Iannis Xenakis: The Analysis of Four Works for Piano Solo

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Music Education – Instrument playing (piano)

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Iannis Xenakis: analýza čtyř sladeb pro sólový klavír

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Declaration

I confirm that this is my own work and the use of all materials from other sources has been properly and fully acknowledged. I agree with storing my work in the library of the Faculty of Education, of Charles University in Prague, Czech Republic, in order to be available for educational purposes.

Constantine Soteriou

Prague, 29 March 2011
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Introduction

Iannis Xenakis is one of the most famous Greek composers of the 20th century and perhaps one of the most important avant-garde composers who sought new ways for creating music. His compositions are based on the principles of mathematics and one of his greatest innovations was his technique of stochastic composition. Xenakis believed that mathematics are fundamental to music. His conception of music was not concentrated on melody or harmony but rather on sound. He perceived music as masses of sound in musical space. Xenakis worked with acoustic instruments and was the creator of the computer music system UPIC, which was used for the composition of some of his works.

I was interested in analyzing avant-garde compositions in my search for understanding the principles and concepts used to structure such complicated and difficult compositions. Being a pianist myself, I was drawn to the compositions of Iannis Xenakis for solo piano. Even though such compositions are perhaps a bit “unfriendly” to the ear of the listener, they require an extensive analysis in order to be understood and subsequently fully appreciated. The aim of this diploma work was to introduce these works to other musicians and enhance their knowledge regarding new techniques required for the analysis of these compositions. The musician cannot rely on his knowledge of the traditional ways for analyzing avant-garde compositions. The reader is required to understand mathematical theories. For this reason I have decided to dedicate one chapter to the compositional concepts and theories used by Xenakis in his four compositions for solo piano. I have tried to simplify these theories so that they could be more easily understood by the musician. I believe that it is important to have a thorough understanding of these theories; otherwise it would be perhaps impossible to analyze such works, whose structure depends on these mathematical principles. For the analysis of Xenakis’s four solo piano compositions, I have mainly relied on the work of Ronald Squibbs, who concentrated on the compositional techniques used by Xenakis and who also provided me guidance for the analysis of the chosen compositions.

I hope that the reader will benefit from this diploma work and use this knowledge as a basis for further study of the work of Iannis Xenakis. The performance of such works makes a great demand on the pianist, as concerns the technique and interpretation. It is a challenge to analyze these compositions and perhaps even greater challenge to perform them.
“...you have the good fortune of being Greek, of being an architect and having studied special mathematics. Take the advantage of these things. Do them in your music.” – Olivier Messiaen referring to Iannis Xenakis.
1 Biography

Iannis Xenakis was one of the most important Greek composers and architects of the 20th century, as he achieved a revolutionary music style after the Second World War. He was the first person who replaced the typical and traditional musical way of thinking with revolutionary new musical concepts in sound composition, although he was – until then of course – an “outsider” in the studied subject of music. His innovative compositional methods that he had developed created a correlation between music and architecture with mathematics and physics through the use of the models of Set Theory, Probability Theory, Thermodynamics, Golden Ratio, Fibonacci sequence, etc. This entirely new way of thinking had a major influence on many younger composers and in general musicians in Europe and around the world.

Iannis Xenakis was the son of a Greek businessman named Klearxos Xenakis from Euboia, and Foteini Paulou from Lemnos. He was born on the 29th of May in 1922, but the exact location of birth is quite ambiguous, and thus almost nobody can tell for sure, as many sources support that he was born in Braila in Romania and some other support the opposite that he was born in Athens in Greece. He spent his entire childhood in Braila and was the eldest child amongst two other siblings, whose names were Iason and Cosmas. At some point in 1927, when he was only five years old, something tragic happened. The harsh and unfair death of his mother, who died from rubeola cost him a lot. Xenakis had first heard from her the beautiful piano playing of classical compositions, and in this way she achieved to implant to him her love for music. He had also said that he developed a “defense mechanism” against certain kinds of compositions due to the sad memories he had when he listened to them because they were linked to the bad childhood he experienced. Xenakis was, in Matossian’s words, “deeply scared by his mother’s death. He clung to the few experiences he had shared with her: the gift of a flute whose sounds had astonished him, her wish that he should enjoy music” (Harley 2004, p.1). He was a very sensitive person, and thus, this led him to his own alienation.

He was tutored in the Greek language and his early schooling was in Romanian. Another side of his sad childhood was that he had faced unfairly the racism as he was teased for being a “foreigner”. Gradually he learned English, French and German as well. When he was ten years old he was sent along with his brothers at a boarding school in Spetses, where he was taught piano and music harmony for the first time, discovering there also the magic of the Greek culture. This led him to a lifelong research and study. Xenakis had also participated as a singer for the very first time in Byzantine liturgical music and learned Greek traditional dances. He was singing also in a school choir, where a major part of the repertoire consisted of music by Giovanni Pierluigi da Palestrina, and when he had the chance he listened to the great masterpieces of classical music from the radio. It is also worthy of note that later, during the Second World War, a Greek comrade introduced him to the music of Béla Bartók, Maurice Ravel and Claude Debussy as well. In Greece they were making fun of his Greek accent and that had as a result, for him to experience his own adolescence as sad, full of problems and isolated as in his childhood. But on the other side, this isolation led him to the library gaining that way more and more knowledge about the philosophy and the Greek history.
After Xenakis finished his scholar duties in Spetses, at the time he was sixteen years old, he went to Athens to give examination in the National Polytechnic Institute of Metsovo, and at the same time he was following music harmony and counterpoint with Aristotle Koundourov, who was a student of Alexander Scriabin, performing thus his first compositional attempts. He had also a big interest in sciences which had led him to the learning of mathematics and physics. Thus, we observe that from that age he had an interest in the relation between mathematics and music, trying to find out how mathematical models could be applied in the “Art of Fugue” of Johann Sebastian Bach, and how the musical structures could be represented by graphics as optical correspondence of music, keeping at the same time his interest he had in Greek philosophy and literature, as one of his favorite philosophers was Plato.

In 1940 by the time he passed the examination, the Italian invasion occurred in Greece and thus the Polytechnic Institute had stopped working. It was the period where the Italians were supplanted by the Germans, who were replaced by the British army, leading to the Civil War. By the end of 1941, Xenakis then, influenced by all these circumstances had entered the Greek resistance, initially in the right-wing and then he became an acting member of the - at the time illegal - Greek Communist Party. Shortly afterwards he joined with others a student group named “Lord Byron” where later he became its commissar. Three years later, in December of 1944, whilst he was fighting illegally against the British authorities, a bomb shell hit him in the face and by miracle he was saved causing him however the loss of his left eye and in general deformation of the left side of his face. He recovered fast and he returned back to his studies in order to finish in February of 1946 with a degree of civil engineer. Later though, the authorities had decided to arrest all those who had participated in the Communist Party, including Xenakis, take them to concentration camps and then send them to exile in Makronissos.

In September of 1947 worrying about the exile, he had achieved to escape from the concentration camp, and with the assistance of his father and a fake passport he travelled to Italy. And because of this, the Greek authorities sentenced him to death as they consider him as a deserter. After his well-done maneuvers and with the assistance of the Italian communists, he winds up, illegally of course, in France on the 11th of November of the same year. Due to his poor interest of the country, and after his post-war concerns he had intended to travel to the United States of America, where his brother Jason was already studying philosophy. But without any money and without the proper papers, this was not efficient at all, although he was later going to work there for five years.

In the middle of all these life-devastated facts and experiences, Xenakis said that if he ever got the chance to devote his own life to music, he would do it at once without a second thought. He, characteristically, once explained, “In my loneliness and isolation I tried to hang on to something – after all, my old life and new circumstances, my old image of the new world and the new experiences, all these were in conflict. I wanted to find out who I really was. In that process, traditional Greek folk music appeared to be in a safe point…” (Harley 2004, p.2). And indeed he acted that way. And so, when Xenakis lived in Paris, he tried to compensate for the musical education he had missed during the war by taking lessons with Honegger and Milhaud. He also attended Messiaen’s analysis course at the Conservatoire from 1950 to 1952. Between 1955 and
1966 he was repeatedly invited to Gravesano by Scherchen, where he was introduced to musicians and experts in electro-acoustics, such as Max Mathews.

Soon and with the assistance of Yorgos Kandylis, Xenakis had been employed at the Le Corbusier’s architectural office, where he was working until 1959. Le Corbusier was one of the persons, from whom Xenakis was influenced and inspired for his later work and thus, this had a major affect in his lifetime carrier. In parallel, he was seeking teachers to continue his studies in composition. The first personalities he appealed to were Arthur Honegger and Darius Milhaud, members of the known group, called “Parisian Six”. Xenakis, though, was not disposed to learn the academic rules of harmony and counterpoint. Soon, he runs against his teachers, who did not respect his innovative ideas. Olivier Messiaen was the first person to respect and understand Xenakis’s musical distinctiveness, telling him that it is not necessary to study the harmony and counterpoint. Messiaen recalled, “I understood straight away that he was not someone like the others... He is of superior intelligence... I did something horrible which I should do with no other student... I said, “No, you are almost thirty, you have the good fortune of being Greek, of being an architect and having studied special mathematics. Take the advantage of these things. Do them in your music”” (Harley 2004, p.4). The only subjects Messiaen suggested to Xenakis to study with him were musical aesthetic and analysis, at Paris Conservatoire. Indeed, Xenakis had started lessons with Messiaen in 1952, while at the same time he was composing. At that period he met Françoise – known today as Françoise Xenakis – where in 1953 they got married and she gave birth to their daughter, named Mache.

From 1960, Xenakis had decided to be truly devoted in music, in composition in particular, after completing a chain of innovative architectural constructions, allocated by Le Corbusier. In particular, Xenakis was responsible for the kindergarten which was constructed on the roof of the residential block in Nantes-Rézé, parts of the government buildings in Chandigarh, India, as well as the rhythmically articulated glass façade of the monastery of St. Marie de La Tourette, near Lyons and the greater part of the chapel there. It is worthy of note that he had also designed the unique shape of the Philips Pavilion at the 1958 Brussels Exposition Universelle, based on a sketch of his employer, Le Corbusier. Most of his later architectural projects were intended to be designed for musical uses like a concert hall and studio for Scherchen’s musical centre in Gravesano in 1961 and the same for the Cité de la Musique in Paris in 1984 but the only design to be realized was the Diatope, one of his invented Polytopes. The space for one of the unique sound-and-light experiences consisted of a tent-like construction which was erected outside the Centre Pompidou in Paris for its opening in 1977 and later re-erected in Bonn for a Xenakis festival.

In 1955 with his orchestral work “Metastasis”, Xenakis received applause amongst the public and thus marking the beginning of “stochastic music”. In parallel, he was publishing his first texts in different kinds of magazines, expressing that way his philosophy about music, creating new terms and musical categories, whilst he criticized the serial music with his text “La crise de la musique sérielle” (English translation: “The Crisis of the Serial Music”), making that way Pierre Boulez and Karlheinz Stockhausen – major personalities of the innovative European music in France and Germany respectively – to become “enemies” of Xenakis, denomining him as
“stupid”. Despite the difficulties he was facing from the official cycle of the innovative European music, from 1960 and then, he made a runaway reputation which had started to be widely and globally spread.

The articles Xenakis contributed to Scherchen’s *Gravesaner Blätter* formed the basis for his book *Formalized Music*, where his first edition appeared in 1963 in French. He had worked from 1957 to 1962 in the so called Schaeffer’s Groupe de Recherches Musicales (GRM) until the year 1958, Studio d’essai de la Radio-Télévision Française, where his early electro-acoustic works had been brought to effect. In 1961, the time when he was invited to Japan, he received there major impressions of the Asian musical culture which strengthened him into the development of his idea, called ‘universal musical structures’. In 1962 something innovative had started while Xenakis had begun to compose a group of instrumental works with the help of a computer at IBM Paris, and thus, in order to extend his research into the nature of sound itself with the help of the computer, in 1966 he founded EMAMu (Equipe de Mathématique et Automatique Musicales), which in 1972 became CEMAMu (Centre d’Etudes de Mathématique et Automatique Musicales). From 1967 to 1972, Xenakis had been teaching at Indiana University in Bloomington US, where he also directed a Center for Mathematical and Automated Music. From 1973 until 1989, he was a visiting professor in Sorbonne, where he was awarded with a doctorate, in 1976, for his deep disciplinary research.

From 1970 until his death Xenakis stayed at the forefront of contemporary European music, working always in the frame of relation of mathematics, music and ancient Greek philosophy, with a personal, innovative and a secluded way as well, leaving fadeless the sign in the contemporary music of the second half of the 20th century. Xenakis died in Paris at sunrise on February 4th in 2001, at the age of seventy eight and after the lingering adventure he had with his health. His corpse was cremated in the underground crypt of the Pere Lachaise cemetery in Paris without any religious ritual, according to his last wish.
2 Xenakis’s Compositional Style

From the beginning of his career, Xenakis sought to understand better the mathematical concepts and theories and figure out how these theories could be applied to his music compositions. Xenakis studied mathematics at the Polytechnic School in Athens. He studied the probability theory, and the works of the founders of modern probability theory (Levy, Borel and Feller). In 1960 he studied algebra and logic with Georges Guilbaud at the University of Paris.

He began to create symbolic music, which means the application of the principles of symbolic logic to his own musical compositions. We can see the application of these principles in his composition *Herma* (1960 – 61) for piano, where he used set theory for his pitch collections. He also applied sieve theory in his composition *à r (Hommage à Ravel)* (1987).

In the mid-1960s, Xenakis was concerned with the idea of determinacy and indeterminacy. He said on the subject: “The two poles, one of pure chance, the other of pure determinacy, are dialectically blended in man’s mind (and perhaps in nature as well, as Epicurus or Heisenberg wished it). The mind of man should be able to travel back and forth constantly, with ease and elegance, through the fantastic wall, of disarray caused by irrationality that separates determinacy from indeterminacy.” (Squibbs 1996, p.19). Thus, Xenakis’s style of composing combined the elements of stochastic and non-stochastic music.

Xenakis used technology to help him calculate complex mathematical equations. Even though in the beginning of the 1960s, computers were only used by large corporations, Xenakis was able to gain access to a computer, thanks to Scherchen’s help at IBM in Paris. Xenakis was thus able to produce his first works using the computer. Some years later with the development of technology for the production of electro-acoustic sounds, Xenakis was able to find an electronic music studio of his own. This studio was called Centre d’Etudes Mathematiques et Automatiques Musicales (CEMAMu). One of his first projects was the use of computer technology for the combination of electro-acoustic sounds and the display of laser lights. Xenakis called this genre “polytope”. One of the first works composed was *Concret PH*, which was performed at the Brussels World’s Fair in 1958. Other works include *Polytope de Montreal*, performed in 1967 in Montréal Expo, *Hibiki Hana Ma*, which means *reverberation-flower-interval* (Harley 2004, p.67) performed at Osaka World’s Fair in 1970, and *Persepolis*, performed in Iran in 1971 and many others.

