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Agent-Based Modeling of the Financial Markets

Bachelor thesis

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Abstrakt

Tato práce se zabývá modelováním finančních trhů pomocí agentů, což v ekonomii reprezentuje takzvaný "bottom-up" přístup, neboli přístup "odspodu". První část práce poskytuje shrnutí vývoje multi-agentového modelování a jeho aplikací v případě finančních trhů. Jádrem práce je jednak implementace již existujícího modelu autorů He, Hemill a Li (2008), ale především implementace rozšíření k tomuto modelu. Základem tohoto rozšíření je aplikace původního modelu na dva trhy, které jsou určitým způsobem korelované, přičemž korelace mezi těmito trhy je reprezentována buď korelovanými výplatami dividend, nebo tím, že ceny na obou trzích jsou upravovány jediným tvůrcem trhu. Na základě výsledků získaných provedením simulací je poté studován vliv obou typů korelací na celkový vývoj modelovaných trhů.

Abstract

The thesis deals with the Agent-based modeling of the financial markets which represent so called "bottom-up" approach in economics. In the first part of the thesis, the brief summary of the development of Agent-based approach and its application in the modeling of financial markets is provided. The main part of the thesis concerns the implementation of an existing asset pricing model of He, Hamill and Li (2008) and also the implementation of an extension to this model. The presented extension lies in the connecting of two sub-markets by a mutual correlation. The considered correlation is represented either by correlated dividends or by the common market maker who adjusts the prices on both markets. The influence of these two types of correlation on the overall performance of both sub-markets is then studied by analyzing the outcomes of performed simulations.

Klíčová slova

multi-agentové výpočetní modely, finanční trhy, fundamentalisté, trendoví obchodníci, tvůrce trhu, korelace trhů

Keywords

agent-based modeling, financial markets, fundamentalists, trend followers, market maker, market correlation

Extent of the thesis: 63769 characters

Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, May 19, 2011

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Chapter 1

Introduction

The financial markets are characterized by their high complexity. This is why it is so difficult for "classical" economic approaches – the theoretical and the experimental approach – to explain, reproduce, or forecast many of the features that can be observed in real financial markets. Some questions are even unfeasible for this methods and both approaches fail in many cases. One can mention, for instance, the quite actual problem of inability to predict the financial crisis. All the problems of "classical" methods together with an expansion of computational technologies lead to the development of an alternative approach called Agent-based modeling (ABM), which is also the methodology used in my thesis.

Agent-based modeling represents so called "bottom-up" approach, i.e. it studies the impact of a behavior of individuals, agents, on the development of a whole market or an economy. Unlike the experimental approach, agent-based models of markets are not populated with real people. Agents are in fact the programs with specified rules and they only imitate the behavior of real individuals. Simulations are then conducted where agents interact, which then determine the behavior of the market. The advantage is that the rules of agents' behavior can be set according to the needs of given experiment, therefore it is easier to more precisely analyze the influence of various specific agents' characteristics to different market phenomena.

However, Agent-based approach has also its limitations. Every agent-based model brings many new parameters that have to be calibrated to obtain a model that corresponds to the data from real financial market.

In the first part of my thesis I provide a brief summary of the development of the agent-based approach and its application in the modeling of financial markets. That is, I create an overview of substantial papers in this field and their findings. The aim of this review is to point out the importance of the "bottom-up" approach in comparison with analytical methods or with experimental markets, especially in cases when these "classical" methods are simply unfeasible. I also point out the variety of questions economists seek to answer by examining the financial markets and a capability of agent-based models to generate the stylized facts observed in real financial markets.

In the second part I implement one of the existing models, specifically, the asset pricing model with market maker presented e.g. by He, Hamill and Li (2008). First, I carry out the numerical analysis and imitate the outcomes obtained in the articles introducing this model. After this, I create an extension of the current model and make the numerical simulation that brings new results.

I create the market with two separate sub-markets. The market structure of each sub-market is the same as in He, Hamill and Li (2008) and traders in each of them can trade only with the other traders in the same sub-market. The difference in comparison to the initial model is such that there is a correlation between the two markets, which can influence the behavior of both markets. I provide outcomes from agent-based simulations of sub-markets joined by two of the possible connections – by correlated dividend process and by one common market maker. The aim of the numerical analysis is to examine the impact of this to the overall performance of the market.

The last part of the thesis concludes and outlines some further questions for possible research.

Chapter 2

Literature Review

Agent-based models are used for studying the role of agents they play in the development of a whole market or an economy. These agents could represent practically anything – such as consumers, companies, or states. The significant features of these models are a heterogeneity and a bounded rationality of agents. Unlike analytical models, there is no need to calculate with rational and homogeneous traders only. Each agent may have her own specific behavioral rules; in other words, there are no "representative agents". Agents can differ in strategies, quantity of available information, speed of learning, risk aversion, etc.

The heterogeneity of agents leads us to one kind of taxonomy of agent-based models of financial markets. So called few-type and many-type models may be distinguished. To the former category models containing only two or three different types of agents are count. The most familiar example of this is a chartist-fundamentalist model. Chartists, called also trend followers or technical traders, believe that the price itself carries information about its future dynamics. Therefore, they build their price forecasts upon the historical price movements. On the contrary, fundamentalists believe that every price converges to its long-run fundamental value and they do not employ any information about past prices in their forecasting.

Many-type models cover more complex configurations with many various groups of agents. This category include also models where every single agent has her own behavior. As a representative of many-type models let us mention Santa Fe artificial market (see LeBaron, Arthur and Palmer,1999, or Arthur at al., 1997).

Santa Fe artificial stock market is one of the most complex artificial markets. Although the market setup is simple, it is the asset pricing model, where agents are choosing between risky assets yielding a random dividend and risk-free bonds paying a constant interest rate, there are quite sophisticated rules for trader's learning and forecasting. The price and the dividend forecasts are created using the classifier forecasting system. This system tries to determine the relevant state of the economy and based on this, traders select between different existing forecasts that are conditioned on certain pieces of market information. A modified Genetic algorithm is used as a learning mechanism that changes the current set of forecasting strategies. The Genetic algorithm works similarly to real organisms - mutations and crossovers are implemented both on the real components and bitstrings of the rules to create new ones, together with the principle of removing the worst performing rules.

Other type of artificial markets is Genoa artificial market, described by Raberto, Cincotti, Focardi, and Marchesi (2001). It simulates a kind of herding behavior; buyers and sellers cluster randomly into larger dependent groups which act together. Agents at each group first decide whether they are buyers or sellers at random, and then place a buy or sell orders determined by their budget constraints. After every period a market-clearing price is given by crossing the supply and demand curves appointed by the current limit orders. The Genoa market generates some facts present in the real world markets, like uncorrelated returns, fat tailed return distributions, or persistent price volatility, however, also some features which are in contradiction with real market occur – e.g. the volatility exhibits an exponential decay, while empirical studies reveal a power law decay.

Another division of agent-based models may be made based on the intelligence of agents. It varies from so called zero-intelligence agents to agents using neural networks (see e.g. Beltratti and Margarita, 1992, or Beltratti at al., 1996). The case of zero-intelligence was examined e.g. by Gode and Sunder (1993). It was shown that even the market with agents behaving almost completely randomly (issuing random bids and offers) converges closely to equilibrium prices and gives results very close to that for humans if some budget constraint are set.

More sophisticated agents can be found for example in Lettau (1997) or Arifovic (1996). Both papers use a genetic algorithm to improve trader's learning and forecasting. The former one employs the setting of the simple asset pricing model and studies how close evolutionary learning mechanisms can get to the optimal solution. It is shown that the genetic algorithm can learn the optimal parameter for the portfolio policy in various specifications, if number of learning periods (S) used for genetic algorithm is going to infinity; with higher S there are better results. Nevertheless, agents tend to hold more risky assets than in optimum for every S finite.

Arifovic (1996) considers a simple foreign exchange model with two countries. Agents optimize their holding of two currencies and their consumption in the first period. The question is whether the genetic algorithm will converge to a single exchange rate. The results of simulations are the same as in experimental markets - they show that the level of consumption converges to a stable value close to the optimum, but the exchange rate never settles to a constant value.

In spite of the doubt if it is the natural way how to imitate the process of evolution of strategies, the Genetic algorithm became a very popular tool in market simulations and was used in many papers. Routledge (2001) studies the question of purchasing a costly additional information. Agents in his asset pricing model with one risky asset and one risk-free bond can decide either to buy a signal about a future dividend payout, or to use only the current price to infer the future dividend and to decide about their portfolio. There exists an analytically deductible rational expectation equilibrium in which a certain fraction of agents purchase the information. The question is, if the application of genetic algorithm for learning whether to buy an information and how to use it, heads to this equilibrium. Routledge shows that the convergence to the equilibrium depends heavily on the rate of selections and mutations in the genetic algorithm. Particularly, when the rate of mutations and selections is high, and therefore there is quite a lot of randomness in the genetic algorithm, it is too difficult for uninformed traders to learn anything from behavior of informed traders and the market converges to a population consisting only of informed traders instead of the rational expectation equilibrium.

In work of Schoreels and Garibaldi (2005), the advantage of adaptive agents is examined. Agents in their model are divided into two populations - static and adaptive one. Both groups employ the genetic algorithm to learn the most profitable strategies. However, there is one significant difference between them. While agents from the adaptive population evolve their strategies set indefinitely (till the end of simulation), the static population learns only in the first (training) period and in the second (testing) period they use strategies learned in the first one. It was shown that in the training period the adaptive approach obtained a clear superiority over the static approach, as the adaptive agents were able to continuously adjust their behavior to current conditions.

Chen and Liao (2005) deal with the so called price-volume relation, i.e. the causal relation between stock returns and trading volume, and study it both on the micro and macro level. The market structure is similar to Routledge (2001). Agents are choosing between one risky asset and one risk-free bond; their expectations about future dividends and prices are modeled by genetic programming. As for the macro level, their results show that nonlinear causality between returns and volumes is in many cases uni-directional (returns-to-volume), while in few other cases it is bi-directional. To the contrary, linear causal relation exists in one direction in some cases, in other cases it is not found any. In micro level they study whether agents incorporated trading volume into their price expectations. It was shown that the number of such agents highly fluctuate during the simulation period.

Also Chen and Yeh (2001) use the genetic programming in their work. They try to develop a suitable model of learning behavior to have a closer connection with human learning and adaptation and to investigate the value of "education". Their model is similar to Santa Fe artificial stock market, but they introduced the common pool of rules - so called "business school" - place where all strategies are stored together and where agents can update their own sets of strategies. Place where, using their words, "unobservable strategies can be actually imitable". Their findings are ambiguous. They obtained some interesting features, like fat tails, that are observable in real markets, however, also some features that disagree with real market time series emerge – e.g. large level of positive skewness.

Some other papers investigate the ability of models to predict time series of financial market or the capability to reproduce the real data. For instance, Gou (2005) presents the mix-game model, employs it to forecasting Shanghai Index and shows that it is a good tool to predict financial time series. The principle of the mix-game is a division of agents into two groups; one group plays the minority game – every agent of the group tries to buy assets when the majority of other agents is selling – and the second group plays the majority game – agents buy assets when majority is buying. Over the given time horizon each agent gives points to strategies that predict a correct outcome in given round and plays the strategy with the highest point. Such strategies are considered as the correct outcome that are in minority in case that the agent is from "minority group" and, analogously, strategies that are in majority in case that the agent is from "majority group". Finally, price time series are calculated based on times series of excess demand. If parameters of mix-game model are chosen optimally, the model is able to simulate a specific financial market. The paper also deals with the problem how to choose these optimal parameters.

Xu and Chi (2007) implemented the pattern-oriented approach in their model. In the stock market with non-rational traders the guidance for agent whether to buy, sell, or hold a stock is not only the stock price but also the "chemical information" of agent's neighbors. That is, how agents around are dealing with their stocks. This model well imitates several macroeconomic phenomena in the real market, especially the ideal moving trend of the market, and cross-correlation between the market price and the cash volume in buyers' hands.

Similar idea as in Xu and Chi (2007) can be found in Chakrabarti and Roll (1999). They try to investigate how the learning from other traders can influence different market phenomena. Their market consists of one single asset paying the unknown dividend, which depends on the set of fundamental variables, and of the population of partially informed traders. Each trader has an information about some of that fundamental variables. No one is completely informed, but agents can observe trades of others and combine their own private information with information possessed by other traders. This market is then compared with an otherwise identical market populated by traders who ignore all information from others. The conclusion of their work is that learning from others usually (but not in all cases) reduces trading volume and volatility, and increases the accuracy of the market price as a forecast of value.

The underlying model for my thesis is a model presented in He, Hamill and Li (2008), therefore I describe it more precisely. The work deals with the question of survivability and profitability of different types of traders under various market conditions.

To be more specific, the model considered in this article is the simple asset pricing model with one risky asset and one risk-free bond paying a constant interest rate. There are two types of agents, fundamentalists and trend followers who differ in their expectations about a future price development. In every trading period these agents try to maximize their expected utility from wealth by choosing the optimal portfolio, i.e. the optimal quantity of the risky asset that they buy. The market price is then adjusted by market maker in response to excess demand for the risky asset at the end of every period.

