One of the basic streams of modern statistical physics is an effort to understand the frustration and chaos. The basic model to study these phenomena is the finite dimensional Edwards-Anderson Ising model. We present a generalization of this model. We study set systems which are closed under symmetric differences. We show that the important question whether a groundstate in Ising model is unique can be studied in these set systems.

Kreweras' conjecture asserts that any perfect matching of the n-dimensional hypercube Q_n can be extended to a Hamiltonian cycle. We prove this conjecture.

The {\it matching graph} $\mg{G}$ of a graph G has a vertex set of all perfect matchings of G, with two vertices being adjacent whenever the union of the corresponding perfect matchings forms a Hamiltonian cycle. We prove that the matching graph $\mg{Q_n}$ is bipartite and connected for $n \ge 4$. This proves Kreweras' conjecture that the graph M_n is connected, where M_n is obtained from $\mg{Q_n}$ by contracting all vertices of $\mg{Q_n}$ which correspond to isomorphic perfect matchings.

A fault-free path in Q_n with f faulty vertices is said to be \emph{long} if it has length at least 2^n-2f-2. Similarly, a fault-free cycle in Q_n is long if it has length at least 2^n-2f. If all faulty vertices are from the same bipartite class of Q_n, such length is the best possible.

We show that for every set of at most $2n-4$ faulty vertices in Q_n and every two fault-free vertices u and v satisfying a simple necessary condition on neighbors of u and v, there exists a long fault-free path between u and v. This number of faulty vertices is tight and improves the previously known results. We also consider much weaker condition of neighbors of u and v. We prove that for every set of at most $(n^2 + n - 4) / 4$ faulty vertices of Q_n, there exists a long fault-free path between any two vertices such that each of them has at most 3 faulty neighbors.