Charles University in Prague

Faculty of Social Sciences

Institute of Economic Studies

Master Thesis

Loan Book Credit Risk Stress Testing - Survey on Practice in the Czech Republic

Autor: Šárka Argayová
Supervisor: Mgr. Magda Pečená Ph.D.
Academic year: 2009/2010

Prohlášení

Prohlašuji, že jsem diplomovou práci vypracovala samostatně a použila pouze uvedené prameny a literaturu.

V Praze dne podpis studentky

I would like to thank my supervisor Mgr. Magda Pečená Ph.D. for valuable comments, all employees from credit risk management departments of banks that agreed to be part of the sample for my survey on stress testing at Czech banks for answers provided and permission to publish them and finally the experts from Czech National Bank (PhDr. Ing. Petr Jakubík Ph.D., PhDr. Adam Geršl Ph.D. and Mgr. Martin Vojtek Ph.D.) for information on stress testing program at CNB and consultation to the empirical model.

UNIVERSITAS CAROLINA PRAGENSIS
založena 1348

Univerzita Karlova v Praze
Fakulta sociálních věd
Institut ekonomických studií

Opletalova 26
110 00 Praha 1
TEL: 222 112 330,305
TEL/FAX:
E-mail: ies@mbox.fsv.cuni.cz
http://ies.fsv.cuni.cz

Academic year 2009/2010

Master Thesis Proposal

Author:	Bc. Šárka Argayová
Main:	Banking and Finance, IES FSV UK
Supervisor:	Mgr. Magda Pečená Ph.D.

Proposed topic:

Stress testing Bank´s Credit Risk

Topic characteristics and methodology:

Stress testing used by financial institutions is a special framework that simulates an institution's portfolio performance under a specific economic scenario. According to Basel II Capital Accord financial institutions do not need to hold just a minimum regulatory capital, but also an additional capital buffer, which can often be determined by stress testing based on hypothetical adverse economic changes.

In this thesis I will mainly focus on capital risk stress testing, which is up till now, in my opinion, the least described in the credit risk area. At the same time stress testing is a very hot topic during today's hard economic situation. Although many papers deal with stress testing, it is not possible to find any whole-covering work, which would sum up possible stress factors and techniques. As a peak of this paper a survey among main Czech banks will be done, after interviews with risk managers I will describe in my thesis how stress testing is used in praxis and how are Czech banks influenced by their foreign parent banks.

Hypothesis:

There are two main parts of my thesis. Firstly, in theoretical part, I will describe possible stress scenarios and stress testing techniques for bank's credit risk. On the basis of that part, I will secondly make a survey on the praxis in Czech banks, which will help me to answer following questions:

1) Are Czech banks influenced in their credit risk stress testing framework by their parents or the supervisory institution?

2) Do the credit risk stress testing techniques and scenarios vary across Czech banks considerably and if they do, how and why?

3) Did the stress scenarios considered by banks in stress testing change before/during the crises?

Outline:

1. Introduction
2. General framework of stress testing – definition of stress testing and its importance, overview of stress scenarios, structure of stress testing
3. Basel II requirements for stress testing
4. Credit risk models for regulatory and economic capital and stress testing techniques
5. Comparison of credit risk stress testing methods in main Czech banks
6. Conclusion

Main bibliography:

BIS Publications (www.bis.org):

Basel II : International convergence of capital measurement and capital standards: A revised framework – Comprehensive version (2006)

Committee on the Global Financial System: Stress Testing at Major Financial Institutions: Survey of Results and Practice (2005)

Committee of European Banking Supervisors publications:

Guidelines on the implementation, validation and assessment of Advanced Measurement (AMA) and Internal Ratings Based (IRB) Approaches (2006)

Technical aspects of stress testing under the supervisory review process, CP 12 (2006)

Monetary Authority of Singapore: Consultative Paper on Credit Stress Testing, 2002,

Saunders, Allen: Credit risk Measurement, New Approaches to Value at Risk and Other Paradigms, Wiley Finance, 2002

Caouette, Altman, Narayanan, Nimmo: Managing Credit Risk, The Great Challenge for Global Financial Markets, Wiley Finance, 2008

Jokivuolle, Virolainen, Vahamaa: Macro Model-Based Stress testing of Basel II capital requirements, Bank of Finland

Mager, Schmieder: Basel II-Type Stress Testing of Credit Portfolios, Queensland University of Technology

Chorafas: Stress Testing for Risk Control under Basel II, Butterworth-Heinemann, 2006

Sobehart : Uncertainty, Credit Migration, Stressed Scenarios and Portfolio Losses, Citigroup Risk Architecture

Höse, Huschens: Worst-Case and Stresses Correlations in the Asymptotic Single Risk Factor Model, Technische Universität Dresden

Term date:	June 2010

Date:

________________ ________________________

Student Supervisor

Abstract

Stress testing is a general term for framework that can assess possible impact of an adverse shock on the financial health and a capital adequacy of a bank, other financial institution or whole financial system. Because credit risk is typically the most important risk for a bank and many international surveys describe the credit risk stress testing as one of the least developed, it is be the main topic of this thesis.

The credit risk stress testing methods developed in the last years very dynamically especially thanks to the requirements on stress testing under the new Basel II regulatory framework, but further improvement of these methods is perceived to be able to withstand severe crisis.

The thesis concentrates on the micro level stress tests that are run by each individual bank. It describes the whole credit risk stress testing procedure, Basel II regulatory requirements, the importance for an institution and offers examples of stress tests.

The first significant contribution to the topic is a survey of the practice in the mayor Czech banks that analyze whether they are influenced in their credit risk stress testing framework by their parents or the supervisory institution, whether the stress techniques and scenarios vary across the Czech banks considerably and whether the scenarios changed in some way before or during the current crisis.

The other contributive part contains a model of stress test on a real corporate credit portfolio of one Czech bank, which uses data on PD for different level of segmentation of this portfolio. The scenarios used are the most actual forecasts of the Czech National Bank. Based on results from this model it is shown how the correctly applied bottom-up approach to stress testing is important for a precise estimation of the minimum capital requirement under stress.

Abstrakt

Zátěžové testování je obecným termínem pro metody, které zkoumají dopad negativního šoku na finanční podmínky a kapitálovou přiměřenost finančních institucí. Jelikož úvěrové riziko je typicky nejvýznamnější bankovní riziko a jak naznačují mezinárodní studie, metody pro jeho zátěžové testování jsou nejméně rozvinuté, je hlavním předmětem práce. Tématem je především zátěžové testování na mikro úrovni, tedy testy prováděné jednotlivými bankami. V teoretické části je podrobně popsán celý proces zátěžového testování, požadavky nové regulace Basel II, význam tohoto testování pro instituci samotnou a příklady nejčastějších testů.

Výrazný přínos k tématu spočívá ve studii na téma zátěžového testování provedené v největších českých bankách a ve zkonstruovaném modelu, který potvrzuje důležitost bottom-up přístupu k testování pro přesnější odhad kapitálového požadavku díky segmentaci.

Table of content

List of Figures

1. Introduction

The financial services offered by banks and other financial institutions are never free from risks. A loss due to these risks can materialize during adverse conditions and hit the financial stability of an institution. As the world becomes more globalized and financial systems are interconnected, problems of financial institutions in one country can easily spread and lead to a severe crisis of one or more regions. For this reason tests for robustness of institutions and financial sectors to adverse shocks became very important tool to ensure financial stability.

Stress testing is a general term for framework that can assess possible impact of an adverse shock on the financial health and a capital adequacy of a bank, other financial institution or whole financial system. Because credit risk is typically the most important risk for a bank and many international surveys describe the credit risk stress testing as one of the least developed, it will be the main topic of this thesis.

The credit risk stress testing methods developed in the last years very dynamically especially thanks to the requirements on stress testing under the new Basel II regulatory framework and perceived need to further improve these methods to be able to withstand financial crisis such as the current one.

In general stress tests are run on two levels. The stress tests on whole banking system, which are usually carried out by a supervisory body or central bank, are important for financial stability of the system considering all interconnections among institutions. The micro stress tests run by banks are used for an assessment if the minimum capital requirement and an additional capital buffer hold by the bank are sufficiently high to withstand adverse but plausible changes in economic conditions that are significant for their portfolio value. The latter type will be the main for this thesis.

In the following chapter I will explain what the stress testing is and what the sources of credit risk are. I will sum main Basel II requirements on stress testing and credit risk measurement and explain why credit risk is one of the most problematic risk for measurement and stress testing. The third chapter will introduce the most common credit risk models, their main logic and assumptions, because the credit risk model used by the institution usually determines the stress testing methods applied. The fourth chapter will cover the stress testing process in the institution and possibilities how a stress test scenario can be defined. In the fifth chapter I will connect the credit risk models and definition of scenarios by description of stress testing techniques and concrete stress tests that an institution can apply to its credit portfolio.

In the chapter six I describe my own survey on stress testing practice at the mayor Czech banks and try to answer three research questions. Based on interviews with experts from banks` credit risk stress testing departments I will analyze whether the Czech banks are influenced in their credit risk stress testing framework by their parents or the supervisory institution, whether the stress techniques and scenarios vary across the Czech banks considerably and whether the scenarios changed in some way before or during the current crisis.

In the seventh chapter I will apply a stress test on a real corporate credit portfolio of one Czech bank, which is quite rare in the literature. The data on PD for different level of segmentation of corporate loan portfolio will be stressed based on the scenarios for economic variables that turned to be significant in a regression model. Finally it will be shown how important is the segmentation of the data on a probability of default for a correct estimation of minimum capital requirement. The last chapter concludes.

2. Definition of credit risk stress testing

2.1 Basel II structure

The actual stress testing requirements for financial institutions within the European Union are encompassed in the Basel II (also called New Basel Capital Accord) and its main document BCBS (2006). This new international recommendation for regulation, which partly changes the earlier principles and initial EU regulation summed in the International Convergence of Capital Measurement and Capital Standards from 1988, became in force on 1.1. 2007[1].

Although Basel I already defined capital adequacy concept, the credit risk was the only one considered. In 1996 an Amendment to the Capital Accord to Incorporate Market Risks was approved and market risk became part of capital adequacy measure.

The Basel II regulation introduces the three pillar structure, in which, according to Andersson, Lindell (2008), under the first pillar the institution is asked to compute minimum capital requirements[2] for the three main financial risks; credit, market and operational risk. For each of these risk types, banks can choose one out of several methods with different level of sophistication, where more advanced methods requiring more investment from the bank's side tends to lead to lower capital requirements.