Different types of traders differ in their future expectations and therefore also in their demands. First of them, fundamentalists, believe that the price of the risky asset converges to some fundamental value in the long run. They have a superior information about this fundamental value and their expectations of future prices are based on it. On the contrary, trend followers predict prices only from their past development and they have no information about the fundamental price.

The profitability and survivability of both traders is studied under various scenarios (diverse levels of fundamentalists' confidence about the convergence to the fundamental value or different market fractions of fundamentalist and trend followers in population). Surprisingly, it is shown that not only fundamentalists but also "irrational" trend followers can survive in the long run in many cases, but their share of total wealth decreases over time. Their profitability increases over time only in case when the level of the fundamentalists' confidence about the convergence to the fundamental value is equal to 0. The higher is the confidence or the share of fundamentalists in population, the higher is the fundamentalists' profitability.

The same model as in He, Hamill and Li (2008) is used in some other articles, nevertheless, they study different phenomena. Namely, it is He and Li (2007) or

He and Li (2005). The former shows the role of the noisy demand, heterogeneity of agents and the noisy fundamental process in the generation of returns. The latter studies the convergence of market prices to fundamental value, the profitability and the survivability of traders and various autocorrelation patterns of returns. It shows that all mentioned phenomena can be characterized by the stability and bifurcations of the underlying deterministic system. The question of stability and bifurcations is studied also by Chiarella and He (2002).

Similar model can be found also in Zhu, Chiarella, He and Wang (2009), where the impact of the market maker on market stability is examined. Unlike the previous models, the market maker plays not only the role of price provider but he also acts as an active investor with his own inventory who tries to maximize his profit. It is shown that in this case the market maker does not necessarily stabilize the market. His impact on stability highly depends on the levels of activity of trend followers and fundamentalists – if trend followers are relatively less active comparing to fundamentalists, the effect of his activity is stabilizing.

The agent-based research field is quite new, but rapidly growing, especially in last years when one can experience the sharp development of computational technologies. There is already too much papers connected to this topic to provide a comprehensive review. Works listed above represent only a little part of the ABM literature and one can find number of another reviews, each of them looking at the problem from different perspective.

The survey of Hommes (2006) deals with the literature on heterogeneous agent-based models of financial markets. Especially, he emphasizes simple models that are to some extent solvable by analytical tools but that are at the same time able to explain important observed stylized facts of financial time series – e.g. volatility clustering, high trading volumes, trend following or fat tails in the returns distribution. LeBaron (2000) and later again LeBaron (2006) are the surveys of the most important works from the agent-based literature existing until that time. The key design issues of each model are described in detail. Unlike the survey of Hommes (2006), they concentrate on cases where analytic solutions are impossible. On the other hand, work of Chen, Chang and Du (2010) provides the first review of the development of agent-based computational economics from an econometric point of view. Another significant survey is for example Samanidou at al. (2007).

Chapter 3

Initial Model

As mentioned above, I use the model described in He, Hamill and Li (2008) as the underlying model. In the following sections I give a detailed account of the model structure. For simplicity, I keep, with minor exceptions, the same notation as in the paper.

3.1 Model Description

3.1.1 Market Structure

Let us recall from the previous chapter that the model considered in this article is the simple asset pricing model with one risky and one risk free asset. The traders at the market are of two types, fundamentalists and trend followers. In every trading period these agents try to maximize the expected utility by choosing the optimal quantity of the risky asset to buy. At the end of the trading period the market price is adjusted by market maker in response to excess demand for the risky asset.

The supply of the risk free asset is considered to be perfectly elastic and the gross return of this asset is equal to $R = 1 + r/K$, where r is a constant annual risk free rate and K stands for the trading frequency (the number of trading periods per year). The value of r is set to 0.05 and K to 250 (i.e. one day trading period) which corresponds to the daily price movements in real financial markets.

As for the risky asset, the excess capital gain at time $t + 1$, which is denoted

R_{t+1}, can be computed as follows:

$$R_{t+1} = P_{t+1} + D_{t+1} - RP_t, \tag{3.1}$$

where P_t is the price per share of the risky asset and D_t is the dividend paid from the risky asset at time t. The dividends D_t correspond to the normal independent identically distributed (i.i.d.) stochastic process with the mean value $\bar{D}$ and the unconditional variance σ_D^2. That is,

$$D_t \sim \mathcal{N}(\bar{D}, \sigma_D^2), \quad \sigma_D^2 = r^2\sigma_1^2, \tag{3.2}$$

where σ_1^2 is the constant unconditional variance of price. It is important to mention that all traders have an unbiased estimate of D_t.

Further, let $z_{h,t}$ be the demand of h type investors in time t, or equivalently the number of shares of the risky asset purchased by h type investors in time t. The index h stands for either f, when the given investor is a fundamentalist, or tf, in the case the investor is a trend follower. The wealth of h type traders in time $t+1$ is then equal to

$$W_{h,t+1} = RW_{h,t} + [P_{t+1} + D_{t+1} - RP_t]z_{h,t} = RW_{h,t} + R_{t+1}z_{h,t}. \tag{3.3}$$

As the priority of agents is to maximize their expected utility of wealth $U_h(W)$, the demand for the risky asset of every trader depends on his expectations about the future gains from this asset. I denote $E_{h,t}$ and $V_{h,t}$ the beliefs of h type investor about the conditional expectation mean and variance of quantities at time $t+1$ based on his information at time t. Then his expectation mean and variance of wealth at $t+1$ are given by

$$E_{h,t}(W_{t+1}) = RW_{h,t} + E_{h,t}(R_{t+1})z_{h,t}, \quad V_{h,t}(W_{t+1}) = z_{h,t}^2 V_{h,t}(R_{t+1}). \tag{3.4}$$

All agents are assumed to have a constant absolute risk aversion (CARA) utility function

$$U_h(W) = -e^{-a_h W}, \tag{3.5}$$

where a_h is the risk aversion coefficient of h type agent. The optimal demand is then determined by maximizing the expected utility of wealth (for details see e.g.

Chiarella and He (2001)). Hence,

$$z_{h,t} = \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})}. \tag{3.6}$$

Then, if I denote n_f and n_{tf} the market fraction of fundamentalists and trend followers, respectively, and define $m = n_f - n_{tf}$ (as both $n_f, n_{tf} \in [0,1]$, obviously $m \in [-1,1]$), the aggregate excess demand per investor is given by

$$z_{e,t} = n_f z_{f,t} + n_{tf} z_{tf,t} = \frac{1+m}{2} \frac{E_{f,t}(R_{t+1})}{a_f V_{f,t}(R_{t+1})} + \frac{1-m}{2} \frac{E_{tf,t}(R_{t+1})}{a_{tf} V_{tf,t}(R_{t+1})}, \tag{3.7}$$

where a_f and a_{tf} are risk aversion coefficients of fundamentalists and trade followers, respectively.

3.1.2 Fundamentalists

Fundamentalists believe that the price of the risky asset converges to some fundamental value in the long run. They have a superior information about this fundamental value P_t^f and their expectations of future prices are based on it:

$$E_{f,t}(P_{t+1}) = P_t + \alpha(P_{t+1}^f - P_t), \quad \alpha \in [0,1], \tag{3.8}$$

$$V_{f,t}(P_{t+1}) = \sigma_1^2. \tag{3.9}$$

Coefficient α indicates the speed with which the fundamentalists expect price P_t to return to its fundamental value P_t^f. Specifically, $\alpha = 1$ means that fundamental traders believe in quick adjustment of price P_t and they expect the next period price to be equal to fundamental value. On the contrary, if $\alpha = 0$ fundamental traders become naive traders, which means that their expectations of future price are derived only from the actual price P_t.

As for the development of fundamental price, it can be presumed that it satisfies a stationary random walk process

$$P_{t+1}^f = P_t^f[1 + \sigma_\epsilon \tilde{\epsilon}_t], \quad \tilde{\epsilon}_t \sim \mathcal{N}(0,1), \quad \sigma_\epsilon \geq 0, \quad P_0^f = \bar{P} > 0, \tag{3.10}$$

where $\bar{P} = \bar{D}/(R-1)$ represents the expected long-run fundamental price of the risky asset.

3.1.3 Trend Followers

In contrast to fundamentalists, trend followers predict prices only from their past development and they have no information about the fundamental price. Precisely, their expectations satisfy

$$E_{tf,t}(P_{t+1}) = P_t + \gamma(P_t - u_t), \quad \gamma \geq 0, \tag{3.11}$$

and

$$V_{tf,t}(P_{t+1}) = \sigma_1^2(1 + bv_t), \quad b \geq 0, \tag{3.12}$$

where u_t and v_t are the sample mean and variance and the coefficient γ measures the rate of extrapolation from trend. The constant $b\sigma_1^2$ measures the influence of the sample variance on the conditional variance estimated by trend followers. The higher the conditional variance, the higher is their belief in more volatile price movement.

Values of u_t and v_t can be estimated by various learning mechanisms. As noted in He, Hamill and Li (2008), I assume that

$$u_t = P_t + \delta u_{t-1} + (1 - \delta)P_t, \tag{3.13}$$

and

$$v_t = \delta v_t + \delta(1 - \delta)(P_t - u_{t-1})^2, \tag{3.14}$$

where $\delta \in [0, 1]$ is a measure of geometric decay rate. The important feature of this mechanism, and hence also the reason for using it, is that it efficiently imitates traders' tendency to put higher weight on the most recent prices while estimating the sample mean and the variance.

3.1.4 Price Determination

Based on equations (3.6), (3.8), (3.9),(3.11) and (3.12) the optimal demand for the risky asset for both fundamentalists and trend followers can be expressed. First, one have to realize that

$$E_{h,t}(R_{t+1}) = E_{h,t}(P_{t+1}) + \bar{D} - RP_t = E_{h,t}(P_{t+1}) + (R - 1)\bar{P} - RP_t, \tag{3.15}$$

and

$$V_{h,t}(R_{t+1}) = V_{h,t}(P_{t+1}) + \sigma_D^2 = V_{h,t}(P_{t+1}) + r^2\sigma_1^2. \tag{3.16}$$

Then, the optimal demand for fundamentalists is

$$\begin{aligned} z_{f,t} &= \frac{E_{f,t}(R_{t+1})}{a_f V_{f,t}(R_{t+1})} = \frac{E_{f,t}(P_{t+1}) + (R-1)\bar{P} - RP_t}{a_f(V_{f,t}(P_{t+1}) + r^2\sigma_1^2)} \\ &= \frac{\alpha(P_{t+1}^f - P_t) - (R-1)(P_t - \bar{P})}{a_f\sigma_1^2(1+r^2)}, \end{aligned} \tag{3.17}$$

and for trend followers

$$\begin{aligned} z_{tf,t} &= \frac{E_{tf,t}(R_{t+1})}{a_{tf} V_{tf,t}(R_{t+1})} = \frac{E_{tf,t}(P_{t+1}) + (R-1)\bar{P} - RP_t}{a_{tf}(V_{tf,t}(P_{t+1}) + r^2\sigma_1^2)} \\ &= \frac{\gamma(P_t - u_t) - (R-1)(P_t - \bar{P})}{a_{tf}\sigma_1^2(1+bv_t+r^2)}. \end{aligned} \tag{3.18}$$

Using these relations for demands and equation (3.7), I have an aggregate excess demand per investor $z_{e,t}$ which is used to determine the price of the risky asset in period $t+1$. At the end of every period t, market maker adjusts the price for the next period in respect to the observed excess demand. I assume the constant speed of adjustment μ.

Furthermore, to capture unexpected market news and an impact of noise traders in the market, another variable is considered in the price determination mechanism - the noisy demand term $\tilde{\delta}_t$. This noisy demand is assumed to be an i.i.d. normally distributed random variable with

$$\tilde{\delta}_t \sim \mathcal{N}(0, \sigma_\delta^2), \tag{3.19}$$

where σ_δ^2 is a constant volatility of the noisy demand. Finally, the market price of the risky asset is given by

$$P_{t+1} = P_t + \mu z_{e,t} + \tilde{\delta}_t. \tag{3.20}$$

3.1.5 Complete Model

Based on the relations presented in the previous parts, I can build the following complete system of stochastic equations characterizing the model which can be used for running numerical simulations:

$$D_t \sim \mathcal{N}(\bar{D}, \sigma_D^2), \quad \tilde{\epsilon}_t \sim \mathcal{N}(0,1), \quad \tilde{\delta}_t \sim \mathcal{N}(0, \sigma_\delta^2),$$

$$
\begin{aligned}
P^f_{t+1} &= P^f_t[1+\sigma_\epsilon \tilde{\epsilon}_t],\\
z_{f,t} &= \frac{\alpha(P^f_{t+1}-P_t)-(R-1)(P_t-\bar{P})}{a_f\sigma_1^2(1+r^2)},\\
u_t &= \delta u_{t-1}+(1-\delta)P_t,\\
v_t &= \delta v_t+\delta(1-\delta)(P_t-u_{t-1})^2,\\
z_{tf,t} &= \frac{\gamma(P_t-u_t)-(R-1)(P_t-\bar{P})}{a_{tf}\sigma_1^2(1+bv_t+r^2)},\\
P_{t+1} &= P_t+\mu\left(\frac{1+m}{2}z_{f,t}+\frac{1-m}{2}z_{tf,t}\right)+\tilde{\delta}_t,\\
W_{h,t+1} &= RW_{h,t}+(P_{t+1}-RP_t+D_t)\,z_{h,t}.
\end{aligned}
\tag{3.21}
$$

3.2 Results

In the Section 3.1 I presented in detail the model characterized in He, Hamill and Li (2008). To recall, one of the goals was to implement the model and to replicate some already known outcomes obtained in connection with this model. These results based on findings introduced in He, Hamill and Li (2008), He and Li (2007), and He and Li (2005) are described in this section.

