Report on doctoral thesis by Jiří Fink "Probabilistic Methods in Discrete Applied Mathematics".

This paper consists of three parts on distinct topics. The first part investigates finite dimensional Edwards-Anderson Ising model for ferromagnetism. It describes phase transitions which arise by a small change of parameters such as temperature or pressure. It is studied purely mathematical methods of combinatorics and theory of probability. The other parts study properties of hypercubes. The second part is devoted to a relation between perfect matchings and Hamiltonian cycles. It is motivated by Kreweras' hypotheses which are proved in this part. The third part is motivated by problems from computer science. It is interested in the existence of long paths and cycles in hypercubes with faulty vertices. The paper contains intelligent and nice introduction connecting these topics. All parts demonstrate the author's competence, in particular the second part which exceeds the standard for a doctoral thesis.

Now in detail about parts of this thesis.

The first part contains mathematical description of nearest neighbor of Edwards-Anderson Ising spin glass model. The author generalizes groundstates for arbitrary graphs and special set systems and he studies them by mathematical tools. This topic is somewhat removed from my interests and therefore I am not able to judge the relevance of these results. Therefore I can say only that author shows his ability to work on this topic. The language of this part deserves more precision. I had some difficulties to differentiate between some definitions and statements. The main drawback of this part is the fact that it lacks discussions of relations between XOR-systems and Abelian groups of order 2, and between σ-XOR-systems and sequential compact Abelian groups of order 2. I think that some results are easy consequences of well-known facts about Abelian groups (for example that σ-closure of an σ-XOR-system is the origin σ-XOR-system).

The second part gives proofs of two Kraweras' conjectures (their importance is illustrated by the fact that they are presented in the famous Knuth's monograph The Art of Computer Programming, Vol.4). The first conjecture says that every perfect matching of a hypercube can be embedded into a Hamiltonian cycle of this hypercube and the second one says that the graph of perfect matchings of a hypercube of dimension greater or equal to 3 is connected. The author proved both conjectures by an intelligent trick. He expands these statements to complete extension graphs of hypercubes and the required statements are then easy consequences of the stronger statements proved here. This part is completed by a survey of results based on these statements. Here the author shows his knowledge and understanding of new results on this topic. I conclude the report of the second part by noting that it is quite readable and contains only a few misprints (with the exception of Theorem 5.3 and the text in its neighborhood where the letter P rather than S is used several times).

The third part is motivated by problems from computer science. Hypercubes provide a very suitable model for parallel computers and distributed networks. In practice it is common that some processors or vertices of networks are faulty. Then it is necessary that these vertices be avoided to interconnect the remaining healthy vertices. This leads to looking for long paths or cycles interconnecting healthy vertices. The author with a coauthor solved these problems for hypercubes. They proved that if the number of faulty

vertices is quadratic relative to the dimension of hypercube then long paths and cycles do exist. Later they strengthened this result and they found the precise bound on the number of faults for which long paths and cycles exist. The proofs of these results use new tools but they are very technical and therefore it is difficult to read them. One cause of reader's difficulties is the fact that this part is a fusion of three articles. For instance, the basic definition are in three places and are used only several pages later. Moreover, this part contains the greatest number of misprints. But it needs to be said that the author gradually improves his presentation: the earlier proofs begin by technical tools but later a proof begins by a description of its idea. This is quite reader-friendly. Nevertheless it would be better to present all proofs in a more readable form, particularly because this part has long proofs involving much thought and technically demanding content.

Summary:

Despite these minor faults, I think that this paper is excellent and each of its three parts is acceptable as a doctoral thesis. The author clearly shows his knowledge and creative abilities. He demonstrates knowledge of the studied topics and his ability to flexibly change them to reach the desired goal. Therefore I unambiguously **recommend that this paper be accepted as a doctoral thesis and the candidate granted the Ph.D. degree**.

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