Under the second pillar according to BCBS (2006) further internal models are required to estimate additional capital buffer that the institution needs to hold, taking into account specific risk profile of its portfolio, that could not be considered in a general formulas applied under the first pillar for minimum capital requirement estimation and further capital needs can be estimated according to other risks run by the institution[3]. The bank must demonstrate to the supervisory institution that all risks not covered under the first pillar are satisfactorily monitored and managed. The last pillar, as in Andersson, Lindell (2008), specifies the disclosure requirements on information of the bank's risk profile, risk management and capital strategies. For Basel II structure see Figure 2.1 below.

[1] In the Czech Republic this regulation is the subject of the CNB decree 123/2007coll. amended by decree 282/2008, which came into force on May 15 2007 and August 15 2008 respectively. It takes effect since 1.7.2007, accepting a possibility that banks can apply this regulation since the beginning of 2008.

[2] The Basel I concepts from 1988 and 1996 had only one pillar specifying the minimum capital requirements for risks mentioned. With Basel II approach to credit risk measurement was changed to be more sensitive to risk. Clients can be differentiated according to credit quality to categories with assigned risk weights.

[3] The stress testing is applied under both, the first and the second pillar.

Figure 2.1: Basel II structure

The First Pillar Minimum capital requirements	**The Second Pillar** Supervisory Review Process	**The Third Pillar** Market Discipline
• Credit risk (changed with Basel II) • Market risk(unchanged) • Operational risk (new with Basel II)	Internal capital adequacy assessment process Review process by supervisory body	Disclosure requirements

Source: based on BCBS (2006) adjusted by author

2.2 Stress testing definition and current stage of development

According to CEBS (2006, pp. 3) stress testing in general, considering any type of risk, is defined as a term for describing the various techniques (quantitative or qualitative) used by institutions to gauge their vulnerability to exceptional but plausible events, while the level and complexity of stress testing used can vary according to the size and level of sophistication of this institution.

Similar definitions can be found in papers by national supervisory bodies as in the Deutsche Bundesbank (2004), where stress testing is defined as an in-depth analysis of a potential impact of a critical event on the institution.

Generally speaking the term stress testing denotes techniques that try to estimate the effect of quite rare but still plausible events, either endogenous or exogenous, on the institution's portfolio, financial prosperity and capital needed to withstand such a negative development. The key problem in stress testing is to detect the most important risk factors or risk drivers that can significantly influence the portfolio value and based on them identify scenarios, which would have the above mentioned characteristics.

Once the scenario is prepared and the relevant changes in risk factors are defined, they can be incorporated into statistical models so that a future portfolio loss for the institution can be estimated. Further the loss conditioned on the event occurrence can be presented to the management, which decides if such losses are still tolerable or if to take any counter measures. In this way, stress testing helps to identify, what kind of future market development can be critical or significantly adverse for a financial institution.

The decision about the stress testing methods and scenarios used depends on the institution itself, but must be finally approved by the supervisory body. The severity of scenarios considered usually depends on the risk tolerance of management, while methods of stress testing hinge on the model used for risk measurement[4].

The main purpose of stress testing and reason why these techniques became part of the European regulation is the improvement of stability of financial institutions and whole financial system. As the financial system is often hit by adverse changes in the financial and other economic variables and financial systems become more global, the financial crises can spread easier and the contagion effect and danger of severe losses increases; the improvement in international regulation to keep financial institutions more sTable was necessary.

High-quality stress testing program on the level of individual institutions can help to see sensitivity of their portfolios to possible adverse economic development and make corrective measures in advance or prepare enough capital buffer. Additionally the stress tests conducted by the supervisory body or central bank on the aggregated financial data can analyze weaknesses and threats for the whole financial system[5] such as contagion effect, which can not be seen on individual level.

During the last years stress testing developed considerably and started to be an integral part of the institutions` risk management accompanied by a commonly used VaR-type analysis. Even though, based on the survey of BIS_CGFS (2005) the usage and sophistication of stress testing on the level of an individual financial institution seems to vary across the portfolios and types of risks considerably.

In the portfolios consisting of market traded securities, stress testing methods tend to be more developed, because of higher availability of data thanks to their regular marking-to-market. Similarly for some types of risks the level of stress testing practice is

[4] In the case of credit risk, under the first pillar the stress testing heavily depend on the rating method used. For the IRB institutions the risk parameters are often stressed according to expected future macroeconomic development and estimated sensitivities of risk parameters to the economic variables or according to their historical volatilities as explained in chapter 4 and 5. Further they can be used in Basel II formulas for minimum capital requirement. Under the second pillar the assumptions of economic capital models are often changed to incorporate risk into the particular model used by the institution.

[5] Such stress tests for whole banking sector were significantly improved within the project of Financial Stability Assessment Program introduced in May 1999 by the IMF and WB, which main aim is to identify the main vulnerabilities and strengths of countries` financial sectors and develop risk management. Nowadays similar stress testing programs are run by many EU supervisory bodies and central banks. The CNB's role in stress testing in the Czech Republic will be described in details in chapter 7 and compared to similar programs in Germany and Austria.

far more described and surveyed then for others. The stress testing for the market risk is considered to be one of the most developed, followed by liquidity and operational risk, while credit risk stress testing in loan book lags, because of the lack in the market data on the products. The least developed stress testing methods are those that could monitor losses due to more risks, such as integration of market and credit risk stress testing[6].

2.3 Sources of credit risk

The credit risk is the most important risk for a bank[7] and as such the risk management for credit risk should be the core stone. Credit risk is actually the risk that an obligor will default on his obligations and will not be able or willing to repay the cash flows as agreed[8]; that can have an adverse impact on the bank's solvency and liquidity.

According to the CNB decree (123/2007 and 282/2008 Coll.) the obligor is in default, if it can be expected that he will not meet his payments when they become due and the bank will not be able to cover his obligations from the collateral or when the repayment of face amount or any connected significant payment is post the due date for more than 90 days.

In Deutsche Bundesbank (2004) we can find a more general definition of credit risk. In a broader sense, credit risk denotes as well the risk of a deterioration in a borrower's creditworthiness (migration risk), which leads to a revaluation of the relevant assets. The debtor's downgrading, either by the external rating agency or within the internal rating system of a bank, due to worsening of its characteristics that are important for his credit quality leads, even if the loan is not traded or priced on the market, to higher capital requirements and higher probability of materialization of loss in case of default event.[9]

---

[6] Considering this problem we can already find steps to integrate market and credit risk stress testing like in Kafetzaki-Boulamatsis, Tasche (2001), who tries to estimate within the Merton type model explained in chapter 4 capital for credit, interest rate and foreign exchange risk.

[7] Considering the Czech banking sector the supportive arguments can be found in Financial market supervision report (2008) (pp. 81), while the same holds also for EU countries according to BIS_CGFS (2005).

[8] This definition of credit risk is sometimes called narrower, because default risk can be considered as just one of sources of credit risk.

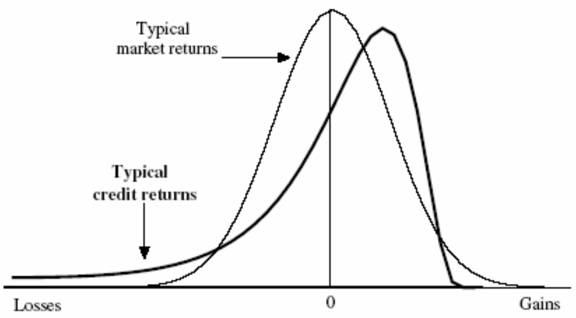
[9] The difference between broader and the narrower definition of credit risk is most evident in the default mode and mark-to-market credit risk models covered in the chapter 3. The first see as credit risk only a risk that an obligor defaults, while in the latter a migration risk is considered as well.

2.4 *Availability of data and probability distribution*

As mentioned, according to the national study of Deutsche Bundesbank (2004) as well as according to the international study by BIS_CGFS (2005) the credit risk measurement and especially stress testing, in comparison with the market risk significantly lags. This feature steams from low availability of market data on credit products, as for example loans. These products are not regularly marked-to-market, so data on their everyday market prices are missing. Furthermore the default events are not so common as for market portfolios, so statistics on defaults is poorer in spite of the fact that credit products have usually longer maturity.

Moreover the distribution of losses for credit products is much more complicated for estimation. Because of the typical characteristics of these products the distribution is non-normal and significantly skewed, as depicted on the Figure 2.2. The asymmetric thicker left tail streams from quite low probable high losses in the case when an obligor defaults and the whole value of the loan is lost, while the right part depicts more probable, but significantly lower than proportional profit that can be earned on interest.

Figure 2.2: Comparison of market and credit returns



Source: CreditMetrics Technical Document

2.5 *Credit risk modeling and credit risk stress testing within Basel II*

As defined in BSBC (2006), under the first pillar of Basel II the minimum capital requirement for credit risk should be computed. For doing this the bank can decide for one of three methods; Standardised, IRB foundation and IRB advanced method, where for the latter two it needs to ask the regulator for a permission to use such method. The methods differ in the quantitative as well as qualitative requirements on the credit rating system[10].

[10] For example, if an institution wants to apply for using IRB for some clients, it has to use rating system for this category of clients already at least for three years and have data on probability of defaults from at least five last years (or two years in special cases). There are special requirements for data storage, equal treatment

Under the standardised rating approach the bank can use ratings from Export Credit Agencies[11] to see the credit quality of the client and assign him the risk weights defined in Basel II, which are needed for further minimum capital requirement computation.

On the other hand under both of the IRB approaches, the clients` ratings are set by an internal rating process, which also assign to each client values of Basel II risk parameters[12], which are further used for risk weighted exposure computation. IRB approaches enable banks to compute regulatory capital more precisely, according to their historical experience on how probable and how big losses were connected with clients from each rating class and with what kind of product. The Standardised method is far less sensitive, because the values of the parameters are determined and can not be adjusted according to the experience of the bank.

Under the first pillar the bank computes so called expected and unexpected loss that must be covered by the bank’s capital. The former should be covered through income received by properly pricing the transactions; for the latter the capital adequacy and reserves are prepared. The expected loss is defined as a mean of the loss expected, in the case of credit risk, in one year horizon. The unexpected loss is the difference between expected loss and maximum loss that can appear within the year with 99,9% probability. The actual capital of the bank should never decrease under the sum of these two measures.

Under the second pillar, in the context of credit risk, so called economic capital is computed. Economic capital is generally defined as capital needed to cover the unexpected loss over given time horizon with given probability. The economic capital usually differs from the regulatory one, because it should take into account more specific characteristics of the portfolio as diversification, concentration or difference between experienced credit risk parameters to those prescribed under the first pillar. For this reason we can see economic capital as a measure of capital that bank itself see as needed for losses from its risky activities to withstand them with some required probability level, which usually corresponds to the rating that the institution wants to reach or stay at. If the regulation is set

---

of clients for rating classification, number of rating classes, which must be at the same time enough granular, for documentation of rating process, stress testing, backtesting etc. more details can be found in BCBS (2006) and CNB decree 123/2007 and 282/2008.