3.2.1 Parameters setting and initial values

I implemented the model in Wolfram Mathematica 6 – the main part of the source code can be found in the appendix. To run the simulations, I need to enter all initial parameters mentioned in Table 3.1 first. It would be very complicated and time demanding work to obtain some good estimations of the values with such a number of required parameters. Therefore I employ the sets of parameters presented in works of He, Hamill and Li (2008) and He and Li (2005) which follow the study of stability and bifurcations presented by He and Li (2007).

r	K	σ_1^2	γ	a_f	a_{tf}	μ	δ	b	σ_ϵ	σ_δ	P_0	$\bar{P}$	$W_{f,0}$	$W_{tf,0}$
0.05	250	1.6	1.0	0.8	0.8	0.4	0.85	4	0.01265	1	100	100	1000	1000

Table 3.1: Parameters setting

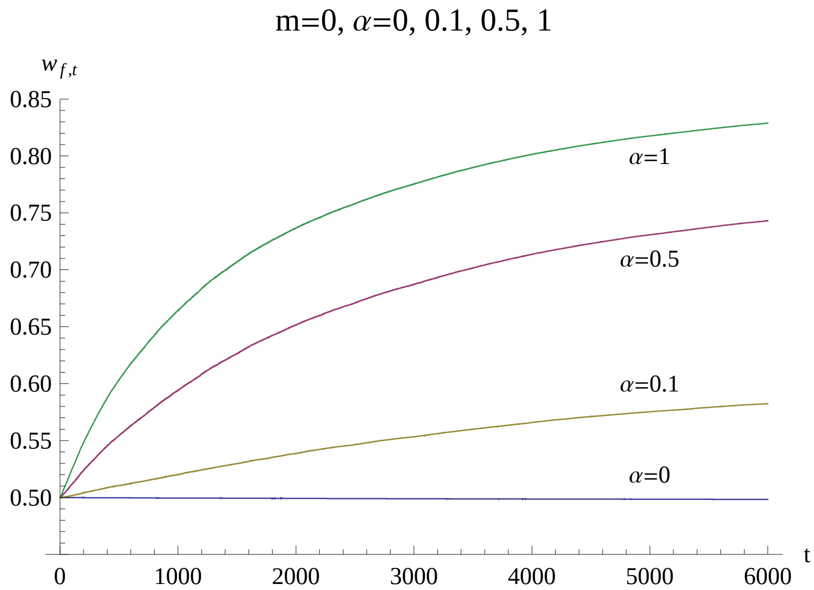

Figure 3.1: Average time series of wealth shares of fundamentalists with the initial set of parameters, $m = 0$, and different α.

3.2.2 Profitability and Survivability

To be able to measure and compare the profitability and the survivability of different strategies in stochastic setting, I introduce first so called absolute wealth shares of fundamentalists $w_{f,t}$ and trend followers $w_{tf,t}$, that are defined by

$$w_{f,t} = \frac{W_{f,t}}{W_{f,t} + W_{tf,t}}, \quad w_{tf,t} = \frac{W_{tf,t}}{W_{f,t} + W_{tf,t}}. \tag{3.22}$$

As the wealth share of trend followers is connected with the wealth share of fundamentalists by the relation $w_{tf,t} = 1 - w_{f,t}$, only the results for wealth share of fundamentalists is depicted in the following figures.

Impact of Speed of Adjustment α

To examine the impact of speed of adjustment α on the profitability and the survivability of fundamentalists and trend followers I have fixed all parameters except for α in the simulations. Thus, I have run 100 independent simulations over 6000 trading periods with $m = 0$, initial set of parameters from Table 3.1 and varying α. The results are demonstrated in Figures 3.1 and 3.2.

Figure 3.1 shows that trend followers can survive in all cases not only in the short run, but also in the long run (their wealth share does not disappear). However, their performance and profitability in confrontation with fundamentalists highly depends on the level of confidence of fundamentalists about the price convergence to its fundamental value. If the level of confidence α is high, the difference between wealth shares of fundamentalists and trend followers is significant and fundamental traders dominate the market. If α is low, fundamentalists still perform better, but the difference between their wealth share and the wealth share of trend followers is not very significant. Only in the situation when $\alpha = 0$, which means that fundamentalists became naive traders without any belief in fundamental value (they do not build in the fundamental value in their price expectations), trend followers are able to slightly outperform them.

In Figure 3.2 the average time series of demand of fundamentalists, demand of trend followers, returns, and wealth share of fundamentalists are demonstrated for different values of α.

Impact of Market Fraction m

I have run 100 independent simulations over 6000 trading periods with initial set of parameters from Table 3.1. But now I fixed α, first at $\alpha = 0.5$ and then at $\alpha = 0$, and examine the effect of varying market fractions of fundamentalists and trade followers represented by m. The results are demonstrated in the following figures.

Figure 3.3 shows $\alpha = 0.5$ that fundamentalists perform better in all the cases. Nevertheless, one can also see that for all values the wealth shares of trend followers do not vanish, thus they survive in the long run. It is also shown that m and α play the similar role concerning the accumulation of wealth – the higher is the market fraction of fundamentalists, the higher is their wealth share. However, the effect of different market fractions is not so conspicuous as the effect of different coefficients α.

In Figure 3.4 one can see the impact of different values of m in the case when $\alpha = 0$, i.e. fundamentalists are naive traders. In this case the wealth shares of both types of traders stay very close to 0.5. However, the wealth share of trend followers

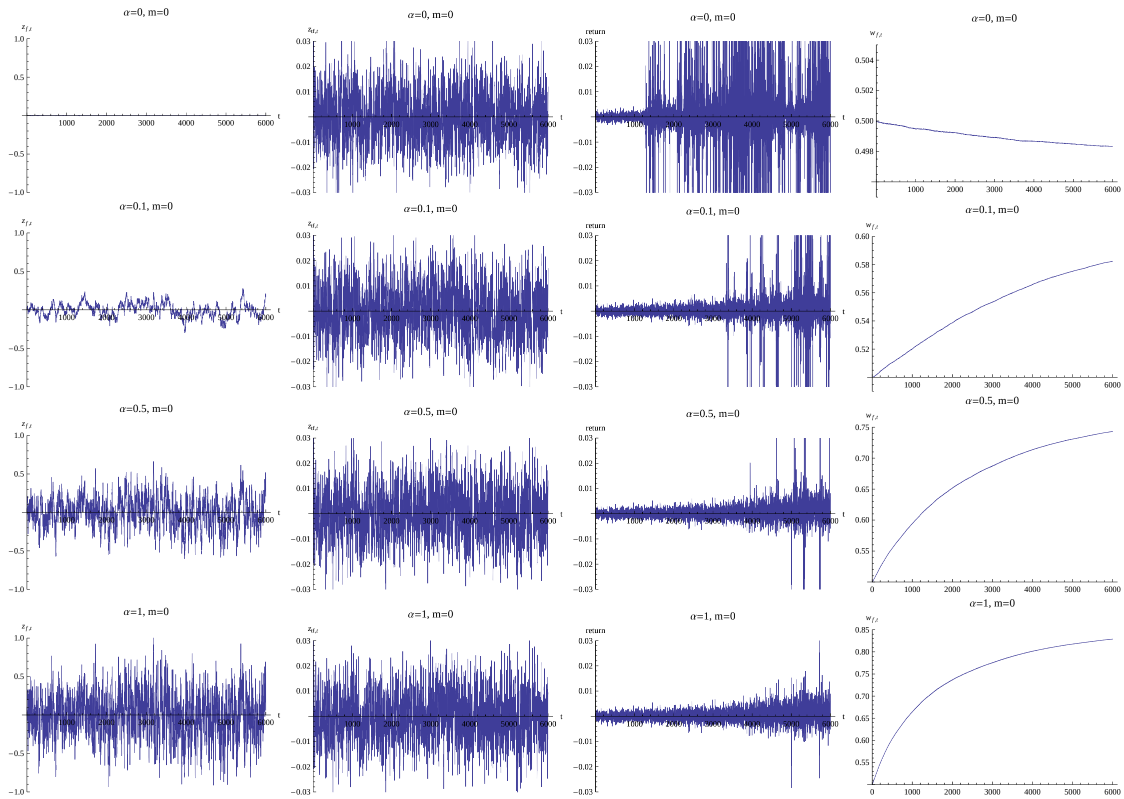

Figure 3.2: Average time series of demand of fundamentalists (first column), demand of trend followers (second column), returns (third column) and wealth share of fundamentalist (forth column) with the initial set of parameters, $m = 0$, and different α. The higher the coefficient α, the higher is the variance of the demand of fundamentalist and the lower is the return volatility. α has no visible impact on the demand of trend followers.

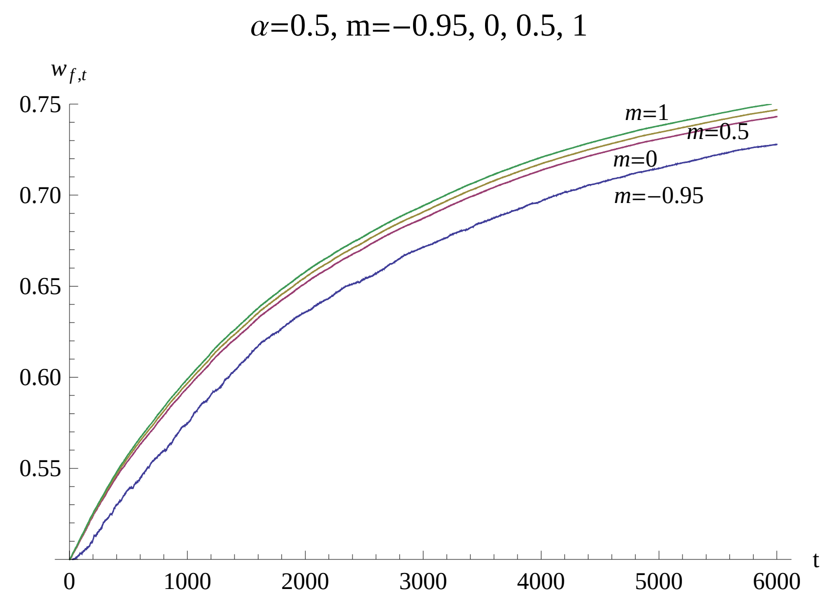

Figure 3.3: Average time series of wealth shares of fundamentalists with with the initial set of parameters, $\alpha = 0.5$, and different m.

is slightly higher for all values of m, except of the case when $m = 1$, i.e. only when the trend followers have no impact on the market price.

In Figures 3.5 and 3.6 the average time series of demand of fundamentalists, demand of trend followers, returns, and wealth share of fundamentalists are demonstrated for different values of m together with $\alpha = 0.5$ or $\alpha = 0$, respectively.