The electro-acoustic music, which was composed for the polytopes was mainly influenced from concrete music. Later, however, a new system was developed for the composition of electro-acoustic music, based on digitally sampled sounds. These compositions were produced in Unite Polyagogique Informatique du CEMAMu (UPIC). Originally the compositions were created on separate sections or pages, on a digital drawing board using an electromagnetic pen. Once those pages were created, the only way to listen to what was composed was with the use of a converter. Later, with the use of the real time technology a playback of the page was created during the compositional process. The works composed using this kind of technology were: *La

One important element of Xenakis’s compositional structure is segments, which contain individual elements. These segments are clearly defined in his compositions since they carry specific characteristics, such as: changes in texture, articulation, density, instrumentation, etc. By understanding these segments in Xenakis’s musical work we are able to perceive the large scale structure of his music. Another important element is duration and the management of the division of time in his compositions. This was inspired by Le Corbusier’s use of proportional systems in his architectural works. It is also worthy of note that pitch was an important structural element in his compositions. He perceived pitch as individual points in a continuous frequency spectrum. This was different from the theories of tonal, atonal, and serial music.

Xenakis made a great contribution to the music of the 20th century. He combined his mathematical knowledge and more specifically set theory, sieve theory, game theory and group theory as well as computer technology for the creation of his musical compositions. Xenakis was influenced by the idea of complex masses of sound from the experience of mass rallies in Greece. He also found inspiration in the techniques used in concrete music. In these aspects his compositional techniques differed from those of his contemporaries.
3 Basic Concepts and Theories used in Xenakis’s Music

Sonic events and their attributes

Sonic events are the independent musical sounds which are found in an abstract musical space. Their attributes include: pitch, duration, and intensity, which are called dimensions. The positions of these musical sounds can be measured relatively to each other, for example: low, high, soft, loud, etc. The attributes of these musical sounds can also be measured in terms of their unit value. Therefore, we can imagine these musical sounds positioned along an axis. We can also imagine several of these musical sounds simultaneously. Thus, we can describe the position and state of a sonic event, based on the coordinates of the axis.

The attributes, dimensions, relative position, and unit values are described in the following table. This table was given by Squibbs in his chapter of sonic events in his work: *An analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works* (1996, p.36).

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Dimension</th>
<th>Relative Position</th>
<th>Unit Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>p-space</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>Sequential time</td>
<td>st-space</td>
<td>before</td>
<td>after</td>
</tr>
<tr>
<td>Time-point</td>
<td>tp-space</td>
<td>early</td>
<td>late</td>
</tr>
<tr>
<td>Duration</td>
<td>d-space</td>
<td>short</td>
<td>long</td>
</tr>
<tr>
<td>Intensity</td>
<td>i-space</td>
<td>soft</td>
<td>loud</td>
</tr>
</tbody>
</table>

The pitch’s (p-space dimension) relative position is in terms of how low or how high it is and its unit value is its particular height (semitone, quarter tone). Sequential time (st-space dimension) refers to the position of a musical sound in relevance to other musical sounds. Thus, its relative position could only be before or after. Its unit value is given by numbers. Time-point (tp-space dimension) concerns with the time or tempo in a musical space and is defined by the composer. Its relative position has to do with the exact time a musical sound is heard. Duration (d-space) has to do with the rhythm of musical sounds and the intensity (i-space) with the dynamic. Xenakis defined these parameters (pitch, sequential time, time, duration and intensity) with numbers.

For example: in *Herma* Xenakis denotes tp-space as being one beat, in this case one-quarter note. \( \text{♩} = 104\). The origin is given the number 0. The origin of the duration is also given the number 0. The unit for pitch is one semitone. Middle C has the number 0. The notes below middle C are found by counting the number of semitones below middle C and the numbers above middle C by counting the number of semitones above middle C. The lowest level of intensity (ppp) is represented by 1.
Herma’s first two measures are illustrated below:

First note: E1, the tp-space is 0, because is the origin, the pitch is -20 (twenty semitones below middle C), the duration is 2 (two quarter-tones) and the intensity is represented by 1.

Second note: F# 5, the tp-space is 1, the pitch is 18 (eighteen semitones above middle C, bearing in mind that the middle C is equal to 0), the duration is 2 (two quarter-tones) and the intensity is represented by $1 + \Delta i$. Xenakis illustrates the crescendo by $\Delta i$, and every time the musical sound that follows, carries an extra number.

Third note: B flat, the tp-space is 2, the pitch is -14 (fourteen semitones below middle C), the duration is 1 (one quarter-tone) and the intensity is represented by $1 + 2\Delta i$.

Fourth note: B, the tp-space is 3, the pitch is -37 (thirty seven semitones below middle C), the duration is 2 and the intensity is represented by $1 + 3\Delta i$.

We can write these four notes as follows:

$S = (tp, p, d, i)$

$S_0$ (first note) = $(0, -20, 2, 1)$

$S_1$ (second note) = $(1, 18, 2, 1 + \Delta i)$

$S_2$ (third note) = $(2, -14, 1, 1 + 2\Delta i)$

$S_3$ (fourth note) = $(3, -37, 2, 1 + 3\Delta i)$

In this manner all notes can be analyzed following the same principles.

**Vector model**

A vector is a way of measuring the distance (direction) and size between two points. In the musical aspect, the vector measures the distance and size between two musical sounds. Vectors are represented in what we call “vector space”. Any member of a particular vector is known as a component. I shall now give an example of the first four notes of Xenakis’s composition *Herma*. I have so far given the analysis of these four notes in terms of sonic events. Now I will attempt to analyze these four notes based on the vector model (the example of *Herma’s* two measures can be seen above).
\[ V = \langle \text{tp}, p, d, i \rangle \]

\[ V_0 \text{ (first note)} = \langle 0, -20, 2, 1 \rangle \]

\[ V_1 \text{ (second note)} = \langle 1, 38, 0, \Delta i \rangle \]

\[ V_2 \text{ (third note)} = \langle 1, -32, -1, \Delta i \rangle \]

\[ V_3 \text{ (fourth note)} = \langle 1, -23, 1, \Delta i \rangle \]

These numbers are derived from the comparison of the distance between the tones. (For example: \( V_0 \) is the same as \( S_0 \) because the initial point is \((0,0,0,0)\). To find the number 38 we have to count the distance in semitones between the first note and the second note. Then, to find the number 32, we count the distance in semitones between the second note until the third. In this way, we can find the distance of all notes, by counting the distance between them. The reason why there is a negative sign before a number is when that number is below middle C.

In order to find tp-space based on the vector model, Xenakis said that the first note being the origin is equal to 0. Since the following three notes appear exactly at the beat the distance between them is only 1.

As concerns duration, we know that one-quarter note equals 1. Therefore, the first note being two quarters equals 2. The second note is also two quarters. We represent, however, its duration with the number 0. The reason for this is that the distance between the first note (two-quarters) and the second note (two-quarters) is 0. Now the distance between the second note (two-quarters) and the third note (one-quarter) is 1. Similarly, we can calculate the distance between the third note (one-quarter) and the fourth note (two-quarters), which is 1.

Interestingly enough, if we add \( V_0 \) and \( V_1 \), we get the result of \( S_1 \). If we add \( V_0 + V_1 + V_2 \) this equals \( S_2 \) and so on.

\[ V_0 = \langle 0, -20, 2, 1 \rangle + V_1 = \langle 1, 38, 0, \Delta i \rangle = \langle 1, 18, 2, 1 + \Delta i \rangle = S_1 \]

**Collections of Sonic Events**

When analyzing Xenakis’s compositional work it is important to understand the attributes associated with the collection of these sonic events. These attributes are pitch (p-space), sequential time (st-space), time point (tp-space), duration (d-space), intensity (i-space), density (d-space) and registral span (r-space). These attributes have already been discussed above with the exception of registral span and density. When we analyze collections of sonic events pitch refers to the minimum and maximum pitch levels that take place within the segments of a composition. The registral span is the size of the interval between the minimum and maximum pitches in a segment. Density is measured numerically in sounds per second (s/s).
The Model for the structure of Sonic Events

Xenakis’s compositions are based on four types of structure:

Outside-time structure includes the dimensions of pitch, intensity, and duration (p-space, i-space, and d-space) and articulation.

In-time structure includes the state and relation of sonic events, which are ordered with respect to time in the composition.

Temporal succession includes the dimension of sequential time (st-space).

Temporal structure includes the dimension of time-point (tp-space).

Set theory

Set theory in mathematics is the study of sets or collection of elements. A sequence is the collection of elements in the order they appear. The spacing of a set (SP) is an ordered listing of the intervals between the elements in a particular set.

For example:

Set A = \{-5, -2, 1, 3, 7, 11, 13, 16, 17\}

SP (A) = <3 2 4 4 2 3 1>

The interval succession (INT) of a sequence is the ordered listing of the intervals of the elements in a sequence.

For example:

Sequence B = <8 -3 4 6 4 19 23 -7>

INT (B) = <-11 7 2 -2 15 4 -30>

There are three basic operations: Union, intersection and complementation.

For example:

A = \{-5, -2, 1, 3, 7, 11, 13, 16, 17\} and E = \{-10, -4, -2, 0, 5, 7, 12, 16, 19\}

The union of A and E is the collection of the elements belonging to both sets.

A \cup E = \{-10, -5, -4, -2, 0, 1, 3, 5, 7, 11, 12, 13, 16, 17, 19\}

The intersection of A and E contains only the common elements from both sets.

A \cap E = \{-2, 7, 16\}

The complement of a set includes the elements that do not belong to the original set, but are contained in a universal set, which includes also the elements of the original set.
For example:

The universal set $U = \{-5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{-5, -2, 1, 3, 7, 11, 13, 16, 17\}$

The complement of $A$ with respect to $U$ is: $A^C = \{-4, -3, -1, 0, 2, 4, 5, 6, 8, 9, 10, 12, 14, 15, 18, 19, 20\}$

A subset is a set that includes the elements that are members of another set.

For example:

If $A = \{-5, -2, 1, 3, 7, 11, 13, 16, 17\}$ and $F = \{1, 7, 13, 16\}$

$F$ is thus a subset of $A$ and is indicated as $F \subseteq A$

A superset is a set that not only includes the elements that are members of another set but also has additional elements of its own. The indication is $A \supseteq F$.

**Sieve Theory and Modular Arithmetic**

Sieve theory is a number theory based on the principles of set theory, as they are applied to modular arithmetic. Before explaining the application of sieve theory in music, it is important to understand what modular arithmetic is. Modular arithmetic in mathematics is a system where integers are numbered until they reach a certain value (modulus), and then they start again from the beginning. An example of modular arithmetic is the twelve-hour clock. The day is divided into twelve hours. Assuming that now is nine o’clock, in five hours it will be two o’clock. But if we add nine and five together the answer will be fourteen. The number fourteen is not however the correct answer; this is because the arithmetical limit for the clock is twelve.

We can apply this knowledge of sieve theory and modular arithmetic in music, if we say, for example, that the smallest interval in music – that is semitone – is equivalent to a number in a series of integers of equal value. In this way, we can create a series of intervals, which form a specific succession and which combined, they can create a scale. Thus, sieve theory can be seen as a method for constructing scales of arbitrary complexity. (Turner 2005, p.1). Xenakis’s conception about sieve theory: “Sieve theory... is applicable to any other sound characteristics that may be provided with a totally ordered structure, such as intensity, instants, density, degrees of order, speed, etc... This method can be applied equally to visual scales and to the optical arts of the future.” (Turner 2005, p.1).

**Sonic Configurations**

Sonic configurations are collections of sonic events in terms of their inside-time structure in a composition (inside-time structure refers to the state and relation of sonic events). There is only
one sonic configuration in each segment. Specific types of sonic configurations were preferred by Xenakis in his career. Some examples are:

Cloud: represented by musical sound in p-space and tp-space. (Examples are seen in *Herma*).

Random walk: characterized by linear melodic movement. (Examples are seen in *à r*, and *Mists*).

Arborescences: represent polyphonic textures. These are inspired by mathematical models and they provide a complex rhythmic and melodic texture. (Examples are seen in *Mists*).

**Probability Theory**

The knowledge of probability theory is important for the understanding of stochastic music. Probability theory in mathematics deals with the question: what is the probability of a possible outcome occurring in a specific situation? A possible outcome is also known as an event. And all possible outcomes if gathered together are known as an outcome set. For example: if we toss a coin in the air, what is the possibility of the outcome being tail or head? The answer is that since there are only two possibilities (tail, head); the possibility of either outcome is one half (\(\frac{1}{2}\)).

**Exponential Distribution**

In order to calculate the intervals between time points, Xenakis used the exponential distribution. The reason for this choice is that the application of exponential distribution has more probability for creating smaller than larger intervals. In this way notes are clustered next to each other and only a few large intervals appear between them. Xenakis also uses the exponential distribution to determine the density (sound/second) of the musical texture.
4 Xenakis’s Piano Works

During his career, Xenakis wrote only seven compositions for solo piano. Four of these have been chosen for analysis in this diploma dissertation. His piano works define a stylistic period of his compositional progress. The first chosen work for analysis is *Herma* (1960 – 1961), which was one of his earliest compositions. *Herma* was composed based on the principles of symbolic music and stochastic elements. *Evryali* (1973) is one of his longest compositions for solo piano, which makes use of the process of stochastic composition and combines four configuration types (time-point sequences, stochastic, types, rests, and arborescences). *Mists* (1981) is the third composition chosen for analysis. While it combines elements explored earlier in the two compositions, Xenakis introduces also the idea of scales, whose pitch material is derived from the theory of sieves. The last work is *à r.* (*Hommage à Ravel*) (1987). Xenakis uses some of the material used in *Mists*, while introducing new ideas, such as random walks and simultaneities. These four piano compositions are examples of Xenakis’s mastery of temporal structure, and his carefully designed proportions from the smallest segments until the larger sections of a composition.

Xenakis also composed fourteen unpublished works for solo piano. These are: *Seven piano pieces without title*, *Menuet*, *Air populaire*, *Allegro molto*, *Mélodie*, *Andante*, all composed between 1949 until 1950; *Suite* composed between 1950 until 1951; and *Thème et conséquences*, composed in 1951.

