The thesis is concerned with the solution of multiphysics problems described by partial differential equations using higher-order finite element method (hp-FEM). Basics of hp-FEM are described, together with some practical details and challenges. The hp-adaptive strategy, based on the reference solution and meshes with arbitrary level hanging nodes, is discussed. The thesis is mainly concerned with the extension of this strategy to monolithic solution of coupled multiphysics problems, where each physical field exhibits different qualitative behavior. In such problems, each physical field is discretized on an individual mesh automatically obtained by the adaptive algorithm to suit the best the corresponding solution component. Moreover, the meshes can change in time, following the needs of the solution components. All described methods and technologies are demonstrated on several examples throughout the thesis, where comparisons with traditionally used approaches are shown.