3.2.3 Analysis of returns

Other already known results, introduced in He and Li (2007), are connected with returns. Returns in the analysis are considered in the following logarithmic form:

$$r_t = \ln \frac{P_t}{P_{t-1}} = \ln P_t - \ln P_{t-1}. \tag{3.23}$$

One typical characteristic of return time series in real markets is the excess kurtosis, and so called fat tails in comparison to normal distribution. I have run again 100 independent simulations over 6000 trading periods, now with the initial set of parameters from Table 3.2 together with $\alpha = 0.1$ and $m = 0$. I concentrate on the behavior of returns to see the ability of the model to replicate these characteristics. Outcomes are illustrated in Table 3.3 and Figure 3.7, where the histogram of r_t

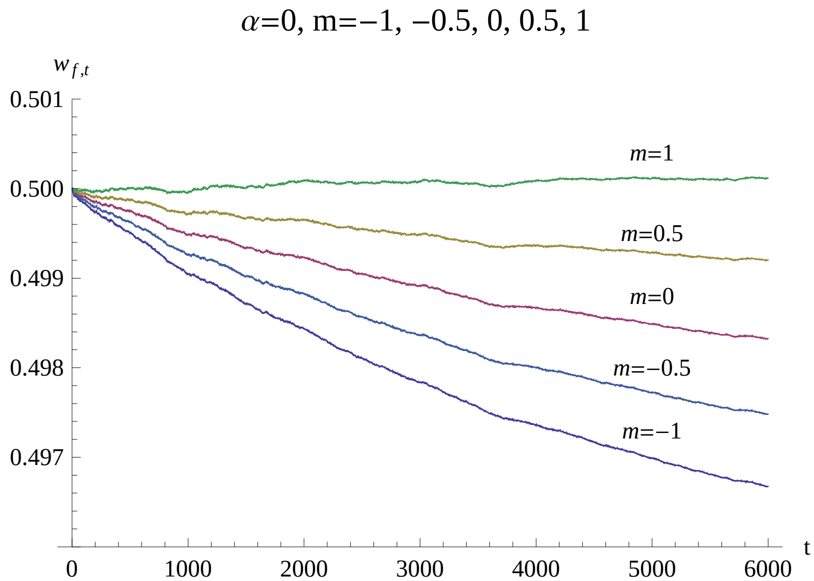

Figure 3.4: Average time series of wealth shares of fundamentalists with with the initial set of parameters, $\alpha = 0$, and different m.

shows that the distribution of simulated returns is more pointed round the mean value and its density is higher at the edges comparing to normal distribution. Also Jarque-Berra test reject the hypothesis of normal distribution and the values in Table 3.3 further confirm these facts observable in real markets.

r	K	σ_1^2	γ	a_f	a_{tf}	μ	δ	b	σ_ϵ	σ_δ	P_0	$\bar{P}$	$W_{f,0}$	$W_{tf,0}$
0.05	250	1.6	0.3	0.8	0.8	2	0.85	1	0.01265	1	100	100	1000	1000

Table 3.2: Parameters setting

mean r_t	max r_t	min r_t	Standard Deviation	Kurtosis	Skewness
-0.00008	0.102942	-0.0992478	0.00950873	74.2434	0.564691

Table 3.3: Characteristics of return r_t time series of the model

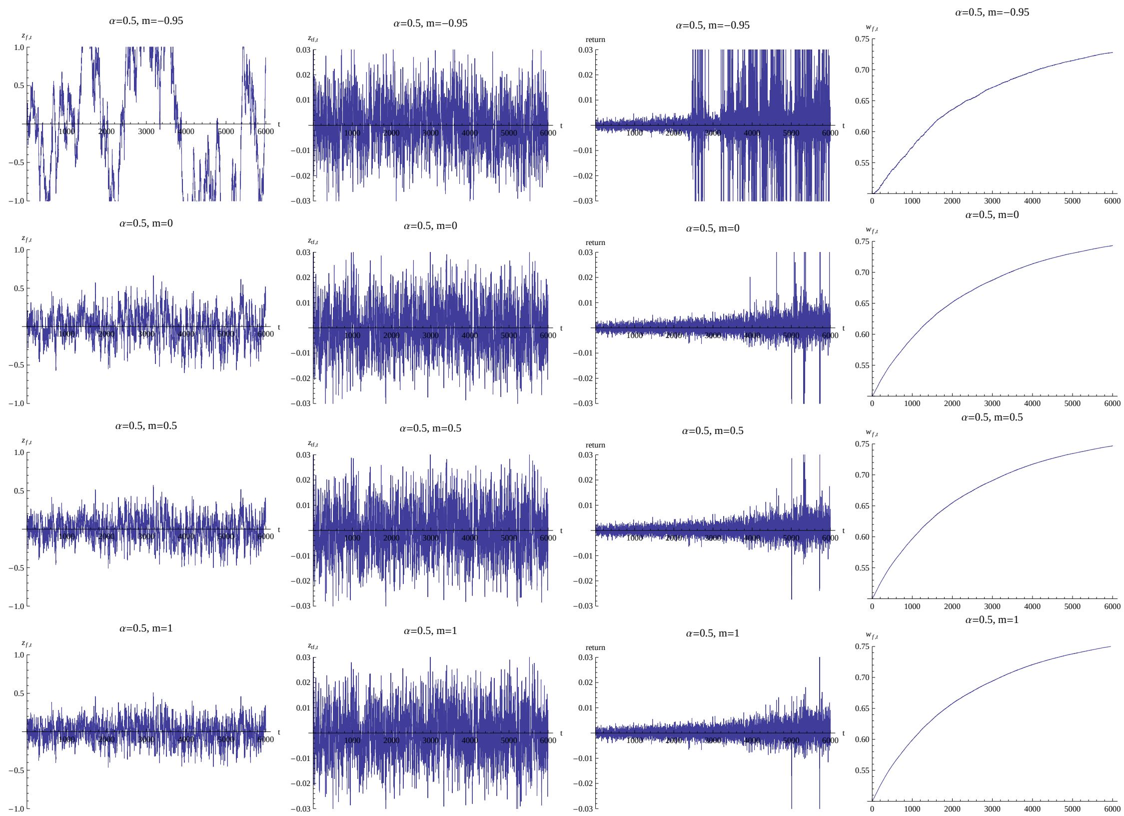

Figure 3.5: Average time series of demand of fundamentalists (first column), demand of trend followers (second column), returns (third column) and wealth share of fundamentalist (forth column) with the initial set of parameters, $\alpha = 0.5$, and different m. The higher the coefficient of market fraction m, the lower are both the volatility of the demand of fundamentalists and the returns volatility. m has no visible impact on the demand of trend followers.

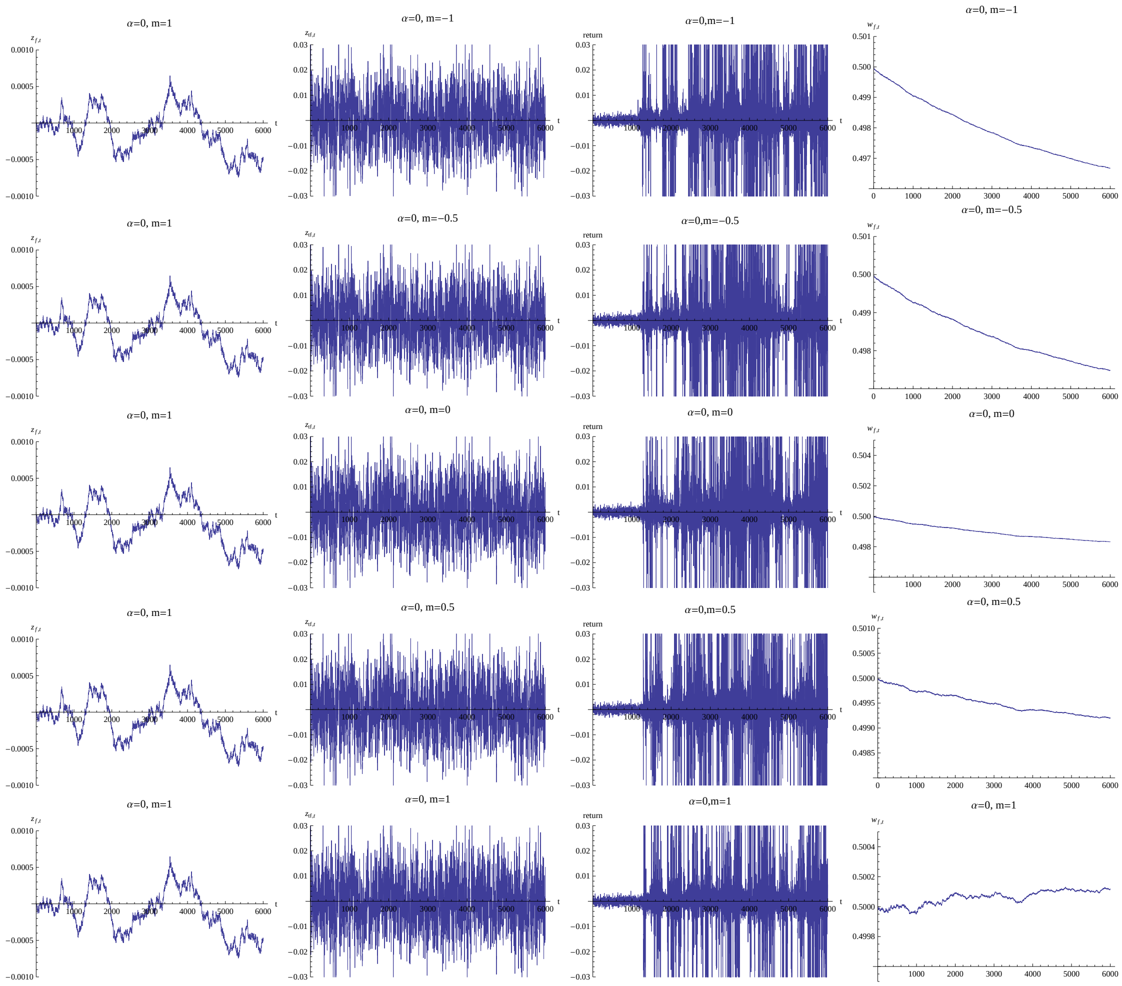

Figure 3.6: Average time series of demand of fundamentalists (first column), demand of trend followers (second column), returns (third column) and wealth share of fundamentalist (forth column) with the initial set of parameters, $\alpha = 0$, and different m. In this case, m has no visible impact on any of the presented time series besides the wealth share of fundamentalists.

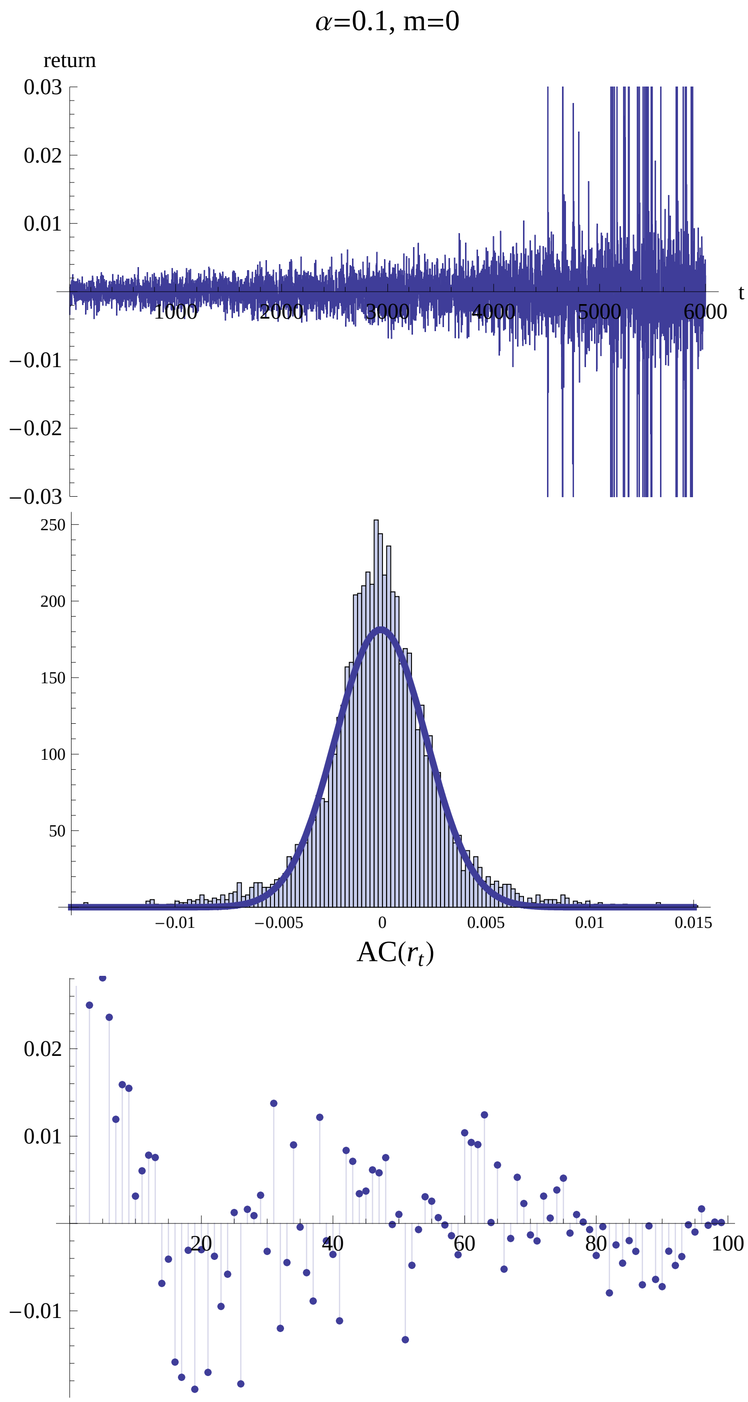

Figure 3.7: Analysis of returns – from the top to the bottom there are the average time series of returns r_t, the histogram of r_t, and the autocorrelation function of r_t.

Chapter 4

Extended Model

In this part of my thesis, I develop the extension of the original model and I also introduce own new results obtained from the numerical simulations. These results deal with the correlation between two markets. According to my best knowledge, the problem of connection of two markets is not very addressed till now, therefore the outcomes of the simulations are interesting, even though the extension is not very complicated.