Other works, which include the piano, are the following: orchestral, choral and vocal compositions, as well as chamber compositions. The orchestral works are: *Synaphaï* (1969), for piano and orchestra; *Erikthon* (1974), for piano and orchestra; *Keqrops* (1986), for piano and orchestra. Xenakis also composed *Zyia* (1952), for male voices, flute and piano. His vocal works include: *Tripli zyia* (1952) for one voice and piano; *Trois poèmes* (1952), on words by poets F. Villon: *Aiés pitié de moy*, V. Mayakovsky: *Ce soir je donne mon concert d’adieux*, Ritsos: *Earini Symphonia* [Spring Symphony], for one voice and piano; *Pour Maurice* (1982), for baritone and piano. Finally Xenakis also wrote chamber compositions: *Morsima-Amorsima* (1956 – 1962), for piano, violin, violoncello, and double base; *Eonta* (1963 – 1964), for two trumpets, three trombones, and piano; *Dikthas* (1979), for violin and piano; *Palimpsest* (1979), for english horn, b clarinet, bassoon, horn, percussion, piano, and string quintet; *Thalleïn* (1984), for piccolo, oboe, clarinet, bassoon, horn, trumpet, trombone, percussion, piano, and string quintet; *Akea* (1986), for piano and string quartet; *Paille in the wind* (1992), for violoncello and piano; *Plektó* (1993), for flute, clarinet, percussion, piano, violin, violoncello.
5 Herma (1960 – 61)

5.1 Introduction

Many composers of the 20th century have been both liked and disliked for their use of abstract compositional procedures. Iannis Xenakis is known for employing mathematical theories in his compositions. Herma was Xenakis’s first published solo work and was composed between 1960 and 1961. Herma means both “bond” or “foundation” and “embryo” or “germ” (Harley 2004, p.26). Herma was subtitled “symbolic music” which means the music based on mathematical principles. Herma is thus an example of Xenakis’s composing technique known as “musique stochastique”. The composition was premiered in Tokyo by the pianist Yuji Takahashi in February 1962. Takahashi writes in a letter to Xenakis: “I received your score and letter. Thank you for such an extraordinary, intense, radical and passionate music. It is very difficult but not too much. I will try to do my best.” (Matossian 2005, p.198). Herma was premiered in France by the pianist Georges Pludermacher, at a concert of the Domaine Musical in May 1963.

5.2 Analysis of the work

Compositional Styles used in Herma

I will try to explain the meaning of Stochastic Music leading to a further discussion of the development of this style. I would like to mention the fact that the end of World War II found France in a period of reconstruction. The same happened as many composers needed to continue the progress of their ideas, not only in music but in any kind of art as well, as an escape from the intrusions of fascist into the cultural life of Western Europe. Neoclassical works such as that of Igor Stravinsky’s, Sergei Prokofiev’s etc, had been in contrast to the works of Arnold Schoenberg and Anton Webern, who worked with serial technique (serialism). Pierre Boulez and Karlheinz Stockhausen tried to apply the elements from serial technique from the second Viennese school in terms of rhythm, instrumentation and form. Xenakis was innovative in applying the probability theory and mathematics into his musical compositions. This he called “stochastic music”. The word stochastic derives from the Greek word “stochos” which means “aim”. Stochastic music is linked with the random procedure of choosing the structural elements of a composition. In stochastic music the various aspects of sounds are manipulated individually in order to form the musical texture. Instead of using series or modes, Xenakis makes use of probability distributions to organize the musical pitches. By using the logical mathematical theories, Xenakis sought to create disorder in music. Herma may not be the first example of Xenakis’s stochastic music, yet it is the first composition for piano solo where the elements of stochastic music are explored, as the previews ones were Metastasis and Pithoprakta for orchestra.

Herma is based on the mathematical concepts of set theory and probability theory. The structure of the composition is based on set theory and the choice of pitches, intervals and durations are based on the random procedure of probability theory.
The structure of the work based on set theory

*Herma* is based on the organization of pitches. All the pitches of the piano keyboard are represented by R, which is the universal set. This is later subdivided by the sets A, B, and C.

\[ R = 88 \text{ pitches of piano keys divided by three sets A, B, C (each have one third of the pitches)}. \]

The pitches in the order they appear in each set:

\[ R = E, F\#, Bb, B, G, D\#, C, C\#, F, G\#, A, D, A\#, Eb \]
\[ A = D, E, F, G, A, G\#, F\#, C, A\#, D\#, B, C\#, Db, Eb, Ab \]
\[ B = A, G, C\#, A\#, B, C, D, D\#, E, Bb, Gb, F, G\#, F\#, Eb \]
\[ C = C, C\#, F, G, A, B, A\#, E, F\#, D\#, D, G\#, Eb, Bb, Dd \]

Set F is the last part of the composition and is based on two equivalent equations:

\[ ABC + ABC + ABC + ABC \]
\[ (AB + AB)C + (AB + AB)C \]

*Herma* is based on sequences derived from the operations in set F.

Example 1: The beginning of set F.

There are three basic relations between the three sets:

Union = the combination of all the pitches found in specific sets. (i.e., if A contains the pitches c, d, f#, g and if B contains the pitches a, b, d, c#, then the union of the A and B is c, d, f#, g, a, b, c#).

Intersection = this includes only the common pitches that are found in two or more sets.

Negation = this includes all the pitches that are do not belong in a selected set.
The composition begins with the introduction of the pitches found in R and then followed by the presentation of the three sets A, B, and C. There is a small transition from one set to another so that a pause will not exist between them. These sets are presented in a similar manner to how themes are traditionally presented in the beginning of a composition. Xenakis develops these “themes” (the three sets) in a way resembling the development of themes found in traditional compositions. The development section of Herma is based on the mathematical principles of set theory, such as the intersection of sets and their complements. These developing “motifs” are called subsets and are smaller than the primary sets (A, B, C). The composition ends with the repetition of these subsets. Thus, we can say that Herma begins with a simple introduction of each set: R, A, B, C and slowly leading to a full intersection of all these sets.

Introduction = R followed by A then B and finally C. The introduction is the first four minutes of the composition.

Development = subsets

Closing section = repetition of the subsets after their original appearances.

The levels of dynamic

In Herma we find not only changes in pitch but also in dynamic. Five different types of dynamic are used in this composition:

Pianississimo (pppp)

Pianissimo (pp)

Forte (f)

Fortissimo (ff)

Fortississimo (fff)

The composition begins with ppp and gradually through a crescendo reaches the highest dynamic, which is fff, in measure 27. The rest of the composition fluctuates in the highest levels of loudness.

Density

Another characteristic of this composition is density. According to Xenakis, density is how many sounds (tones) occur every second in the music. We can see in the introduction that the density gradually increases and reaches its peak before the arrival of the A section in measure 30. The composition ends at the highest level of dynamic and density.
Terms “linear” and “cloud”

These terms appear throughout the composition and have a special meaning. Linear means the spatial succession of notes in high dynamic. Cloud means the densely succession of notes in low dynamic (pp). When the cloud effect appears, the pianist employs the damper pedal.

Linear = low density and high dynamic.

Cloud = high density and low dynamic.

Overlapping of sets

The overlapping of sets is another characteristic of the composition. When the intersection of these sets takes place, the density is increased causing also the pitches from the different sets to come closely with one another. This overlapping characterizes the latter part of the composition which forms set F.

The application of Probability Theory

The basis of the probability theory regarding its application in Herma, is the interval (the distance between two pitches). The music is based on the succession of intervals, which are generated from two basic characteristics of sound. These two characteristics are: the pitch, and the time
when the sound begins, known as “attack time”. Xenakis composed these intervals based on three probability distributions:

1. **Exponential distribution** = this is used for the composition of intervals between attack times. It determines *when* the sound begins. As the size of an interval increases linearly, the time interval decreases exponentially.

2. **Linear distribution** = this helps to determine the intervals between pitches (size). As the size of an interval increases linearly, the probability of appearing decreases linearly.

3. **Uniform distribution** = it determines whether the interval will move up or down. As the size of an interval increases linearly, the probability of appearing remains constant.

Xenakis worked with probability theory in the beginning manually, calculating everything by hand and the help of mathematical equations, and then with the help of computer program for the composition of stochastic music. The way Xenakis originally worked with probability theory was the following: he divided the range of possible values equally and then with the help of mathematical equations he found the probability of the occurring intervals. A result close to 0 signified that there was less probability of the interval occurring, whereas a result close to 1 signified that there was more certainty of that happening. Xenakis would then choose a number of intervals, place them end to end, and thus, receive a random succession of intervals.

**Golden Ratio**

The arrangement of the sets in *Herma* is not only based on a random procedure. Xenakis structured the sets in the composition in a way respecting the golden ratio (0.618), which is based on Le Corbusier’s use of the golden ratio in his own architectural works. According to Squibb’s calculations, the beginning of the AB set commences at 4’07”, which is approximately thegolden ratio compared to the duration of the whole composition which is 6’44” (excluding the periods of silence between the sets). The approximate golden ratio can be found using the following calculation: $\frac{247}{404} = 0.611$. 
The use of the golden ratio contributes to the creation of a new kind of symmetry and dynamic in the whole work. It is worthy of note that Xenakis also carefully calculates the duration of each set in the composition based on the mathematical equation of exponential distribution.

Example 5: The beginning of the set AB which signifies the golden ratio of the work.

5.3 Conclusion

Mathematical logic is used as a basis for the structural development of Herma. The composition is based on the concepts of set theory and probability theory and is a great example of the architecture of sound. Although what sounds to the ear is a chaotic enumeration of pitches, in reality is a rational calculation of sound, duration and intervals. In terms of interpretation, the pianist is required to play fast and evenly on all registers of the keyboard. Takahashi writes to Xenakis about his impressions after the first performance of Herma: “It made some excited and wonder, others feel painful, totally I think. They were deeply impressed with your music which is absolutely unique and intense. I wish to play it again in the near future.” (Matossian 2005, p.200).
6 Evryali (1973)

6.1 Introduction

It is written in 1973 and it is the longest in duration in comparison to the other solo piano compositions of Xenakis (Herma, Mists, a r.). The word “Evryali” is the Greek word for the “open sea” and also “Medusa”. The name Medusa could possibly refer to the use of arborescences in the composition which resemble the tangling of Medusa’s hair, because of their branching structure.

6.2 Analysis of the work

Configurations

The work is divided into fifty segments. Each segment has a different type of configuration. There are four types of configurations:

Time-point sequences (tpseqs)

Stochastic types (ST)

Arboresences (A)

Rests

Time-point sequences

Time-point sequences derive from Xenakis’s application of the Sieve Theory. Twenty-three of the fifty segments of this composition are time-point sequences. An example of a tpseqs is in the opening measures (1 – 4) of Evryali, where we have seven pitches: C4, D4, D#4, E4, F4, G4, and A4. Their time-points represent the exact time of appearance of the specific pitch. Their exact time of appearance in the range of 0 to 60 of each of the seven pitches is calculated by Squibbs in his work An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works (1996, p.65) and is given below. The calculation of the numbers within each group, which corresponds to a specific note, is given based on the value of the sixteenth note. For example the first note to appear in the measure is F4 and has the value number 0. The following note is D4 and has the value number 3. This is because the distance between the first note and the second is the duration of three sixteenth notes. The third note is F4 and has a value number 4 because it appears on the fourth sixteenth note. The limit is 60 because in these four measures we have 60 sixteenth notes.

C4 = <19, 24, 27, 31, 36, 41, 46, 50, 53, 57, and 60>

D4 = <3, 7, 8, 14, 16, 18, 21, 24, 28, 31, 35, 38, 43, 46, 51, 54, 58, and 60>

D#4 = <29 32 48 52 55 57>

E4 = <12 16 21 26 29 34 39 44 48 52 55 60>
F4 = <0 4 6 13 18 21 23 26 30 34 36 38 41 45 47 49 53 56 58 60>

G4 = <6 11 15 20 23 34 40 43 48 52 55 58 60>

A4 = <10 14 18 24 28 30 33 38 42 47 51 56 60>

Thus we can conclude that C4 appears eleven times, D4 eighteen times, D#4 six times, E4 twelve times, F4 twenty-one times, G4 thirteen times, and A4 thirteen times as well.

Stochastic types

The stochastic types (ST) refer to the way Xenakis stochastically composed some of the measures in *Evryali*. The model of stochastic composition is based on two aspects: the temporal aspect (the specific time a sound appears) and a non-temporal aspect (pitch, duration of specific notes etc.). Xenakis used for the stochastic modeling of these passages in *Evryali* the mathematical distributions: the exponential and linear distribution. We should also bear in mind that both aspects of stochastic composition (temporal and non-temporal aspect) include random structures (the word stochastic means “random”).
In *Evryali* the stochastic types appear four times. The first ST is found in segment 2 (measures 5 – 35). Within segment 2 there are two types of configuration that appear simultaneously. Measures 16 – 18 are segment 3, which is a time-point sequence; measures 25 – 28 are segment 4, which is also a time-point sequence; measures 28 – 31 are segment 5, and measures 33 – 35 are segment 6, both a time-point sequence. We can distinguish the stochastic types from the time-point sequences in three ways (Squibbs 1996, pp.151 – 152):

The density of the stochastic type is less than the time-point sequences.

The intensity of the stochastic type varies from *ppp* until *ffff*, whereas the intensity of the time-point sequences remains constant at *ffff*.

The last difference between them is the use of the damper pedal, which is employed only where the stochastic type appears. It is not used for the time-point sequences.

The next stochastic type appears in segment 7 (measures 36 – 40). The intensity of this segment remains between *mf* until *ffff*. This segment has, thus, a higher dynamic than in the first stochastic type in segment 2. Also the half-pedal is employed throughout this segment. In this way it resembles segment 2.
The third stochastic type is in segment 23 (measures 136 – 146). The pitch material of segment 23 uses the whole register of the keyboard. The highest pitch is C8, which is also the upper limit of the keyboard’s register. The lowest pitch is A0, which is the lowest limit of the keyboard’s register. The rhythm of this segment is constant since we have only sixteenth notes. The dynamic increases dynamically from $p$ to $fff$ and then decreases to $ppp$ at the end of segment (measure 145).
The fourth stochastic type is found in segment 24 (147 – 179). Like in segment 2, we have the overlapping of another configuration type in this segment. The configuration type that appears simultaneously with the stochastic type is the configuration of arborescences. Arborescences are found in the following measures: segment 25 (measures 148 – 149), segment 26 (measures 150 – 151), segment 27 (measure 152), segment 28 (measure 154), segment 29 (measure 155), segment 30 (measures 158 – 160), segment 31 (measures 159 – 160), segment 32 (measures 161 – 166), segment 33 (measures 165 - 170), and segment 34 (169 - 171). We can distinguish between the stochastic type and the arborescences as concerns their density. The stochastic type has a lower density than the arborescences. The half-pedal is held throughout most of the segment from measure 147 until the end of measure 171.

Arborescences

Arborescences are linear lines of musical notes that have a polyphonic structure. Each line of notes is dependent on the previous lines of notes. This means that the beginning note of a line is the same note that exists also in the previous line. This creates an effect of continuity. Arborescences appear for the first time between measures 46 – 60. Arborescences are also used in Xenakis’s solo piano composition *Mists* (see chapter 7.)

