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A Review of "Logic of Questions" by Michal Peliš

My opinion on the dissertation written by Michal Peliš is positive. The dissertation provides an important contribution to the logic of questions. It has weak points, but certainly the virtues prevail. I was impressed by some insights and solutions proposed.

In order to justify my claim let me comment on the consecutive chapters of the dissertation.

Chapter 1 is of an introductory character. Michal Peliš faced a difficult task of writing a high-level overview which, on the one hand, is accessible to non-specialists and, on the other, simplifies matters to a degree acceptable to specialists. Moreover, since he was supposed to write a Ph.D. thesis, he had to demonstrate his acquaintance with recent developments in the field. In my opinion Peliš found a very good key to accomplish these, partly incommensurable, objectives. Usually the so-called Hamblin's Postulates serve as the points of reference in telling the story of what the logic of questions is. Peliš, however, used here, in addition, Harrah's "axioms" and some general distinctions made by Belnap. Both Harrah and Belnap are among the Founding Fathers of the logic of questions. Yet the papers referred to are not widely known. It was a very good idea to apply the distinctions introduced in these papers in the overview, and I envy Peliš his priority here. Another good idea was to scatter the more detailed comments on recent developments (relevant to the subject of the dissertation) over the remaining chapters.

The nature of the enterprise carried on in the introductory chapter required simplifications, and the author, in the majority of cases, did not cross the tiny line between a simplification and a blatant falsity. However, there are a few statements in the chapter which I am not able to agree with. For example, the following are, say, not quite right.

> *The IEL methodology makes it possible to transform the derivability of a declarative formula into a sequence of questions and produce an analytic-tableaux style calculus—socratic proofs, see [49] for classical propositional logic and [21] for some normal modal propositional logics.* (page 11)

My comments: First, what is transformed is a question about derivability. Second, the "IEL methodology", like any methodology, is not powerful enough to transform a logical relation into a sequence of questions.

Partitioning of logical space is the intension of a question. (page 12)

My comment: I guess that this statement is supposed to pertain to the Groenendijk-Stokhof account of questions. I hope that the author meant something like "The intension of a question is a set of sets of sets of possible worlds - a set of propositions - such that these propositions constitute a partition of the logical space".

Chapter 2 is devoted to Inferential Erotetic Logic (IEL). By and large, IEL is a logic that analyzes inferences in which questions play the role of conclusions and proposes criteria of validity of these inferences. IEL was developed in the nineties of the 20th century as an alternative to the received view in the logic of questions, which situated the answerhood problem in the centre of attention, and to the Interrogative Model of Inquiry, elaborated on by Jaakko Hintikka.

Chapter 2 is not only a plain exposition of the basics of IEL. Peliš does not present IEL just by quoting and paraphrasing, but develops a new conceptual setting. The foundational concepts of IEL had been initially defined for a class of first-order languages enriched with questions and supplemented with model-theoretic semantics. Then a more general framework, based on Minimal Erotetic Semantics (MES), with the core concept of admissible partition, was developed. Peliš proposes a new general setting, in which the concept of model (broadly understood, or, to be more precise, construed accordingly with regard to a background logic just considered) plays the key role. The setting is as general as that based on MES, but presumably is more comprehensible. Yet this is not the only advantage of the new setting. It enables to define some concepts which are both intuitive and useful; the concept of *range* of a question[1] is a paradigmatic example here. Moreover, it enables to simplify some proofs of (the counterparts of) already known theorems, and some known facts become almost apparent.

Chapter 2 contains also some new results and theorems. Let me list them in the order of appearance (I omit those which are obvious consequences of definitions and for this reason have not been noted in other papers; Theorem 4, page 32, Facts 15 and 16, again page 32, or Fact 20, page 36, are cases in point here).

