že od určité optimální míry velikosti velkého věřitele je jeho efekt na úspěšné financování projektu záporný. Zjistil jsem, že žádný z výsledků nedává přesvědčivou odpověď na tento problém. Navíc řešení mého modelu naznačuje, že efekt velikosti velkého věřitele by měl být ještě pozitivnější. Tudíž usuzuji, že toto empiricky pozorované chování může být přisouzeno kvazi-rentám, které si velcí věřitelé přisvojují, jak bylo prokázáno v článku od Hubert and Schaefer [2000], jelikož žádný ze mnou uvažovaných modelů toto chování není schopen vysvětlit.

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Declaration

I declare that I wrote this thesis myself and used only the literature listed in References.

In Prague, date
signature

Contents

Chapter 1

Introduction

The problem we pretend to solve is the problem of creditor coordination. This problem can be interpreted in a following way - debtor who is solvent so he could repay the loan with interest is cut off from financing from his creditors so he is illiquid and he defaults. This is different to the case when the debtor could not repay the loan even if all the creditors remained lending. In the creditor coordination problem the problem are not the economic fundamentals, but herd behavior of the creditors. In the original paper on debt pricing by Merton [1974], namely in its empirical part, we can find such an anomaly for highly risky debts where the effect of creditor coordination problem is reflected the most. Therefore the problem of solvent but illiquid debtors who fail only because of coordination failure of creditors is an example of irrational decision made by rational decision makers.

To answer this puzzle at least theoretically we adopt novel approach of global games. This branch of game theory was devised by Carlsson and VanDamme [1993] and further developed by Morris and Shin [2003] into an applicable tool how to tackle real world problems. Basic idea of global games is that players do not cooperate in their actions, they cannot observe true state of the world and they do not know what the others are doing. This is fairly relaxed assumption on behavior of the people in real world situations.

My objective is to answer question that arises from discrepancy between empirical findings and theoretical models, namely to build a model that could solve following mismatch. The problem is that in Krahnen and Brunner [2000] we observe contradictory behavior of creditors - basic insight from the theory of global games, as I will show below, is that with increasing size of the large creditor, creditor coordination should increase. In this paper, the results are not supportive for this theoretical view.

But what is important to note is the fact that solutions of creditor coordination problems are only one side of the coin - even though results from global games theory generally favor large creditor over small creditors, opportunistic behavior of large creditors is well documented fact , for details see Hubert and Schaefer [2000].

Chapter 2

Theory and Solution concepts

In this part, I present basic concepts from game theory, which I further use in explaining of global games. I start with definitions from game theory and then I continue with explanation of chronological development of global games theory used in this thesis.

2.1 Elements of Game Theory

Here I present brief overview of non-cooperative game theory. The sources I used are Fudenberg and Tirole [1991], Gibbons [1992] and Yildiz. Notations I used for unifying is that from Yildiz.
First, I begin with definition of the game we are playing - it is a Bayesian Game, as the players are uncertain about outcomes of the game. But global game has also some other properties, as I will present later.

Bayesian Game
A Bayesian game is a list (N, A, Θ, T, u, p) where:

- N is the set of players
- $A = (A_i)_{i \in N}$ is the set of action profiles (with generic member $a = (a_i)_{i \in N}$)
- Θ is a set of payoff parameters θ
- $T = (T_i)_{i \in N}$ is the set of action profiles (with generic member $t = (t)_{i \in N}$)
- $u_i : \Theta \times A \to \mathbb{R}$ is the payoff function of player i, and
- $p_i(.|t_i) \in \triangle(\Theta \times T_{-i})$ is the belief of type t_i about (θ, t_{-i})

Next, consider what are the possibilities of the players. In the language of game theory, this is called strategy, which is defined by

Strategy
A strategy of a player i is a function $s_i : T_i \rightarrow A_i$

And strategies of particular interest in the theory of global games are namely Strictly Dominant Strategies, which are strategies that we define as

Strictly Dominant Strategy
First denote list of all strategies played by players other than player i by S_{-i}. Then we have following definition of Strictly Dominant Strategy : A strategy s_i^* strictly dominantes strategy s_i if and only if $u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i})$,for all $s_{-i} \in S_{-i}$.

Strictly dominant strategies are used in proving that the global game has unique strategy that survives iterated elimination of strictly dominated (dominated, not dominant) strategies, which means that the game is dominance solvable.

The objective of global games is finding of Bayesian Nash Equilibrium, which is defined as

Bayesian Nash Equilibrium
A strategy profile $s^* = (s_1^*, ..s_n^*)$ is a Bayesian Nash Equilibrium in an n-person static game of incomplete information if and only if for each player i and type $t_i \in T_i$,
$s_i^*(t_i) \in \underset{a_i}{argmax} \sum_{t_{-i}} u_i[s_1^*(t_1), ..s_{i-1}^*(t_{i-1}), a_i, s_{i+1}^*(t_{i+1}), ...s_n^*(t_n), (t_i, t_{-i})] \times p_i(t_{-i}|t_i)$,
where u_i is the utility of the player i, and a_i denotes his action.

Therefore, for each player i, each type t_i this player chooses action optimal under conditional beliefs $p_i(t_{-i}|t_i)$, that is conditionally on the other player type and action.
But definition of Nash Equilibrium should not be omitted at least for completness.

Nash Equilibrium
A strategy vector $s \in S$ is said to be a Nash equilibrium if for all players i and alternate strategy $s_i' \in S$, we have that
$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$
Please note that $-i$ reffers to the other player (different from player i)

Finally, concept used for proving of uniqueness of the strategy is Iterated Elimination of Strictly Dominated Actions, where the idea is kept in its name.

Iterated Elimination of Strictly Dominated Actions
Consider a Bayesian game $B = (N, A, \Theta, T, u, p)$. For each $i \in N$ and $t_i \in T_i$, set $S_i^0[t_i] = A_i$, and define sets $S_i^k[t_i]$ for $k > 0$ iteratively, by letting $a_i \in S_i^k[t_i]$ if and only if

$$a_i \in \underset{a_i^{'}}{argmax} \int u_i(\theta, a_i^{'}, a_{-i}) d\pi(\theta, t_{-i}, a_{-i})$$

for some $\pi \in \triangle(\Theta \times T_{-i} \times A_{-i})$ such that

$$marg_{\Theta \times T_{-i}} \pi = p_i(.|t_i)$$

and

$$\pi = (a_{-i} \in S_{-i}^{k-1}[t_{-i}]) = 1.$$

This is what is essential from the game theory and now I pass on to the theory of global games. First I present chronological overview of this theory, as I consider this as the easiest way how to present key ideas and next I continue with solution concepts, where I propose a manual how to solve the global games for private information only.

2.2 Overview of theory of global games

Theory of global games is built around two major papers, the first is the article by Carlsson and VanDamme [1993], which initiated the theory of global games, based on 2×2 games, the second is the article by Morris and Shin [2003] who generalized this theory to n players case and continuous players case. What is essential about approach adopted by Carlsson and VanDamme is the fact that they removed assumption of common knowledge about economic fundamentals and and introduced players´ uncertainty about other players behavior in equilibrium. For approach preceding global games see Obstfeld [1986], which is perfect example of this approach. Here the author assumes that economic fundamentals are commong knowledge, i.e. information perfectly observed by everyone, and that everyone knows how would everyone behave. This results in multiple equilibria, which is what can be avoided by adopting approach of global games. For evidence, see part dealing with symmetric information model, where it is shown that it indeed results in multiple equilibria.