The arborescences appear in two ways. They appear either with branching structures or without. Even though this is not common for arborescences, the reason why some arborescences without branching structures are classified as belonging to this configuration is their similar motion in pitch-space. Another reason is that these non-branching types of arborescences are not at all
similar with the other types of configurations (time-point sequences and stochastic types). An example of an arborescence without a branching structure is segment 9, where is already, partially discussed above. This arborescence expands in the whole register of the keyboard. Its lowest pitch is A#0 and its highest is F#7. Segment 12 (measures 66 – 69) is also an example of a non-branching arborescence. It begins at a high register and gradually descends. Its lowest pitch is also like in segment 9 A#0 and its highest F#7. In the end of measure 68 the pitches gradually ascend.

The appearance of a complex arborescence, which has a branching structure is found in segment 14 (measures 75 – 87). This arborescence begins with A0 and gradually ascends until it reaches C8, which is also the upper limit of the keyboard’s register. Thus, the pitch material covers the whole register of the keyboard. This arborescence begins with only one line of pitches and gradually builds up until it reaches as many as five lines.
Measures: 75 – 80, segment 14

After the arborescence in segment 14 two more arborescences appear in segment 16 (measures 192 – 195) and in segment 18 (measures 97 – 100). These two segments occupy the higher register of the keyboard. Between them there is a brief appearance of a time-point sequence configuration in measures 95 – 97. Segment 18 is also followed by the brief appearance of a time-point sequence in measures 100 – 102. This alternation between the arborescences and the time-point sequences prepare segment 20, which is one of the longest configurations, since it covers more than thirty measures (measures 102 – 136).

The arborescences that are found in segments 25 – 34 (measures 148 – 171) are part of the stochastic type of configuration in segment 24, which covers measures 147 – 179. The seven arborescences in segments 25 – 31 are short in length and have a low dynamic (pp). The following three arborescences (segments 32 – 34) are longer in length than the previous arborescences have a higher dynamic (mf) and cover a wider pitch-space from the lowest pitch of the keyboard until the highest pitch. The arborescence that follows in segment 35 (measures 179 –
188) is of great length and covers the entire range of the keyboard. As concerns its intensity it increases gradually from the lowest dynamic (pp) to the highest (ffff).

The three last arborescences that appear in Evryali are found in segments 47 (measure 207), 48 (measures 207 - 212), and 50 (measure 213). The time signature for these segments changes. Each measure does not have four-quarter notes, but rather each measure has indefinite length. The arborescence in measure 207 begins from the central register of the keyboard with only two lines of pitches and gradually ascends in pitch building up as to many as eleven tones, which are played simultaneously. It is important to note that segment 48 includes the measure 207 that was regarded as an independent segment, in this case segment 47. The reason for dividing the segments in such a way is that the arborescences can be seen either as independent, like in measure 207, or as a whole from measure 207 – 212. Segment 48 begins from the central register of the keyboard and gradually ascends. In the middle of this segment the tones descend from the upper register towards the center and then ascend again towards the upper register from the end of measure 211 until the end of 212. The final arborescence in measure 213 is also the last segment of the composition and appears after a ten-second pause. This final segment 50 begins at a slower tempo with the indication plus lent.

Measure: 207, segment 47
Measures: 208 – 212, segment 48
The application of Sieve Theory in Arborescences

The arborescence in segment 9 (measures 46 - 60) derives its pitch material from the mathematical application of sieve theory. The pitches in this segment are the following:

Register 0: A#0

Register 1: C1 C#1 D1 E1 F1 F#1 G1 A1 B1

Register 2: C2 C#2 D#2 F2 F#2 G#2 A#2 B2

Register 3: C3 D3 E3 G3 F3 A3 A#3 B3

Register 4: C#4 E4 D#4 F4 F#4 G#4 A#4 A4

Register 5: C5 C#5 D5 D#5 F5 F#5 G5 G#5 A5 B5
Register 6: C#6 D6 E6 F#6 G6 G#6 A#6
Register 7: C7 C#7 D#7 F7 F#7 G7 A7 B7
Register 8: C8

We can represent each of these pitches with a number. If we say that middle C (C4) = 0, then if we put them in order from the lowest pitch until the highest we get the following numbers:

\{-38, -36, -35, -34, -32, -31, -30, -29, -27, -25, -24, -23, -21, -19, -18, -16, -14, -13, -12, -10, -8, -7, -5, -3, -2, -1, 1, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14, 15, 17, 18, 19, 20, 21, 23, 25, 26, 28, 30, 31, 32, 34, 36, 37, 39, 41, 42, 43, 45, 47, 48\}

\{-38=A#0, -36=C1, -35=C#1, -34=D1, -32=E1, -31=F1, -30=F#1, -29=G1, -27=A1, -25=B1, -24=C2, -23=C#2, -21=D#2, -19=F2, -18=F#2, -16=G#2, -14=A#2, -13=B2, -12=C3, -10=D3, -8=E3, -7=F3, -5=G3, -3=A3, -2=A#3, -1=B3, 1=C#4, 3=D#4, 4=E4, 5=F4, 6=F#4, 8=G#4, 9=A4, 10=A#4, 12=C5, 13=C#5, 14=D5, 15=D#5, 17=F5, 18=F#5, 19=G5, 20=G#5, 21=A5, 23=B5, 25=C#6, 26=D6, 28=E6, 30=F#6, 31=G6, 32=G#6, 34=A#6, 36=C7, 37=C#7, 39=D#7, 41=F7, 42=F#7, 43=G7, 45=A7, 47=B7, 48=C8\}

The distance between the pitches can also be represented in numbers. Number 1 represents the minimum distance between two notes, which is the semitone. The distance between the pitches can be written as follows:

<21121112 2112212 21122112 21121111 21112212 2112212 211221>

**Rests**

The fourth type of configuration is rests (R). A rest appears for the first time in measure 65 and lasts twelve seconds (12”). The second time a rest appears is in segment 36 in measure 189 and lasts six seconds (6”). The pedal, which was held down from the previous arborescence continues to be held throughout the silence. Thus, the sound of the previous arborescence continues to resonate throughout the rest. The next rest appears in segment 49 at the end of measure 212 and has duration of ten seconds (10”). Rests or silence play an important role in the structure of the composition.
The rhythmic structure of *Evryali*

The basic unit value of the whole composition as it has already been discussed above (the section on time point sequences) is the sixteenth note. The rhythmical structure varies throughout the composition. The tempo is chosen by Xenakis and it is $\square=60$. The rhythmic values that predominate in time-point sequences are the sixteenth and thirty-second notes.

It is important to note that the first arborescences that appear in segments 9, 12, 14, 16, 18, and 20 and from 25 until 34 are more rhythmically varied than in the following arborescences in segments 35, 47, 48, and 50, where we have the steady repetition of sixteenth notes.

**Intersections between time-point sequences**

By the word intersection we mean how many notes appear simultaneously within a segment, which has the configuration of time-point sequences. This creates the characteristic of “order” and “disorder”. Disorder is created in the case where we have a small number of intersections within a segment. Thus, there exists more variety as concerns the rhythm in this specific segment. For example in segment 8 (measures 40 – 46) we have the following pitches in order of appearance: A#4, A3, F#4, F3, D#4, G3, B3, and C#4. No more than four notes are played simultaneously. The only time four notes (B3, C#4, D#4, and F#4) sound together is in measure 45. In contrary to segment 8, segment 46 (measures 198 - 206) is an example of a big intersection with eight notes sounding together. The pitches in segment 46 are the following: D3, F3, E4, F#4, G#4, C#5, A3, D#4, and Bb3. The eight notes (F#3, G3, Bb3, C#4, D4, D#4, F4, and A4) are played simultaneously and are repeated from the end of measure 200 until 206.
Measures: 40 – 46, segment 8
Segment 13 (measures 69 – 74) includes the following pitches in order of appearance: A#5, A5, C#6, F6, G5, B5, D#6, and F#5. In contrast to the previous two segments (8 and 46), which are already mentioned above, segment 13 begins directly with the intersection of six pitches (F#5, A5, C#5, D#5, F5, and A#5). The pattern of notes gradually becomes simpler until only one note is played in the end of measure 74.
Measures: 69 – 74, segment 13

Segment 15 (measures 87 – 92) follows the same pattern as segment 13. As many as eight pitches sound simultaneously from measure 87, but gradually less and less notes sound together until only three notes are played in the end of measure 92.
Time-point sequences in pitch space

The time-point sequences include pitches that occupy a certain register on the keyboard. There is a small distance between the lowest and their highest pitch. This makes it easy for us to see where the different segments appear in pitch space or rather what part of the register of the keyboard they occupy.

Segment 1 (measures 1 – 4) → it has a central position. Its lowest pitch is C4 and the highest is A4. (See example above).

Segments 3 – 6 (measures 16 – 35) → Segment 3 begins in measure 16 at a very high register (C#7) and gradually descends towards the lower range of the keyboard until measure 35, which is also the end of segment 6. These segments occupy the entire range of the keyboard. The lowest pitch is A#0 and the highest is C#7.
Measures: 16 – 35, segments 3 - 6

Segment 8 (measures 40 – 46) → it has a central position. Its lowest pitch is F3 and the highest is A#4. (See example above).

Segment 10 (measures 60 – 64) → it occupies a higher register than segment 8. Thus, we have a gradual ascending of pitches. Its lowest pitch is B4 and the highest is D#7.
Measures: 60 – 64, segment 10

Segment 13 (measures 69 – 74) → it occupies a high register but the pitches are lower than segment 10. Its lowest pitch is F#5 and the highest is F6. (See example above).

Segment 15 (measures 87 – 92) → it occupies an even higher register than the previous segments. Its lowest pitch is G#1 and the highest is G#6. (See example above).

Segment 17 (measures 95 – 97) → it begins at a lower register than segment 15, but gradually ascends in pitch. Its lowest pitch is G#3 and the highest is C5.
Segment 19 (measures 100 – 102) → like segment 17, segment 19 begins at a lower register and gradually ascends in pitch. Its lowest pitch is B3 and the highest is G#6.
Segments 21 and 22 (measures 107 – 114) → in these segments we reach the highest pitch in comparison to the previous ones. Both segments occupy a very high register. The lowest pitch is F1 and the highest is E7.

Segments 37 – 46 (measures 190 – 206) in these segments include the lowest pitch in the register of the keyboard. They occupy a high register, which begins to descend until it reaches the chord F#, F#3, G3, Bb3, C#4, D4, D#4, F4, and A4, which is repeated until the end of measure 206. The lowest pitch within these segments is B0 and the highest is D#7. We can also say that the time-point sequences in these segments begin at a high register, but descend and reach a central position like segments 1 and 8.
The Temporal Structure of *Evryali*

*Evryali* can be divided into two major parts. The division of the work occurs in the end of segment 20 in the beginning of measure 136. Segment 20 is one of the most complex and longest segments in this composition. The reason for this division is not only because of the importance of this segment but also because segment 20 ends on pitch E4 (beginning of measure 136), which is at the center of pitch-space. This pitch is also the beginning of segment 23. Thus, part one is segments 1 – 20 (segments 21 and 22 are part of segment 20) and part two is segments 23 – 50.

Measures: 135 – 136, end of segment 20

In the beginning of each part we have the appearance of two consecutive stochastic types. Part one begins with the appearance of the time-point sequence in segment 1 (measures 1 – 4), which is immediately followed by two stochastic types in segment 2 (measures 5 – 35) and segment 7 (measures 36 – 40). Part two begins with two consecutive stochastic types in segment 23 (measures 136 – 146) and segment 24 (measures 147 – 179). It is important to note that segment 2 in part one includes segments 3, 4, 5, and 6, which are time-point sequences. Segment 24 in part two includes segments 25 – 34, which are arborescences.

After the appearance of the stochastic types in part one we have an alternation of time-point sequences and arborescences, with the exclusion of the rest in segment 11 (measure 65), from segment 8 until segment 20. In contrast to part one, the arborescences and time-point sequences in part two are grouped together according to their type, whereas in part one, as we have already seen, they appear alternatively.

Part one

Time-point sequences (segment 1, measures 1 – 4).

Two stochastic types (segment 2, measures 5 – 35 and segment 7, measures 36 – 40).

Time-point sequences (segment 8, measures 40 – 46).

Arborescence (segment 9, measures 46 – 60).
Time-point sequences (segment 10 measures 60 – 64).

Rest (segment 11, measure 65).

Arborescence (segment 12, measures 66 – 69).

Time-point sequences (segment 13, measures 69 – 74).

Arborescence (segment 14, measures 75 – 87).

Time-point sequences (segment 15, measures 87 – 92).

Arborescence (segment 16, measures 92 – 95).

Time-point sequences (segment 17, measures 95 – 97).

Arborescence (segment 18, measures 97 – 100).

Time-point sequences (segment 19, measures 100 – 102).

Arborescence (segment 20, measure 102 – 136).

Part one = TPS, ST, ST, TPS, A, TPS, R, A, TPS, A, TPS, A, TPS, A, TPS, A.

Part two

Two stochastic types (segment 23, measures 136 – 146 and segment 24, measures 147 – 179).

Arborescence (segment 25, measures 148 – 149).

Arborescence (segment 26, measures 150 – 151).

Arborescence (segment 27, measure 152).

Arborescence (segment 28, measure 154).

Arborescence (segment 29, measure 155).

Arborescence (segment 30, measures 158 – 160).

Arborescence (segment 31, measures 159 – 160).

Arborescence (segment 32, measures 161 – 166).

Arborescence (segment 33, measures 165 – 170).

Arborescence (segment 34, measures 168 – 171).

Arborescence (segment 35, measures 179 – 188).

Rest (segment 36, measures 188 – 189).
Time-point sequences (segment 37, measures 190 – 192).

Time-point sequences (segment 38, measures 191 – 194).

Time-point sequences (segment 39, measures 192 – 194).

Time-point sequences (segment 40 and 41, measures 194 – 196).

Time-point sequences (segment 42 and 43, measures 196 – 197).

Time-point sequences (segment 44 and 45, measures 197 – 198).

Time-point sequences (segment 46, measures 198 – 206).

Arborescence (segment 47, measure 207).

Arborescence (segment 48, measures 207 – 212).

Rest (segment 49, measure 212).

Arborescence (segment 50, measure 213).


The composition’s total duration is 471 seconds (7’ 51”’). According to Squibbs in his work An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works (1996, p.163) the duration is derived from Xenakis’s indication of the tempo □=60. The duration of part one is 280.125 and the duration of part two is 190.875. Thus, part one is 3/5 of the total duration, whereas part two is 2/5 of the total duration.

If we group together the configurations of the same types we get the following results the arborescences grouped together have a total duration of 233.625, the stochastic types grouped together have a total duration of 156.750, the time-point sequences grouped together have a total duration of 134.375 and finally the total duration of rest is 28.875. Thus, arborescences are 2/5 of the work’s total duration, whereas, 3/5 of the whole work is occupied by time-point sequences, stochastic types, and rests.

Part one can be further subdivided into two sections. Section A occupies segments 1 – 11 and its total duration is 140”’. Section B occupies segments 12 – 22 and its total duration is 140.125”’.