- Lemma 3 (page 26) states, *int.al.*, that if a normal question is evoked by a set of declaratives, it is also evoked by the set of presuppositions of the question. This gives a nice consequence: each evoked normal question is proper (however, Peliš does not notice this fact).
- Theorem 5 (page 33) states that regular e-implication "preserves" evocation.
- The concept "a question Q_1 gives a (direct) answer to [a question] Q_2" is introduced at page 34. The concept is, to some extent[2], natural, and generalizes Kubiński's concept of "being weaker". Then Facts 17-19, Lemma 8, and Theorem 6 (*cf.* pages 35 and 36) generalize the corresponding, already known results pertaining to the behavior of question evocation and e-implication in the presence of Kubiński-style relations of being weaker, stronger, and equipollent.

[1] Roughly, a range of a question is the class of all models in which the question is sound.
[2] But only to some extent. We get the following consequence of Definition 11 (page 34): (a) any question gives answer to a question which has a tautology between its direct answers, and (b) a completely contradictory question (i.e. a question whose each direct answer is a contradiction) gives answer to any question.

- Theorem 7 (page 36) establishes a nice connection between evocation, the relation of "giving answer", and regular e-implication.
- The concept "a declarative gives a partial answer to a question" is introduced at page 38. Again, a certain already known concept of partial answerhood is weakened by the definition.[3] Anyway, Fact 22 presented at page 38 is worth to be noted.
- The concept of (generalized) reducibility of a question to a set of questions, introduced at page 42, is new, but, as a matter of fact, results by a simple weakening from the corresponding concept already analyzed in IEL.[4] Let me stress, however, that Peliš does not hide this connection. What comes next is an analysis of what properties of the "old" relation are retained by the "new" relations. In general, the most important *are* retained, although some results have smaller import now; the claim of Fact 24, page 44, gives an example here. Sometimes "one-step" generalizations of known facts are presented (cf. Fact 25 at page 45). What is certainly new is the analysis of connections between reducibility and regular e-implication (see Lemma 9, page 41, and Theorem 10, page 42). Finally, Theorem 11 at page 43 provides a new result.[5] Let me also add that the proof of Theorem 12 (pages 43-44) does not repeat the line of though present in the proof of the known analogue of the theorem.

So far I have been talking about virtues. Now it is time for vices. The author aims at generality (and, presumably, originality). At page 14 one reads:

In the correspondence with Section 1.2 a question is the following structure

$$? \{\alpha_1, \alpha_2, ...\}$$

Two problems arise. First, what are $\alpha_1, \alpha_2, ...$? We go to Section 1.2 and find out that:

"(...) $\alpha_1, \alpha_2, ...$ *are formulas of the extended language.*" (page 8).

It is unclear whether any of $\alpha_1, \alpha_2, ...$ can be a question itself (in the context of "standard" IEL certainly not!), but let us leave this issue unresolved. The second problem emerges: Is a question, $? \{\alpha_1, \alpha_2, ...\}$, an expression of the object-level language, or not?

If yes, infinite expressions seem to be permitted, but there is no trace of such an approach in the dissertation.

If not, what are linguistic representations of questions?

What I am aiming at is: the issue of reduction of questions to sets of answers is subtle. Peliš knows that, but it looks like he does not want to reach a definite decision here. Sometimes he *identifies* questions with sets of direct answers, *viz.*:

> *We want to be very liberal and this leads us to considering questions* to be [*emphasis added, AW*] *sets of formulas, which play the role of direct answers.* (page 8)

Sometimes, however, he is more cautious, e.g.:

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[3] As a paradoxical consequence of the current definition one gets: any declarative gives a partial answer to a question which properly includes a safe question. In particular, any declarative gives a partial answer to a question which has a tautology among its direct answers.

[4] What is omitted is the requirement that the relevant direct answer to the "reduced" question should not be entailed by Γ alone.

[5] I doubt, however, if the first result announced just after the theorem, at page 43, is correct, although I can be mistaken here.

In our set-of-answers methodology (questions are defined [*emphasis added, AW*] *by sets of direct answers*) (...) (page 34, footnote 16).

Of course, it suffices to assume that questions, construed as set-theoretic entities, have *some* linguistic representation(s); in my above remarks concerning virtues I simply presupposed this. But the syntactic level cannot be completely ignored in general considerations concerning IEL, even conducted within the "model-based" framework proposed by Peliš. The reason is: the truth of some theorems and claims is dependent upon syntax. For example:

In classical propositional logic the disjunction of all direct answers of a question is a presupposition of this question. (page 23)

True, but on the condition that no question with infinite set of direct answers is permitted.