In explaining theory of global games, it is beneficial to stick to this chronological order. First, we explain two-player global game :

Carlsson and Van Damme paid attention to the problem of Nash equilibrium, namely the problem of non-uniqueness of Nash equilibrium, in noncooperative games.

They give an example of following game :

	α	β
α	x x	0 x
β	x 0	1 1

where (0<x<1)

and each player has choice between two strategies - α is a safe strategy, that yields always x, no matter what the other player does. β is a strategy that yields 1 only if the other player also chooses β , but if the other player chooses α, then this strategy yields 0. Therefore this game has two strict Nash equilibria - $\bar{\alpha}$if both players choose α, and $\bar{\beta}$ if both players choose β. This bring the element of strategic uncerntainty - the dilemma is should the players choose higher payoff in equilibrium $\bar{\beta}$, bearing risk that they can receive 0 as well, or should they favor safe action in equilibrium $\bar{\alpha}$? The result is obtained by adding small amount of noise to the game, namely considering that there is incomplete information about x.

Then we move to the article by Morris and Shin [2003], and modify their definitions to suit the models I will solve later.

The symmetric binary action global game we will use in solving the first proposed model has following properties :

There is a continuum of players of measure 1 - what I should point out here is the fact that every single player is considered negligble in comparison to the others. This was corrected by Heinemann [2000], who also presents corrected statement regarding the original paper.

Each of the players has to choose an action $a \in \{0,1\}$, where 1 stands for rollover and 0 for foreclosure. These two actions would be interpreted in later.

All players have the same payoff function $u : \{0,1\} \times [0,1] \times \mathbb{R} \to \mathbb{R}$, where $u(a,l,x)$is a player's payoff if he chooses action a, proportion l of his opponents choose action 0, and his "private signal" is x.

We assume that his payoff is independent of which of his opponents choose action 1.

Best response is defined only as payoff gain from choosing one action and not the other - for best response itself, we do not care for behavior of the others.

The utility function is parametrized by a function $\pi : [0,1] \times \mathbb{R} \to \mathbb{R}$ which has following form

$\pi(l,x) \equiv u(1,l,x) - u(0,l,x)$

Finally, we say that the action is Laplacian if it is best response to a uniform prior over the oponents' choice of action. Thus the action 1 is the Laplacian action at x if

$\int\limits_{l=0}^{1} u(1,l,x)dl > \int\limits_{l=0}^{1} u(0,l,x)dl,$

equivalently

$\int\limits_{l=0}^{1} u(1,l,x)dl > 0$

$\int_{l=0}^{1} u(0,l,x)dl < 0$

Generically, a continuum player, symmetric payoff, two action game will have exactly one Laplacian action.

A state θ is drawn according to the improper uniform density on the real line - therefore we have only posterior distribution as a consequence of. Simply speaking, improper prior is the case when the prior density goes to zero.

Player i observes a private signal $x_i = \theta + \sigma\varepsilon_i$, where $\sigma > 0$ is a real constant, which defines precision of information.

The noise terms ε_i are distributed in the population with continuous symmetric single-peaked probability density function f , with support on the real line.

The uniform prior on the real line is improper , i.e. has infinite probability mass, but the conditional probabilites are well defined: a player observing signal x_i puts density $\frac{1}{\sigma} f(\frac{\theta - x_i}{\sigma})$ on state θ, for details, see Hartigan [1983].

We impose five properties on the payoffs :

Action monotonicity:$\pi(l,\theta)$ is non-increasing in l.

State monotonicity:$\pi(l,\theta)$ is non-decreasing in θ.

Strict laplacian state monotonicity: There exist a unique θ^* solving $\int_{l=0}^{1} \pi(l,\theta^*)dl = 0$.

Limit dominance : There exist $\underline{\theta} \in \mathbb{R}$ and $\bar{\theta} \in \mathbb{R}$ such that $\pi(l,x) \leq 0$ for all $l \in [0,1]$and $x \leq \underline{\theta}$; and $\pi(l,x) > 0$ for all $l \in [0,1]$and $x \geq \bar{\theta}$.

Continuity : $\int_{l=0}^{1} g(l)\pi(l,x)dl$ is continuous with respect to signal x and density g.

For the model with large creditors, either static or sequential, it was proved by Corsetti et al. [2004] that the results concerning dominance solvability of the game can be expanded also to the class of asymmetric binary action global games.

Chapter 3

Application to Creditor Coordination Problem

In this chapter, I apply theory of global games to the problem of creditor coordination. First, to a static model of creditor coordination, where we allow for uniform improper prior about fundamentals. This results in a model with explicit solutions and therefore more educative effect, but it removes aspect of public information from the model. This can be seen as trivial from the point of view of global games literature, because it follows exactly the same logic as Corsetti et al. [2004] and Schuele and Stadler [2005] . I found that this model was already solved by Takeda [2003], but I decided to keep it because the author omitted several minor issues(namely comparative statics away from the limit and considerations about nature of the information) and also for explanatory reasons. I extend model setup from this paper, because I consider that some more realistic assumptions could explain puzzle that I proposed in the introduction - why do the creditor pools work best only for mid-size, why are they superior to either fragmented creditor structure or to larger creditor pools. My reflection is that richer action profile and more realistic nature of information could explain this. Therefore I build a new sequential move model, which reflects strategy structure adopted by novel work by Bjoenes et al. [2011], where the authors user this reasoning for a model that captures empirically observed behavior in currency attacks. In my conception this model with some modifications regarding logic of actions can also capture bargaining aspect of creditor coordination, where the large creditor can prosper from his size and capabilities. The third model is the basic sequential move model as was devised by Corsetti et al. [2004], which was already solved by Schuele and Stadler [2005] and I only use for comparison of the results. Next we move to illustrative results from one empirical study that directed me to my research topic.

In Krahnen and Brunner [2000] we can find evidence that creditor's structure is fundamental for solvent debtors to stay liquid. The authors present evidence from German banks where they observe that crucial aspect for succesfull creditor coordination, attaining best possible outcome for either creditors or debtors, is formation of so-called creditor pool. A creditor pool is an alliance that some of the small creditors form, and therefore they create the large creditor. But the question is why the empirical evidence contradicts theoretical insights that with larger creditor pool the results are worse than for the smaller ones. This contradicts basic observations from global games literature, which shows that there is a linear relation between size of the large player in general and consecutive effect on the result of the game. This is because of the fact that with increasing size of the large player the strategic uncertainty in the game vanishes, as can be seen in the original paper on the asymmetric binary action global game by Corsetti et al. [2004], where the effect of size the large currency speculator results in higher probability of currency attack. But in this paper the effect of signalling (to let the others know what you are doing) and relative information structure (what would the large player do, if he believes that he is better informed than the small players?) is considered as well and the sequential move model is built on this considerations. This is what is what I consider in this thesis and I compare the results that completely reflect this reasoning with results that I obtain if I adopt more realistic assumptions on both the information structure and the allowed actions. Now I explain what is the basic idea of creditor coordination problem in broader sense.