Each of these sections occupies 30% of the whole work’s duration (471/140). Section A can be equally subdivided into section 1 (segments 1 – 6, 70”’ of the work’s total duration) and section 2 (segments 7 – 11, 70”’ of the work’s total duration). Section B can also be subdivided into section 3 (segments 12 – 19, 72.125”’ of the work’s total duration) and section 4 (segments 20 – 22, 68”’ of the work’s total duration). The reason for these subdivision of sections A and B is that sections 1 and 4 both include complex and long segments (segment 2 and 20 respectively), whereas sections 2 and 3 include the successions of short or brief segments. Section 3 is further subdivided into section a (segments 12 – 14, 43.75”’ of the work’s total duration) and section b
(segments 15 – 19, 28.375”’ of the work’s total duration). The reason for the subdivision of
section 3 into subsections a and b is that in the end of segment 14 we have a rapid descending
motion towards the center of the register as in the case of segment 20, which divides the
composition into two major parts. The duration of section a, is 60% (43.75 / 72.125 = 0.61) of the
duration of section 3, and section b is 40% (28.375 / 72.125 = 0.39) of the duration of section 3.

Part 2 is subdivided into section 5 (segments 23 – 36, 111.875”’ of the work’s total duration) and
section 6 (segments 37 – 50, 79”’ of the works total duration). Part two is subdivided in such a
way because of the rest that appears in segment 36 and naturally discontinues the flow of the
music. It is also important to note that the duration of section 5 is 60% (111.875”’/190.875”’= 0.59) of the total duration of part 2 and section 6 is 40% (79”’/190.875”’= 0.41) of the total
duration of the part two.
The application of the Exponential Distribution in Evryali

The exponential distribution was applied by Xenakis in order to determine the duration of segments. If we group together the configurations of the same type we are able to calculate their total durations in terms of Xenakis’s outside-time structure.

Total duration of arborescences = 233.625"

Total duration of stochastic types = 156.750"

Total duration of time-point sequences = 134.575"

Total duration of rests = 28.875"

Thus, the total duration of these four types of configuration is 553.825". If we divide the number of segments (50) by the total duration we have the result of a mean density of 0.09 (segments/second). Using the mean density as one of the parameters of exponential distribution we are able to calculate the expected probability of segments, whose duration is within the range of specified values x. Squibbs in his work An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works (1996, p.167) gives these probabilities in the following table:

<table>
<thead>
<tr>
<th>duration (sec)</th>
<th>probability (G= 0.09)</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ x &lt; 10</td>
<td>0.593</td>
<td>0.680</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>0.241</td>
<td>0.200</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>0.098</td>
<td>0.060</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>0.040</td>
<td>0.000</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>0.016</td>
<td>0.000</td>
</tr>
<tr>
<td>50 ≤ x &lt; 60</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>60 ≤ x &lt; 70</td>
<td>0.003</td>
<td>0.060</td>
</tr>
<tr>
<td>70 ≤ x &lt; 80</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>80 ≤ x &lt; 90</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Example 1
If we divide the total number of segments (50) by the total duration of *Evryali* (inside-time), which is 471" we get the result of 0.11 (segment/second). If we use this number (0.11) as a parameter in an exponential distribution we are able to calculate the probability of time-point intervals, whose duration is within the range of specified values x. Squibbs in his work *An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works* (1996, p.167) gives these probabilities in the following table:

<table>
<thead>
<tr>
<th>duration (sec)</th>
<th>probability ($\theta = 0.11$)</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq x &lt; 10$</td>
<td>0.667</td>
<td>0.660</td>
</tr>
<tr>
<td>$10 \leq x &lt; 20$</td>
<td>0.222</td>
<td>0.220</td>
</tr>
<tr>
<td>$20 \leq x &lt; 30$</td>
<td>0.074</td>
<td>0.100</td>
</tr>
<tr>
<td>$30 \leq x &lt; 40$</td>
<td>0.025</td>
<td>0.000</td>
</tr>
<tr>
<td>$40 \leq x &lt; 50$</td>
<td>0.008</td>
<td>0.020</td>
</tr>
<tr>
<td>$50 \leq x &lt; 60$</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>$60 \leq x &lt; 70$</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>$70 \leq x &lt; 80$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$80 \leq x &lt; 90$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Example 2

Xenakis uses the exponential distribution to calculate the duration of segments (outside-time structure) and their time positions in the compositions (inside-time structure). When Xenakis composed his stochastic compositions, the intervals calculated by the application of the exponential distribution were placed randomly in the composition, whereas in his works *Herma* and *Evryali*, works of great temporal structure, the intervals were not placed randomly but rather according to the taste of the composer. The application of the exponential distribution in these compositions creates a “statistical homogeneity” (Squibbs 1996, p.169).

**Comparison of the segments in *Evryali***

We can compare the segment in *Evryali* based on their four dimensions: duration, density, registral span, and intensity. Squibbs in his work *An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works* (1996, p.170) gives the minimum and maximum values for each dimension. In this way we are able to calculate in numbers each dimension. The result of the four dimensions of each segment will be added and then compared to the other segments.

**Duration:** minimum is 0 seconds and maximum 68 seconds.

**Density:** minimum sounds/second is 0 and maximum 43.36 seconds.

**Registral span:** minimum is 0 semitone and maximum 87 semitones.

**Intensity:** minimum is 0 (silence) and maximum is 8 (ffff).
When comparing these segments we will classify them between the values 0 to 10. The longest and loudest and the more the registral span and the density of a segment, the closest it is to the number 10. Each of the four dimensions are considered equal, and therefore, if we divide the maximum value 10 by the number 4 (because there are four dimensions) we get a normalization constant (term used by Squibbs 1996, p.171) 2.5.

For example, in segment 1 (measures 1 – 4) the following results are obtained:

Duration = 8”. If we divide this value by the maximum duration, which is 68’’ we have the following result: 0.118.

Density = 11.63 sounds/second. If we divide this value by the maximum density, which is 43.36’’ we have the following result: 0.268.

Registral span = the highest pitch is A4 (position on the keyboard is 9 semitones above C4) and the lowest pitch is C4 (position on the keyboard is 0). Thus, the registral span is 9. If we divide this value to the maximum value of the registral span, which is 87 semitones, we have the following result: 0.103.

Intensity = the intensity is ffff. This is equal to value 8, since this is the maximum value of density in the composition. If we divide this value by the maximum value, which is 8 we have the following result: 1.

If we multiply each number by 2.5, which is the normalization constant, we get the following results:

Duration = 0.118 multiplied by 2.5 = 0.295.

Density = 0.268 multiplied by 2.5 = 0.67.

Registral span = 0.103 multiplied by 2.5 = 0.258.

Intensity = 1 multiplied by 2.5 = 2.5.

If we add these four values (0.295, 0.67, 0.258, 2.5) we have the result of: 3.723. Thus, we can classify this segment in comparison to other segments as being close to average.

The results of the longest and most complex segments can be summarized as follows:

The value of the four dimensions in segments 2 – 6 (measures 5 – 35) = 6.447.

The value of the four dimensions in segments 20 – 22 (measures 102 – 136) = 8.413.

The value of the four dimensions in segments 37 – 46 (measures 190 – 206) = 7.809.

These three groups of segments are the three peaks in the composition. Segments 2 – 6 are found in section 1 of part 1. Segments 20 – 22 are found in section 4, which constitutes the end of part 1 and segments 37 – 46 are found in section 6 of part 2, which is also the final part of the composition.
The pitch structure of *Evryali*

There are two characteristics of how Xenakis applies and organizes the pitch-sets in his compositions. The first characteristic is when large pitch-sets appear in a composition but they do not make a complete use of their pitch material. Rather their pitch material is represented by their subsets. A pitch-set is called a subset if its pitches intersect with all the pitches of the larger pitch-set. If there is only an intersection of some of the pitches or no intersection at all between the subset and the larger set, then the subset is regarded as the transposition of the larger pitch-set.

The second characteristic of the pitch structure of Xenakis’s compositions is that sometimes pitches that are not a part of a specific pitch-set appear in the composition. These are called *stray* or *rogue* pitches.

We can categorize the pitch-sets that appear in *Evryali* in terms of large pitch-set models. Models refer to large pitch-sets, which include smaller pitch-sets used in a composition. These models are only hypothetical. Xemakis makes use of the following models in *Evryali*:

**Middle C (C4) = 0**

Model A: \{-39, -38, -37, -36, -35, -33, -31, -30, -28, -27, -26, -25, -23, -21, -20, -19, -18, -17, -16, -15, -13, -11, -10, -8, -7, -6, -5, -3, -1, 0, 1, 2, 3, 4, 5, 7, 9, 10, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 27, 29, 30, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 45, 47\}

Each number’s equivalent pitch:

-39 = A0, -38 = A#0, -37 = B0, -36 = C1, -35 = C#1, -33 = D1, -31 = F#1, -30 = F1, -28 = G#1, -27 = A1, -26 = A#1, -25 = B1, -23 = C#2, -21 = D#2, -20 = E2, -19 = F2, -18 = F#2, -17 = G2, -16 = G#2, -15 = A2, -13 = B2, -11 = C#3, -10 = D3, -8 = E3, -7 = F3, -6 = F#3, -5 = G3, -3 = A3, -1 = B3, 0 = C4, 1 = C#4, 2 = D4, 3 = D#4, 4 = E4, 5 = F4, 7 = G4, 9 = A4, 10 = A#4, 12 = C5, 13 = C#5, 14 = D5, 15 = D#5, 17 = F5, 19 = G5, 20 = G#5, 21 = A5, 22 = A#5, 23 = B5, 24 = C6, 25 = C#6, 27 = D6, 29 = F6, 30 = F#6, 32 = G#6, 33 = A4, 34 = A#4, 35 = B6, 37 = C#7, 39 = D#7, 40 = E7, 41 = F7, 42 = F#7, 43 = G7, 44 = G#7, 45 = A7, 47 = B7

The distance between pitches in terms of semitones is:

<111122121111111122121111111112212111111221211111122121111111221211111118>

The repeated interval pattern in this model is:

<11112212111111112212111111112>

Model B: \{-38, -36, -35, -34, -32, -30, -29, -27, -25, -24, -23, -21, -19, -18, -16, -14, -13, -12, -10, -8, -7, -5, -3, -2, -1, 1, 3, 4, 6, 8, 9, 10, 12, 14, 15, 17, 19, 20, 21, 23, 25, 26, 28, 30, 31, 32, 34, 36, 37, 39, 41, 42, 43, 45, 47, 48\}

Each number’s equivalent pitch:

-38 = A#0, -36 = C1, -35 = C#1, -34 = D1, -32 = E1, -30 = F#1, -29 = G1, -27 = A1, -25 = B1, -24 = C2, -23 = C#2, -21 = D#2, -19 = F2, -18 = F#2, -16 = G#2, -14 = A#2, -13 = B2, -12 = C3, -10 = D3, -8 = E3, -7 = F3, -5 = G3, -3 = A3, -2 = A#3, -1 = B3, 1 = C#4, 3 = D#4, 4 = E4, 6 = F#4, 8 = G#4, 9 = A4, 10 = A#4, 12 = C5, 14 = D5, 15 = D#5, 17 = F5, 19 = G5, 20 = G#5, 21 = A5,
23=B5, 25=C#6, 26=D6, 28=E6, 30=F#6, 31=G6, 32=A#6, 36=C7, 37=C#7, 39=D7, 41=F7, 42=F#7, 43=G7, 45=A7, 47=B7, 48=C8

The distance between pitches in terms of semitones is:

<21122121221122122122122122102122122122122122122>

The repeated interval pattern in this model is:

<2112212>

Model C: {-38, -36, -33, -31, -30, -29, -27, -25, -23, -20, -18, -17, -16, -14, -12, -10, -7, -5, -4, -3, -1, 1, 3, 6, 8, 9, 10, 12, 14, 16, 19, 21, 22, 23, 25, 27, 29, 32, 34, 35, 36, 38, 40, 42, 45, 47, 48}

Each number’s equivalent pitch:-38=A#0, -36=C1, -33=D#1, -31=F1, -30=F#1, -29=G1, -27=A1, -25=B1, -23=C#2, -20=E2, -18=F#2, -17=G2, -16=G#2, -14=A#2, -12=C3, -10=D3, -7=F3, -5=G3, -4=G#3, -3=A#3, -1=B3 1=C#4, 3=D#4, 6=F#4, 8=G#4, 9=A4, 10=A#4, 12=C5, 14=D5, 16=E5, 19=G5, 21=A5, 22=A#5, 23=B5, 25=C#6, 27=D#6, 29=F6, 32=G#6, 34=A#6, 35=B6, 36=C7, 38=D7, 40=E7, 42=F#7, 45=A7, 47=B7, 48=C8

The distance between pitches in terms of semitones is:

<232112232112232112232112232112223211222321>

The repeated interval pattern in this model is:

<2321122>

Model D: This model includes all the pitches of the range of the piano (88 pitches). The interval distance between the pitches is of course number 1, which indicates the smallest distance between the pitches (semitone).

The segments 1 – 4 and 6 – 8 include the pitches of model A. Segments 2 and 7 have the wider registral span in comparison to the previously mentioned segments and thus constitute the majority of pitches in model A. The pitches in segment 5, on the other hand, are not all included in model A, but only two of its five pitches (F3, and F#3 from A#2, C3, D#3, F3, and F#3). If we transpose, however, the pitches of segment 5 by a semitone lower, they do correspond entirely with model A.

Segments 8 – 10 are part of model B. We notice that segment 8 intersects both with model A and model B. In the case of model A, segment 8 has the majority of the pitches in common, whereas in comparison to model B, segment 8 intersects entirely. Thus, segment 8 is like a link between model A and model B. Segment 9 includes all of the pitches of the keyboard’s register. The pitches of segment 10 (B4, C#5, D5, E5, F#5, G5, G#5, and D#6) also like the previously mentioned segment 5, if transposed a semitone higher they correspond entirely with model B. Segment 11 includes a rest and of course it does not belong to either model. Segment 12 is the only segment whose pitches belong in model C.
The remaining segments (13 – 50) include pitches that intersect with both three models and in some cases to model D, which includes all the pitches of the keyboard. The segments which have a registral span of 88 pitches are segments 14, 16, 18, 20, 23, 25 – 34, and 35.

6.3 Conclusion

*Evryali* is the longest of Xenakis’s solo piano compositions in terms of its duration and has thus required a more detailed and perhaps complex analysis. We have seen the four configurations that construct the whole composition. These configurations are stochastic types, time-point sequences, arborescences, and rests. These configurations have been compared according to their basic characteristics, which are their density, registral span, intensity (dynamic), and duration. The application of mathematics in this composition has been seen in the use of sieve theory for the choice of pitch material and the exponential distribution was used for calculating the duration of segments. The temporal structure of *Evryali* has been analyzed not only when grouping together segments of similar content, but also in terms of the pitch structure of the different segments. The pianist is once again facing a difficult or perhaps challenging task of not only understanding and analyzing this work, but also meeting its demands, as concerns performance and interpretation.
7 Mists (1981)

7.1 Introduction

*Mists* was written in 1981 and it is one of the major works of Xenakis for piano. This composition contains ideas formerly developed in his compositions *Herma* and *Evryali*. *Herma* was composed early in his career and it is an example of symbolic music, whereas in *Evryali* Xenakis combines the use of arborescences with percussive rhythmic patterns. In *Mists*, Xenakis makes use of several mathematical concepts including sieve theory, his own theory of musical time and stochastic composition. The textural elements of random walks and arborescences are also explored.