4.1 Extension

As mentioned above, the extension of the initial model is based on creation of a market with two separate sub-markets. The structure of each sub-market fully corresponds to the description presented in Chapter 3 – i.e. there is a market maker and two groups of traders, fundamentalists and trend followers, that choose between the risk-free bond and the risky asset paying a random dividend. Traders in both sub-markets are isolated; it means that each agent can buy risky assets only in his own sub-market and does not have any information about the second sub-market. Nevertheless, there is a need of the existence of some correlation between the sub-markets to obtain the desirable outcome. To reach that, I take into consideration two possible connections between the markets – first of them is through the dividend process, the other one is mediated by the market maker. Although in an simplified way, both considered linkages are inspired by real markets.

Before I start with a more detailed description of the extended model, let me present the following notation: index $^{(1)}$ (or $^{(2)}$) indicate that the given variable or parameter is specific for sub-market 1 (or 2), while variables and parameters without any index are considered the same for both markets. Further, unless stated otherwise, all initial parameters are set to the same value for both sub-markets.

4.1.1 Correlation of Dividends

Let me explain the motivation of connecting the sub-markets by correlation of dividend processes on an example. Imagine the case of two markets for two different stocks or commodities in one economy. Even though these markets are not connected, i.e. each stock has its own potential buyers, they are both partially influenced – and it is important to say that they are both influenced in the same way – by the overall performance of the economy. In plain english, when economy enjoys high prosperity, all parts of economy tend to prosper, which can lead to higher dividends in all markets. On the contrary, if economy is in crisis, one can hardly believe in high dividend payments in any of the markets. Thus, one can observe some kind of correlation of dividends in a real market. It is more complicated in reality but for the sake of building the feasible numerical model, I assume a simplified correlation process.

Recall from Chapter 3.1.1 that the dividend process D_t in the initial market model corresponds to the normal distribution with mean $\bar{D}$ and unconditional variance σ_D^2, i.e. $D_t \sim \mathcal{N}(\bar{D}, \sigma_D^2)$. Using this, dividend processes corresponding to two different uncorrelated markets, market 1 with dividends $D_t^{(1)}$ and market 2 with dividends $D_t^{(2)}$, are

$$D_t^{(1)} \sim \mathcal{N}(\bar{D}, (\sigma_D^2)^{(1)}) \tag{4.1}$$

for market 1, and

$$D_t^{(2)} \sim \mathcal{N}(\bar{D}, (\sigma_D^2)^{(2)}) \tag{4.2}$$

For market 2. On the other hand, to capture the correlation between market 1 and 2 I consider dividends $D_t^{(1)}$, $D_t^{(2)}$ as a one vector $D_t = (D_t^{(1)}, D_t^{(2)})^T$. Then both dividends follow the one stochastic process

$$D_t \sim \mathcal{N}(\bar{D}, \sigma_D^2), \quad \sigma_D^2 = ((\sigma_D^2)^{(1)}, (\sigma_D^2)^{(2)})^T. \tag{4.3}$$

4.1.2 Common Market Maker

In this case, there is an assumption of only one market maker for both markets. In every trading period he adjusts prices of the risky assets $P_{t+1}^{(1)}$ and $P_{t+1}^{(2)}$. The adjustment equations are similar to equation (3.20) but now the market maker does not take into consideration only the excess demand on the given sub-market but adjust the price in response to the observed excess demands on both markets. However, he gives to the demands the different weights – i.e. he adjusts the price to different excess demands with different speeds of adjustment. Specifically,

$$P_{t+1}^{(1)} = P_t^{(1)} + \mu_1 z_{e,t}^{(1)} + \mu_2 z_{e,t}^{(2)} + \tilde{\delta}_t^{(1)}, \tag{4.4}$$

$$P_{t+1}^{(2)} = P_t^{(2)} + \mu_1 z_{e,t}^{(2)} + \mu_2 z_{e,t}^{(1)} + \tilde{\delta}_t^{(2)}. \tag{4.5}$$

The formulas are motivated by the fact that suppliers who provide liquidity to two different markets with one stock exist. When supplier chooses the optimal price of his stock in one market, he takes into consideration demands on both markets because he assumes that some of potential buyers of his stock can fluctuate from one market to another to find a better price. Thus, both demands are important in the price formation process. Albeit the fluctuation of traders between the sub-markets is not permitted in the model, the above described scenario of the market maker behavior is still possible. One can suppose, for instance, that the market maker has a misleading information about the fluctuations.

4.2 Results

Since I have implemented the model for a single market before, it remains to perform the modifications to capture the time development of two sub-markets together and to implement the above mentioned correlation processes. In simulations, I use the set of initial parameters presented in Table 4.1. To see the impact of different types of connections of sub-markets, I have run the set of simulations with sub-markets with correlated dividends only, i.e. with $\mu_2 = 0$, another set of simulations with sub-markets connected by market maker and without correlated dividends, and finally the simulations, where markets are connected by both processes. The results are described in the following subsections. I concentrate particularly on the analysis of return series.

r	K	α	γ	a_f	a_{tf}	m	δ	b	σ_ϵ	σ_δ	P_0	$\bar{P}$	$W_{f,0}$	$W_{tf,0}$
0.05	250	0.1	0.3	0.8	0.8	0	0.85	1	0.01265	1	100	100	1000	1000

Table 4.1: Parameters setting

4.2.1 Impact of Correlated Dividends

To study how the correlated dividends affect the behavior of sub-markets I have run 100 independent simulations over 6000 trading periods with the parameters set from Table 4.1, together with $\mu_1 = 2$, and $\mu_2 = 0$ for various values of dividend variances $(\sigma_D^2)^{(1)}$ and $(\sigma_D^2)^{(2)}$. For every pair of values I have run one simulation where markets are correlated and one simulation with uncorrelated markets. This helps to distinguish between the effect of higher values of variance and the effect of the correlation of markets. The results are demonstrated in Tables 4.2 and 4.3 and in the Figures 4.1, 5.1, and 5.2.

Values in Table 4.2 indicate that there is no significant difference between the cases of correlated and uncorrelated dividends; there are the same results for both cases. It is due to the fact that the payment of dividend is only one of more random processes that influence the price formation and the development of market. Probably, other random variables such as the random part of demand influence the market behavior much more. Thus, I further concentrate only on the impact of various values of variances and will not differentiate between the correlated and uncorrelated dividends.

The forth rows of Figures 5.1 and 5.2 shows that different values of variances have an impact on the wealth share of fundamentalists and trend followers in the both markets. In all cases fundamentalists' wealth share is higher than 0.5, however, their final wealth share reaches lower values if the variance of dividends is higher. The level of variance effects also the demands of both trend followers and fundamentalists (see Figures 5.1 and 5.2 first and second rows). It is caused by the fact that the variance of dividends is closely connected with the variance of price σ_1^2 (recall that $\sigma_D^2 = r^2\sigma_1^2$), and σ_1^2 is a component of the formation of price expectations of both groups of traders. Therefore the dividend variance influences also optimal demands – the higher is the variance, the lower are the optimal demands of both types of agents.

The impact of the size of the dividend variance σ_D^2 on a correlation of returns is depicted in Figure 4.1. The left figure shows the dependence of the autocorrelation of returns, i.e. the correlation between the shifted values of returns in the same sub-market $r_t^{(1)}$ and $r_{t-1}^{(1)}$, while the right picture shows the cross-correlation between the returns $r_t^{(1)}$ and $r_t^{(2)}$ on markets 1 and 2. Both pictures indicate an inverse relation, i.e. the level of correlation between returns is lower with a higher σ_D^2. Nevertheless, both the autocorrelation and the cross-correlation are negligible – for given parameters they move around zero. For exact values see Table 4.2.

Other interesting characteristics of return series on the both sub-markets (mean value, minimum, maximum, standard deviation and also kurtosis and skewness of the return distribution) are shown in Table 4.3.

4.2.2 Impact of Common Market Maker

I have run 100 independent simulations over 6000 trading periods with the parameters set from Table 4.1 and $\mu_1 = 2$ for different values of coefficient $\mu_2 \in [0, \mu_1)$. I do not take into consideration the results for $\mu_2 > \mu_1$ out of two reasons – first, it does not make much sense for market maker to take the excess demand from other sub-market with a higher weight than the excess demand from the corresponding market in the process of price formation, and second, markets seem to be unstable for $\mu_2 \geq \mu_1$.

The outcomes of simulations are shown in Tables 4.4 and 4.5 and in the Figures 4.2, 5.3, 5.4, 5.5 and 5.6. The first observation concerns the wealth share of

model	$\mathrm{Corr}(r_t^{(1)}, r_{t-1}^{(1)})$	$\mathrm{Corr}(r_t^{(1)}, r_t^{(2)})$
$(\sigma_D^2)^{(1)} = 1.6, (\sigma_D^2)^{(2)} = 1.6$	0.05393	-0.00313
$\sigma_D^2 = (1.6, 1.6)$	0.05393	-0.00313
$(\sigma_D^2)^{(1)} = 3, (\sigma_D^2)^{(2)} = 3$	0.02336	-0.00406
$\sigma_D^2 = (3, 3)$	0.02336	-0.00406
$(\sigma_D^2)^{(1)} = 5, (\sigma_D^2)^{(2)} = 5$	0.00533	-0.00511
$\sigma_D^2 = (5, 5)$	0.00533	-0.00511
$(\sigma_D^2)^{(1)} = 10, (\sigma_D^2)^{(2)} = 10$	-0.00823	-0.00614
$\sigma_D^2 = (10, 10)$	-0.00823	-0.00614
$(\sigma_D^2)^{(1)} = 16, (\sigma_D^2)^{(2)} = 16$	-0.01364	-0.00741
$\sigma_D^2 = (16, 16)$	-0.01364	-0.00741

Table 4.2: Correlations of returns for both the correlated and uncorrelated dividends.

fundamentalists. If you examine the graphs of wealth shares of fundamentalists for different values of μ_2 (forth rows of Figures 5.4 – 5.6), you can see that in all the cases the evolution of the fundamentalists' wealth share follows the same path. Thus, the wealth shares of both groups of traders do not change with adding an excess demand from other market to the price adjustment mechanism.

Nevertheless, changes in the structure of demand of fundamentalists can be observed – see the first row of Figures 5.4 – 5.6. While for low value of μ_2 the fundamentalists' demand switches quickly between positive and negative values, with higher values of coefficient μ_2 the demand starts to move differently – periods of either positive or negative demand emerge.

The impact of the coefficient μ_2 on a correlation of returns is described in Table 4.4 and in Figure 4.2. Similarly to the case of correlated dividends, the left picture shows the dependence of the autocorrelation of returns $r_t^{(1)}$ and $r_{t-1}^{(1)}$, while the right picture shows the cross-correlation between the returns $r_t^{(1)}$ and $r_t^{(2)}$. Unlike the previous case with correlated dividends, both the pictures show a direct relation, i.e. the level of correlation between returns is higher with a higher coefficient μ_2 which is in accordance with intuition. However, Table 4.4 shows that again all values of the autocorrelation are not very significant; the highest value of the correlation is

model	market	mean r_t	$\max r_t$	$\min r_t$	StDev	Kurt	Skew
$\sigma_D^2 = (1.6, 1.6)$	1	-0.00008	0.057	-0.093	0.0028	222.5	-4.322
	2	-0.00007	0.103	-0.104	0.0035	427.9	-1.870
$\sigma_D^2 = (3, 3)$	1	-0.00008	0.090	-0.099	0.0044	248.8	-1.126
	2	-0.00007	0.107	-0.100	0.0061	184.4	0.933
$\sigma_D^2 = (5, 5)$	1	-0.00008	0.097	-0.101	0.0075	114.2	-0.001
	2	-0.00007	0.107	-0.102	0.0080	115.0	0.293
$\sigma_D^2 = (10, 10)$	1	-0.00008	0.104	-0.129	0.0134	39.8	-0.111
	2	-0.00007	0.107	-0.100	0.0106	64.9	0.286
$\sigma_D^2 = (16, 16)$	1	-0.00008	0.105	-0.112	0.0166	26.9	-0.021
	2	-0.00007	0.157	-0.174	0.0142	42.3	0.180

Table 4.3: Characteristics of return r_t time series of the sub-markets 1 and 2 for different values of σ_D^2 in the case of correlated dividends and $\mu_2 = 0$.

between $r_t^{(1)}$ and $r_t^{(2)}$ for $\mu_2 = 0.9\mu_1$ and it is equal to 0.11.

Another effect of the coefficient μ_2 is on the variance of returns. If you look at the values of standard deviation of returns in Table 4.5 you can see that the standard deviation (and therefore also the variance) is higher with higher values of μ_2. It it visible also in Figures 5.4 – 5.6, where the time series of returns are depicted on the third row.

The same characteristics of return series as in the case of correlated dividends (mean value, minimum, maximum, standard deviation, kurtosis and skewness of the return distribution) are shown in Table 4.5.