3.1 Motivation for Global Games Approach

As I mentioned in the introduction, there are issues in debt pricing, namely why standard debt pricing models fails in pricing high-risk corporate bonds. To illuminate this, I propose following highly stylized balance sheet, which is helpful for further explanations.

assets	liabilities
cash	short-term debt
risk assets	long-term debt
	equity

Standard approach based on this highly stylized balance sheet is that the credit risk of the company is measured as

$$\frac{\text{risk assets}}{\text{equity}}$$

i.e. the more risky assets in comparison to equity the firm has, the higher is the credit risk. If this ratio is high, we say that the firm is insolvent with high probability - it can easily happen that you will not get paid if you keep your investments in this firm. This is what we basically call fundamental risk in this work.

But there are also some other risks, namely risks that creditors would see their short term too high in comparison to cash (and its equivalents) of the firm, therefore believing that there is substantial probability that some creditors will not get paid if they would not collect their collateral early

$$\frac{\text{short-term debt}}{\text{cash}}$$

i.e. there is some probability that you would be better off if you would cash out your collateral early, instead of rolling over the debt. This is what we call strategic risk in this work.

Please note that the major impact of global games literature is that it has focused at the addition of strategic risk components instead of focusing on fundamental risk only. This is the basic concept of creditor coordination problem, which is reflected in defaultable debt pricing, and this is what I further put to use for explanation of behavior of the creditors under various assumptions on their structure, information and possible actions.

3.2 Basic setup of the model

First, we set up a basic model of creditor coordination. We have one large creditor and a continuum of small creditors who face uncertainty about yield of debt. They can either chose to withdraw their investment, thus cashing out collateral k , or wait until maturity of the debt and obtain liquidation value v, which depends on success of the investment. Therefore we have three states of debt - best outcome is that loan succeeds and creditors are paid liquidation value L - this is uncertain, worse outcome is that the creditors foreclose on a loan a receive their collateral K^* - this is certain, and the worst outcome is that creditors roll over the loan until maturity but the loan fail and they are paid only K_* - this is uncertain as this is the worse state of possibility of receiving L.

$$K_* < K^* < L$$

What is essential is the fact that there are two aspects that have effect on either success or failure of the project - not only economic fundamentals θ, but also proportion of creditors l, which denotes proportion of creditors who foreclose on a loan. This results in a payoff function v

$$v(\theta, l) = \begin{matrix} V \Leftrightarrow l \leq \theta \\ K_* \Leftrightarrow l > \theta \end{matrix}$$

Where V is a constant greater than L. If we normalize $L = 1$ and $K_* = 0$ (interpretation is not important, but you can imagine this is a percentage of best possible payoff), we get matrix of possible payoff, where $k = \frac{K^* - K_*}{L - K_*}$.

	project succeeds	project fails
Roll over loan	1	0
Foreclose on loan	k	k

This matrix of payoffs summarizes posibble outcomes of the game in this model.

We also assume that either large or small creditors are risk neutral. Risk neutrality means that decision maker's Bernoulli utility function of money is linear.

Basic insight of the model is as follows : if we remember that we wrote 1 for rollover and 0 for foreclosure, we have what

$$u(1,l,\theta) = \begin{array}{c} 1 \Leftrightarrow l \leq \theta \\ 0 \Leftrightarrow l > \theta \end{array}$$

which denotes utility from the action "rollover" and

$$u(0,l,\theta) = k$$

which denotes utility from the action "foreclosure" and which holds for all states of the world θ. This is what is often described in the global games literature as the "status quo", the safe action.

Next consider what is the utility under various states of the world - this is simply utility in the form

$$\pi(l,\theta) = u(1,l,\theta) - u(0,l,\theta) = \begin{array}{c} 1-k \Leftrightarrow l \leq \theta \\ -k \Leftrightarrow l > \theta \end{array}$$

. And if we apply Strict Laplacian State Monotonicity we already know from the setup that

$$\int\limits_{l=0}^{1} \pi(l,\theta)dl = \begin{array}{c} -k \Leftrightarrow \theta \leq 0 \\ \theta - k \Leftrightarrow 0 \leq \theta \leq 1 \\ 1-k \Leftrightarrow \theta \geq 1 \end{array}$$

which means that $\theta^* = k$ is the equilibrium condition, which we will derive in the models later, this holds for the case small creditors.

As the last remember I should point out that the primary insight of the model is that value of collaterilazed debt is

$$V(k) = kF(k) + 1 - F(k) = 1 - (1-k)F(k)$$

,where F and f are distribution a probability function of normal distribution, and his value increases with value of collateral k

$$\frac{dV(k)}{dk} = F(k) - (1-k)f(k)$$

Those are basic insights from the introductory paper by Morris and Shin [2003] and now, we turn our attention to various interpretations this basic setup allows.

3.2.1 The model with asymmetric information

The first model I solve is the static model of creditor coordination, where the information is asymmetric. This means that information perceived by each player is different, but the distribution function of the underlying random variable is the same for all. This allows the player to assess the actions of the others - he does not know what the others know, but this is sufficient. Please consider that this model was also partially solved by Takeda [2003], so my own input is that when I solved it previously on my own I considered some extra questions, that I will mention during the progress of solution.

Information

Information perceived by large and small creditors varies, which we will consider later, is a realization of random variable. To formalize the information structure we have that small creditors observe realization of random variable x_i about fundamentals $\theta$with precision σabout random variable ε_i ,formally

$$x_i \quad = \quad \theta + \sigma\varepsilon_i \tag{3.1}$$

where ε_i is a random variable distributed accordingly to smooth symmetric density function f with mean 0 and σ, precision of information, is a real constants greater than zero. Cumulative distribution fuction of this probability density function f is denoted F.

Large creditor observes realization of random variable y about fundamentals $\theta$with precision τ about random variable η ,formally

$$y \quad = \quad \theta + \tau\eta \tag{3.2}$$

where η is a random variable distributed accordingly smooth symmetric density function g with mean 0 and τ , precision of information, is a real constants greater than zero. Cumulative distribution fuction of this probability density function g is denoted G.

Assume also that ε_i is independent and identically distributed random variable and each ε_i is independent of η.

3.2.1.1 Small creditors only

We have a continuum of small creditors of measure 1. Small creditor i receives signal x_i and foreclose on a loan if his signal is lower than threshold value x^*. What is important is that small creditors do not observe any public information, which means that there is no source of multiple equlibrium in the settings. This is what was proved by Morris and Shin [1998].