[11] Including external rating agencies such as S&P, Moody`s etc.

[12] Under the IRB foundation approach at least the probability of default is set for all clients and for retail clients all parameters as probability of default, loss given default, maturity, exposure, and conversion factor, are set within the internal rating system.

correctly, the regulatory and economic capital should be the same or at least very close to each other, because they both should estimate the same; the capital needed to cover the risks run by the institution.

According to Bangia et al. (2002), if the economic capital is higher than the regulatory one, the Basel II concept undervalues the risks run by the institution; additional capital buffer must be hold by the institution to cover its losses that materialize with certain probability. If the economic capital is lower than the regulatory, the institution anyway needs to hold enough capital to meet regulatory requirements even though the regulatory rules overestimate the actual risk. The capital for undertaken risks, as perceived by the institution, is lower which may motivate the institution to increase its risk profile.

According to BCBS (2006) as in Rösch, Scheule (2008) a bank must have in place sound stress testing processes for use in the assessment of capital adequacy. These stress measures must be compared against the measure of expected positive exposure and considered by the bank as part of its internal capital adequacy assessment process. Stress testing must also involve identifying possible events or future changes in the economic conditions that could have unfavorable effects on a firm’s credit exposure and assessment of the ability to withstand such changes. Examples of scenarios that could be used are economic or industry downturns, market-place events or decreased liquidity conditions.

The stress testing is required to be part of the risk management framework and should be conducted under both the first and the second pillar. It should search for loss and changes in the capital, either minimum required capital or economic capital, in case of adverse future development. Under the first pillar the changes in minimum required capital thanks to changes in risk parameters for different segments of the institution’s portfolio are estimated; under the second pillar all differences from the assumptions made for the model under the first pillar and changes in the economic capital for future extreme events should be covered.

The requirements for stress testing according to the CNB decree (No. 123/2007 Coll. as amended by Decree No. 282/2008 Coll.) are very similar to the Basel II ones. The financial institutions should identify all possible events or future economic changes, which could adversely influence the value of the credit position and assess the ability to withstand such changes. Stress testing should be reasonable and conservative, should consider at least mild recession and according to the 2006/48/EC also migration in rating and deterioration in credit quality of protection providers. Furthermore stress tests should cover most of the risky positions. The scenarios and stress testing methods are defined by the institutions and approved by the CNB.

The stress tests for credit risk should be according to the requirements based on CEBS (2006), run as frequently as necessary according to the risk drivers and volatility in stress testing framework, but at least once a year. With this frequency it should be evaluated if the methodology and scenarios are still adequate for current conditions and portfolio structure and should be up-dated if necessary. It is also recommended for stress tests for credit risk to take into account longer time periods than for example for market risk, because of the longer maturity and lower possibility to change the portfolio structure[13].

The complexity of stress testing program is allowed to differ according to the size and complexity of the institution, while key control over the whole institution's stress testing program is assigned to the supervisory body that must review its appropriateness. The supervisory authority can also prescribe an ad hoc test at a specific point in time e.g. when there is a significant change in environment or financial markets or wants to asses an impact of the same stress scenario on the wide range of institutions[14].

---

[13] Further requirements and recommendation of Basel II connected with credit risk stress testing and modeling can be found in BCBS (2006).

[14] This rule is used regularly since 2009 for the bottom-up stress tests run by the CNB as explained in the chapter 7.

3. Credit risk management models

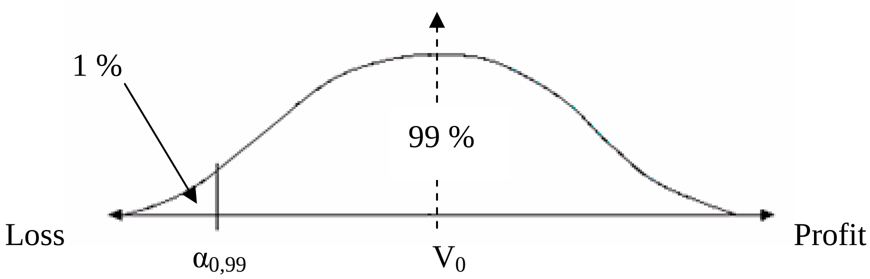
Credit risk management in a financial institution can be seen as two-stage problem. First, the decision about the composition of the portfolio is done according to the standard methods, which are commonly used in credit risk portfolio management like Value at Risk or its alternative for credit risk, based on the same logic or other well known models explained further. As in Singapore (2002) the second problem in the credit risk portfolio management is analysis of the reaction of portfolio value during unsTable times, when factors that were considered during the VaR analysis to behave in the similar way as in the past observation start to be unsTable.

3.1 Value at risk

The VaR types of models are based on the basic logic of estimation of loss that will materialize within a given confidence interval over some period of time under normal conditions. At the level of an individual instrument, the assessment of the counterparty or instrument risk as its volatility plays the key role as well as the assumption on the distribution of rates of return on the instrument. The maximum loss over one year under the standard normal distribution that will not be exceeded in 99 % of cases can be computed as follows.

$$VaR_{1-Y} = \alpha_{0,99} * \sigma_{1=Y} * W_0$$

Figure 3.1: VaR model



Source: author

On the portfolio level, the VaR can be computed based on the correlations between each two instruments or counterparties.

$$VaR_P = \sqrt{\sum_{i=1}^{N} VaR_i^{\ 2} + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} VaR_i * VaR_j * \rho_{ij}}$$

The similar logic is used by some other models like economic capital models described further, which compute the expected price of the loan and its volatility due to credit risk and apply the similar logic to estimate Capital at Risk (CaR)[15].

The biggest disadvantage of VaR-types of models is that they often use an assumption on the distribution of the future value, which is usually supposed to be at least symmetric with thin tails like on the Figure 3.1. This according to empirical findings described above does not hold for credit products. The assumptions can be eased by using historical observations that can much better approximate the truth probability distribution of the future value of a portfolio and according to this observed distribution the more precise VaR value can be set as in Soberhart (2008).

Even though the second disadvantage of this model stays unsolved. The VaR analyses expect that the past behavior of the instruments` values can characterize their development during the horizon of the analysis. This can be true for calm periods, when market factors do not change considerably and the forecast based on past behavior can be accepTable at least for a short horizon. On the other hand in more turbulent periods or even in crisis, when economic factors become unsTable, even such short run forecasts of future value of the portfolio will be wrong.

3.2 Difference of stress testing and VaR, importance of stress testing for an institution

According to Schachter (2001) there are few imperfections in modeling abnormal, more extreme events by the VaR-types of models, which can be addressed by usage of the stress testing programs. Firstly, the VaR models see extreme events as tail observations from the same distribution as every other normal day observation, but as argued, the crisis observations should be more likely from a different return distribution with different characteristics than those considered by VaR for normal market situation. Secondly, if they were from the same distribution, VaR analysis would not be able to estimate the exposure to such extreme market moves as it is a function expressing the risk as function of changes

[15] Like in CreditMetrics, where expected price of a bond is computed based on the future cash flows discounted by relevant discount factors and volatility of the price can be estimated as its standard error. For the portfolio CaR we again need to estimate correlations among individual instruments. CreditMetrics is described in details in 3.3.2.

in normal market rates and prices and finally this analysis does not give any information on losses once the VaR measure is exceeded.

All these shortages of the VaR can be solved by broadening the credit risk management program with stress testing. Stress tests are built to estimate exposures and losses under extreme situations and even in BCBS (2006) and BIS_CGFS(2005) are said to complement VaR. The latter further highlights the importance of stress testing in comparison with VaR for understanding firm`s risk profile, verification of adequacy of limits and capital allocation or evaluation of business risks.

As mentioned in FSA (2005) the stress tests can help to identify big sensitivities of parts of portfolio or whole business lines to some risk factors and set limits or monitor new products for example on new emerging markets, which are more volatile and where few data are available. It further helps the management to determine it optimal risk/return tradeoff and see, whether they would be satisfied with the value of this ratio under stress conditions or some adjustments in portfolio structure or institution's policy must be done.

On the other hand also stress testing has its disadvantages, for which it is often criticized. As for example in Berkowitz (1999), who sees two results for losses as confusing and recommends to incorporate stress events into the VaR analysis as described in details in the following part. Further the stress testing program is highly subjective as it is very difficult to judge if all relevant risks are included and assessed, when scenarios are prepared. Moreover the results from stress test expressed as loss under some extreme scenario do not have any probability of occurrence, which can question the importance of taking any corrective measures[16]. To solve shortcomings of both methodologies, there already exist some attempts to incorporate stress testing into the standard VaR analysis as described in the following part.

3.2.1 Methods for unification of VaR and stress testing

According to Schachter (2001) there are two main theoretical approaches, how historical observations during normal periods and stress scenarios can be unified. The first applied by Cherubini, Della Lunga (1999) uses Bayesian statistics for combining actual observed historical data with certain mean, variance and correlation with subjective stress scenario. After adding a noise factor to given scenario, conditional as well as unconditional

[16] Remedial measures as listed in the CEBS (2006) are discussed in chapter 6.

distribution for historic and expected data can be derived. This distribution already encompasses the analysis based on VaR and stress scenario logic.

Second theoretical way, how these two concepts can be connected can be found in Berkowitz (1999), who directly combine "typical" historical distribution of factors` values with subjective stress distribution using as a weight some subjective probability to get a combined distribution describing joint behavior of factors under all circumstances. Further for forecasting the portfolio return distribution a valuation function describing the portfolio value under certain factor setting can be used. Except of these very theoretical approaches, there is one more applicable concept; the extreme value theory that tries to adjust tails of the classical portfolio value distribution and related block maxima and peak over threshold approaches.