4.2.3 Impact of Both Processes

As it was stated in the beginning of this section, I have performed also simulations with sub-markets with both common market maker and correlated dividends. Nevertheless, as I have written in Subsection 4.2.1, the correlation of dividends has only very small impact on the behavior of the market and therefore the results of these simulations appeared to be the same as in the case of the common market maker only. Hence, I do not present here any new tables or figures, all can be seen in the part dealing with results of the case of the common market maker, i.e. in Subsection 4.2.2.

model	$\mathrm{Corr}(r_t^{(1)}, r_{t-1}^{(1)})$	$\mathrm{Corr}(r_t^{(1)}, r_t^{(2)})$
$\mu_2 = 0$	0.05393	-0.00313
$\mu_2 = 0.1\mu_1$	0.05496	0.01128
$\mu_2 = 0.3\mu_1$	0.05768	0.03969
$\mu_2 = 0.4\mu_1$	0.05834	0.05327
$\mu_2 = 0.5\mu_1$	0.05945	0.06614
$\mu_2 = 0.7\mu_1$	0.06061	0.09045
$\mu_2 = 0.9\mu_1$	0.06219	0.11222

Table 4.4: Correlations of returns for uncorrelated dividends and different values of μ_2.

model	market	mean r_t	$\max r_t$	$\min r_t$	StDev	Kurt	Skew
$\mu_2 = 0$	1	-0.00008	0.057	-0.093	0.0028	222.5	-4.322
	2	-0.00007	0.103	-0.104	0.0035	427.9	-1.870
$\mu_2 = 0.1\mu_1$	1	-0.00008	0.012	-0.016	0.0024	4.8	-0.038
	2	-0.00007	0.081	-0.101	0.0030	300.5	-2.818
$\mu_2 = 0.3\mu_1$	1	-0.00008	0.013	-0.015	0.0024	5.3	-0.044
	2	-0.00007	0.101	-0.101	0.0042	314.0	-0.004
$\mu_2 = 0.4\mu_1$	1	-0.00008	0.097	-0.097	0.0041	267.1	-0.128
	2	-0.00007	0.101	-0.101	0.0048	255.6	0.415
$\mu_2 = 0.5\mu_1$	1	-0.00008	0.101	-0.101	0.0055	184.3	1.414
	2	-0.00007	0.101	-0.101	0.0062	172.0	-0.750
$\mu_2 = 0.7\mu_1$	1	-0.00008	0.109	-0.099	0.0103	68.1	0.445
	2	-0.00007	0.102	-0.103	0.0104	63.4	-0.015
$\mu_2 = 0.9\mu_1$	1	-0.00007	0.194	-0.183	0.0241	21.2	0.280
	2	-0.00007	0.112	-0.113	0.0168	26.7	0.161

Table 4.5: Characteristics of return r_t time series of the sub-markets 1 and 2 for different values of μ_2 and for uncorrelated dividends with variances $(\sigma_D^2)^{(1)}, (\sigma_D^2)^{(2)} = 1.6$.

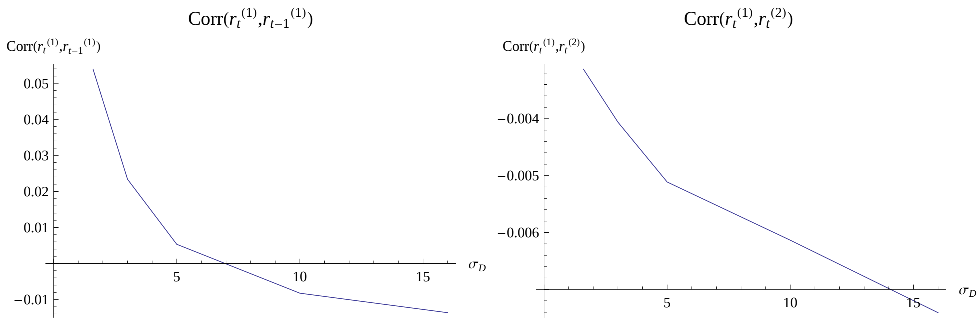

Figure 4.1: Correlations of returns, effect of σ_D.

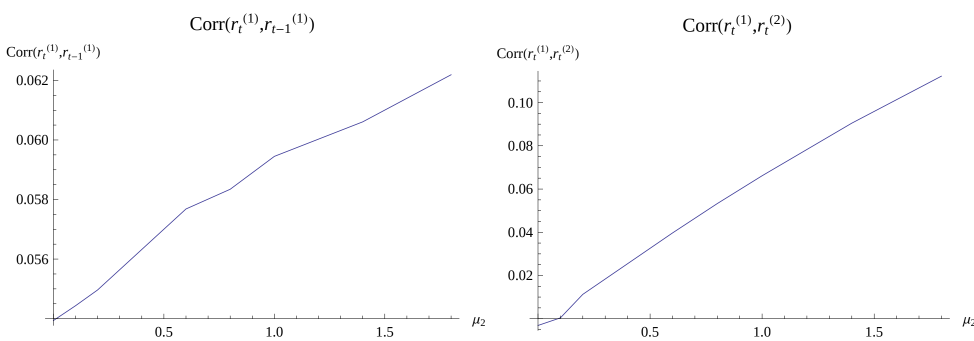

Figure 4.2: Correlations of returns, effect of μ_2.

Chapter 5

Conclusion

In the thesis, I have fully implemented the already existing asset pricing model with market maker presented by He, Hamill and Li (2008) to which I have added my own extension. Both the initial model and the extension were fully implemented in Wolfram Mathematica 6. Then I have performed a number of simulations to gain data for further analysis of a behavior of the market model. Besides the already known results, that I obtained from the running the initial model, I have provided my own new results connected to the model extension.

From the already known results I replicate the ones characterizing the profitability and survivability of fundamentalists and trend followers, and the results concerning some of the stylized fact of return time series. It was shown that for all tested sets of parameters both groups of trading agents survive in the long run, even the "irrational" trend followers. In most of the cases fundamentalists appear to be more profitable but the combinations of parameters for which the trend followers perform slightly better also exist. The analysis of return time series shows the excess kurtosis and fat tails in comparison to the normal distribution, which is characteristic for return time series of real markets.

The extension of the model lies in the connecting of two sub-markets by a mutual correlation which is represented either by correlated dividends or by the common market maker. The analysis of both the cross-correlation of sub-markets and the autocorrelation of return in one market shows on the one hand the inverse relation between the correlation of returns and dividends, on the other hand the direct relation of the correlation of returns and the coefficient of the effect of an excess demand from the non-corresponding market.

All the characteristics of return series (autocorrelations, fat tails, excess kurtosis, volatility clustering, etc.) were studied before from many different angles for many different models settings. I mention here some of them to have a comparison to results obtained in this thesis. From works dealing with the same initial model, He and Li (2007) study the role of the noisy demand, heterogeneity of agents and the noisy fundamental process on the autocorrelation of return, squared return and absolute return time series. The same question is partially examined also in He and Li (2005). Further, Zhu, Chiarella, He and Wang (2009) examine the impact of the market maker on market stability and different autocorrelation patterns. The contribution of the thesis lies in the presented extension of the given model to two markets and the results gained from this extension. The case of the agent-based model of two correlated markets have not been studied yet. Thus, the results obtained for return series, describing the dependence of return correlation on the correlation of the sub-markets are new in the literature and can be the inspiration for a further agent-based research.

It is important to say that the presented extension represent only a first step of the work connected with creating the fully functional model which could be able to replicate and predict the development of real data obtained from real financial markets. It would be very time demanding and difficult work, to calibrate all the parameters of the model to gain data corresponding to some of the real market data and it was not the goal of this thesis. However, it could be one of the possible ways of further investigation.

Other possibilities of further research are the simulations of other various connections of sub-markets. For example, it would be an interesting extension to allow the fluctuations of agents between the sub-markets. Another option is to introduce the third group of trading agents which dispose of the information about the second sub-market. It may be interesting as well to examine the situation when agents can buy the costly information about the other sub-market.

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Appendix A

Each figure in Apendix A describes the average time series of the both sub-markets, obtained by simulation with a specific combination of dividend variances $(\sigma_D^2)^{(1)}$, $(\sigma_D^2)^{(1)}$ and coefficient μ_2. The first column of every figure contains the time series from sub-market 1, the second column includes the time series from sub-market 2 gained in the same run. In the every column, time series are arranged as follows: fundamentalists' demand in the first row, trend followers' demand in the second row, returns in the third row, the wealth share of fundamentalists in the forth row. In the last row, there is a histogram of returns.

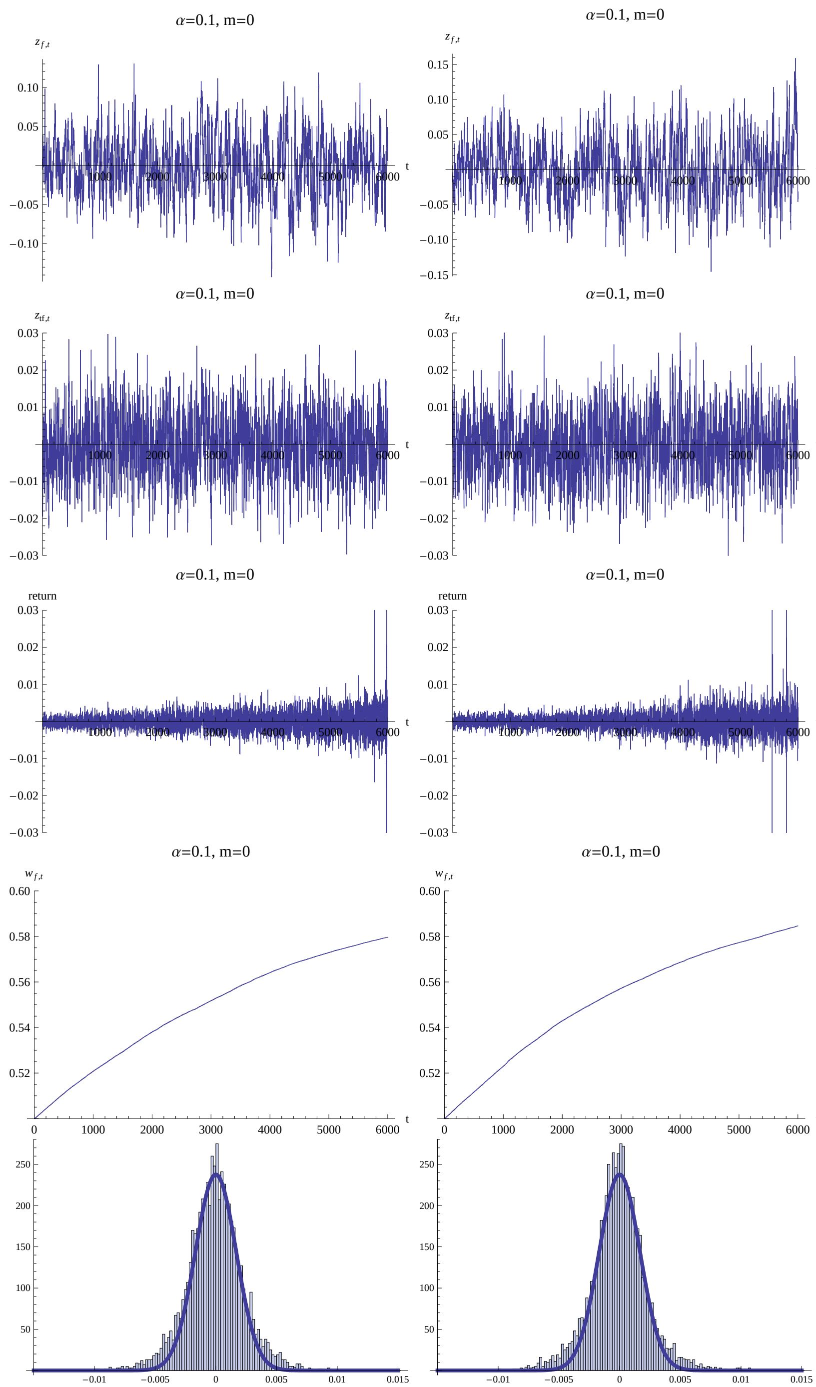

Figure 5.1: Model with $\mu_2 = 0$, dividends correlated, $\sigma_D^2 = (1.6, 1.6)$.