First, we consider decision of small creditor based on their private signal $x_i = \theta + \sigma\varepsilon_i$, which defines proportion of foreclosing creditors l.

$$P(x_i \leq x^*|\theta) = F(\frac{x^* - \theta}{\sigma}) = l \tag{3.3}$$

From payoff function we know that critical level of foreclosure with given θ depends on participation of creditors l , therefore rollover of the debt is not allowed if

$$l > \theta \tag{3.4}$$

.
This gives us condition for critical volume of creditors who received signall less than threshold value x^*, which is

$$\theta^* = l \tag{3.5}$$

and defines switching point between rolling over the debt and foreclosing.
Therefore we obtain the first equilibrium condition for foreclosure

$$F(\frac{x^* - \theta^*}{\sigma}) = \theta^* \tag{3.6}$$

which results in switching strategy x^*.

$$x^* = \theta^* + \sigma F^{-1}(\theta^*) \tag{3.7}$$

This defines critical signal below which every single small creditor foreclose on a loan because he thinks that everyone else would do so.
Next we consider the question of behavior of creditor who received signal $x_i = \theta + \sigma\varepsilon_i$ about fundamentals θ^*.This defines conditional probability of project's success and the small creditor observing signal $x_i = \theta + \sigma\varepsilon_i$ assigns following probability about fundamentals strong enough for rolling over the debt

$$P(\theta \geq \theta^*|x_i) = 1 - F(\frac{\theta^* - x_i}{\sigma}) = F(\frac{x_i - \theta^*}{\sigma}) \tag{3.8}$$

.
Then the small creditor considers if the expected gain from rolling over the debt is higher than collateral k.

$$F(\frac{x_i - \theta^*}{\sigma}) \geq k \tag{3.9}$$

This is because he is risk neutral and chooses higher payoff irrespectively of risk.

Therefore his second equilibrium condition for rollover is given as expected payoff not lower than collaterally k , formally

$$F(\frac{x^* - \theta^*}{\sigma}) = k \tag{3.10}$$

which we solve for explicit switching strategy x^*

$$x^* = \theta^* + \sigma F^{-1}(k) \tag{3.11}$$

Results above give use following proposition :
Proposition 1. If $\lambda = 0$,the equilibrium is defined by these two equations

$$x^* = k + \sigma F^{-1}(k) \tag{3.12}$$

$$\theta^* = k \tag{3.13}$$

Note that for precision $\sigma \to 0$ we have that

$$\lim_{\sigma \to 0} x^* = k \tag{3.14}$$

as k - value of collateral, is some real constant and $F^{-1}(k)$ is a real constant as well.

To summarize behavior of small creditors, we can say that they will foreclose on the debt if realization of fundamentals θ is lower than k, and individual creditors will foreclose on the debt if their private signal x_i about this realization of fundamentals θ was lower than $k + \sigma F^{-1}(k)$. But note that with increasing precision of information individual creditors will foreclose on the debt if their signal is lower than k, which equals foreclosure of creditors as a group.

3.2.1.2 Large creditor only

For large creditor only, the problem of coordination simplifies into a simple decision about expected payoff as there is no coordination problem. Hence the large player who observes signal y decides about foreclosure only by considering his threshold value y^*, which results from his private information $y = \theta + \tau\nu$.

As he is the only one creditor, he is better off if expected gain from lending is higher than the value of collateral

$$P(\theta \geq 0|y) = 1 - G(\frac{0-y}{\tau}) = G(\frac{y}{\tau}) > k \tag{3.15}$$

Which gives us optimal condition for foreclosure on loan

$$G(\frac{y^*}{\tau}) = k \tag{3.16}$$

And threshold value of y

$$y^* = \tau G^{-1}(k) \tag{3.17}$$

This gives us following statement about behavior of large creditor :

Proposition 2. If $\lambda = 1$, the equilibrium is defined by equation

$$y^* = \tau G^{-1}(k)$$

And if we consider limiting cases, then

$$\lim_{\tau \to 0} y^* = 0 \tag{3.18}$$

as k - value of collateral, is some real constant and $G^{-1}(k)$ is a real constant as well.

Resumé is that large creditor forecloses on the debt if his private signal y about fundamentals θ is lower than $\tau G^{-1}(k)$. Note that as precision of his information goes to zero, i.e. his information is very precise, he decides to foreclose only if his observed private signal is lower than 0.

Conclusion on benchmark cases

In this simplified analysis, we showed the idea of coordination - for the small creditors, where the result is determined by behavior of many, it is fundamental, but for the large creditor there is none.

Motivation for small and large creditors problem

But next we show what was meant by this line from Prime Minister of Malaysia, Mohammad bin Mahathir in response to speculative attack by George Soros during South Asia's currency and financial crisis in 19996. For full article see Friedman [1997].

> "We thought they were helping us to prosper. We conducted road shows to encourage them to invest in our share and financial markets. We will continue to do so," he said, adding: "We still believe there are sincere investors out there. But there are also quite a few rogues who can cause an avalanche forcing others to run for cover."

If we translate what Mr Mahathir said into language of global, we have following model of large and small players - in this case not speculators, but creditors. And in the end we will show it does not matter. Even though it was targeted mostly to currency attacks, we can find analogy in the end. I chose to present this particular statement instead of statement directly related to creditor coordination as it is related in principle and well known from public debate.

3.2.1.3 Large and small creditors

If we consider case where there both large and small creditors, situation change. The large creditor pays attention to behavior of small creditors as he is no longer determining equilibrium alone. The small creditors also pay attention to existence of large creditor as there is major change if the large player is participating or not.

First, we consider small players and signal they perceive. Consider that small players represent $1-\lambda$of continuum of creditors with measure 1. Remaining $\lambda$is volume of large creditor. Therefore for volume of small players observing signal higher than x^*we have that

$$(1-\lambda)P(x_i \geq x^*|\theta) = (1-\lambda)(1-F(\frac{x^*-\theta}{\sigma})) = (1-\lambda)F(\frac{\theta-x^*}{\sigma}) \quad (3.19)$$

for the loan to be rolled over with only small creditors participating, it must be that

$$(1-(1-\lambda)F(\frac{\theta-x^*}{\sigma})) < \theta \quad (3.20)$$

and consequently critical value of fundamentals for only small creditors participating is

$$\overline{\theta} = (1-(1-\lambda)F(\frac{\overline{\theta}-x^*}{\sigma})) \quad (3.21)$$

. This determines value of fundamentals where lending only by small creditors is sufficient for rolling over the debt.
Next we move to the case where also the large creditor takes part in rolling over the debt. For large and small creditors participating simultaneously it must be that

$$(1-\lambda+(1-\lambda)F(\frac{\theta-x^*}{\sigma})) < \theta \quad (3.22)$$

which gives us second critical value of fundamentals for both large and small creditors participating is

$$\underline{\theta} = (1-\lambda-(1-\lambda))F(\frac{\underline{\theta}-x^*}{\sigma}) \quad (3.23)$$

. This determines value of fundamentals that is sufficient for rolling over the debt if the large creditor participates too.

As both critical values of fundamentals are functions of switching strategy $x^*$of small creditors, which consequently depends on switching strategy y^* of large creditor, we must solve for this switching point simultaneously by considering equilibrium condition of large creditors given his expected gain from foreclosure.

$$P(\theta \geq \underline{\theta}|y) = (1-G(\frac{\underline{\theta}-y^*}{\tau})) = G(\frac{y^*-\underline{\theta}}{\tau}) > k \quad (3.24)$$

which gives us switching strategy of large creditor in an implicit form

$$G(\frac{y^* - \underline{\theta}}{\tau}) = k \tag{3.25}$$

and explicitly

$$y^* = \underline{\theta} + \tau G^{-1}(k) \tag{3.26}$$

. This defines value of fundamentals given precision of information, $\tau \to 0$ above which the large creditors wishes to roll over the debt.

Please note that with increasing precision of information we have that

$$\lim_{\tau \to 0} y^* = \underline{\theta} \tag{3.27}$$

. Which also applies for the small creditors, which have under $\sigma \to 0$

$$\lim_{\sigma \to 0} x^* = \underline{\theta} \tag{3.28}$$

. Implications of this behavior will follow in the comparative statics in the limit section.