Model-based approach introduces normal questions as questions with semantic range delimitated by models of maximal presuppositions. (page 25)

OK, but on the condition that syntax provides, for all the normal questions considered, formulas which can perform the roles of maximal presuppositions.

Chapter 3 brings in certain new, interesting ideas. This time a modal setting is used. All the basic concepts, both syntactic and semantic, are carefully defined. At the syntactic level one starts with the language of (propositional, multimodal) logic **K** and then supplements the language with questions. At the next step compounds which involve questions are constructed. This enables questions to be arguments of connectives, both truth-functional and modal. A standard Kripke-style semantics for (multimodal) **K** is employed, with the concept of a frame as the basic one. What is peculiar is the way in which truth conditions for questions are introduced. Questions are not reduced to modalities, neither syntactically nor semantically. The truth condition pertaining to questions specifies what it means that a question is *askable* (at a state). Askability is a normative concept. Roughly, a question Q is askable at state $\boldsymbol{s}$ of model $\mathbf{M}$ iff (a) no direct answer to Q is "known" at $\boldsymbol{s}$, (b) each direct answer to Q is epistemically possible at $\boldsymbol{s}$, and (c) the (prospective) presupposition of Q is known at $\boldsymbol{s}$. This is a general scheme: questions are syntactically ascribed to agents, and so is askability.

At first sight, the above concept of askability seems intuitive. It has no direct counterpart in IEL.

Some concepts of IEL have their counterparts in the modal setting. In particular, this pertains to the concepts of safety and riskiness (see page 55)[6]. I guess that at least one of the concepts of relativized askability introduced at pages 56 - 58 was thought of as a counterpart of IEL-evocation. This is not explicitly stated, however. What about e-implication? Chapter 3 includes a subsection titled "Epistemic erotetic implication" and this suggests that the concept introduced there is a "rival" of IEL's e-implication. So let me comment on this concept.

Let Q and Q^* be questions of the considered language.[7] In view of the proposed semantics a *valid* formula of the form:

(1) $Q \rightarrow Q^*$

carries the following piece of information:

(1') for each model $\mathbf{M}$ and each state $\boldsymbol{s}$ of $\mathbf{M}$: if Q is askable at $\boldsymbol{s}$, then Q^* is askable at $\boldsymbol{s}$.

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[6] By the way, the definition of safety gives the following consequence: as long as we stay in **K**, no question is safe. So safe questions can occur at the level of **D** or above. Peliš does not notice this.

[7] For brevity as well as generality I omit indices which refer to agents.

Similarly, valid formulas of the form:

(2) $\neg Q \rightarrow \neg Q^*$

(3) $\neg Q \rightarrow Q^*$

(4) $Q \rightarrow \neg Q^*$

carry the following pieces of information, respectively:

(2') for each model **M** and each state *s* of **M**: if *Q* <u>is not</u> askable at *s*, then *Q** <u>is not</u> askable at *s*.

(3') for each model **M** and each state *s* of **M**: if *Q* <u>is not</u> askable at *s*, then *Q** is askable at *s*.

(4') for each model **M** and each state *s* of **M**: if *Q* is askable at *s*, then *Q** <u>is not</u> askable at *s*.

This is, I must admit, intuitively appealing: the old problem of what the "laws" of the logic of questions should describe is solved in an original way.[8]

So far so good. But there is a problem with the concept of askability.

If I understand well the definition of the notion "a question is partially answered in a state **s** of model **M**" (definition 26, page 60)[9], a question is partially answered at *s* just in case the negation of a certain direct answer to the question is "known" at *s*. It follows that once a question is partially answered at *s*, it is not askable at *s*. My intuition is different here: if a question is *only* partially answered at *s*, it still *remains* askable at *s*, because it is not fully answered at *s*. Generally, and regardless of whether Peliš' definition of giving a partial answer is good or not: a partially answered question remains askable until the information request it expresses is completely satisfied, and this manifests in the fact that a certain direct answer to the question becomes a piece of questioner's knowledge. Partial answers, whatever defined, are not enough to cancel the askability of a question - or, if you prefer, to suppress the question.