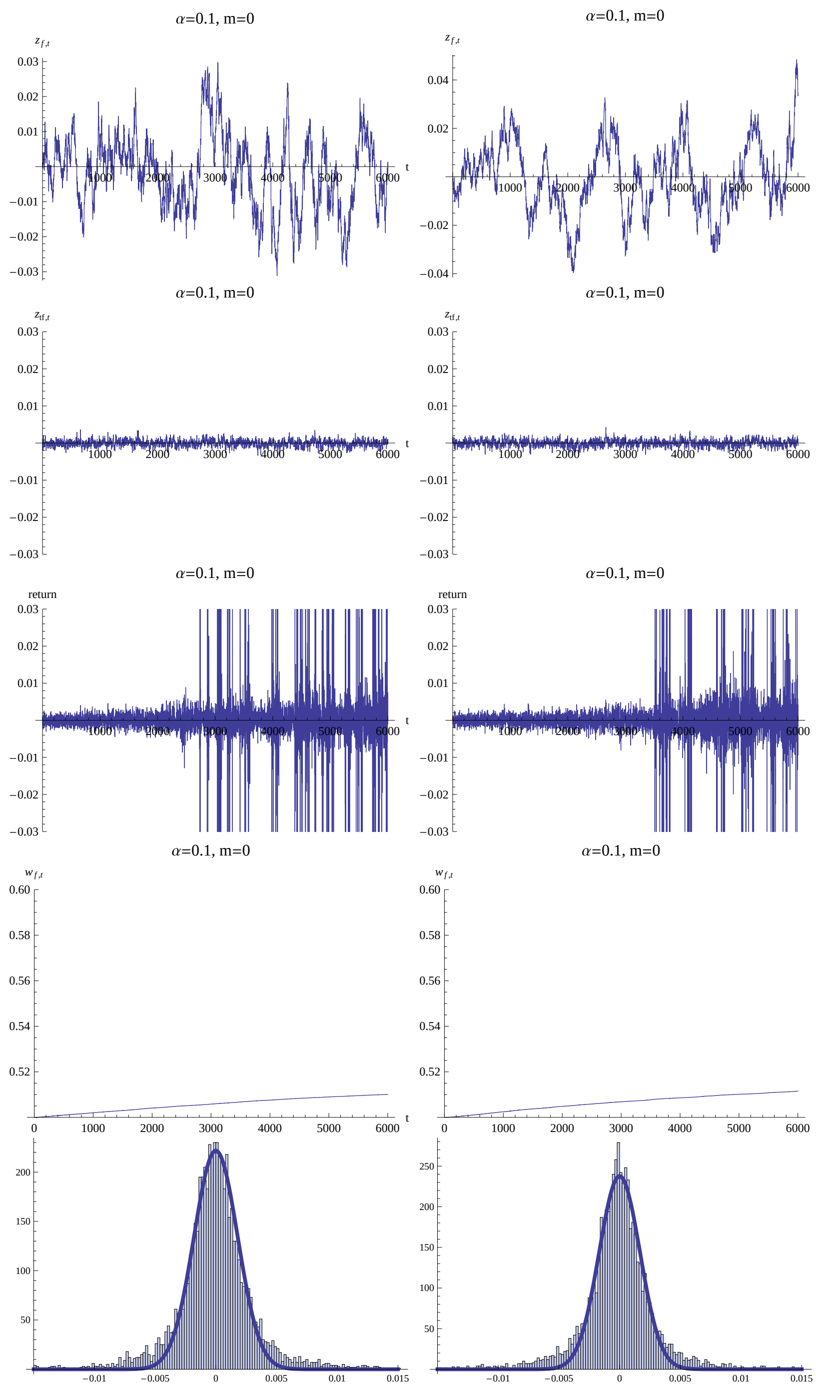

Figure 5.2: $\mu_2 = 0$, dividends correlated, $\sigma_D^2 = (16, 16)$.

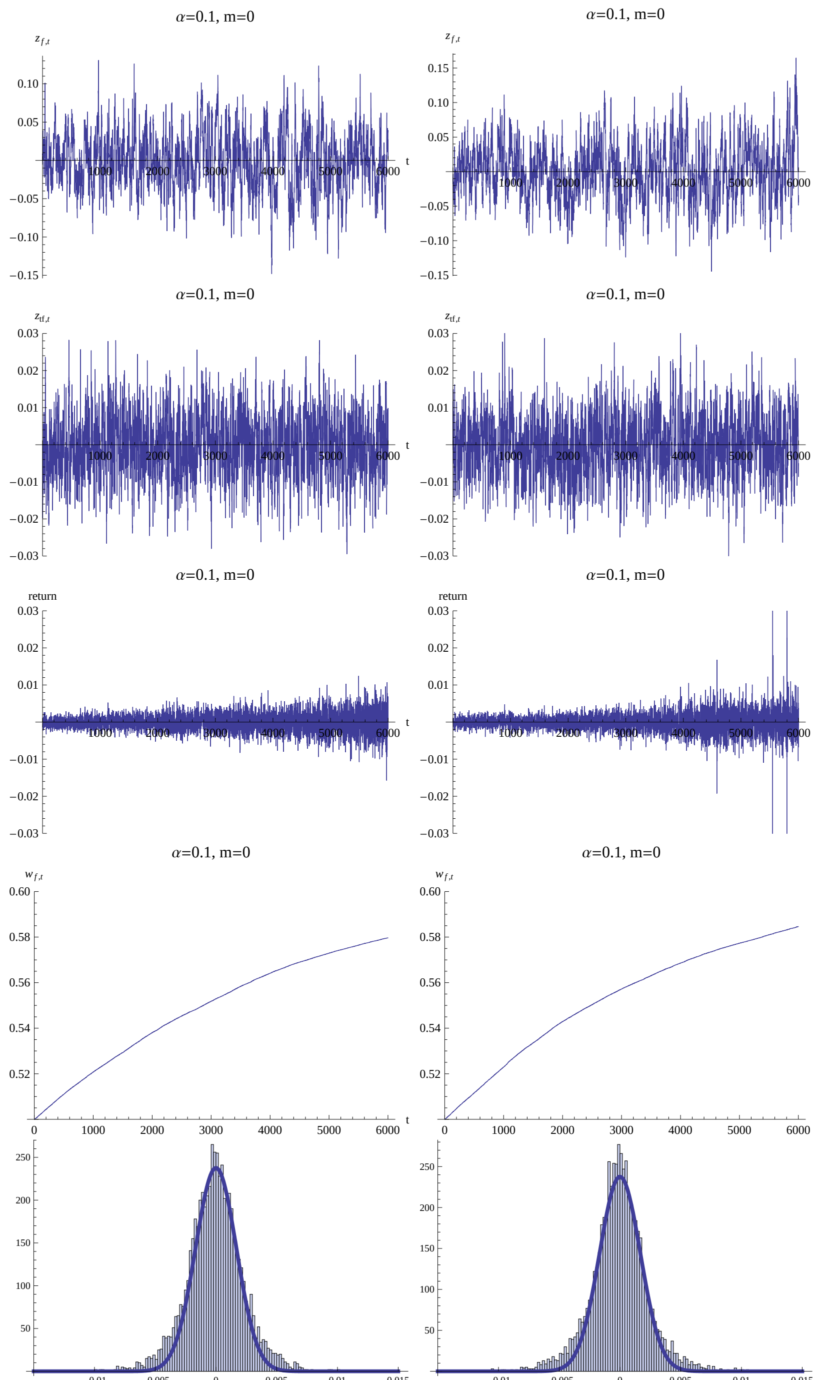

Figure 5.3: $\mu_2 = 0.1$, dividends correlated, $\sigma_D^2 = (1.6, 1.6)$.

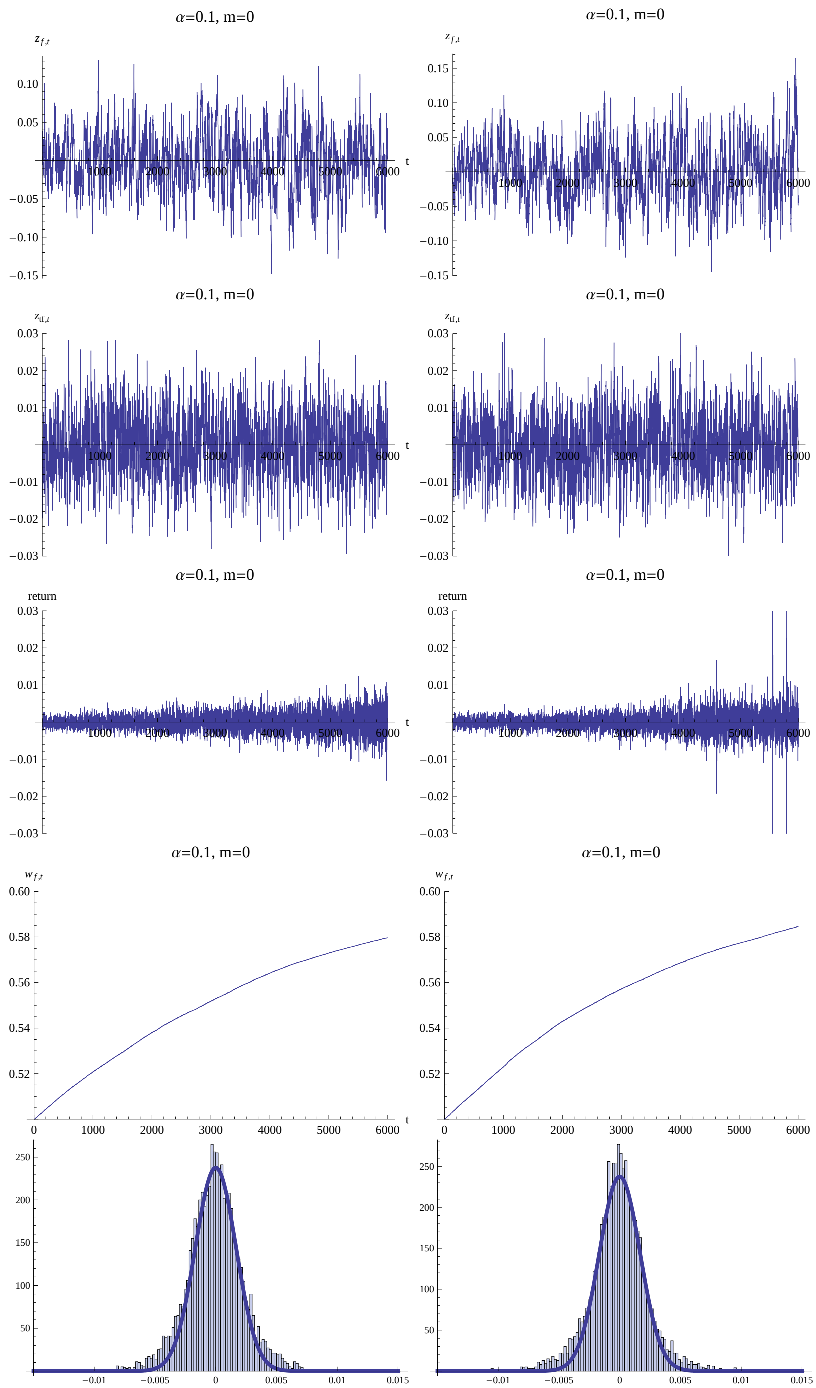

Figure 5.4: $\mu_2 = 0.1$, dividends not correlated, $(\sigma_D^2)^{(1)} = 1.6$, $(\sigma_D^2)^{(2)} = 1.6$.

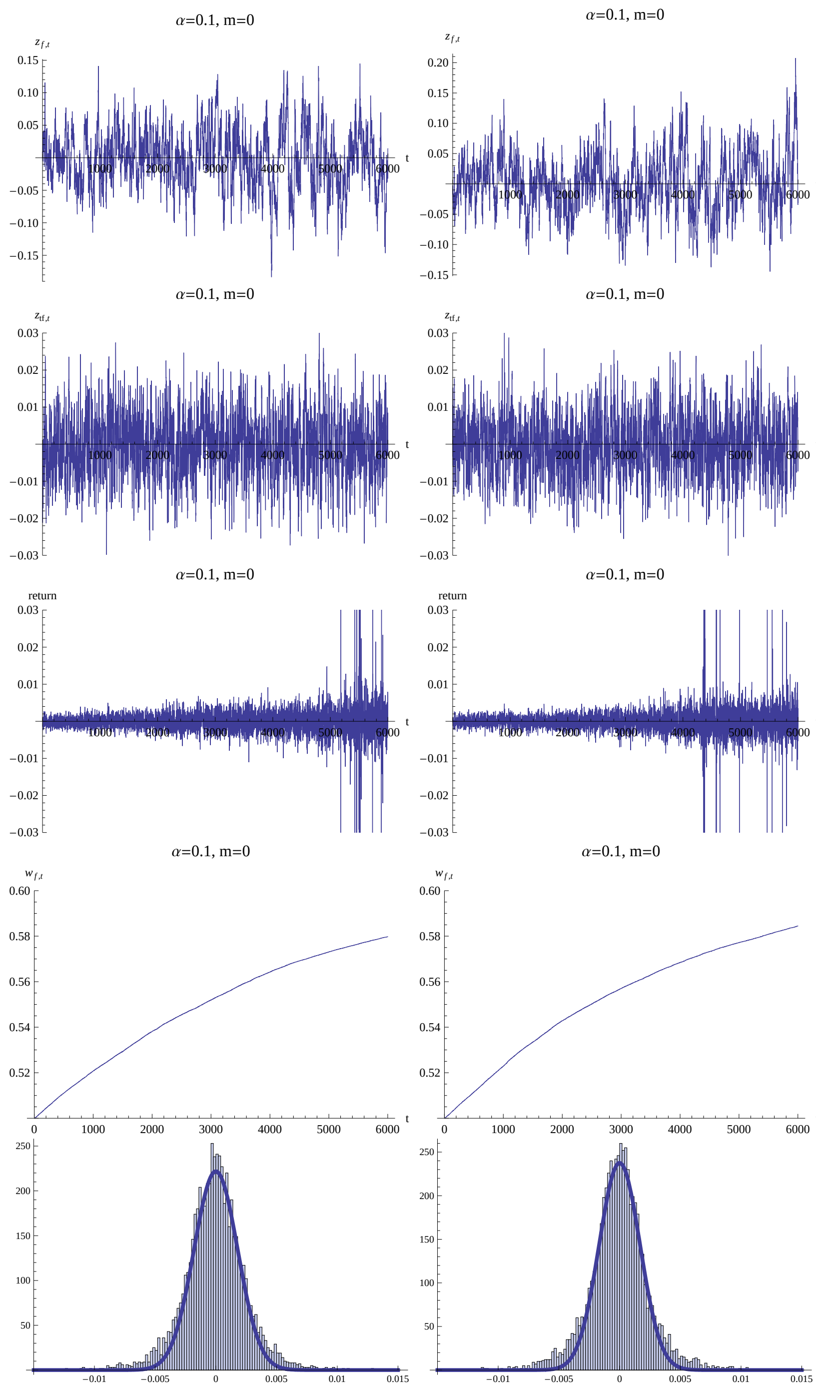

Figure 5.5: $\mu_2 = 0.5$, dividends not correlated, $(\sigma_D^2)^{(1)} = 1.6$, $(\sigma_D^2)^{(2)} = 1.6$.