Next we move to the question what would be the small creditor's action for values of fundamentals above $\underline{\theta}$. If we consider probability that small creditor continues lending conditional on θ we have that

$$F(\frac{\theta - x_i}{\sigma}) \tag{3.29}$$

and if we derivate this we get the probability density of small creditor

$$\frac{dF(\frac{\theta - x_i}{\sigma})}{d\theta} = \frac{1}{\sigma} f(\frac{\theta - x_i}{\sigma}) \tag{3.30}$$

where it holds that for fundamentals $\theta \geq \bar{\theta}$, the small creditors alone are sufficient for successful rolling over, and for $\bar{\theta} > \theta \geq \underline{\theta}$ it depends on the large creditor - success comes only if large creditor takes part in lending too, and for fundamentals$\theta < \underline{\theta}$ the debt would be always foreclosed.

This jointly gives us this equation for expected payoff from lending

$$P(\underline{\theta} \quad \leq \quad \theta \leq \bar{\theta}, y > y^* | x_i) + P(\theta \geq \bar{\theta} | x_i) =$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f(\frac{\theta - x_i}{\sigma}) G(\frac{\theta - y^*}{\tau}) d\theta + \int_{\bar{\theta}}^{\infty} \frac{1}{\sigma} f(\frac{\theta - x_i}{\sigma}) d\theta \tag{3.31}$$

which must be higher than collateral k

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f(\frac{\theta - x_i}{\sigma}) G(\frac{\theta - y^*}{\tau}) d\theta + \int_{\bar{\theta}}^{\infty} \frac{1}{\sigma} f(\frac{\theta - x_i}{\sigma}) d\theta \geq k \quad (3.32)$$

and therefore in the equilibrium we have that

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f(\frac{\theta - x^*}{\sigma}) G(\frac{\theta - y^*}{\tau}) d\theta + \int_{\bar{\theta}}^{\infty} \frac{1}{\sigma} f(\frac{\theta - x^*}{\sigma}) d\theta = k \quad (3.33)$$

. This equation determines expected payoff from rolling over the debt, where the first part of the left hand side of the equation is expected payoff from behavior of small and large creditors jointly and the second part of this equation denotes expected payoff from small creditors only. As the large creditor's switching strategy is already defined, we proceed to switching strategy of the small creditors. To do so, we create following substitutions
$z = \frac{\theta - x^*}{\sigma}$,$\bar{\delta} = \frac{\bar{\theta} - x^*}{\sigma}$,$\underline{\delta} = \frac{\underline{\theta} - x^*}{\sigma}$

$$\int_{\underline{\delta}}^{\bar{\delta}} f(z) G(\frac{\theta - y^*}{\tau}) dz + \int_{\bar{\delta}}^{\infty} \frac{1}{\sigma} f(z) dz \quad (3.34)$$

$$\int_{\underline{\delta}}^{\bar{\delta}} \frac{1}{\sigma} f(z) G(\frac{x^* + \sigma.z - \tau G^{-1}(k)}{\tau}) dz + \int_{\bar{\delta}}^{\infty} \frac{1}{\sigma} f(z) dz \quad (3.35)$$

and the same condition for equilibrium applies too, total expected payoff must be equal to k

$$\int_{\underline{\delta}}^{\bar{\delta}} \frac{1}{\sigma} f(z) G(\frac{\sigma}{\tau}(z - \underline{\delta}) + G^{-1}(k)) dz + \int_{\bar{\delta}}^{\infty} \frac{1}{\sigma} f(z) dz = k \quad (3.36)$$

But the problem is that this equation has not got any closed form solution. This will be solved as a comparative statics in the limit and away from the limit as was proposed in Corsetti et al. [2004].
To summarize the results, we have that

$$\overline{\theta} = (1 - (1 - \lambda) F(\frac{\overline{\theta} - x^*}{\sigma})) \quad (3.37)$$

$$\underline{\theta} = (1 - \lambda - (1 - \lambda)) F(\frac{\underline{\theta} - x^*}{\sigma}) \quad (3.38)$$

$$G(\frac{y^* - \underline{\theta}}{\tau}) = k$$

$$\int_{\underline{\delta}}^{\bar{\delta}} \frac{1}{\sigma} f(z) G(\frac{\sigma}{\tau}(z-\underline{\delta}) + G^{-1}(k))dz + \int_{\bar{\delta}}^{\infty} \frac{1}{\sigma} f(z)dz = k$$

what is interesting now is the question what would be the effect of change of the critical signal in comparison to the boundaries of this integral,

$$\frac{d\bar{\delta}}{dx^*} = \frac{d\frac{\bar{\theta}-x^*}{\sigma}}{dx^*} = -\frac{1}{(1-\lambda)f(\frac{\bar{\theta}-x^*}{\sigma}) + \sigma} = \frac{1}{(1-\lambda)f(\bar{\delta}) + \sigma} < 0$$

$$\frac{d\underline{\delta}}{dx^*} = \frac{d\frac{\underline{\theta}-x^*}{\sigma}}{dx^*} = -\frac{1}{(1-\lambda)f(\frac{\underline{\theta}-x^*}{\sigma}) + \sigma} = \frac{1}{(1-\lambda)f(\underline{\delta}) + \sigma} < 0$$

as we can see, the effect is negative, so that for sufficiently small x^* it holds that

$$\int_{\underline{\delta}}^{\bar{\delta}} \frac{1}{\sigma} f(z) G(\frac{\sigma}{\tau}(z-\underline{\delta}) + G^{-1}(k))dz + \int_{\bar{\delta}}^{\infty} \frac{1}{\sigma} f(z)dz < k$$

and for sufficiently large x^* it holds that

$$\int_{\underline{\delta}}^{\bar{\delta}} \frac{1}{\sigma} f(z) G(\frac{\sigma}{\tau}(z-\underline{\delta}) + G^{-1}(k))dz + \int_{\bar{\delta}}^{\infty} \frac{1}{\sigma} f(z)dz > k$$

. As the equation $\int_{\underline{\delta}}^{\bar{\delta}} \frac{1}{\sigma} f(z) G(\frac{\sigma}{\tau}(z-\underline{\delta}) + G^{-1}(k))dz + \int_{\bar{\delta}}^{\infty} \frac{1}{\sigma} f(z)dz$ is continuous in x^*, we know from , namely from mean-value theorem and from the fact that this function is strictly monotonic, that there is a unique solution for x^*, and once x^* is solved, we can consequently determine y^*, as y^* is a response to x^*. But the problem remains that in general we do not know any of them explicitly. Once we have proved that there exist unique solution (for proof of iterated dominance argument for asymmetric binary global games see Corsetti et al. [2004], this is extension of proof from Morris and Shin [2003]), we move to what is the objective of these computation - explicit solution for $\bar{\theta}$ and $\underline{\theta}$.
We have two options - first, we impose some limiting properties on precision of information, and the second is the possibility to compare possible states of $\bar{\theta}$ and $\underline{\theta}$ away from the limit.