7.2 Analysis of the work

**Pitch**

The pitches used in *Mists* are based on a scale that was created by Xenakis specifically for this work. The scale includes ninety semitones, which exceeds the range of the standard piano keyboard, which has eighty-eight semitones. Therefore, Xenakis uses in *Mists* only twenty-nine of the scale’s thirty pitches.

Example 1

Measures: 1 – 4, present all the tones of the original scale.
The choice of pitches for the creation of this scale is based on the mathematical theory of sieves. The scale is based on the cycle of two, five and nine semitones. The reason why the duration of the scale is ninety semitones is because it is equivalent to the lowest common multiple of its interval cycles (2, 5, and 9). The lowest common multiple of two, five and nine is ninety, which constitutes the period of the scale.

Xenakis also uses other scales based on the cyclic transposition of its principle scale. The transpositions, however, are restricted by the limits of the piano keyboard. The examples of three such transpositions are shown below:

Example: 2

Example: 3

Example: 4

All four examples, which are listed in this diploma work, are extracted from Squibbs 2002, p. 94).
The scale of example 2 is the transposition of thirty semitones up from the lowest pitch of our original scale. The lowest pitch of the original scale was Bb0 and the transposition begins from E3. E3 appears in position 10 even though it is the beginning tone of the transposition. The reason for this is that the lowest tones from C0 to C#3, which are shown now in positions 0 – 9, are in reality extending beyond the keyboard’s range. Xenakis thus, has situated these pitches (C0 – C#3) in the lower part of the scale.

The scales in examples 3 and 4 are created in a similar manner. The scale in example 3 is transposed thirty-eight semitones from the lower tone of the original scale, whereas the scale in example 4 is transposed ten semitones from the lower tone of the original scale. We can see that in example 3 the beginning tone of the transposition is C4 and it is found in position 14. In example 4 the beginning tone of the transposition is G#1.

The pitches in these transposed scales, which exist beyond the keyboard’s range are the notes C#8 and D8 for example 2 and C#8 for example 3. These pitches are shown in brackets. For this reason only twenty-eight of the thirty pitches in example 2 and only twenty-nine of the pitches in example 3 are represent in the score. The transposed scale in example 4 has no tones that extend beyond the keyboard’s range.

Between these transpositions we have a number of common tones. The more the common tones between them, the greater the similarity in the sound of these scales. This was one of the reasons for Xenakis’s choice of the specific transpositions that he used in Mists. We should bear in mind that the number of transpositions of the original scale devised by Xenakis it is equivalent to its period. The original scale has a period of ninety semitones, thus it has ninety transpositions. Xenakis chose eleven transpositions for this composition, three of which are shown above. Among these scales the common tones vary from 0 to 16. The scales shown in examples 3 and 4 have the most common tones. An example is shown in measures 39 and 40, where Xenakis writes all the common notes that exist between the two scales.

Measures: 39 – 40

It is worthy of note that the spacing between the intervals remains the same for all transpositions, beginning from the original tone of transposition for each scale. The spacing for each scale is shown below the line of pitches in the previews examples. The spacing of the original scale begins in position 0. The spacing of the first transposition shown in example 2 begins in position
Thus, we can observe that the spacing which began in position 0 for the original scale now begins in position 10 for the first transposition. The spacing of the first ten intervals of the first transposition (positions 0 – 9) is the same as the spacing of the last ten intervals of the original scale (20 – 29). We can conclude that the spacing of the intervals remains the same for all transpositions.

These scales where used by Xenakis either in their entirety or only partially, in order successions (ascending or descending) or in random successions. These scales are an important part of the form. *Mists* has the ternary form of ABA’ including a small transition between A and B. The original scale and the first transposition of the scale (example 2) are the only ones to appear in sections A and A’. The section A includes the measures 1 – 30 and the section A’ includes the measures 122 – 134. The original scale appears in the transitional section (measure 31 – 40), whereas the first transposition of the scale links the end of the A section to the transition and the transition to the beginning of the B section (measures 41 – 121). The transposed scales of examples 3 and 4 are first introduced in the transition and they also appear in the beginning part of the B section. The other seven transpositions which are not displayed as examples above appear only in B section. The original scale appears before the end of the B section and links it to the introduction of the A’ section.

**Texture**

There are three distinct types of textures used in *Mists*:

Linear texture or continuous.

Non-linear or pointillistic or discontinuous.

Quasi-polyphonic or arborescences.

The first two types of textures involve the use of random walks. In physics the random walk is a concept used to describe “unpredictable motions in space”. (Squibbs 2002, p.96). In the case of this composition, continuous random walks refer to the stepwise motion of a scale, whose speed and direction can change in an unpredictable or random way. A discontinuous random walk refers to a motion where there are unpredictable leaps between pitches. The effect created by discontinuous random walks is described by Xenakis as clouds. Arborescences consist of linear lines, which create a polyphonic or “branching structures”. The lines in an arborescence are interdependent. This means that the beginning tone of a line is the same tone that is found in a previous line. Xenakis wished in this way to create the effect of continuity of pitch.

We can observe the entrance of five continuous random walks in the introductory part of *Mists* between measures 1 – 6. Each random walk begins in the lowest register of the piano with the note Bb0. The first random walk appears in the first measure, the second in the end of the first measure, the third in the beginning of measure four, the fourth in the beginning of measure five, and the fifth in measure six. All these continuous random walks derive their pitch material from the original scale. The points in the score marked with a vertical dashed line signify the points of temporal intersection between these random walks. Each random walk moves upwards. Its
direction and speed, however, can change. We can observe that in measures 1 – 6 the random walks are more rhythmically differentiated than the random walks between measures 7 – 11. We can see a two-voice random walk in measures 7 – 9 and a four-voice random walk in measure 9 – 11. These four-voice random walks continue until measure 30. The following section until measure 40 is in a much faster tempo where the continuous random walks move in continuous line of thirty-second notes. The end of this section sees the simplification of the random walks, which appear in the succession of ascending eighth-notes. The continuous random walk appears in the closing section of *Mists* in measure 122. Until the final measure of this composition the random walks move in a fluent descending motion in four voices.
Measures: 7 – 9

Measures: 9 – 11
Discontinuous random walks are found in the middle section of the composition in measures 41 – 121. This middle section creates a contrast between the first and last sections of the composition. The notation of these discontinuous random walks is rather unique. We can see from the score...
that we have independent beams and stems as well as unattached note heads. The musical pitches are thus notated in their exact time-points based on mathematical precision. In the first measure of the second section (measure 41) the beginning note C2 appears in the middle of the two sixteenth note stems. G#2 appears on the second sixteenth note. A5 follows immediately and B5 soon after and A#0 occurs between the second and third note stem.

Measures: 41 - 42

In *Mists*, the time between intervals and their pitches have been chosen stochastically. Xenakis mentions in his preface to the composition, that he used two distributions: Cauchy and hyperbolic cosine. These two distributions generate both positive and negative values, which result in ascending and descending intervals. The pitch material for the discontinuous random walks in the middle section was chosen by calculating the intervals between the position numbers in the scales. For example the succession of pitches of the first discontinuous random walk in measure 41 is: C2, G#2, A5, B5, A#0, D#6, F7, G#7, F#6. These pitches are part of the second transposition of the original scale (see example 3). The numbers of these pitches as regard their position are: 5, 4, 20, 21, 12, 26, 28, and 23. The interval succession of these pitches starting from number 5 is: -1, 16, 1, -20, 21, 4, 2, and -5.

In the middle section of *Mists* we find sixty-four random walks, which are interrupted by rests and arborescences. The discontinuous random walks appear in both the upper and lower registers. In measures 50 – 53 the movement concentrates on the high register. The discontinuous random walks in the following measures appear in both high and low registers until the pitches concentrate in the treble clef in a range of two and a half octaves (measures 63 – 76). This movement is interrupted by the introduction of the bass clef in measure 76. The discontinuous random walks that follow descend gradually until the beginning of the final section in measure 122.

Measures 51 – 52
Arborescences appear throughout the composition and provide contrast with the continuous random walks in the first and last sections and with the discontinuous random walks in the middle section. The first appearance of the arborescences is in measures 14 – 16. The pitch material is based on the original scale. The arborescence begins on F4 and the first branching proceeds from F#4 to B4 upwards and to D#4 downwards. The arborescence is to be played legato. The second appearance of the arborescence is found between measures 22 – 24 and derives its pitch material from the original scale. The third arborescence occurs in measures 28 – 30 and uses the pitches from the first transposition of the original scale (see example 2). It is the larger of the two. The fourth arborescence appears in measures 36 – 38 and it is the last before the middle section. Its pitch material derives from the third transposition of the original scale as shown in example 4.
Example: Arborescence

(Squibbs 2002, p. 100).

The remaining arborescences use the pitches of the chromatic scale. The first to appear is in the middle section in measures 80 – 83. The rest of the arborescences are found in measures: 93 – 94, 109 – 110, 115 – 116, 129 – 130, and 133 – 134. The non-coinciding rhythms help to distinguish between their different lines. Between the set of these five arborescences we notice a change in register. The arborescence in measures 93 – 94 appears in high registers, whereas the two arborescences, which follow (measures 109 – 110 and measures 115 – 116 respectively) gradually move towards the lower registers. We also notice that the fourth and fifth arborescence (measures 129 – 130 and measures 133 – 134 respectively) move gradually towards the higher registers.
We can also observe that between the first arborescence in measure 93 - 94 and the beginning of the next there are fifteen measures between them. There is also a huge gap between the arborescences found in measures 109 - 110 and 115 - 116 respectively. This gap becomes smaller between the last two arborescences, which are separated by only two measures of rests.
Form and Structure of the work

It is possible to divide the work into segments based on one or more of the following characteristics: pitch collection, dynamics, articulation, density, changes in tempo, and texture. We have observed so far that the continuous random walks are found in *Mists* in the first and final sections of the composition. In the transitional section, which leads to the middle section, we have a change to a faster tempo. In the middle section the discontinuous random walks are predominant. In measure 41, which is the beginning of the middle section, we have the return of the original tempo. Thus, we can divide the composition into three parts. The middle part appears to be the longest in duration in comparison to the outer sections of the composition.

Xenakis’s temporal structures in music are based on his theory of musical time. This theory distinguishes between “inside-time” musical structure and “outside-time” musical structure. We can better understand these two structures by giving an example: a melody based on a scale can be consider as having an inside-time structure, whereas the scale itself, which can become the basis of other melodies has an outside-time structure. In *Mists* the various scale transpositions are regarded as having an outside-time structure. The ways by which Xenakis works with these scales is through the creation of continuous and discontinuous random walks and arborescences. These are considered inside-time structures.

The division of *Mists* based on the duration of each part is considered an inside-time aspect of the whole work’s structure. The types of texture in which the random walks and arborescences belong are part of an outside-time musical material. The total duration of each texture is independent of the temporal structure of the composition. The discontinuous random walks occupy the larger amount of time in comparison to the continuous random walks and arborescences. The rests have a short duration in comparison to the other three types of texture. We observe that the composer carefully proportioned the three parts of the composition as well as its inner elements, which play an important role in the structure of the whole work.

Golden Ratio as applied in *Mists*

The middle part of the composition occupies the 62% of the composition’s total duration. This number is close to the golden ratio, which is 0.618. The first part is 28% of the work’s total duration and the last part is 10%. Thus, we see that the golden ratio is located in the middle of the composition. This is contrary to its more traditional application, where it divides the composition into a larger and a smaller part. We also remarkably observe that the combined duration of the discontinuous random walks and rests is 0.63 of the whole work’s duration, which is close to the golden ration. The remaining parts, which are the continuous random walks and arborescences occupy the remaining 0.37 of the total duration.
7.3 Conclusion

*Mists* is an important work of Xenakis’s middle period. It is based on the mathematical theory of sieves, on the theory of stochastic composition and Xenakis’s own theory of musical time. We have also noticed the composer’s use of random walks and arborescences, which constitute the basic structural elements of his composition. This work is not only demanding in terms of its analysis but also in terms of its performance. The pianist is required to have a clear understanding of the structure of the work and the intentions of the composer as well as a solid technique, precision and accuracy in execution and a careful rendering of the rhythmical aspect of the composition.
Chapter 8: À R (Hommage à Ravel) (1987)

8.1 Introduction

The last composition of this diploma work is a short piece which its duration is approximately two minutes and a half, its title is à r., its complete title is Hommage à Ravel, and is a tribute to Maurice Ravel. Written in 1987, it was a request of the French Radio Broadcast for the remembrance of the 50 years from Maurice Ravel’s death, who died in 1937. It is a characteristic piece of the last period of Xenakis, that is the end of 80’s, beginning of 90’s. In relation with other pieces, it is a very simple piece, which alternates melodic movement on the non-recurrent scales, similar to those which were used in Mists by Xenakis; that is, when the right hand is ascending, the left hand is descending, and conversely. This idea has been alternated with a static idea, which consists of usually strong held chords.

This chosen work for analysis belongs to the category of Xenakis’s miniature works composed between early 1970s and late 1990s. These miniature works were composed for solo instruments or small ensembles. These compositions even though are short in length; their analysis is still a long and complex process, which demonstrates Xenakis’s mastery of form and structure.

Even though the title of the chosen work indicates a tribute to Maurice Ravel, it bears no resemblance with the compositional style of Maurice Ravel, except perhaps for its virtuosity and brilliance.

8.2 Analysis of the work

Types of Texture used in à r.

In à r. there are two types of textures: simultaneities and random walks. Simultaneities in music mean the combination of musical textures. Pitch simultaneity is more than one pitch or pitch class all of which occur simultaneously. Mihu Iliescu says about Xenakis’s use of simultaneities in his music that they are “a transformation of the idea of sound masses – formerly manifested as collections of disconnected, individual sonic elements – into large chords, resulting in “vertical blocks” of “harmonic/timbral color” (Squibbs 1996, pp.121-122). Random walks symbolize the movement of sound in successive steps, length and direction. The decision for each step is taken randomly. These melodies are example of stochastic waveforms. Xenakis’s first example of random walks is found in his composition for violin Mikka (1971). The random walks are represented with continuous glissando movements with changes of direction and speed. The application of the random walks to piano music is represented by rapid, continuous, and linear movements creating melodies of great collection of tones. The melodies in à r. which are in the form of scales are referred to as random walks.
The random walks in \( \text{à } r. \), appear in pairs. Thus we have a two-voice texture. Both random walks have the same rhythm (triplets of 32\textsuperscript{nd} notes) but their direction is independent of the other. They can both move in the same direction or in contrary motion. In some cases in the random walk there are repetitions of pitches. We can see this in the beginning of the composition in measure 1. The upper staff begins with A4 and moves to G#4 which is repeated eight times and then moves upwards to G6 which is repeated three times. We can also notice that the random walks of the upper and lower staff are coordinated as regards dynamic and harmony. This balance between the two random walks is maintained throughout the composition.