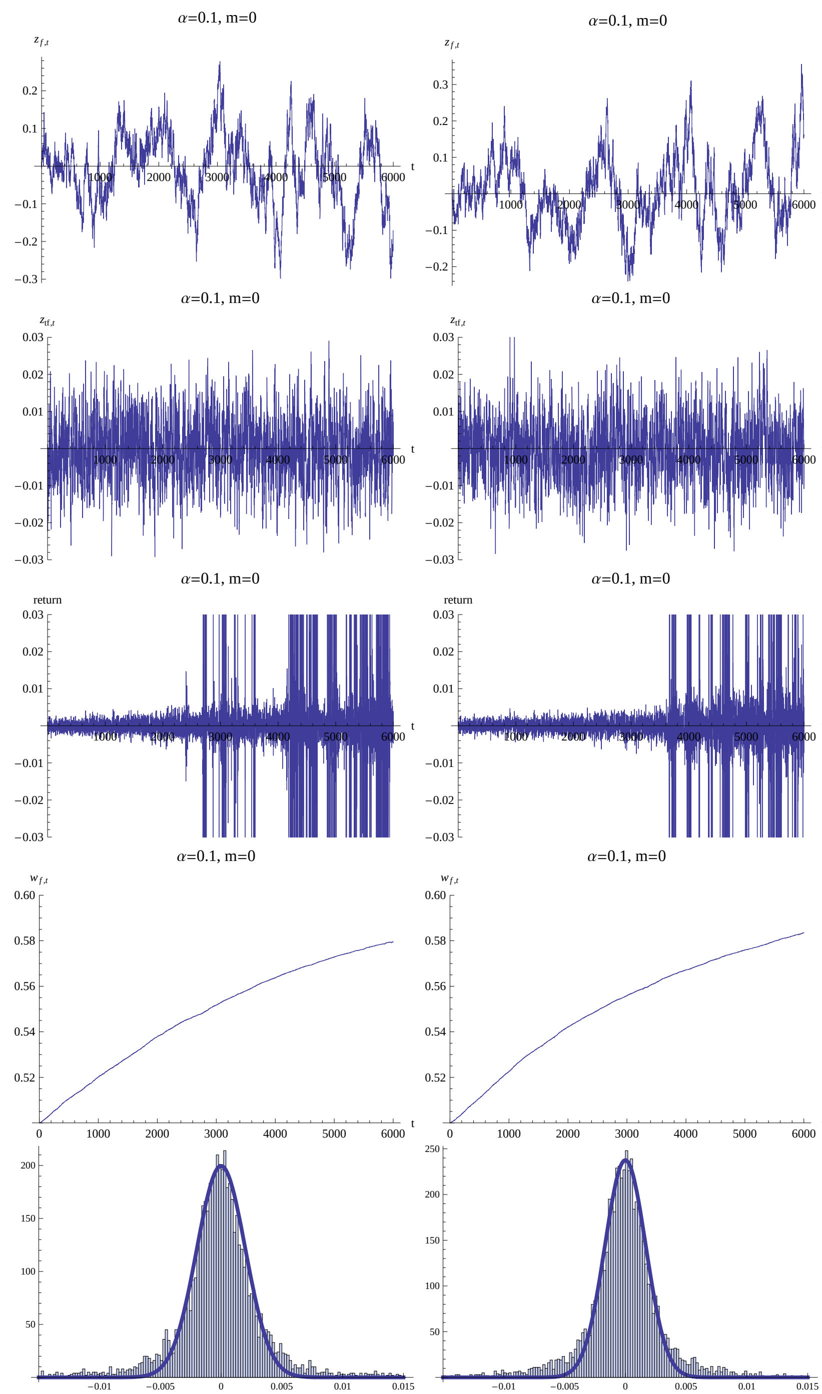

Figure 5.6: $\mu_2 = 0.9$, dividends not correlated, $(\sigma_D^2)^{(1)} = 1.6$, $(\sigma_D^2)^{(2)} = 1.6$.

Appendix B

Source code

```
(*arrays of prices on the sub-markets 1 and 2*)
(*e.g. P1[[1,5]] ... price on the sub-market 1 at period 1 of run 5*)
P1 = Array[0, {6000, 100}, {1, 1}];
P2 = Array[0, {6000, 100}, {1, 1}];
(*arrays of absolut wealth of fundamentalists on the sub-markets 1 and 2*)
Wf1 = Array[0, {6000, 100}, {1, 1}];
Wf2 = Array[0, {6000, 100}, {1, 1}];
(*arrays of wealth shares of fundamentalists on the sub-markets 1 and 2*)
wf1 = Array[0, {6000, 100}, {1, 1}];
wf2 = Array[0, {6000, 100}, {1, 1}];
(*arrays of demands of fundamentalists on the sub-markets 1 and 2*)
zf1 = Array[0, {6000, 100}, {1, 1}];
zf2 = Array[0, {6000, 100}, {1, 1}];
(*arrays of demands of trend followers on the sub-markets 1 and 2*)
ztf1 = Array[0, {6000, 100}, {1, 1}];
ztf2 = Array[0, {6000, 100}, {1, 1}];
(*arrays of excess capital gains on the sub-markets 1 and 2*)
Ret1 = Array[0, {6000, 100}, {1, 1}];
Ret2 = Array[0, {6000, 100}, {1, 1}];
(*arrays of log returns on the sub-markets 1 and 2*)
logR1 = Array[0, {6000, 100}, {1, 1}];
logR2 = Array[0, {6000, 100}, {1, 1}];
```

```
(*setting of seed for random number generator*)
SeedRandom[1000];

(*Initial setting of parameters:*)
alpha = 0.1;
m = 0;
mu = 2;
mu2 = 0.9*mu;
r = 0.05;
K = 250;
delta = 0.85;
gamma = 0.3;
Af = 0.8;
Atech = 0.8;
B = 1.0;
sigmaDelta = 1.0;
Wealth = 2000;
SigmaEpsilon = 0.01265;
prumerP = 100.0; (*long run fundamental value*)
(*sigma1=0.2;
sigma2=0.3;*)
 R = 1 + r/K;
 sigma211 = 1.6;
 sigma2D1 = sigma211*r*r; (*0.004*)
sigma212 = 1.6;
 sigma2D2 = sigma212*r*r;
NumberOfRuns = 100;

(*The main cycle, 100 runs, every run 6000 trading periods*)
For[run = 1, run <= NumberOfRuns, run++,
 (*inicialization of variables for the first period of each run:*)
  t = 1;
```

```
  fundamentalP1 = prumerP;
  fundamentalP2 = prumerP;
  zt1 = 0;  (*aggregate excess demand on sub-market 1 at time t*)
  zt2 = 0;  (*aggregate excess demand on sub-market 2 at time t*)
  Ut1 = 100;  (*sample mean at time t, sub-market 1*)
  Vt1 = 0; (*sample variance at time t, sub-market 1*)
  Ut2 = 100; (*sample mean at time t, sub-market 2*)
  Vt2 = 0; (*sample variance at time t, sub-market 2*)
  wf1[[t, run]] = 0.5;
  Wf1[[t, run]] = Wealth*wf1[[t, run]];
  (*absolute wealth of trend followers at time t, sub-market 1*)
  Wtt1 = Wealth*(1 - wf1[[t, run]]);
  zf1[[t, run]] = 0;
  ztf1[[t, run]] = 0;
  P1[[t, run]] = 100;
  Ret1[[t, run]] = 0;
  wf2[[t, run]] = 0.5;
  Wf2[[t, run]] = Wealth*wf2[[t, run]];
  (*absolute wealth of trend followers at time t, sub-market 2*)
  Wtt2 = Wealth*(1 - wf2[[t, run]]);
  zf2[[t, run]] = 0;
  ztf2[[t, run]] = 0;
  P2[[t, run]] = 100;
  Ret2[[t, run]] = 0;
  (*6000 periods of one run:*)
  For[t = 1, t <= 5999, t++,
   epsilont1 = RandomReal[NormalDistribution[0, 1]];
   epsilont2 = RandomReal[NormalDistribution[0, 1]];
   (*random walk process of fundamental prices:*)
   fundamentalP1 = fundamentalP1*(1 + SigmaEpsilon*epsilont1);
   fundamentalP2 = fundamentalP2*(1 + SigmaEpsilon*epsilont2);
(*computing of the optimal demand of fundamentalists in markets 1 and 2:*)
   zf1[[t,
```

```
  run]] = (alpha*(fundamentalP1 - P1[[t, run]]) - (R -
       1)*(P1[[t, run]] - prumerP))/(Af*(1 + r*r)*sigma211);
zf2[[t,
  run]] = (alpha*(fundamentalP2 - P2[[t, run]]) - (R -
       1)*(P2[[t, run]] - prumerP))/(Af*(1 + r*r)*sigma212);
(*computing of the optimal demand of trend followers*)
(*in markets 1 and 2:*)
Vt1 =
 delta*Vt1 +
  delta*(1 - delta)*(P1[[t, run]] - Ut1)*(P1[[t, run]] - Ut1);
Vt2 =
 delta*Vt2 +
  delta*(1 - delta)*(P2[[t, run]] - Ut2)*(P2[[t, run]] - Ut2);
Ut1 = delta*Ut1 + (1 - delta)*P1[[t, run]];
Ut2 = delta*Ut2 + (1 - delta)*P2[[t, run]];
ztf1[[t,
  run]] = (gamma*(P1[[t, run]] - Ut1) - (R - 1)*(P1[[t, run]] -
       prumerP))/(Atech*(1 + r*r + B*Vt1)*sigma211);
ztf2[[t,
  run]] = (gamma*(P2[[t, run]] - Ut2) - (R - 1)*(P2[[t, run]] -
       prumerP))/(Atech*(1 + r*r + B*Vt2)*sigma212);
(*computing of the excess demands on market 1 and market 2:*)
zt1 = ((1 + m)*zf1[[t, run]] + (1 - m)*ztf1[[t, run]])/2;
zt2 = ((1 + m)*zf2[[t, run]] + (1 - m)*ztf2[[t, run]])/2;
(*adjustments of prices:*)
P1[[t + 1, run]] =
 P1[[t, run]] + mu*zt1 + mu2*zt2 +
  sigmaDelta*RandomReal[NormalDistribution[0, 1]];
P2[[t + 1, run]] =
 P2[[t, run]] + mu*zt2 + mu2*zt1 +
  sigmaDelta*RandomReal[NormalDistribution[0, 1]];
(*computing ofexcess gain from the risky asset on markets 1 and 2:*)
random1 = RandomReal[NormalDistribution[0, 1]];
```

```
 random2 = RandomReal[NormalDistribution[0, 1]];
 Ret1[[t + 1, run]] =
  P1[[t + 1, run]] -
   R*P1[[t, run]] + (prumerP*(R - 1) + Sqrt[sigma2D1]*random1);
 Ret2[[t + 1, run]] =
  P2[[t + 1, run]] -
   R*P2[[t, run]] + (prumerP*(R - 1) + Sqrt[sigma2D2]*(random1));
 (*Computing of the wealth of fundamentalist and trend followers:*)
 Wf1[[t + 1, run]] =
  R*Wf1[[t, run]] + Ret1[[t + 1, run]]*zf1[[t, run]];
 Wf2[[t + 1, run]] =
  R*Wf2[[t, run]] + Ret2[[t + 1, run]]*zf2[[t, run]];
 Wtt1 = R*Wtt1 + Ret1[[t + 1, run]]*ztf1[[t, run]];
 Wtt2 = R*Wtt2 + Ret2[[t + 1, run]]*ztf2[[t, run]];
 wf1[[t + 1, run]] =
  Wf1[[t + 1, run]]/(Wf1[[t + 1, run]] + Wtt1);
 wf2[[t + 1, run]] =
  Wf2[[t + 1, run]]/(Wf2[[t + 1, run]] + Wtt2);
 (*returns:*)
 logR1[[t + 1, run]] = Log[P1[[t + 1, run]]/P1[[t, run]]];
 logR2[[t + 1, run]] = Log[P2[[t + 1, run]]/P2[[t, run]]];
 ];
];
```