3.2.1.3.1 Comparative statics in the limit This was already solved by Takeda, so I restrict from repeating his solutions. Following original work by Takeda [2003], we have that for

$\sigma \to 0, \tau \to 0, \frac{\sigma}{\tau} \to r$

, that is for very precise information of both groups of creditors, we have that

$$\frac{d\underline{\theta}}{\underline{\lambda}} > 0 \tag{3.39}$$

This boundary condition is the only one that is interesting, because it defines range of fundamentals below which the foreclosure will occur.

3.2.1.3.2 Comparative statics away from the limit This is what the author omitted, so I present this solution. Results away from the limit are more complex than those in the limit. There are additional beliefs about probabilities of successful and unsuccessful rollovers, so that we are always unsure what could happen in the interval $\bar{\theta} > \theta \geq \underline{\theta}$, but we know for sure that debt would be always rolled over for fundamentals $\theta > \bar{\theta}$ and always foreclosed for fundamentals $\theta < \underline{\theta}$. What is important for results in the interval $\bar{\theta} \geq \theta \geq \underline{\theta}$ is the fact that the lending is successful only if the large players is participating. Our results are summarized in the following table.

probability of rollover	state of fundamentals
1	if $\theta > \bar{\theta}$
$G\left(\frac{\underline{\theta}-\theta}{\tau} + G^{-1}(k)\right)$	if $\bar{\theta} \geq \theta \geq \underline{\theta}$
0	if $\theta < \underline{\theta}$

Probability of rolling over the debt is given by

$$G(\frac{y^* - \theta}{\tau}) \tag{3.40}$$

and if we consider that switching strategy for the large creditor is

$$y^* = \underline{\theta} + \tau G^{-1}(k) \tag{3.41}$$

we get the probability that the large player continues lending

$$G\left(\frac{\underline{\theta} - \theta}{\tau} + G^{-1}(k)\right)$$

3.2.2 The Model with Symmetric Information

Here I present the model of creditor coordination where symmetric information. This brings us back to the beginning, where I explained motivation for global games approach - here I show consequences of symmetric information assumption in the model of creditor coordination. This is what would be the result of adopting approach similar to Obstfeld [1986], which he used for explanation of currency crises.

In the setup of the model, the only change from asymmetric information model is that we have information symmetric for all creditors i

$$a_i = a \quad = \quad \theta + \rho\chi \tag{3.42}$$

, so that small creditors observe realization of random variable a_i about fundamentals $\theta$with precision ρabout random variable χ. χ is a random variable distributed accordingly to smooth symmetric density function h with mean 0 and ρ, precision of information, is a real constants greater than zero. Cumulative distribution fuction of this probability density function h is denoted H.

Remember that for $\theta > 1$, even if no one is lending, the project succeeds, for $\theta < 0$ the project fails irresptively what the creditors do and for $0 \leq \theta \leq 1$ the project succeeds if less than l of mass of creditors foreclose. If we consider our previous results where we have two Bayesian-Nash equilibria, we have that everyone either foreclose or rollover the debt, based on value of fundamentals.

Therefore if we consider expected gain from lending for individual creditor, conditional on $l = 0$, everyone is lending, we have that

$$P(\theta \geq 1|a) - k.P(\theta < 1|a) = H(\frac{a-1}{\rho}) - k \tag{3.43}$$

which is implicit condition for critical signal condition, which we solve for

$$a_0^* = 1 + \rho H^{-1}(k) \tag{3.44}$$

.

This results in condition for critical signal the individual small creditor has to receive so that he foreclose on the debt. If he receives higher signal, he continues lending, and if he receives lower signal, he forecloses.

But we also have to account for the second possibility, that individual creditor believes that $l = 1$,i.e. everyone foreclosed on the debt. For this condition we have that expected gain from lending

$$P(\theta \geq 0|a) - k.P(\theta < 0|a) = H(\frac{a-0}{\rho}) - k = H(\frac{a}{\rho}) - k \tag{3.45}$$

and resulting equilibrium condition for critical signal conditionally on $l = 1$ is

$$a_1^* = \rho H^{-1}(k) \tag{3.46}$$

.

Therefore we have two equilibrium conditions which have the same rationalization - each of them is optimal strategy for individual small creditor conditional on his beliefs about behavior of the others. This is the major result of global games, where (albeit under some restrictions, for details on uniqueness conditions see Morris and Shin [1998]) there is unique equilibrium due to assymetricity of information.

If we consider what could happen, we have two ranges of signal where we know the action undertaken - for signal higher than a_0^*, all creditors rollover, as this is also higher than a_1^*. For signal lower than a_0^*(which is also lower than a_1^*) all creditors foreclose. This is true for sure, but now consider that critical signal is not higher than a_0^* and not lower than a_1^* - there is no decision rule, so in this case, the decision is purely random. This is stronger claim than in the case of self-fulfilling prophecy, where the outcome is determined by beliefs of players, this is only random as the creditors do not have any decision rule.

3.3 Extended creditor coordination model with sequential moves

Then, we solve creditor coordination game with delay allowed. This brings interesting strategy structure, as the players do not consider only actions of the others, but also timing of the other players. Here we adopt approach based on work from Bjoenes et al. [2011], where they adopted novel approach for solving currency attack model with sequential moves. They argue that in previous sequential moves models, such as can be found in Schuele and Stadler [2005] and Corsetti et al. [2004], there is a perfect information about behavior of the large player, and furthermore, those models allow only for strict structure of delay in actions - large player either moves first, or does not move at all. Bjoenes et al. [2011] also stress the fact that in empirical studies, large currency traders often move as the first, inducing attack, and then they move again after the small currency traders attack the currency, so they do not bear any risk and cash in on results of their early speculation.

I adopt basic idea of this model in a model of creditor coordination, where the interpretation of my model is as follows - the large creditor continues lending to the debtor, but he does not lend all he could lend, he waits for small creditors to participate on rolling over the debt and if there is some extra need for credit, he provides debtor with any requested amount of credit as the large creditor has easy access to credit and knows that he will gain profit in lending. Inspiration for doing this was found in the article by Krahnen and Brunner [2000], where they emphasize impact of banks and creditor pools. A creditor pool is a coalition of small creditors who behave as a large creditor, should we interpret this in language of global games. They underline that negotiations about sharing risk from debtor´s bankruptcy are much easier if there is a large creditor (a bank or a creditor pool), and those negotiations are often lengthy, resulting in gradual lending, where creditors behave very cautiously until the success is almost sure.

To formalize this, I create this model :

First, consider that order of actions is as follows -

- period 1 - $\theta$is chosen by nature
- period 2 - creditors receive information
- period 3 - large creditor decides to roll over or foreclose with a view to induce further lending by small creditors
- period 4 - small creditors decide to roll over or foreclose based on information that was amended by large creditor´s early move, their joint action until period 4 determinates either success or failure of the lending
- period 5 - if the early lending by large and small creditors is successful, large creditor decides to lend more, if he has the possibility

Then, I denote volume of lending by each respective creditor at each period in a following way :

- Volume of early lending by large creditor α is the volume of credit that large creditor wishes to lend with a view of influencing small creditors´ behavior
- Volume of lending by small creditors γ , which denotes volume of lending by the small creditors after they observes information about state of the debt, which was previously influenced by large creditor´s lending.
- Volume of late lending by large creditor β is the volume of lending that large creditor wishes to lend after he is affirmed in his views on debtors solvency and profitability. Note that it is also possible that large creditor may, or may not have chance to lend in period 5, after small creditors´ action, which we denote by probability q that there is no such oportunity.

