

Report on the Doctoral Thesis:

"Truth between Syntax and Semantics"

by Karel Procházka, Department of Philosophy, Charles University

What does it mean to provide semantics for a formal language? What distinguishes semantics from syntax? In what sense are the notions of meaning and truth essentially semantic? These questions, which are among the most fundamental and important in analytical philosophy and logical theory, form the initial motivation and guiding theme of this careful and thorough study. The method of inquiry is partly historical, partly systematic, and draws extensively on a wide range of secondary literature from philosophy, mathematics, and logic, as well as a penetrating original analysis of the technical works and topics at issue.

The thesis is structured in three main parts, each addressing a range of issues related to the possibility of stating a semantic definition of truth, in three different settings: Russell's (ramified) theory of types, Zermelo's set theory, and Carnap's Logical Syntax. These well-chosen formal systems (and their variants) indeed represent the most important examples of the many different sorts of formal systems encountered in logical research. They also typify the three main approaches to foundations of mathematics: logicist, set-theoretic, and axiomatic. They are distinguished by having been particularly influential in the development of logic - a fact which has also resulted in a large volume of commentary. The author's mastery of the field is impressive, with authoritative expositions of some of the most subtle and difficult aspects of the primary sources, as well as obvious familiarity with the analysis and debates in the extensive secondary literature. With respect to the latter, one finds a welcome clarity and precision often lacking in the literature, including numerous insightful elaborations and corrections.

Russell's theory of ramified types remains among the most difficult subjects in the history of modern logic, and the treatment given here is exceptionally lucid. The idea of providing a contemporary semantics and the associated possibility of a truth definition draws on the ground-breaking work of Church, and concludes with a view of the ramified theory as admitting a "stratification", in which each "level" - suitably defined - admits a truth definition in the next higher level, which can be seen as serving as a meta-language for the previous level(s). As pointed out by the author, the resulting conception, which can be made entirely rigorous, agrees with Russell's own informal understanding of the nature of truth in his theory.

(A minor technical point of correction in this chapter may be noted: in the treatment of the "Antinomy of the Proposition" on p. 34, the 4<sup>th</sup> displayed formula, i.e.:

$$w(f(w)) \leftrightarrow \exists m[f(w) = f(m) \wedge \neg m(f(w))]$$

does not, together with (S-IdP), immediately imply  $\neg w(f(w))$ , as asserted. Rather, one requires some further assumptions regarding intensional propositional identity, such as:

$$(p \leftrightarrow q = r \leftrightarrow s) \longrightarrow (p = r) \wedge (q = s).$$

Indeed, as stated the derivation of the contradiction doesn't even use the specific form of the special function  $f$ .

As a result, the suggestion that simple type theories involving propositional quantification are inconsistent is incorrect - although nothing further depends on that point.)

The two chapters on set theory are the most technical ones of the thesis, dealing with a subject of great mathematical sophistication and difficulty. The author handles the technical matters clearly and precisely, considering various possibilities for the specification of a notion of set-theoretic truth, including large cardinal assumptions, partial truth predicates,

class theories, and second-order set theory.

There is in the literature a common suggestion, or at least a plausible inference to the idea, that one could perhaps arrive at a total theory of truth for set theory by successive approximation by partial notions "from below". Especially the Zermelo quasi-categoricity result, together with the possibility of defining truth for an initial segment by using the resources of a system extended by a further inaccessible ordinal, suggests that one might arrive at a total definition of truth as the "limit" of partial definitions - i.e. definitions for restricted systems. However appealing this conception might be - and it does seem to be in harmony with some remarks of Gödel regarding successive approximations to arithmetical truth by adding higher types - it should be resisted. There is no clearly defined "limit" to such a procedure, since the supposed steps in a sequence of successively more powerful systems are not uniquely determined by foregoing ones, nor need such a sequence necessarily be compatible in such a way that one could speak of a "limiting theory of truth". It is to the author's credit that this common, but unwarranted, philosophical error is avoided - despite being so close at hand.

The final main section on Carnap and the systems of the Logical Syntax is in many ways the best of the three: the exposition of the historical context is thoroughgoing, accurate, and illuminating; the treatment of the intricate details of Carnap's various systems in the Logical Syntax is authoritative and precise; the logical analysis is penetrating and clear - and often more accurate than that given by some of the best commentators; and the philosophical conclusions reached are compelling and well-founded - again, superior to that found in many respected commentators. The picture of Carnap's position that emerges is not only accurate and indeed charitable to Carnap's own intentions, even where these are occasionally

obscure, but it is also a compellingly coherent and powerful view in its own right. It presents Carnap at his best, as few commentators have sufficiently appreciated him.

One issue that could perhaps be addressed in more detail is the question of the essential dependence on the choice of a metalanguage in the definition of analyticity - and the resulting conventional aspect in logical truth - which is of course related to Carnap's Principle of Tolerance, and arguably is even the motivation for it. The fact that there is no uniquely determined or canonical extension of a given language in which to define truth, but rather a range of overlapping and even mutually distinct possibilities leads to a conception quite different from that of the "semantical absolutism" sometimes attributed to Carnap, especially in his later phase. It also shows clearly that one cannot hope to approach a total theory of truth "in the limit" in any reasonable sense. The recognition of this point is clearly there, but the argument could be made much more forcefully on the basis of the evidence already collected.

The historical discussions in each section are wide-ranging and engrossing, touching on various related topics in a way that is both illuminating with respect to the main line of inquiry and informative in its own right as historical and conceptual analysis. The treatment of Frege's logic in the Russell chapter, the separate chapter on Zermelo's conception of set theory, and the knowledgeable discussion of Carnap's interactions with Gödel and Tarski - as well as the Wittgensteinian elements in his philosophical outlook - are all worthwhile elaborations that give the work a valuable additional historical and philosophical breadth and depth beyond the central line of inquiry.

Mathematically competent and indeed admirably clear, without being overly-technical, the author has achieved that difficult balance of providing sufficient detail to carry the argument

forward in a precise and rigorous way, without overwhelming the reader with so much detail that the argument is obscured. The treatment of the difficult set-theoretic material is especially noteworthy.

Philosophically, finally, the position arrived at in the conclusion is both novel and subtle: the difference between syntax and semantics is not absolute, but relative; a truth definition for a given language requires the availability of some methods or means beyond that language, but these need not be of an essentially different nature - distinctly "semantic" - nor is there anything distinctively "semantical" involved in the use of a "metalanguage" beyond the fact that it allows the given "object language" to be treated as a closed system and object of study from without. This conclusion - which has a decidedly deflationary character with respect to some tendencies in semantic theory and contemporary analytic philosophy, is well-supported by the extensive case studies and logical analyses that precede it. And it lends support to a conception of logical theory that is at once more clear, precise, and straightforward than more intricate philosophical accounts emphasizing the absolute or fundamental character of semantics. The author cites Kant as an antecedent to this conception, but the younger Wittgenstein's doctrine of the possibility of saying clearly that which can be said at all certainly also comes to mind.

All in all, this is an exceptionally good piece of historical and mathematical scholarship in the service of a philosophically sophisticated position of originality and depth.

Professor Steve Awodey  
Philosophy Department  
Carnegie Mellon University  
Pittsburgh USA  
31 July 2010