3.2.1.1 Extreme Value Theory (EVT), Block Maxima and Peak over Threshold approach

The EVT deals with modeling the extreme situations exceeding those considered by VaR as depicted on the Figure 3.2. It looks for a statistical distribution of extreme values of return respecting the correlations between risk factors; either historical or correlations conditional on extreme events can be used to depict the distinct parts of distribution of returns[17]. The EVT has an asymptotic univariate and a multivariate version, the first one is used for a tail distribution modeling for portfolios with few assets and sTable positions, while the latter better analyzes complex positions with many assets and risks. Basically, according to Longin (2000), we can describe the univariate approach of EVT as:

Let's observe returns over n periods of time and denote the extremes (here minimum, but can be derived in the same way for maximum) as:

$Z_n = \min\{R_1, \ldots, R_n\}$

Where $R_1, \ldots, R_n$ *are independent, taken from the same distribution* F_R

Then the distribution of minimum value is:

$F_{Z_n}(z) = 1 - (1 - F_R(z))^n$ ……..*in reality* F_R *is not known*

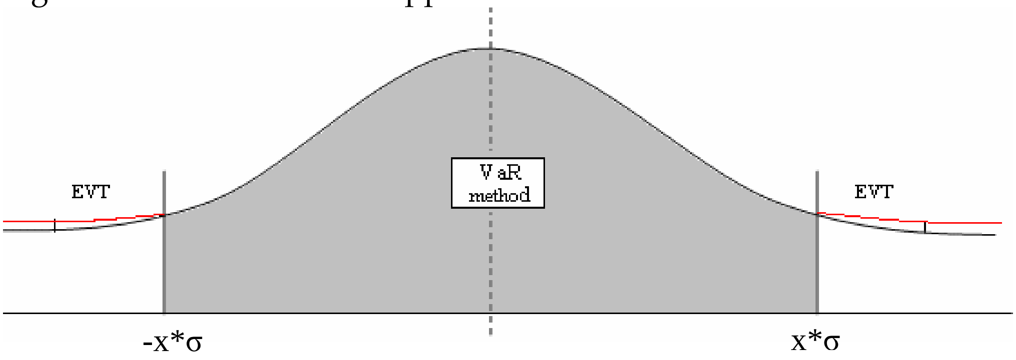
We can use a scale parameter α_n *and a location parameter* β_n *to get a limiting distribution of minimal returns (for n→∞, T→∞)*

$Z = (Z_n - \beta_n) / \alpha_n$

[17] The detailed analysis of normal and stress time correlation among risk factors can be found in the section 4.3.2.1.

$F_Z(z) = 1 - \exp\left(-(1+\tau * z)^{\frac{1}{\tau}}\right)$ ....*distribution of tails for* $z < -1/\tau$ and τ <0 or $z > -1/\tau$ and τ >0[18].

Figure 3.2: Univariate EVT approach



Source: by autor

As depicted on the Figure 3.2 the main problems of EVT is to find distribution relevant for the tails and edge, from where the tails of normal distribution considered for the classical VaR method should be replaced (on the Figure defined as x multiple of standard deviation). The trade off in this problem is following, the tails must start far enough from the mean to describe extreme event only, and on the other hand enough tail observations are necessary for estimation of parameters of the tail distribution.

The multivariate approach differs in the dimension of the vector of extremes, but can be derived similarly.[19]

---

[18] τ is a tail index, which describes fatness of tails as well as what kind of distribution the tails have. (τ <0 means Fréchet distribution, τ =0 Gumbel and τ >0 Weibull distribution). Further a shape parameter defined as k= $-1/\tau$ describes the moments of the distribution of returns (if k>1 the mean of distribution exists, if k>2 variance is finite and if k>3 skewness is well defined)

[19] Let's have q-dimensional vector of random variables, i.i.d., where realization of i-th component in time t is R_t^1:

$R = \left(R^1, R^2, \ldots, R^q\right)$

$Z_n = \left\{\min\left(R_1^1, \ldots R_n^1\right), \min\left(R_1^2, \ldots R_n^2\right) \ldots \min\left(R_1^q, \ldots R_n^q\right)\right\}$ .....vector of minimal returns

$(Z_n - \beta_n)/\alpha_n$

A q-dimentsional distribution F_Z is limiting to extreme value distribution if and only if:

1) univariate margins $F_Z^1, F_Z^2 \ldots F_Z^n$ have Gumbel, Weibull or Fréchet distribution

2) there is dependence function d_{F_Z} which satisfies

$F_Z\left(z^1, z^2, \ldots z^q\right) = 1 - \left(F_Z^1\left(z^1\right) F_Z^2\left(z^2\right) \cdots F_Z^q\left(z^q\right)\right)^{d_{F_Z}\left(z^q - z^1, z^q - z^2, \ldots z^q - z^{q-1}\right)}$

Two other concepts described in Schachter (2001) are related to the univariate EVT; the Block maxima and Peak over threshold approach, both based on statistical limit laws for extreme observations[20]. The Block maxima approach searches for loss that is expected to be exceeded once in given number of years k.

Let's observe returns over n periods of time and define maximum as:

$M_n = \max\{R_1, \dots, R_n\}$

Where $R_1, \dots, R_n$ *are independent, taken from the same distribution* F_R

Then the distribution of maximum value using a scale parameter α_n *and a location parameter* β_n *to get a limiting distribution of minimal returns (for n→∞) can be described by*:

$\Pr\left\{\frac{M_n - \beta_n}{\alpha_n} \le x\right\} \to F(x) = \exp\left(-\left(1 + \xi * \frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right)$*.........F(x) is generalized extreme value distribution with parameters* μ, σ, ξ.

Then the loss x expected to be exceeded once in k years (with probability 1/k), where n=1year (we have annual observations) can be expressed as:

$$1 - \frac{1}{k} = \exp\left(-\left(1 + \xi \frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right)$$

$$x = \mu - \frac{\sigma}{\xi}\left(1 - \ln\left(1 - \frac{1}{k}\right)^{-\xi}\right)$$

While Peak over threshold describes, how high loss can be expected once the VaR is exceeded or otherwise it expresses the sum of a VaR measure and mean loss from generalized distribution.

$EL_P = VaR_P + E(X + VaR_P \big|_{X > VaR_P})$*...where* $E(X + VaR_P \big|_{X > VaR_P})$ *is mean of the excess loss distribution for* $u = VaR_P$.

$$EL_P = \frac{VaR_P}{1-\xi} + \frac{\sigma - \xi u}{1-\xi}$$

The Peak over threshold approach of EVT can be seen as a special case of Expected Shortfall theory, which also measure the average loss above a given quantile of loss distribution (α) considered by VaR, but do not use generalized distribution for extreme values. According to Kalkbrener et al. (2004) the expected shortfall can be expressed as:

$$E(L \big| L > VaR_\alpha(L)) = (1-\alpha)^{-1} E(L * 1_{\{L > VaR_\alpha(L)\}})$$

[20] For precise description of limits laws used see Schachter (2001).

3.3 Economic capital credit risk models

Further I will explain main ideas of the most famous credit risk models that are commonly used for economic capital estimation. The economic capital stress testing heavily depends on the credit risk model used by the bank, because for stress testing some of the settings of the model can be changed according to the scenario, make dependent of some economic factors or the assumptions can eased in some other way. Some of the possibilities, how credit risk model can be adjusted for stress testing, will be covered in the chapter 5.

Models that are used for estimation of the volume of economic capital needed can be divided into default models and mark to market models. Default models consider just two stages of the world; either the obligor defaults and the bank looses part of the loan or does not default and the whole value is repaid. These models see the changes in the credit quality of the obligor as market risk and concentrates just on a prediction of defaults. Because of this fact, according to Allen, Saunders (2002), it is better to use DM models, when the asset is held till the maturity and meanwhile the market value of the asset is not important; the credit quality deterioration is not considered for capital reserves.

The mark-to-market models use the broader definition of the credit risk, because they take into account also changes in the credit quality of obligors. The models see as credit risk not only the default of the obligor but also a migration to a different credit quality class and connected change in spreads that influence the value of the loan. For this reason it is believed that these models track the value of the asset and predict the capital needs in more general manner and more precisely especially for longer maturities.

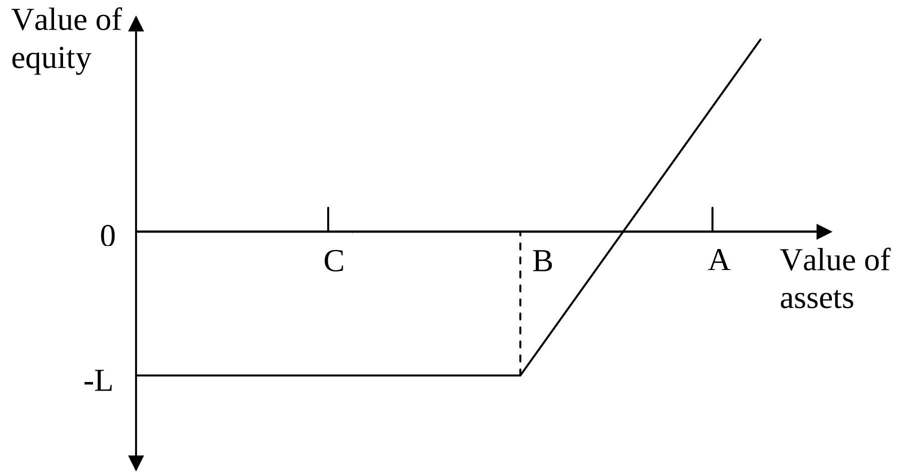
Further there are two approaches to the default modeling. In structural approach models an analysis of variables influencing the default of the obligor is typically included, while in reduced form approach models, the most important variable is a probability of default and reasons for defaults are not explored.

3.3.1 Moody's KMV model – Credit Monitor model

The main goal of this model is to determine the default probability of an obligor, which is in this model called expected default frequency (EDF). The mechanism of the model is based on the equivalence of a loan to a firm and option. In the comparison with the Merton model, which first came with this option concept, is the option logic taken from the exactly opposite point of view, from the firm's side.

The payoff for a shareholder in a levered firm can be expressed as buying a call option on its assets. According to Allen, Saunders (2002) as displayed on the Figure 3.3, if the firm initially borrows OB and the value of their assets at the end of the period is higher (say OA) the firm will repay its debt and the residual value will be available for shareholders. On the other hand, if at the end of the period the market value of its assets will be lower (e.g. OC), the firm will become insolvent, the loan will not be repaid and the bank will loose principal as well as interest, while the shareholders will pay the downside limited value of their initial stake in the firm L.

Figure 3.3: Equity of a levered firm as a call option on assets in the KMV model



Source: Allen, Saunders (2002)

According to this concept the position of shareholders can be seen as holding a call option on the assets, which can be priced in the same way as any other option, based on the Black-Scholes-Merton model.

$$\overline{E}_i = h(A_i(t), \sigma_{Ai}, \overline{B}_i, \overline{r}_{Ai}, \overline{T}_i)$$

The value of the equity depends on the market value of assets (A_i), its volatility, the borrowed amount (B), rate of return (r), and the maturity of the option (T) or default horizon of the analysis, which is for credit risk usually taken as one year. The exercise price B is recommended by KMV to be measured as half of long term liabilities increased by all short term liabilities. As the bar denotes the observable variables, we can see that two variables in the one equation stay unsure. This is in the KMV model, as in Crosbie, Bohn (2003), solved by additional equation defining a link between equity and asset volatility[21].