The first simultaneity appears in the beginning of measure 2. Throughout the course of the composition we can observe that there is an alternation between random walks and simultaneities. This shows the contrast between the two textures. In the following measures: 6 - 7, 13 - 14, 16 - 17, 21, the simultaneities appear in succession.
Division of sections

The whole composition is divided into six sections. The sections are represented by the letters A and S. Letter A stands for “alternating” textures and letter S for “similar” textures. Each section is divided into segments. A segment is only one type of texture. In the case of simultaneities the ligatures are not included.

Section A1 includes the following segments: 1 (random walk), 2 (simultaneity), 3 (random walk), 4 (simultaneity), 5 (random walk), 6 (simultaneity), 7 (random walk), 8 (simultaneity), 9 (random walk).

Section S1 includes the following segments: 10 (simultaneity), 11 (simultaneity), 12 (simultaneity), 13 (simultaneity), 14 (ambiguous texture), 15 (simultaneity), 16 (random walk).

Section A2 includes the following segments: 17 (random walk), 18 (simultaneity), 19 (random walk), 20 (simultaneity), 21 (random walk).

Section S2 includes the following segments: 22 (simultaneity), 23 (simultaneity), 24 (simultaneity), 25 (simultaneity), 26 (random walk), 27 (random walk), 28 (random walk), 29 (random walk).

Section S3 includes the following segments: 30 (simultaneity), 31 (simultaneity), 32 (simultaneity), 33 (simultaneity), 34 (random walk), 35 (random walk), 36 (random walk).

Section A3 includes the following segments: 37 (random walk), 38 (simultaneity), 39 (random walk), 40 (simultaneity).

Thus the order of the sections appears in the following way: A1, S1, A2, S2, S3, A3.

It should be noted that sections with the letter A all begin with random walk, and the sections with the letter S all begin with simultaneities.

The ambiguity of segment 14 found in measures 7 – 8 is due to the reason that the motion and the small rhythmic values of the notes reminds us of the random walk but at the same time it consist of 15 simultaneities. This texture can be characterized as a “random walk of simultaneities”.

Examples Measure 7 (Ambiguous)
We also observe in the composition that we have a change of tempo and articulation in segment 17. The original tempo was $J=46$ and then changes to $J=36$. The articulation in the random walks changes from single attack of the key to the double repetition of each note. At the end of measure 12 we return to the original articulation. This change of articulation is also noted in segments: 31, 33, 38, and 40.

Example: Measure 10 – 12.

The random walks in sections S2 (segments 26, 27, 28, 29) and S3 (segments 34, 35, 36) ascend in parallel motion, they are written in sixty-fourth notes and are much shorter in duration than the previous random walks found in sections A1, S1, and A2. The continuous movement of the random walks is for a moment interrupted by an eighth note rest in segment 27 after the first pair of random walks in S2. Segment 37 found in the beginning of the final section A3, is the longest and the most complex pair of random walks in comparison to the whole work. The buildup of the energy finds its climax in segment 37. The energy, however, decreases steadily until the end of the composition.
The characteristic features of segment 39 (the random walk before the last simultaneities) are:

The return to slower note values (triplet thirty-second notes).

The indication of *ralentir*, meaning a slowing down of tempo, similar to *ritardando*.

The dynamic is also lower, resulting to *mp* in the simultaneity in segment 40 (last measure of a composition).
It is also worthy of note that the simultaneity in segment 40 is the lowest in pitch in comparison to all of the simultaneities in the composition.

In order to analyze this composition it is very important to understand the structure of the sections and their segments. The number of segments in the A sections decreases as the composition progresses. A1 has nine segments, A2 has five segments and A3 has four segments. Sections A1 and A2 end with a random walk, whereas A3 ends with simultaneity. In general the sections which end with a random walk are considered as being structured in an “open” manner, while those sections that end with a simultaneity are considered as being structured in a “closed” manner.

We can notice from the score that the composer does not indicate any specific nuances or marks of interpretation. There are no rests, ritards or fermatas, except for the silence of the segment 27 and the ritards in the end of the composition. The listener does not understand when one section begins and when one section ends. This feeling of continuity and the lack of indications of any articulation on the score is typical of the non-thematic nature of Xenakis’s music.

**Sieve theory and pitch-class sets**

Pitch-class sets refer to the pitch collections or rather scales (random walks) used in this composition. The choice of pitches for the creation of these pitch-class sets are based on sieve theory and its principles. This theory of musical structures was developed by Xenakis in the mid-1960s. Sieve theory enables the creation and analysis of the structure of musical spaces, which include a series of pitches of equal durational values. We can observe in à r. that the random walks include pitches of the same rhythm and durational value.

The first two pitch-class sets, which appear in à r. extend over the entire piano keyboard (88 semitones). There are two pitch-class sets in set class 1. We should note that the number in the label of a set (for example set 1.0) signifies the class to which it belongs. The second integer represents the level of transposition in comparison to the first note of its class. For example in the case of set 1.0, 1 represents class 1 and the number 0 that this set is the original scale. In the case of set 1.64, the number 1 shows that it belongs to set class 1, whereas the number 64 represents that this pitch-class set is transposed 64 semitones from the original tone of the scale.
Example: Set class 1.0 and 1.64.

The original scale (set 1.0) begins with A0 and follows the interval succession of, <2 1 1 3 2 1…>.

The second set (1.64) is the transposition of the original scale, beginning from C#6. We can observe the same succession of intervals as the original scale. The interval succession of the beginning of set 1.64 is the same as the interval succession of set 1.0, starting from A#2. Both the original scale and its transpositions follow the same intervallic succession. We always begin to count the intervals from the beginning note of each transposition. When the notes exceed the keyboard’s range, those notes are then moved to a lower register at the beginning of the transposition.

The transpositions (set 2.82 and set 2.52) are produced based on the same principles. They however contain ninety semitones, which exceed the piano’s range. These notes are shown in brackets. The intervals of these two sets are larger than the intervals of sets 1.0 and 1.64. Therefore they contain a smaller number of pitch-class sets.

Example: Set class 2.82 and 2.52.

Set 1.0 appears only partially in the first measure of à r. In measure 2 we find the structure of set 1.64. Segments 16, 17 (measures 8-11) derive their pitch material from the set 2.82. Set 2.52 is found in segment 19 (beginning of random walks in measure 11).
Example: Measures 1 and 2

Example: Segments 16 – 19.

It is worthy of note that segments 16, 17 and 19 were also used in Xenakis’s work *Mists* in measures 31-34. This quote however is not exact. The borrowed segments from *Mists* are reordered in à r. and are also played in a more slowly tempo than in *Mists*. In à r. segment 16 begins at J=46 and progresses to J=36 at measure 10, whereas the passage in *Mists* is played at
the tempo $J=72$. Another difference is also the articulation between the segments in à r. and the corresponding passages in *Mists*.

Example: Measures 31 – 34 from *Mists*.

The random walk in segment 16 in à r. corresponds to the first beat of measure 31 in *Mists* until the third beat of measure 32. The random walk however begins on C#5 whereas the corresponding walk in *Mists* begins with G#4. Even though we can see that the random walk in the segment 16 of à r. begins one position later than the random walk in *Mists*, their rhythmical structure is maintained. Whereas measure 31 in *Mists* begins with a single random walk, segment 16 begins with a two simultaneous random walks. Furthermore, in the third beat of measure 31 in *Mists* the random walk in the lower staff forms a second voice and thus creating arborescence. We also notice that compared to *Mists* in the beginning of measure 10 in à r., we have a change of articulation (double-notes). Segment 17 in à r. corresponds to the fourth beat of measure 32 in *Mists* until the third beat of measure 33. Segment 17 begins with the sustained note C4, which changes the rhythm of the original walks. Segment 19 begins with F4 and E6 which corresponds to the third beat of measure 34 in *Mists* and progresses in a retrograde way towards E3 and F4, which correspond to the fourth beat of measure 33 in *Mists*.

Xenakis also uses more segments from *Mists* into à r. The two random walks that appear in *Mists* from the end of measure 34 until the beginning of measure 36 appear in a retrograde movement in à r. in the following segments: segment 5 (measures 3 – 4) and segment 7 (measures 4 – 5).
Example: Measures 34 – 36 From *Mists*.

Example: Segment 5.
Example: Segment 7.

The segments 1, 3, 5 in the à r., belong to the pitch-class sets 1.0, 1.64, and 1.0 respectively. Thus their structure is aba. The pitch-class sets are part of a bigger ternary form ABA. A stands for set class 1 and B for set class 2. Pitch-class sets in segments 1, 3, and 5 are part of A and their duration combined is 14% of the whole work’s duration. B includes segments 7, 9, 16, 17, and 19 and their duration is 38% compared to the total duration. A returns again with segments 21, 26, 27, 28, 34, 35, 36, 37, and 39 and makes up the remaining 48% of the total duration.

Ronald Squibbs in his scientific work on Xenakis’s à r. *Xenakis in Miniature: Style and Structure in À R. (HOMMAGE À RAVEL) for Piano (1987)* gives a full list of the scales and their transpositions which were used in this composition.

<table>
<thead>
<tr>
<th>Segments</th>
<th>Pitch-class sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>1.64</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>2.0</td>
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<tr>
<td>9</td>
<td>2.02</td>
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<tr>
<td>16</td>
<td>2.82</td>
</tr>
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<td>17</td>
<td>2.82</td>
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<td>19</td>
<td>2.52</td>
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<td>21</td>
<td>1.24</td>
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<td>26</td>
<td>1.0</td>
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<td>39</td>
<td>1.83</td>
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<tr>
<td></td>
<td>1.7</td>
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<td></td>
<td>1.19</td>
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</tbody>
</table>

**Temporal Structure**

Xenakis intentionally structured a temporal proportion of this composition. The duration of a segment can be calculated as the distance between the time-point of the first musical sound in the segment and the time-point of the first musical sound in the following segment. The duration of a section is the sum of the duration of all the segments it includes.

<table>
<thead>
<tr>
<th>Sections</th>
<th>Duration in seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>28.00</td>
</tr>
<tr>
<td>S1</td>
<td>20.50</td>
</tr>
<tr>
<td>A2</td>
<td>21.67</td>
</tr>
<tr>
<td>S2</td>
<td>16.67</td>
</tr>
<tr>
<td>S3</td>
<td>17.50</td>
</tr>
<tr>
<td>A3</td>
<td>22.50</td>
</tr>
</tbody>
</table>

We can notice from the table above that the A sections are longer in duration than the S sections. The whole composition is based on a constant proportion of sections. We can observe the following facts:
A1 / S1 (28.00 / 20.50) = 1.36
A2 / S2 (21.67 / 16.67) = 1.30
A3 / S3 (22.50 / 17.50) = 1.29

Thus, the temporal structure of the composition is based on the consistent relation in terms of duration between the sections. The proportion can be summarized as 1.3:1. Similarly, if we divide all sections A by all sections S, we also get an average answer of 1.32 (A1 + A2 + A3 / S1 + S2 + S3). It is also remarkable to notice that if we divide all segments that include random walks (W) by all segments that include simultaneities (S) we also get the answer of 1.31 that is near to 1.32.

The proportion 1.3:1 can also be seen in the pitch-class sets. The pitch class set 1 has duration of 39 seconds, while the pitch-class set 2 has duration of 30.5 seconds. Therefore, the proportion between them is 1.28:1 (close to 1.3:1). In the same manner if we compare the duration of the quarter note beat of the fast tempo j=46, which is 1.67 seconds, to the duration of the quarter note beat of the slower tempo j=36 (this appears in measure 10), which is 1.3 seconds we get the result of 1.28:1 (close to 1.3:1).

We can therefore conclude that Xenakis intentionally structured the work proportionally. Temporal proportions play an important role in understanding and analyzing the structure of Xenakis’s work.

**Golden Ratio**

Xenakis’s composition à r. follows the proportional system of the golden ratio. The golden ratio divides the length of the composition so that the proportion of the smaller part to the larger part is equivalent to the proportion of the larger part to the whole composition. The larger parts of the golden ratio are called “major” and are equal to 0.618, whereas the smaller parts of the golden ratio are called “minor” and are equal to 0.382. The minor part of the golden ratio in à r. takes place in the of segment 16 and the beginning of segment 17 and more precisely in measure 10 between the first quarter note Bb3 and the subsequent quarter note C4.

Example: Measure 10.
We can also observe this proportional system if we divide the duration of the segments 7 – 20, which are found in the middle part of the composition, by the remaining segments of the outer part of the composition. Segments 7 – 20 are the minor part of the golden ratio, whereas the rest of the segments (1 – 6 and 21 – 40), are the major part of the golden ratio.

**Exponential Distribution**

The exponential distribution was also used for the temporal structure of the composition. In exponential distribution shorter time intervals occur more frequently than longer time intervals. According to Squibbs the use of the exponential distribution in a composition creates a temporal structure where most of the musical sounds are grouped close together, whereas a few of these musical sounds are placed far apart from each other. Xenakis used the exponential distribution in a series of stochastic compositions written between 1956 and 1962 with the help of a computer program. Xenakis was thus able to calculate with the help of a computer and using the exponential distribution to decide of segment durations and the attack times of individual sounds. When he was writing these compositions the segments of music were created randomly by the computer program and then were revised by the composer to create the finished composition. In the case of à r., the organization of the segments does not appear to be the result of a random procedure. The reason for this was that Xenakis intentionally organized these segments based on their textural content (for example the random walks occur in the longer segments, whereas the simultaneities appear in the shorter segments). Thus, we can conclude that the segment durations are not based on a stochastic process and were not chosen randomly. However, if we observe the durations of these segments collectively we can see that they have similar characteristics to an exponential distribution.

**8.3 Conclusion**

We can notice in à r. that the development of the thematic material does not happen in a traditional way. “The form is clearly the function of the musical materials, which are handled in a consistent and musically logical manner.” (Squibbs 1996, p.130). This miniature shows Xenakis’s skill in the creation of the temporal structure of the composition. It is very impressive how he clearly determined the duration of segments and the organization of the parts based on the golden ratio. This work also shows a complex application of mathematical theories, which can only be understood after a thorough analysis of the work. The challenge that the performer faces is great. The pianist has to master the technical difficulties of the piece and achieve the task of memorization but most importantly he or she has to understand the form. In comparison to the previous piano compositions *Herma, Evryali*, and *Mists*, is the shortest in duration and perhaps the less challenging in terms of reading the score and ultimately performing it. The rhythmical values and the continuity of the melodic lines in à r. are more familiar to the pianist than the notation of the other solo piano compositions. In à r., the pianist needs to play the scales with brilliancy, accuracy, and speed. The chords need to be played forcefully and with great energy.
Conclusion

Analyzing these four solo piano compositions by Iannis Xenakis has been a great challenge for me. I have learnt a lot about new techniques and principles of analysis. The more I discovered how Xenakis incorporated mathematical theories in his compositions, the more I admired and appreciated this great composer. Understanding, however, these mathematical theories and new ways of analysis has not been at all easy. It required a great effort to see how these theories were implemented in the musical score. I had to change my way of thinking and perceive like Xenakis has, the pitch as sound in the musical space, and thus understanding music as spatial. I believe that through this diploma work musicians will have an opportunity to get to know Iannis Xenakis and his contribution to the music of the 20th century. This was also one of my aims. I hope that other musicians will be interested in analyzing further Xenakis’s compositions, and thus make his work better known.

