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Report on 'Probabilistic Methods in Discrete Applied Mathematics' by Jiří Fink

This thesis contains many interesting results. It is divided into two parts.

Part 1 concerns groundstates in Ising model. It focuses on the nearest-neighbour Edwards-Anderson Ising Model on an arbitrary graph $G = (V, E)$, and on a special class of set systems. The definition of a groundstate of a graph is extended to the so-called XOR-systems, which are closed under (finite) symmetric differences. By considering limits of sequences of sets, compactness arguments can be used to obtain results about existence and uniqueness of groundstates according to the weight distribution. This part ends with a conjecture, which illustrates the fact that serious research always leads to new questions and problems.

Part 2 is longer and deals with several problems in hypercubes.

Inspired by links with Gray codes, Mr. Fink studied Hamiltonian cycles in hypercubes. Every Hamiltonian cycle can be split into two disjoint perfect matchings, so it is natural to ask whether every perfect matching of the hypercube can be extended into a Hamiltonian cycle. This was conjectured to be the case by Kreweras in 1996. This thesis confirms this conjecture. The proof is neat and short. Actually, Mr. Fink managed to find a stronger statement that allowed him to apply induction. His proof is another illustration of the importance (and the difficulty) of finding an induction hypothesis that captures the essence of the problem.

The next chapter is concerned with the matching graph of hypercubes : the matching graph $M(Q_d)$ of the d-dimensional hypercube Q_d has vertex set the set of perfect matchings of Q_d, two vertices of $M(Q_d)$ being adjacent if the union of the two corresponding perfect matchings is a Hamiltonian cycle of Q_d. Kreweras defined M_d to be the graph obtained from $M(Q_d)$ by contracting isomorphic classes of perfect matchings (under isomorphisms of Q_d). He conjectured in 1996 that the graphs M_d are connected for $d \geq 3$. Despite the known fact that, contrary to M_3, the graph $M(Q_3)$ is not connected, Mr. Fink proved that $M(Q_d)$ is connected whenever $d \geq 4$, thereby proving (again in a stronger form) another conjecture of Kreweras.

The rest of the thesis deals with the concept of faulty vertices. The goal is to find long cycles or paths between two given vertices that avoid a fixed set of "faulty" vertices of the hypercube. Let F be a subset of the vertices of the hypercube Q_n. A cycle is long if its length is at least $2^n - 2|F|$. What is the maximum size of F for which $Q_n - F$ always contain a long cycle? This question has been studied by several researchers, who obtained linear lower bounds (in n). Mr. Fink, in a joint work with P. Gregor, provided the first quadratic lower bound using induction. Next, they improved this result and proved a conjecture of Castañeda and Gotchev by showing that F may have order as large as $\binom{n}{2} - 2$, which is optimal. To obtain theses results, an in-depth study of long paths in faulty hypercubes is made, which includes several questions and results of independent interest (some of which are joint work with T. Dvořák, P. Gregor and V. Koubek).

Overall, the thesis contains attractive results, which gave rise to publications in international journals. Mr. Fink's work provides significant advances and solutions to several conjectures. There is plenty of evidence of understanding, and new approaches—as that of potentials—have been successfully used. His work demonstrated his capacity for independent mathematical research. The presentation is excellent, and shows the ability of Mr. Fink to summarise his work and explain the essence of his arguments. It makes his thesis very pleasant to read. I am happy to recommend strongly that Mr. Fink be awarded the degree of PhD.

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