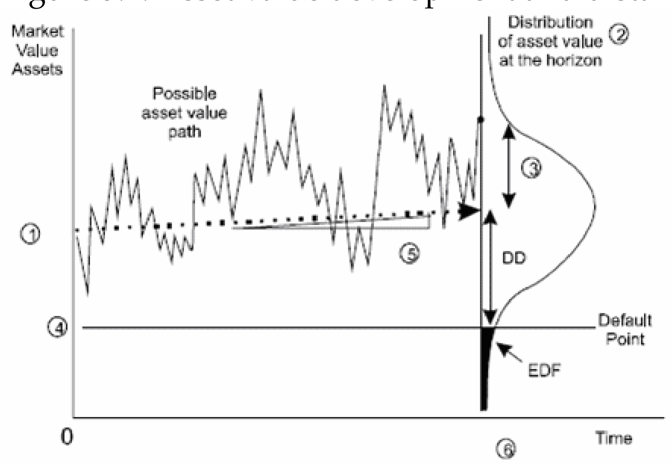
$$\overline{\sigma} = g(\sigma_{Ai})$$

Or

Equity volatility = Option function($A_i(t), \sigma_{Ai}, \overline{B}_i, \overline{r}_{Ai}, \overline{T}_i$)

Further the EDF for one borrower can be estimated based on the assumption that the asset value follows a log normal distribution. Some additional assumptions about its growth in time (or the asset value drift rate denoted as r) can be done.

Figure 3.4: Asset value development and distance to default in the KMV model



Source: Crosbie, Bohn (2003)

According to Crosbie, Bohn (2003), there are six variables that determine the default probability, denoted in the Figure 3.4 by numbers, they are respectively – the current asset value A(0), the distribution of the asset value at the end of the period A(T), volatility of the asset value at T, the level of the default point B, expected growth of assets over the horizon and the length of the horizon T. The development of asset value and the distribution of the asset value at time t can be described as:

$$dA_i = r_{Ai} A_i(t)dt + \sigma_{Ai} A_i(t) dW_i(t)$$
$$\ln A_i(t) = \ln A_i(0) + (r_{Ai} - \sigma_{Ai}^2 / 2)t + \sigma_{Ai} \sqrt{t} W_i(t)$$

where W_i denotes Wiener process of random changes in the firm's assets value.

The distance to default (DD) and EDF can be computed as follows:

[21] The functional form differs for different authors.

$$DD_i = \frac{\ln(A_i(0)/B_i) + (r_{Ai} - \sigma_{Ai}^2/2)T}{\sigma_{Ai}\sqrt{T}}$$

$$EDF_i = P(A_i(T) < B_i) = P\left(\frac{W_i}{\sqrt{T}} \le -\frac{\ln(A_i(0)/B_i) + (r_{Ai} - \sigma_{Ai}^2/2)T}{\sigma_{Ai}\sqrt{T}}\right)$$

$$= \Phi\left(-\frac{\ln(A_i(0)/B_i) + (r_{Ai} - \sigma_{Ai}^2/2)T}{\sigma_{Ai}\sqrt{T}}\right)$$

The KMV model advantage is that it can be easily applied on all listed firms, as it links the observable financial variables with default probability. For non-listed firms the EDF can be estimated by comparison with firms with similar rating and financial conditions. The biggest disadvantage, the normality assumption, can be outweighed thanks to the empirical EDF, which is a statistic that depicts observed default frequency as a function of distance to default. In this way for each firm's distance to default an empirical expected default frequency, based on historical observation of firms with similar financial characteristics, can be found. Even if firm's assets do not follow the log normal distribution, empirical results for similar firms are available in this statistics. Similar statistics for empirical EDF is also offered according to the standard firm's ratings as e.g. from S&P.

The bank's expected loss will be equal to LDG times EAD times the expected default frequency weighted by the probability of default. Unexpected loss is for this model usually defined as standard deviation for two possible stages of the world, default and non-default of an obligor.

$$UL = EAD * LGD * \sqrt{PD * (1 - PD)}$$

For *n* loans with weights *w* and correlations of defaults[22] *ρ*:

$$EL = w_1 EL_1 + w_2 EL_2 \ldots + w_n EL_n$$

$$UL = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij} * w_i * w_j * UL_i * UL_j}$$

3.3.2 CreditMetrics

This mark-to-market model estimates the distribution of future value of particular portfolio, which can be further used for estimation of expected and unexpected loss or VaR for different quantiles. The downgrading and upgrading of the obligor during the life of the

[22] In the KMV model correlation of default of two assets is usually approximated by asset correlations.

loan is considered as part of credit risk, so the economic capital is estimated to cover losses caused by defaults as well as by other rating changes.

According to Saunders, Allen (2002), the CreditMetrics specialize on non-traded assets as loans or privately listed bonds. The model assumes that rating system contains one class for defaulted debtors and each obligor is assigned to one rating class according to clearly defined criteria. Further the probabilities of rating migrations are assumed to be given. Moreover for each non-default rating class there is a forward zero curve and spreads for each rating, while the LGD for each credit is known.

Under these quite strict assumptions the future value of a bond (or alternatively a loan) after one year can be computed, based on the basic discounting by zero rates and spreads related to the end-of- period rating, in the following way:

$$FV_i = \sum_{i=0}^{T-1} \frac{CF_{i+1}}{(1+r_i+s_i)} = CF_1 + \frac{CF_2}{\left(1+{}_1r_1+s_1\right)} \dots + \frac{CF_T + M}{(1+{}_1r_T+s_T)}$$

As the first cash flow comes at the end of first year it is not discounted. The forward zero rate curves are derived from zero coupon risk free instruments such as government bonds. An example from Saunders, Allen (2002) for U.S. Treasury notes with semiannual coupons is following (r denotes interest rate, ${}_0z_2$ is one year forward zero rate and ${}_1z_1$ is six month zero coupon rate six month from now etc.)[23]:

$$100 = \frac{C+F}{1+{}_0r_1} = \frac{C+F}{1+{}_0z_1}$$

$$100 = \frac{C}{1+{}_0r_2} + \frac{C+F}{(1+{}_0r_2)^2} = \frac{C}{1+{}_0z_1} + \frac{C+F}{(1+{}_0z_2)^2}$$

$$(1+{}_0z_2)^2 = (1+{}_0z_1)(1+{}_1z_1)$$

Further the zero coupon rates are increased by spread required for particular rating class, where the borrower can end up in the one year horizon and used for computation of the value of the loan in one year, which are finally weighted by the probability of migration to this rating during the year, which can be found in credit rating migration matrix. The expected (mean) value in one year and a volatility of any loan can be computed as:

$$E = p_{AAA}P_{AAA} + p_{AA}P_{AA} + \dots + p_{CCC}P_{CCC} + p_D P_D$$

$$\sigma^2 = p_{AAA}(P_{AAA}-E)^2 + p_{AA}(P_{AA}-E)^2 + \dots + p_{CCC}(P_{CCC}-E)^2 + p_D(P_D-E)^2$$

[23] For credit risk the annual rates are important.

where p denotes the probability that an obligor will shift to a given rating class and P is the value of loan as discounted cash flows with relevant discount factor. The value for default class (P_D) is given by recovery rate (or RR=1-LGD) as a percentage of EAD.

For more loans the joint probabilities of migration must be calculated according to the individual migration matrices and correlation among obligors, which can be estimated econometrically according to observed sensitivities to returns in linked industries. In CreditMetrics Technical document (1997), there is offered the following example of correlation of A-rated bond of a universal bank sensitive to banking and insurance industry and BB-rated bond of a chemical factory sensitive to chemical industry climate.

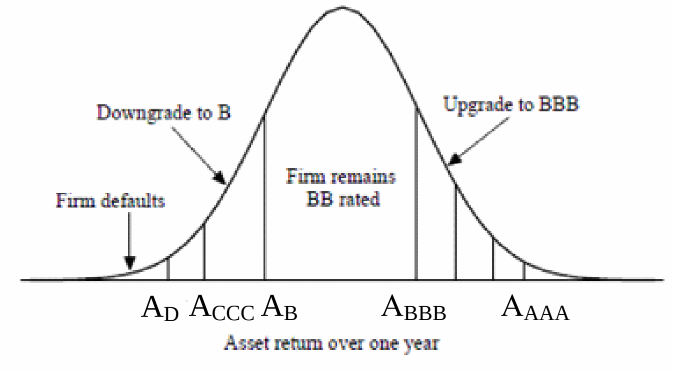
$$R_A = 0{,}15 R_{bank} + 0{,}74 R_{INS}$$
$$R_{BB} = 0{,}9 R_{chem}$$
$$\rho_{chem,ins} = 0{,}16$$
$$\rho_{cham,bank} = 0{,}08$$
$$\rho(A, BB) = \rho(0{,}15 R_{bank} + 0{,}74 R_{INS}, 0{,}9 R_{chem}) = 0{,}15 * 0{,}9 * \rho_{cham,bank} + 0{,}74 * 0{,}9 * \rho_{chem,ins} = 0{,}1174$$

The migration from one category to another is estimated based on the rate of return on the assets. This process should be of Markov type, which means that development of an obligor's rating in the next year does not depend on its past development[24]. For each rating category the threshold values of rate of returns on assets are set according to the probability of such shift in the rating as on the Figure 3.5[25].

[24] The test for Markovian process for transition matrices from S&P database can be found in Bangia et.al (2002). The test is based on the analysis of eigenvalues and eigenvectors and analysis of path dependence, where the eigenvalues of transition matrices for increasing time horizons must decay exponentially and the set of eigenvectors must be the same for matrices for different horizons.

[25] Usually the assumption that the asset value change ($\Delta W / W$) is normally distributed is accepted and the rate of return on assets is standardized to have zero mean and volatility one. The probabilities of rating changes assigned to the bond are equal to the value of cumulative standard normal distribution for the thresholds (α_i) . Threshold levels can be computed from inverse standard normal distribution.