I also assume, that large creditors lending is not limited, i.e. he has access to credit which is not limited, if he considers that lending is not profitable. The only limitation is the fact that α will be never more (or equal to) 1, as it contradicts basic setup of the model.

As we mentioned in introduction to this model, information perceived by small creditors is not based on fully credible large player as in Schuele and Stadler [2005], but it depends on real effects of large creditor on economic fundamentals, increasing small creditor's signal by large creditor's volume of early loan α

$$x_i = \theta + \alpha + \sigma\varepsilon_i \tag{3.47}$$

.

But information perceived by large creditor remains the same,

$$y = \theta + \tau\eta$$

as the large creditor´s views are not changed, he has only richer set of strategies.

Next we impose following condition on success of the project, where $\alpha + \gamma$ denotes total volume of lending available to debtor. The project fails if

$$(\alpha + \gamma) \geq \theta$$

which gives us condition for foreclosure

$$(\alpha + \gamma)^{*} = \theta \tag{3.48}$$

As in the static model, we first solve for small creditors' switching strategy x^*. In doing so, we first derive condition for small creditors who continue lending γ . To continue lending, small player must observe signal x_i about fundamentals $\theta + \alpha$ higher than x^*, formally

$$P(x_i \geq x^*|\theta + \alpha) = P(\theta + \alpha + \sigma\varepsilon_i \geq x^*) =$$

$$P(\varepsilon_i \geq \frac{x^* - (\theta+\alpha)}{\sigma}) = 1 - F(\frac{x^* - (\theta+\alpha)}{\sigma}) = F(\frac{(\theta+\alpha) - x^*}{\sigma}) \quad (3.49)$$

.

This defines volume of creditors who continue lending. We denote this γ, volume of lending by small creditors.

$$F(\frac{(\theta+\alpha) - x^*}{\sigma}) = \gamma \quad (3.50)$$

.

And considering small players' perceived information about fundamentals, we have that

$$(\theta+\alpha) \leq F(\frac{(\theta+\alpha) - x^*}{\sigma}) \quad (3.51)$$

which defines condition under which lending by small creditors after early lending by the large creditor succeeds.

This implies that the first equilibrium condition - critical mass of small creditors - is

$$(\theta+\alpha)^* = F(\frac{(\theta+\alpha)^* - x^*}{\sigma}) \quad (3.52)$$

.

Next, we move to small creditors´ expectations about strength of fundamentals which ensures that the expected gain from lending is higher than expected gain from foreclosure. Note that in the previous section we showed that in the simple static game critical signal which defines foreclosure is generally higher than critical fundamentals which defines foreclosure as well.

For small creditors' perceived expected gain from foreclosing on the debt we have that

$$P(\theta+\alpha \leq (\theta+\alpha)^* \,|x_i) = P(x_i - \sigma\varepsilon_i \leq (\theta+\alpha)^*) =$$
$$= P(\varepsilon_i \geq \frac{x_i - (\theta+\alpha)^*}{\sigma}) = 1 - F(\frac{x_i - (\theta+\alpha)^*}{\sigma}) = F(\frac{(\theta+\alpha)^* - x_i}{\sigma}) \quad (3.53)$$

and for small creditors to favor foreclosure on the debt instead of rolling over the debt it must be that

$$F(\frac{(\theta+\alpha)^* - x_i}{\sigma}) \leq k \quad (3.54)$$

which defines our second equilibrium condition, which is implicitly given by small creditors' indifference condition

$$F(\frac{(\theta+\alpha)^* - x^*}{\sigma}) = k \quad (3.55)$$

.

This condition says that for value of fudamentals less than $(\theta+\alpha)^*$ the expected gain from foreclosure is higher than expected gain from rolling over the debt.

If we solve this implicitly given condition, we get this switching strategy

$$x^* = (\theta+\alpha)^* - \sigma F^{-1}(k) \tag{3.56}$$

which defines critical signal, but there this still implicitly depends on $(\theta+\alpha)^*$, so we solve for this and if we substitute this condition for switching strategy into equation

$$(\theta+\alpha)^* = F(\frac{(\theta+\alpha)^* - x^*}{\sigma}) \tag{3.57}$$

we have that

$$(\theta+\alpha)^* = F(\frac{(\theta+\alpha)^* + \sigma F^{-1}(k) - (\theta+\alpha)^*}{\sigma}) = F(-F^{-1}(k)) = F(F^{-1}(k)) = k \tag{3.58}$$

and we obtain following switching strategy

$$x^* = k - \sigma F^{-1}(k) \tag{3.59}$$

and critical value of fundamentals

$$\theta^* = k - \alpha \tag{3.60}$$

and therefore debt is foreclosed only if perceived value of fundamentals falls below critical value of fundamentals

$$\theta < \theta^* = k - \alpha \tag{3.61}$$

.This is all we need for small creditors strategy profile and we turn our attention to the large creditor.

The large player cares only about his expected gain, not about how many peers join the attack - he has no strategic uncertainty. But there is strategic uncertainty in the group of small creditors.

Thus we evaluate what is the large creditor's volume of lending in early phase of "credit talks"

$$P(\theta \geq \theta^*|y) = P(y - \tau\eta \geq k - \alpha|y) = P(\eta \leq \frac{y-k+\alpha}{\tau}|y) = G(\frac{y-k+\alpha}{\tau}) \tag{3.62}$$

And afterward we consider his decision regarding lending in late phase of "credit talks" - but there is some probability that there is no need for late lending - either is debtor comfortable with volume of credit, or large creditor comes too late. This condition is imposed to show that late lending is not that safe and large player has to consider risks.

$$E(\pi) = G(\frac{y-k-\alpha}{\tau})\,(L(1-q)+q\alpha) - k\alpha \tag{3.63}$$

with first order condition for interior solution being

$$\frac{dE(\pi)}{d\alpha} = \frac{1}{\tau}g(\frac{y-k+\alpha}{\tau})q - k \stackrel{!}{=} 0 \tag{3.64}$$

$$g(\frac{y-k+\alpha^*}{\tau}) = \frac{\tau k}{q} \tag{3.65}$$

.

Note that if we impose condition of impossibility of late lending, setting $q = 1$, we have a model parallel to that of Schuele and Stadler [2005], except for information structure, but strategy profiles remain the same.

$$E(\pi) = G(\frac{y-k+\alpha}{\tau})\alpha - k\alpha \tag{3.66}$$

3.4 Creditor coordination model with sequential moves

Here I present work by Schuele and Stadler [2005], which completes this trio of models. The authors assume following work by Corsetti et al. [2004] that the large player benefits from signalling his decision to small player, who benefit from waiting because the only information that could affect their behavior is the move by the large player. Therefore timing of their action is as follows

- period 1 - $\theta$is chosen by Nature
- period 2 - large creditor decides to roll over or foreclose with a view to induce further lending by small creditors
- period 3 - small creditors decide to roll over or foreclose based on information that was amended by large creditor´s move.
  Results from this model setup are provided in the following section, but the were also able to figure out other results, including threshold, which I omit as it is not for the problem I attempt to answer.
  Unconvincing nature of this model inspired me to adopt approach from Bjoenes et al. [2011], where the authors suggest much more realistic set of assumptions. This includes richer time structure, concretely large player can move either before the small players move or even after they move, and the large player has also mostly unlimited access to credit.

