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Report on PhD-Thesis by Martin Scholtz

Dear Prof. Jagrova,

It is a pleasure for me to report on the thesis "Helical symmetry and the non-existence of asymptotically flat periodic solutions of in general relativity" by Martin Scholtz.

The topic of the thesis is an interesting question about the behaviour of self-gravitating physical systems, in particular electromagnetic and scalar fields of several varieties coupled to gravity. The question considered is whether there are any solutions of the Einstein field equations coupled to these matter fields which exhibit a helical symmetry. The question was raised (again) in the context of self-gravitating binary systems for which a helically symmetric solution could be a good approximation to the early stages of the evolution when the two partners are still far away from each other.

The thesis consists of two parts, the first giving the necessary background material leading up towards the statement of the theorems while the second consists of two original publications giving the proof of the theorems. The main result is that analytic, asymptotically flat solutions of the Einstein equations coupled to Maxwell or scalar fields which are periodic on null-infinity are necessarily stationary. This means that there is a time-like Killing vector which reduces to a translation on null-infinity.

The first step in the proof consists of conformally rescaling the physical space-time into a conformally related unphysical space-time in order to have access to null-infinity. The Einstein equations and the matter field equations are suitably transformed to obtain a regular system of equations in the unphysical space-time. Introducing appropriate coordinates and null-tetrads, which are adapted to the situation one obtains a large system of equations for the metric, the spin-coefficients and the matter fields.

The main tool in the argument is the Bondi mass loss, which is known to be negative, thereby leading to a non-increasing Bondi mass. However, assuming periodicity this implies that the Bondi mass must be constant, its flux and hence the grav-

itational news function and the field values must vanish on null-infinity. This argument gives the starting point for a quite elaborate inductive procedure to show that the radial derivatives of the time derivatives of all geometric quantities vanish on null-infinity. Due to the analyticity they vanish everywhere, so that the space-time is stationary.

The thesis is very nicely written and I enjoyed the discussions of helical symmetry in various other contexts. The background material, which consists of the NP formalism, spinors, the structure of asymptotically flat space-times and their conformal properties is carefully described. The necessary equations are very complicated even though the gauge freedom in the setup has been used completely in order to reduce the complexity. The author has written an interesting MATHEMATICA application which allowed him to project the field equations to the null-tetrad, thereby eliminating possible sources of error. Even with the help of this tool the inductive argument requires a good grasp of the structure of the equations and it is clear that the author has mastered this very well.

Two questions come to mind:

- Is there a way to get rid of the assumption of analyticity? A possible way to achieve this might be the consideration of an appropriate form of the asymptotic characteristic initial value problem as discussed by Friedrich and Kannar.
- Is it possible to extend the results to include other physically relevant matter models such as Yang-Mills fields, Higgs fields, Dirac or neutrino fields and fluids?

To summarise, I think that this thesis provides a very deep discussion of helically symmetric space-times. The proof of the non-existence of asymptotically flat and periodic gravitating systems is very well presented in spite of the high complexity of the formalism. Therefore, I strongly recommend that this work be accepted as a doctoral thesis.

Yours sincerely

J. Frauendiener