Figure 3.5: Rating change thresholds for BB rate



Source: CreditMetrics Technical Document (1997)

$$\Delta W / W = N(\mu, \sigma)$$

$$Z = \frac{\Delta W / W - \mu}{\sigma}$$

$$p_D = P(\Delta W / W < A_D) = P(\mu + Z\sigma < A_D) = P(Z < \frac{A_D - \mu}{\sigma}) = P(Z < \alpha_D) = \Phi(\alpha_D)$$

$$p_{CCC} = P(A_D < \Delta W / W < A_{CCC}) = P(\frac{A_D - \mu}{\sigma} < Z < \frac{A_{CCC} - \mu}{\sigma}) = \Phi(\alpha_{CCC}) - \Phi(\alpha_D)$$

The joint probabilities for correlated bonds that the first bond will be in the rating category x+1 and second in y+1 can be found as integral of bivariate normal distribution.

$$P(A_X < \Delta W_1 / W_1 < A_{X+1}, A_Y < \Delta W_2 / W_2 < A_{Y+1}) = \Phi(\alpha_X < x < \alpha_{X+1}, \alpha_Y < y < \alpha_{Y+1}) =$$

$$= \int_{\alpha_X}^{\alpha_{X+1}} \int_{\alpha_Y}^{\alpha_{Y+1}} (x - \mu_1)(y - \mu_2) f(x, y) dx dy$$

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp[-\frac{1}{2(1-\rho^2)}(\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2^2})]$$

As soon as we know the joint migration probabilities among each two obligors as well as individual migration probabilities the distribution of the portfolio value, mean value and volatility can be estimated. Finally the CVaR can be computed.

In the CreditMetrics model for future value of portfolio of loans, at first, based on the assumption of normal distribution of returns on assets, the threshold values according to given migration matrices are set and the correlations (ρ) are estimated for all couples of obligors. Further it is possible to use a Monte Carlo simulation for estimation of distribution of the future value of the portfolio; within the simulation a rating category is assigned to each obligor and the future value of the loan is computed based on discounted cash flows, forward zero rates and relevant spreads.

3.3.3 CreditRisk+

The CreditRisk+ is one of the models by Credit Suisse Financial Products based on the application of an insurance logic to the credit risk measurement. As a default mode model just two states of the obligor are considered, default, which is taken as continuous variable with some probability distribution[26] and non-default. The goal of CreditRisk+ is to get the distribution of future losses related to a given portfolio, consisting of models for frequency of defaults (default probabilities) and severity of losses, in comparison to Credit Metrics, where changes in the portfolio's value and VaR were the main results. In the CreditRisk+ changes in the credit rating are considered to be part of a market risk; just a narrow definition for credit risk is used.

It is assumed that each obligor has a small probability of default and that each default is independent of the defaults on other loans[27]. The portfolio of loans is at first divided according to the severity of losses in case of default into bands to make the measure of this random variable easier. The narrower the bands the more precise will be the model for future losses.

According to the Saunders, Allen (2002) the main advantage of this model is its simplicity and low data demand. The severity of losses together with the frequency of defaults relevant for given band are the only information needed for finding the distribution of losses connected with loans included in this band. Once we know loss distribution of all bands it can be summed to get the loss distribution of the whole portfolio.

As discussed in the book mentioned above, according to the CSFP (1997) the loss distribution in CreditRisk+ is symmetric and close to normal distribution, however the both random variables in this model (the default frequency and loss severity) typically have fatter tails as they are not sTable with the economic cycle. The observed mean default frequency is higher than the variance, which are in this model assumed to be equal, because of the Poisson distribution. So the final result for the basic CreditRisk+ model is just approximation and for better results one more random variable of changes in the mean value of default frequency can be added.

---

[26] in comparison with the CreditMetrics, where default was just one of the rating classes and final rating was a discrete variable.

[27] Under these assumption the probabilities of default follows Poisson distribution.

According to Saunders, Allen (2002) this basic CreditRisk+ model tends to underestimate economic capital requirement as the mean value of defaults changes with the economic cycle of each country, the distribution will have fatter tails and the required quantil of unexpected loss will be further from the mean value (i.e. from expected loss). Furthermore default in the bands does not seem to be independent as they will probably be dependent on some systematic risk factors[28].

3.3.4 Comparison of economic capital models

Table 3.6: Comparison of economic capital models

	KMV	**CreditMetrics**	**CreditRisk+**
model type	DM/structural	MTM/structural	DM/reduced
logic of the model	loans priced as options		model is similar to insurance ones
What is the source of credit risk?	changes in the asset value during one year, default occurs, when asset value fall below the default point	changes in the value of loan due to rating migration connected with changes in the asset value	default rates and severity of credit events
Which data are required?	data for BSM formula observable for listed companies, correlation of assets, exposures	transition matrix, credit spreads for each rating class, forward zero yield curves, LGD for each loan, correlation among loans, exposures	severity of losses in case of default (exposure and LGD), default rates, macro factors for more advanced version, where default frequency changes with the cycle
What influences the probability of credit event?	distance to default	probability of changes in the rating	default rates (considered to be a random variable)
main assumptions	asset value follows log normal distribution, default point is estimated as some portion of liabilities of the company	transition matrix is sTable in time, correlation between each two loans can be estimated econometrically, rate of return on assets follow normal distribution	probabilities of default follow Poisson distribution, default events are rare and mutually independent
goal, main result	EDF	distribution of future value, VaR etc.	distribution of future losses

Source: by author

The KMV and CreditRisk+ are default mode models, which consider the narrow definition of credit risk and economic capital is measured to cover losses from defaults of obligors. The CreditMetrics on the other hand sees also the credit migration of the obligors

[28] Models that link default frequency to the economic cycle (i.e. systematic risk factors) will be discussed in the chapter 5.

as part of credit risk. The KMV and CreditMetrics are structural type models, because they estimate changes in the value of assets and see probability of default as a result of financial characteristics of the firm or macroeconomic conditions. The CreditRisk+ in the basic version do not consider reasons for default risk, in the more advanced version the default risk can be linked to a development of macroeconomic variables and independence of defaults can be replaced by default correlations as defaults in each band will depend on economic cycle.

The KMV is based on the option pricing model, which helps to price the loans and model the development of its price under the restrictive assumption of log normal distribution of asset value. The CreditRisk+ use the logic that is typical for insurance models considering large amount of loans with low probability of default, where default are independent of each other. Also here a crucial assumption of Poisson distribution of probabilities of default is made, which according to the empirical findings leads to underestimation of economic capital.

However the CreditMetrics model leads directly to the distribution of future value of portfolio, which can be easily used for estimation of any CVaR quantiles, its main disadvantages are the data requirement as well as the assumption of normal distribution of asset return needed to divide obligors into the rating classes. This complexity streams from the type of this model, which connects the MTM approach with structural view on rating migration.

For each model there exist more advanced versions that ease some of the assumptions and enable more sensitive modeling of credit risk capital. For example in the case of CreditMetrics the rating migration can be linked to the development of macroeconomic variables instead of assumption of its stability in time. According to the empirical findings as described in details in the chapter 5 the downgrading in more probable especially for a lower rating quality loans during the contraction, while upgrades are more probable during the economic expansion. Because these changes in assumptions are usually used for stress testing the estimated capital needs, the broad usage of these models in the literature will be summed in the mentioned chapter.

4. Methodology of credit risk stress testing

4.1 Stages of Stress testing procedure

The Basel II regulation CEBS (2006), impose special requirements on the accomplishment of stress testing program in a financial institution. At first, according to the mentioned document, the institution should start with an assessment of its portfolio and identification of risks that it is most exposed to. Further it is asked to determine those risk factors that are material to include them into stress testing. The identification of risks should be based on the analysis of the portfolio characteristics as well as on the external environment in which the institution is operating. Assessment of all material risk drivers/factors that can affect the institutions solvency, profitability or compliance with the regulatory requirements is key for appropriateness of the stress testing program[29]. The whole responsibility for stress testing program lies on a management body, which should approve scenarios as well as should be informed about the results of the stress tests and decide about usage of corrective measures, while individual stages of stress test can be and usually are delegated to institution's departments.

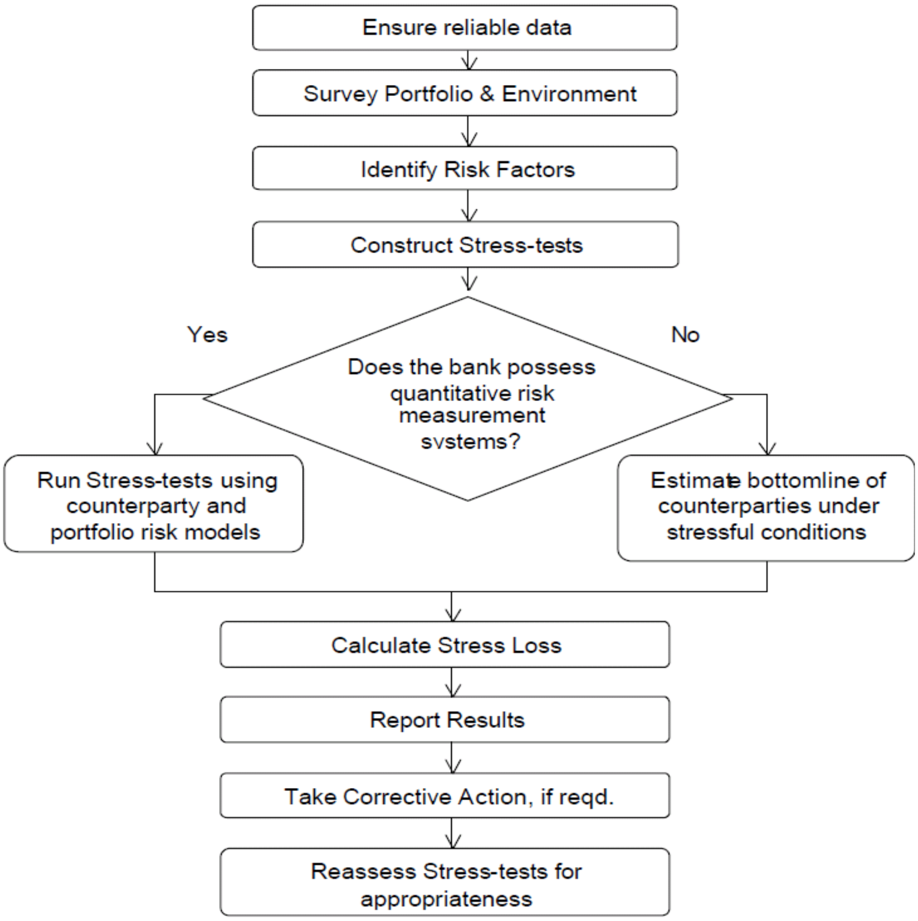
The stress testing process can be divided into two main parts, the identification of key factors finished with definition of a scenario as combination of changes in risk factors and assessment of the results and impact on the institution. According to Čihák (2007), once the portfolio specific risk drivers are identified the institution should define a scenario consisting of a shift in these factors that will fulfill the Basel II requirements and have economic sense considering correlations among factors as discussed further. Further the impact of such a scenario on the portfolio value, institution balance sheet, income statement and capital adequacy should be judged. Finally the management receives report about the whole procedure and decides if such a future development would be too adverse for the institution and some type of corrective measures should be applied or if the risk is still bearable.