3.5 Comparison of various models of creditor coordination

Finally I present all the three solutions to the models of creditor coordination, a static one, with sequential moves and with extended model with sequential moves.

In the static model, it holds that

$$\overline{\theta} = (1 - (1 - \lambda)F(\bar{\delta})) \tag{3.67}$$

$$\underline{\theta} = (1 - \lambda - (1 - \lambda))F(\underline{\delta}) \tag{3.68}$$

and particularly in two comparative statics cases, $\bar{\theta}$is increasing in the size of creditor in the limitting case, and away from the limit it holds that

probability of rollover	state of fundamentals
1	if $\theta > \bar{\theta}$
$G\left(\frac{\underline{\theta}-\theta}{\tau} + G^{-1}(k)\right)$	if $\bar{\theta} \geq \theta \geq \underline{\theta}$
0	if $\theta < \underline{\theta}$

Where I could not figure out closed form solution, so the possibility that the size effect of large creditor is not monotonic is not excluded.

Here I present the results from Schuele and Stadler [2005] , where the authors adopt analogical approach as is in the Corsetti et al. [2004]. I.e. they assume that small creditors wait for the large one to move first as this is mutually beneficient, as I have mentioned before.

	relatively informed	relatively uninformed
investment decision is	$\sigma/\tau \to \infty, \sigma \to 0$	$\sigma/\tau \to 0, \sigma \to 0$
Unobservable	$\theta^* = k(1 - \lambda)$	$\theta^* = \begin{matrix} k\left(\frac{1-\lambda}{1-k}\right) \Leftrightarrow \lambda > k \\ k \Leftrightarrow \lambda \leq k \end{matrix}$
Observable	$\underline{\theta}^* = 0$ and $\overline{\theta}^* = 1$	$\underline{\theta}^* = k(1 - \lambda)$ and $\overline{\theta}^* = k(1 - \lambda) + \lambda$

And the final result is from the extended sequential moves model , we have only one switching strategy profile for fundamentals

$$\theta^* = k - \alpha \tag{3.69}$$

.

None of the models captures empirically observed detrimental effect of too big large creditors, that is none of the equations concave in parameter λ,the size parameter. The only model that allows this interpretation as I was nto able to dismiss this possibility is the static model, namely when you consider its comparative statics in the limit. But as this result is mostly obscure, I consider it highly unlikely.

Next, I present a short historical perspective on global games.

Chapter 4

Economic Perspective of Global Games

The problem of coordination is one of the many social phenomena ranging across social sciences. One of the major examples of early thoughts on coordination problems can be found in Jean Jacque Rousseau´s Discourse on inequality.

> “In this manner, men may have insensibly acquired some gross ideas of mutual undertakings, and of the advantages of fulfilling them: that is, just so far as their present and apparent interest was concerned: for they were perfect strangers to foresight, and were so far from troubling themselves about the distant future, that they hardly thought of the morrow. If a deer was to be taken, every one saw that, in order to succeed, he must abide faithfully by his post: but if a hare happened to come within the reach of any one of them, it is not to be doubted that he pursued it without scruple, and, having seized his prey, cared very little, if by so doing he caused his companions to miss theirs!”

This intuitive example is what we nowadays call coordination game – this is basically what the paper of Carlsson and Van Damme turned into formal model. What is appealing, albeit not important, about the history of global games is the fact that one of the most important people in formation of this theory was Robert K. Merton, a sociologist, who coined the term of self-fulfilling prophecy in early 20th century in his paper The Unanticipated Consequences of Purposive Social Action and even though it is nonessential, the theory he helped to establish explains the problem of empirical validity of his son´s major paper and one of the most important papers of finance. For details, see Merton [1936]
.

To stress importance of global games for economic theory as such, I would like to present a few major answers that I found in currency crises literature, because the most impressive results were achieved in the field of currency attacks.

Currency attack is an attack to currency peg with a view of reaping benefits from doing so. Early models of currency attacks, called First generation models, based on work by Krugman [1979], completely omit any self-fulfilling nature of currency crises. They consider only expectations about government response to speculative attacks - in other words, they only care about probability that the government will run out of foreign reserves due to running excessive deficit. This was criticized by Obstfeld [1986], who is founder of so called Second generation models of currency attacks. His model finally incorporates aspect of self-fulfilling nature of currency crises, but in his view investors and government both observe perfect information, and therefore his model leads to multiple equilibrium, which somehow decreases impact of his work on economic policy. This was finally solved by Morris and Shin [1998], who finally create a model with deserved property of uniqueness of equilibrium where it was possible to track sources of attack and give sound economic policy advice. This was made possible by observing that dissemination of public information is a major factor of currency attack success, because central bank can render attacking speculators less aggressive by greater transparency and more precise information.

Chapter 5

Conclusion

In the beginning I posed a question if the empirical evidence from the German banks which indicates that with increasing size of the creditors the probability of foreclosure decreases.

I found that any of these setups I used or referred to cannot explain this behavior, apart from one that is highly unlikely, but I was not able to dismiss the possibility of non-monotonic effect of the large creditor. In each of these models, even in the extended sequential moves model, decreasing foreclosure rate is present, and especially in the model I built to capture potential misbehavior of the large creditor the foreclosure level of economic fundamentals is very favorable for the debtor. But what I consider positive is the fact that the result I obtained in the extended sequential moves model suggests that under those assumptions large creditors could be much more helpful regarding foreclosures. That is, based on their easy access to credit, they could easily finance every project that is threatened by the fact that creditor coordination problem captures - it decreases strategic risk as it allows small creditors to behave much more confidently. The fact that empirically observed data are contradictory can be explained by Hubert and Schaefer [2000], where the authors show that the large creditors use their power for enforcing of quasi-rents.

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Akademický rok 2009/2010

TEZE BAKALÁŘSKÉ PRÁCE

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Garant studijního programu Vám dle zákona č. 111/1998 Sb. o vysokých školách a Studijního a zkušebního řádu UK v Praze určuje následující bakalářskou práci

Předpokládaný název BP:

Coordination Games and its Applications in Economics

Charakteristika tématu, současný stav poznání, případné zvláštní metody zpracování tématu:

Coordination games are nowadays considered as a hugely promising field of mathematics with applications in economics, modelling coordination problems which arise in multiple pure Nash equilibria strategies. A typical example is the currency attack, where each group of players takes into account coordination problem.

Struktura BP:

1. Mathematical background – elements of probability and game theory
2. Analysis of a concrete model (currency attack, possibly also another one)

Seznam základních pramenů a odborné literatury:

Steiner, J.: Coordination cycles, CERGE Working Paper No. 274, 2005 (ISSN 1211-3298)

Martin J. Osborne & Ariel Rubinstein: *A Course in Game Theory*, Cambridge, Massachusetts: MIT Press, 1994 (ISBN 0-262-65040-1)

Morris, Stephen Edward and Shin, Hyun Song ,Global Games: Theory and Applications(August 2001). Cowles Foundation Discussion Paper No. 1275R. Available at SSRN: http://ssrn.com/abstract=284813

Other literature if necessary.

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Podpisy konzultanta a studenta:

V Praze dne