Xenakis is one of the most significant composers of the 20th century. His innovations and new compositional techniques opened the way to further exploration of sound and like James Harley says: “He (Xenakis)…pointed the way to new ways of understanding music and of organizing it” (2007, p.254). Xenakis believed that his compositions also partially reflected life itself. In one of his interviews he regards the composition of music as a mission. Xenakis says: “For years I was tormented by guilt at having left the country for which I’d fought. I left my friends – some were in prison, others were dead, some had managed to escape. I felt I was in debt to them and that I had to repay that debt. And I felt I had a mission. I had to do something important to regain the right to live. It wasn’t just a question of music – it was something much more significant.” (Harley 2007, p.253).
Summary

This diploma work is concerned with the analysis of four compositions for solo piano by Iannis Xenakis. In the first chapter I give a complete biography of the composer from his early years in Greece, his education, his life and career in France until his death in Paris in 2001. Chapter 2 deals with Xenakis’s compositional style and influences and the following chapter 3 concentrates on the basic concepts and theories used by Xenakis in his piano compositions. Chapter 4 provides a description of all of Xenakis’s compositions for piano. These compositions include works for solo piano, as well as works for piano and orchestra, chamber compositions, choral and vocal works. The subsequent chapters 5, 6, 7, and 8, provide an analysis of the following compositions for solo piano, Herma, Evryali, Mists, and à r (Hommage à Ravel). The appendices include an abstract from Xenakis’s interview “Monogram – Iannis Xenakis”, which was broadcasted on January 1, 1983 from the Digital Archive of the Hellenic Broadcasting Corporation (the Greek ERT, which stands for Greek Radio Television), a list of the most important interprets of Xenakis’s pianistic works, and a complete list of Xenakis’s compositions.
Résumé

Tato diplomová práce se zabývá analýzou čtyř skladeb pro sólový klavír Iannise Xenakise. V první kapitole představuji životopis skladatele. Zaměřují se na první létá v Řecku, na jeho vzdělání a pokračuji pohledem na jeho činnost ve Francii až do smrti v Paříži v roce 2001. Druhá kapitola pojednává o Xenakisově kompozičním stylu a vlivem na utváření jeho osobitého hudebního jazyka. Následující kapitola se soustřeďuje na teoretické základy Xenakisovy tvorby zejména v jeho klavírních skladbách. Čtvrtá kapitola poskytuje analytické pohledy na Xenakisovy sklady pro klavír, a to jak pro klavír sólový, tak pro klavír a orchestr, dále na komorní skladby, na dílo sborové a na vokální díla. Následující kapitoly (5, 6, 7 a 8), představují analýzu následujících skladeb pro sólový klavír: Herma, Evryali, Mists, a À r (Hommage à Ravel). Dále uvádím přepis z rozhovoru se skladatelem “Monogram – Iannis Xenakis”, který byl vysílaný v 1. ledna, v roce 1983 z Digitálního archivu helénského rozhlasu, seznam nejdůležitějších interpreterů Xenakisova klavírního díla a kompletní seznam Xenakisových skladeb.
Bibliography


WIKIPEDIA – Iannis Xenakis

SCORES, PHOTOS & WEBSITES USED IN DIPLOMA WORK

Iannis Xenakis – http://www.gold.ac.uk/ccmc/xenakisinternationalsymposium/IannisXenakisBWprofile.jpg

Iannis Xenakis and Olivier Messiaen – http://www.annis-xenakis.org/images/photos/Messiaen.jpg


For interview search on Youtube.com - Απόσπασμα από συνέντευξη του Γιάννη Ξενάκη.
Appendices

1. “Monogram – Iannis Xenakis”, Abstract from Interview of Xenakis for the Hellenic Broadcasting Corporation

In this appendix I include an abstract from interview of Iannis Xenakis, which was broadcasted on January 1, 1983 for the Hellenic Broadcasting Corporation. In this interview Xenakis speaks about how he constructs his compositions. The interview was in Greek language and it was translated by me into English. I would like to mention that due to the fact that Xenakis lived most of his life in France, some words he mentions have ambiguous meaning. For this reason, I will be writing in parentheses what he probably wanted to say.

“The music I write is an occlusive result, based on thinking of life experiences and thought and thus it is too difficult to say that this idea is built for the corresponding work, and another idea is built for another work. It is a continuous path just like a railway (motorway), where there we have a variety of vehicles, but often, more and more vehicles appear into the railway. So, it is difficult to say how the whole development of a work it is constructed, but what I can say for sure is this: that all together are connected. Everything is in amalgam, like theoretic ideas or the philosophical, i.e., the idea of definite or the randomness, quantum mechanics, philosophical aspect, physics, and so it is in music as well. All these aspects are finally getting into some frames, in rational forms, forms that sometimes I don’t know if they are even rational, are concentrated into forms of thinking, and of movements as Plato would say, of mental movements which are translated almost automatically into sounds which are symbols already, symbols of things and facts that have behind them all these whole thoughts. So, I will give you an example. It is like when you see in museums, relics of ancient civilizations – I mention the word relics, because are broken, aside from some parts – and you are trying with the help of these parts to figure out how to solve the puzzle in order to construct the whole work which lays behind. So, music is these parts which are obvious and prepared. And at the back side (...) are the parts of thinking where sometimes I usually can’t express, can’t analyze, although I have written books on my own compositional theories. Or another example is like icebergs, where you find only the one eighth, seventh or so above the surface of the sea and all the rest of it is beneath. And indeed, the lower parts are the ones that hold that one eighth. Music is that one eighth. I don’t know if I am definite enough on this. In general, in order to talk about some works, I could have possibly done a reference, saying how a work is constructed, mentioning its causes and effects, its functions either with mathematical or physics relations or even without a relation at all, not with mathematics, nor with physics. What guides me though is for the work to be interesting and to have more general meaning than a work with an apparent effect. And I mention the word apparent because it is something that you already know, otherwise it wouldn’t be apparent. Something you don’t know, which is a prototype, you find it strange, awkward, out of place or even you reject it – because perhaps it might not have an importance, although something that is unpredictable, or the incongruous is always interesting – you are trying to figure out how it is done, how it is been analyzed and you are taking it a step further, you create new things. Music does not have to bear neither screams or desolation or so, but it is a music, which is written
without us trying to make them have such beginnings, as in Nychtes (Nights) that I have written for a twelve-member mixed choir, which is dedicated to the whole world’s civil convicts, is a homage to those civil convicts and it is – at least it was, because it been fourteen years since I wrote it, thirteen actually – and it is a music, which is extracted from that moment of my work and of various problems, musical problems, of aesthetical, which is linked, connected with deep problems and of course, physical, that is of the world’s, which surrounds us with the thinking, that is the philosophy. An example again is the certainty and uncertainty, the problem of repetition, of periodicity, of duplication, of reduplication, which determines the whole world, the universe. It determines our genetics, isn’t it? The continuation of the kind from the descendants and so on, this is based on the problem of periodicity, of the strict periodicity, or of the less strict periodicity. This is of the repetition that is of the reduplication. This is found in music, is found in the universe, everywhere.”

2. Interprets of Xenakis’s works for solo piano

Yūji Takahashi (1938) is a composer and pianist from Japan. During his time in Europe between 1963 – 1966, he met with Xenakis and gave the premiere to his works *Herma* and *Eonta*. He is influenced by Xenakis’s stochastic compositions and is famous for his interpretations of avant-garde music.

Aki Takahashi is the sister of Yuji Takahashi. She also specializes in the performance of composers of the 20th century including works by John Cage, Olivier Messiaen, Pierre Boulez, Iannis Xenakis, as well as works by contemporary composers. She has given many tours in Europe, Asia, and the United States of America.

Marie-Françoise Bucquet (1937) is a French pianist who interprets mostly works by composers of the 20th century. The composers Betsy Jolas, Xenakis, and Bussotti wrote compositions especially for her. She is a teacher at the Paris Conservatory and is also in the jury of many international piano competitions.

Claude Helffer (1922 – 2004) was a pianist from France who also specializes in the music of the 20th century. He gave the premiere to Xenakis’s work *Erikhthon* in 1974. He was also the dedicatee to many works by many contemporary composers.

Ermis Theodorakis (1979) is a Greek pianist, composer, and musicologist. After graduating from the Athens Music Society Conservatory he received the first prize for the interpretation of Xenakis’s works *Mists*. He performs music of contemporary composers and he has recorded the complete works for solo piano by Iannis Xenakis and Xenakis’ piano concerto *Synaphaï* with the Orchestra of Colours. The Greek Union of Music and Drama Critics has awarded Ermis Theodorakis with a prize for his recording of the complete piano solo works by Iannis Xenakis. He has also received a prize by UNESCO for his contribution to Greek Contemporary music. Iannis Xenakis has written in a recommendation letter for Ermis Theodorakis, that he regards Theodorakis as an excellent interpreter of his piano compositions.
3. Complete list of compositions by Xenakis

Orchestral

_Anastenaria_ (1953)

_Metastaseis_ (1953 – 1954)

_Pithopracta_ (1955 – 1955)

_Achorripsis_ (1956 – 1957)

_Duel_ (1959)

_Syrmos_ (1959)

_Stratégie_ (1959 – 1962)

_ST/48_ (1959 – 1962)

_Akrata_ (1964 – 1965)

_Terretektorh_ (1966)

_Polytope_ (1967)

_Nomos gamma_ (1967 – 1968)

_Kraanerg (ballet)_ (1968)

_Synaphaï_ (1969)

_Antikhthon (ballet)_ (1971)

_Eridanos_ (1973)

_Erikhthon_ (1974)

_Noomena_ (1974)

_Empreintes_ (1975)

_Jonchaies_ (1977)

_Aïs_ (1980)

_Pour les baleines_ (1982)

_Lichens_ (1983)

_Shaar_ (1983)
Alax (1985)
Horos (1986)
Keqrops (1986)
Ata (1987)
Tracées (1987)
Kyania (1990)
Tuorakemsu (1990)
Dox-Orkh (1991)
Krinōūdi (1991)
Roāī (1991)
Troorkh (1991)
Mosaïques (1993)
Koīranoī (1994)
Ioolkos (1995)
Voile (1995)
Sea-Change (1997)
O-Mega (1997)

Choral
Zyia (1952)
Anastenaria (1953)
Polla ta dhina (Sophocles: Antigone) (1962)
Hiketides: les suppliées d’Eschyle (1964)
Oresteïa (incid music/concert work, Aeschylus) (1965 – 1966)
Medea (incid music, Seneca) (1967)
Nuits (1967 – 1968)
Cendrées (1973 – 1974)

A Colone (Sophocles) (1977)

A Hélène (1977)

Anemoessa (phonemic text) (1979)


Pour la Paix (Xenakis) (1981)

Serment-Orkos (Hippocrates) (1981)

Chant des Soleils (Xenakis, after P. du Mans) (1983)

Idmen A/Idmen B (phonemes from Hesiod: Theogony) (1985)

Knephas (phonemes by Xenakis) (1990)

Pu wijnuej we fyp (A. Rimbaud) (1992)


Other Vocal

Tripli zyia (1952)

Trois poèmes (F. Villon: Aiês pitié de moy, V. Mayakovsky: Ce soir je donne mon concert d'adieux, Ritsos: Earini Symphonia [Spring Symphony]) (1952)

La colombe de la paix (1953)

Stamatis Katotakis (table song) (1953)

N’shima (1975)

Pour Maurice (1982)

Kassandra (Aeschylus) (1987) [second part of Oresteïa: see choral].

La déesse Athéna (Aeschylus) (1992) [scene from Oresteïa: see choral].

Chamber

Dipli Zyia (1951)

ST/10 (1956 – 1962)

Morsima-Amorsima (1956 – 1962)

Analogique A (1958)

Amorsima-Morsima (1962)

Atrées (1962)

Eonta (1963 – 1964)

Anaktoria (1969)

Persephassa (1969)

Arour (1971)

Charisma (1971)

Linaïa-Agon (1972)

Phlegra (1975)

Epeï (1976)

Retours-Windungen (1976)

Dmaathen (1976)

Akanthos (1977)

Ikhoor (1978)

Dikhthas (1979)

Palimpsest (1979)

Pléïades (1979)

Komboï (1981)

Khal Perr (1983)

Tetras (1983)

Thalleïn (1984)

Nyûyô [Setting Sun] (1985)

Akea (1986)

A l’Ile de Gorée (1986)
Jalons (1986)
XAS (1987)
Waarg (1988)
Echange (1989)
Epcycle (1989)
Okho (1989)
Ophaa (1989)
Tetora (1990)
Paille in the wind (1992)
Plektô (1993)
Ergma (1994)
Mnemas Xapin Witoldowi Lutoslawskiemu [In Memory of Witold Lutosławski] (1994)
Kaï (1995)
Kuîenn (1996)
Hunem-Iduhey (1996)
Ittidra (1996)
Roscobeck (1996)
Zythos (1996)

Solo Instrumental

Seven piano pieces without title, Menuet, Air populaire, Allegro molto, Mélodie, Andante (1949 – 1950)

Suite (1950 – 1951)

Thème et consequences (1951)

Herma (1960 – 1961)

Nomos alpha (1965 – 1966)

Mikka (1971)

Evryali (1973)
Gmeeoorh (1974)
Psappa (1975)
Theraps (1975 – 1976)
Khoai (1976)
Mikka ‘S’ (1976)
Kottos (1977)
Embellie (1981)
Mists (1981)
Naama (1984)
Keren (1986)
À r. (Hommage à Ravel) (1987)
Rebonds (1988)

Tape

Diamorphoses (1957 – 1958)
Concret PH (1958)
Analogique B (1958 – 1959)
Orient-Occident (1960)
The Thessaloniki World Fair (film score) (1961)
Bohor (1962)
Hibiki Hana Ma (1969 – 1970)
Persépolis (1971)
Polytope de Cluny (1972)
Polytope II (1974)
La legende d'Eer (Diatope) (1977)
Mycenae alpha (1978)
Mycenae (1978)
Taurhiphanie (1987)

Voyage absolu des Unari vers Andromède (1989)

GENDY3 (1991)

S 709 (1994)