Similar description of stages of the stress testing process as summarized in the Consultative Paper on Credit Stress Testing (2002) follows on the Figure 4.1, where in the

[29] The institution can decide to run hypothetical as well as historical scenarios, if they are relevant; can implement scenario and/or sensitivity tests as they are defined in the last part of this chapter. The stress tests should be repeated as frequently as necessary according to the nature of the risk drivers and their volatility. Further requirements on frequency of stress tests, definition of scenarios etc. are listed in the chapter 2 and 6.

bottom part a reassessment of the adequacy of the stress test scenario is added to adjust the actual scenarios to meet changes in the external environment and portfolio structure for the next round of stress test as it is required at least once a year under the Basel II.

Figure 4.1: Flowchart for building a stress-testing program

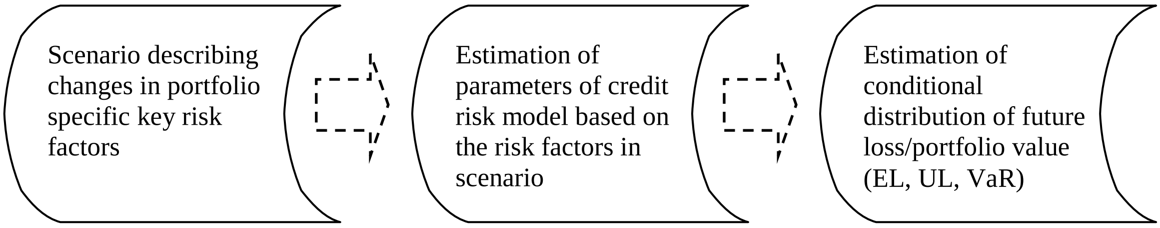


Source: Consultative Paper on Credit Stress Testing (2002)

3.2 General framework of credit risk stress testing under pillar I and II

Concentrating on the most technical part of the stress testing program, on the methods how an impact of given scenario on the institution’s portfolio value can be estimated, we can find a general feature among stress tests conducted under the first and second pillar, because all stress tests, by definition, search for a distribution of loss under the scenario. Such a conditional distribution of loss depends on the change in credit risk model parameters caused by adverse shift in risk factors considered in the scenario. The basic mechanism of stress can be described by following Figure 4.2.

Figure 4.2: Estimation of conditional distribution of loss for a given scenario

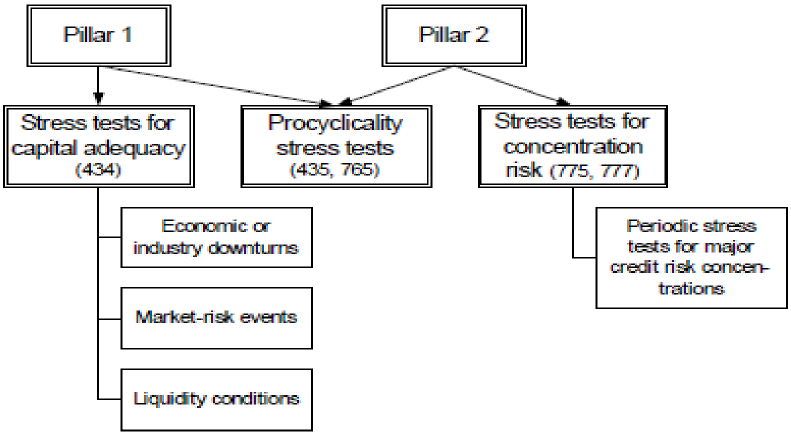


Source: based on consultation with the expert from KB and Bonti et al. (2005)

In spite of this similarity, the goal of stress tests under the pillar one and two significantly differ. According to Rodriguez, Trucharte (2007), under the first pillar the parameters and conditions that influence the minimum capital requirement are stressed, while under the second pillar it is tested whether this requirement increased by the capital buffer held by the institution is enough to absorb a possible adverse shock.

More concrete is the analysis of stress tests required under the Basel II by Mager, Schmieder (2008), who defines the first pillar credit risk stress tests as those that denote adjustments to IRB parameters for stressed conditions, an assessment of their robustness and a periodical evaluation of the impact of an economic recession on the regulatory capital requirements called cyclicality stress tests. The global impact of adverse events or changes in market conditions on the bank's risk profile and capital adequacy is assessed under the pillar two. Similar distinction is showed on the Figure 4.3 bellow.

Figure 4.3: Stress tests conducted under the first and second pillar



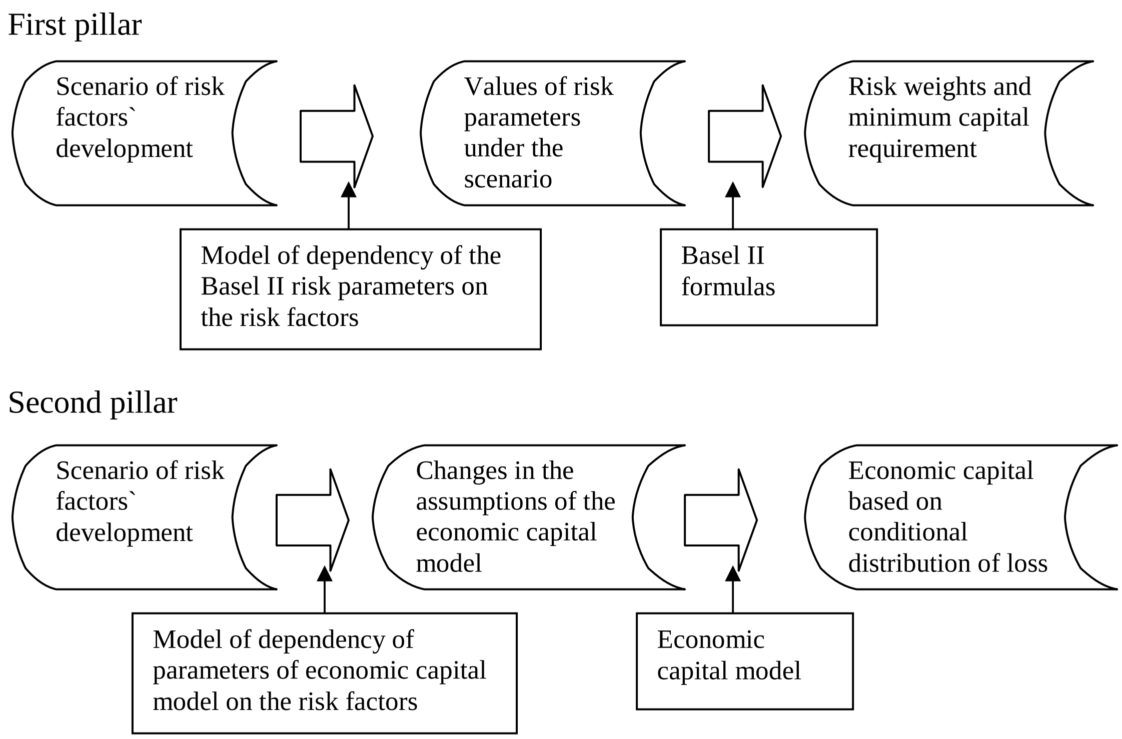
Source: Bonti et al. (2005) based on the BIS (2004)

In a similar manner we can differentiate between tests under the first and second pillar according to the credit risk model used. As Mager, Schmieder (2008) further adds, under the former the stress tests usually search for impact of the scenario on the Basel II parameters such as PD, LGD, EAD, M, or asset correlations and regulatory formulas are used for final step of computation of expected and unexpected conditional loss. Under the latter the economic capital models[30] are usually used, while some of their assumptions are changed to fit the scenario conditions.

For both pillars the stress testing can be seen, as the arrows on the Figure 4.2 denotes, as a two-stage problem. At first one must identify a link between the values of risk factors in the scenario and variables in the credit risk model, later the conditional loss must be computed based on the model itself.

For example, as in Rodriguez, Trucharte (2007) under the IRB approach the effect of a change in the variables like unemployment rate or interest rate on mortgages, which are factors identified as influencing probability of default of mortgage loans to households can be through an econometric model linked to PDs. Further the risk weights and capital requirement can be computed based on the stressed parameters through the IRB formulas[31]. According to these arguments, the Figure 4.4 shows the main logic of stress testing methods under both pillars.

Figure 4.4: Mechanism of credit risk stress testing under first and second pillar



Source: by author based on generalization of methods used in literature mentioned in chapter 5 and concretization of general model on the Figure 4.2.

[30] The most famous economic capital models are KMV, CreditMetrics, CreditRisk+ and Multifactor risk model, which were described in the chapter 3 and chapter 5.

[31] Chapter 5 includes more examples of stress testing methods used in current literature.

4.3 Possible types of scenarios

4.3.1 Sensitivity analysis vs. Scenario stress test

On a basic level we can distinguish two types of stress tests according to the number of factors included in the scenario. Firstly, sensitivity analysis, sometimes called univariate stress test, as defined in CEBS (2006) assesses the impact of large move in one risk factor on the financial conditions of the institution, where magnitude of its change is set based on some expert judgment. An example of sensitivity analysis can be a test for decline in real estate prices or increase in effective exchange rate. These tests are less demanding to run, can be repeated regularly and can serve as a first approximation of loss caused by such a movement in this risk factor or for identification of risk factors relevant for given portfolio.

On the other hand this approach does not catch the effect on the loss fully. In the real world risk factors are interconnected by correlations and a shock in one factor will cause a shift in a large number of other risk factors that may be also relevant for portfolio loss. Such a secondary effect is omitted by this approach so the sensitivity analysis can underestimate real loss. This can make according to Deutsche Bundesbank (2004) a false impression that the shock in particular factor with a given severity is unproblematic for a financial institution.

Secondly, scenario stress tests or multivariate stress tests, as mentioned in CEBS (2006), are more realistic, because they look for an impact of a simultaneous move in a number of risk drivers while the stress event causing such a movement is usually well-defined. These tests are more complicated to run and the setting of movements in risk factors in a realistic way can be problematic; the correlations among factors are in some of these tests respected[32], in others they are completely ignored.