The above remark does not mean that Peliš' proposal is wrong. It only means that the proposal, and hence the emerging "logic of questions" as well, do not fit some intuitions, at least my intuitions.

Chapter 3 contains some further ideas and results as well. Yet I must confess that in some cases I had problems with comprehension. Let us consider Definition 23 (page 57).

Definition 23. *A question Q is askable* (*by an agent i*) *in* (**M**, **s**) *with respect to a set of formulas* Γ iff $(M, s) \models [i]\Gamma$ and $(M, s) \models Q^i$. *By* $[i]\Gamma$ *we abbreviate the set* $\{[i]\gamma \mid \gamma \in \Gamma\}$. *Then we write* $(M, s) \models (\Gamma, Q)_i$.

I guess that askability *for* an agent is defined; this is not a problem. The problem arises when $(\Gamma, Q)_i$ is used as a (metalinguistic scheme of a) formula. For consider Fact 32 (page 63). One reads:

Fact 32. $(\Gamma, Q)_i \rightarrow Q^i$ *is valid formula.*

[8] Regardless of possible difficulties that may arise when other truth-functional connectives and modalities are taken into consideration (they must be taken, by the way).

[9] The reservation stems from the fact that the clause "$(\mathbf{M}, s) \models \bigvee_{a_j \in dQ^i}([i]\neg a_j)$" does not comply with previous explanations.

The antecedent is not a formula, so the whole implication is not a formula either. I guess that something like the following is meant:

$[i]\gamma_1 \wedge ... \wedge [i]\gamma_n \wedge Q^i \rightarrow Q^i$ *is a valid formula.*

If Γ is infinite, there is no corresponding formula, however. Inscriptions of the form $(\Gamma, Q_j)_i \rightarrow Q_k^i$ are also present in Fact 33 and Lemma 11 (page 64). A similar - syntactical - problem occurs in the case of Fact 31, page 61.

Chapter 4 is the most interesting part of the dissertation. The setting is based on multimodal **S5**, enriched, first, with the operators of common knowledge and distributed knowledge, and second, with the public announcement/update operators. Then the appropriate "erotetic" concepts are brought into the picture. Askability becomes group askability, and similarly for "being answered". Distributed knowledge is considered and this results in the distinctions between "being implicitly answered" by a group, "being commonly answered" by the group, and "being answered" by an agent (a member of the group). Theorem 18 (page 72) illustrates the role of implied questions in the ("distributed") group answering.[10] In general, Theorem 18 shows that the consecutive concepts form a coherent net. Next, the public announcement operators are characterized semantically (in the standard manner), and both successful and unsuccessful updates (in a state of a model) are defined. As it should be expected, askable questions are successful updates (cf. Fact 38, page 76). The concept of informativeness of a formula with regard to a question (relativized to: an agent, a model, and a state) is explicated in a natural way by applying, *int.al.*, the dual of the announcement operator (*cf.* Definition 33, page 77). Lemma 18 and Fact 39 (*cf.* page 77) confirm that the explication is proper. Finally, it is shown that the conceptual apparatus can be used in a (general) modeling of the phenomenon of eliciting answer(s) from distributed knowledge of a group. Details are presented in subsection 4.3.3, and some interesting comments are provided in the subsection 4.4.

It should be stressed that although there are some affinities between the proposal of Peliš and the (recently elaborated on) "issue management" logic of van Benthem and Minică, Peliš' account clearly goes further. I have no detailed knowledge as to who influenced who and when, but I can say that, for sure, these are different proposals.

Conclusion. Michal Peliš wrote an interesting Ph.D. dissertation. Despite its weak points, the dissertation constitutes a substantial contribution to the logic of questions: it contains new results and introduces many concepts and insights which, in my opinion, are important to the field. I am fully convinced that the dissertation is a sufficient basis for granting Michal Peliš the doctoral degree.

[10] Although, strictly speaking, for syntactic reasons the theorem deals with the finite case only.