4.3.2 Historical and hypothetical scenarios

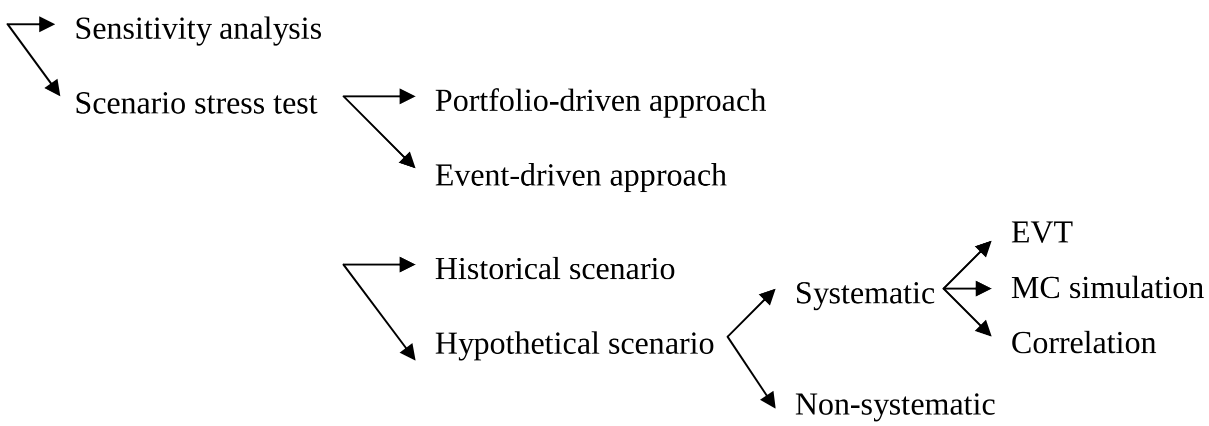
As shown on the Figure 4.5 scenario tests can be further divided into those based on historical and hypothetical scenarios according to the way how the decision on the magnitude of the shock in the core risk drivers is made. In historical scenarios the risk

[32] For historical scenarios the correctness of correlations among factors is often met by setting all relevant factors according to their values during the particular past event. In hypothetical scenarios historical correlation can be used as well as correlation observed during stress periods. Sometimes correlations among factors are completely ignored, as described further.

factors are set according to some past crisis[33]. An example of historical scenario test considering past crisis in the Czech Republic could be a currency crisis in 1997. Such scenario would include increase in nominal interest rates, depreciation of home currency and downturn in many industries.

Although this method takes as scenario, by definition, a plausible event, it may fail to capture the nature of market changes and new products that have appeared since that crisis, so it may not be relevant for stress testing the current portfolio for likely future stresses. Moreover, as commonly mentioned, the same crisis is highly improbable to reoccur again and most probably some different crisis than past ones will come. On the other hand, this method usually respects the correlations among the risk factors, because their values are set equal to some real past crisis ones. Further an observation of correlations during historical crisis can provide us useful information about the joint behavior of factors in stress events.

Figure 4.5: Types of stress test scenarios



Source: Consultative Paper on Credit Stress testing (2002)

Hypothetical scenarios rely in risk factors´ setting more than historical scenarios on expert judgments and usually also on some macroeconomic models[34]. In cases when

[33] In general for all risks, in BIS_CGFS (2005) we can see that stress tests often applied by large financial institutions are based on historical events like Asian crisis in 1997, the terrorist attack in 2001 (connected with interest rate and credit risk), the oil price change during the Iraque war in 2003 or the oil price shock in 1973-74 (accompanied with some industries downturn and loss in banks loan books).

[34] A hypothetical scenario can be for example an economic downturn expressed as a slowdown of the GDP growth accompanied by increase in unemployment rate and decline in real estate prices. Other hypothetical scenarios that were prepared by the CNB are used in chapter 7.

macroeconomic models are not used properly, the correlations among risk factors may not fit the commonly observed comovements. Then it can happen that scenario defining some development in the market variables will loose economic sense and it will be problematic to explain such unusual behavior of variables. On the other hand hypothetical scenarios can help the institution to see new combinations of risk factors that may fit the current portfolio shortcomings better than any historical scenario based on a past event.

Further according to BIS_CGFS (2005) the scenario tests can be divided into those arising from portfolio-driven approach and event-driven approach. In the former, a creation of a scenario starts with an identification of vulnerabilities in the portfolio, continues with searching for factors´ changes that can cause losses according to these vulnerabilities and ends with constructing scenario consisting of these factors. In the latter approach, the mechanism goes exactly the other way round; it starts with formulation of a plausible event, continues with its decomposition to factors that change during this event and ends with analyzing the effect of these changes on given portfolio.

4.3.2.1 Systematic and non-systematic hypothetical scenarios

Hypothetical scenarios can be differentiated according to Consultative Paper on Credit Stress testing (2002) based on the fact, if correlations among the risk factors are taken into account or not when the scenario is being defined. In this manner we can divide hypothetical scenarios into systematic and non-systematic. The latter do not consider correlations among risk factors, but set the changes in risk factors according to some different rule, as for example, the most adverse value for each factor that appeared during some period of time is taken and they are combined into one scenario. Further examples of this type are scenarios that contain a change in most important (core) risk factors with magnitude decided by experts, while other factors (peripheral) are kept constant or that subjective judgment on factor changes is applied, ignoring correlations among factors completely.

On the other hand systematically built scenarios try to incorporate all relevant risk factors and use appropriate correlations among them as we can see for example in the case of extreme value theory, Monte Carlo simulation and other approaches that respect correlations. In favor of these methods speaks the fact that defining a realistic scenario is a core part of whole stress testing procedure. If correlations among factors are omitted and just a set of some arbitrary decided changes in factors is used for building a scenario, the link among factors is interrupted. The scenario will probably not have much economic sense and results of portfolio loss analysis will not match with possible real event effect. There is a great discussion considering what kind of correlations should be used, (if historical or some other), because as shown in Kim, Finger (2000) correlations during stress events tend to differ from normal historical ones. But one can surely say that it is always better to use at least some[35] type of correlations to set factor changes in realistic manner, because otherwise the scenario may contain such unusual comovements of factors that it will loose economic sense and become improbable to occur.

Historical and special turbulent time correlations

In more advanced methods for a scenario construction correlations among factors are respected not just for decision about the change in core assets but also for setting the peripheral factors. For this purpose historical correlations can be used as well as the peripheral factors can be set according to their values during the past event when core assets moved in the same way as in this scenario[36].

Naturally for getting as precise result for loss as possible we should set the peripheral factors so that they will follow the natural co-movements with core assets; only in this way secondary effect on loss will be analyzed correctly. The question is what kind of correlations should be used, historical or some special ones conditional on stress events, because historical correlations according to Consultative Paper on Credit Stress testing (2002) tend to break down in turbulent times, and if such a conditional correlations are used, will the results change significantly.

To address this problem, we can see Kim, Finger (2000), where stress tests results using conditional correlations on turbulent times can be compared to those with historical unconditional correlations. At first bivariate normal distribution parameters were estimated based on all data in given period (unconditional parameters), then the data were divided into two groups; for quiet and hectic periods; and conditional parameters of two multivariate normal distributions were estimated (for core x and peripheral assets y)[37].

---

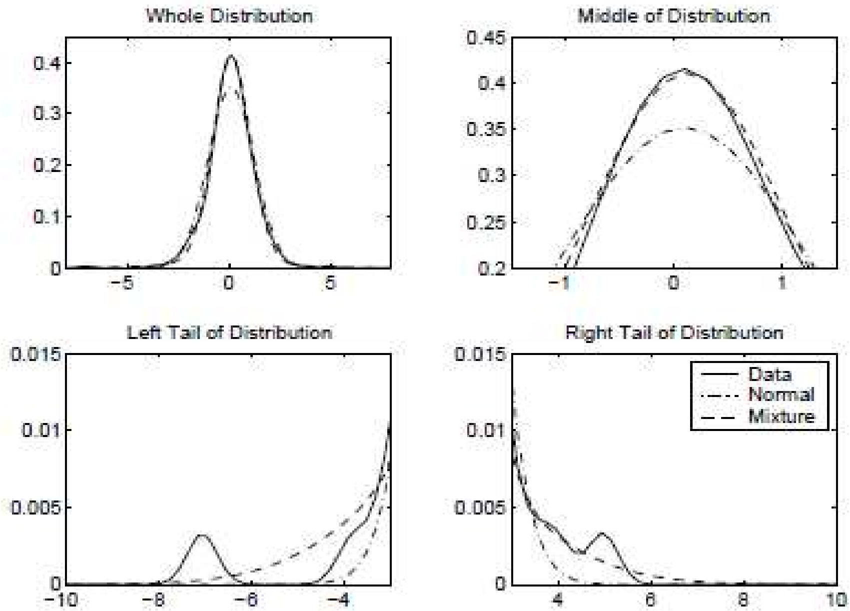
[35] Deeper discussion on the possible types of correlations for building stress scenario will follow further.

[36] For example, if we want to create a scenario containing 3% decrease in GDP accompanied by 4% increase in unemployment, we can search for the same situation in the past and set peripheral factors as they were that year.

[37] In Kim, Finger (2000) the risk factors are called assets, the core assets are market factors for which the magnitude and duration of stress is set in the scenario and peripheral assets are other factors, that are set according to correlations with stressed factors.

$$(x,y) \sim \begin{cases} MVN\left[\begin{pmatrix}\mu_{x1}\\ \mu_{y1}\end{pmatrix};\begin{pmatrix}\sigma_{x1}^2 & \sigma_{x1}\rho_1\sigma_{y1}\\ \sigma_{x1}\rho_1\sigma_{y1} & \sigma_{y1}^2\end{pmatrix}\right] \ldots\ldots\ldots quiet\_days \\ MVN\left[\begin{pmatrix}\mu_{x2}\\ \mu_{y2}\end{pmatrix};\begin{pmatrix}\sigma_{x2}^2 & \sigma_{x2}\rho_2\sigma_{y2}\\ \sigma_{x2}\rho_2\sigma_{y2} & \sigma_{y2}^2\end{pmatrix}\right] \ldots\ldots\ldots hectic\_days \end{cases}$$

Figure 4.6: Goodness of fit – one normal vs. mixture of two normal distributions



Source: Kim, Finger (2000)

In the application on a return on S&P index used in the mentioned paper the combination of these two multivariate normal distributions fit real data distribution better than the single bivariate distribution for all data (Figure 4.6). Further the hypothesis that correlations for calm and turbulent times are the same is rejected; the correlations are proofed to differ for these two subsets. Finally a stress is applied on core assets in the magnitude of -3σ, while peripheral assets are set in the following four ways;

- zeroed-out[38],
- according to their values when core assets changed in the same way in the past (historical approach),
- according to unconditional correlations (predictive stress test),
- according to conditional correlations based on hectic periods (called broken arrow approach).

[38] Set so that returns on peripheral assets are equal to zero.

The drawbacks for each method and more details on the setting of peripheral assets are described in Table 4.7.

Table 4.7: Benefits and drawbacks of different methods of setting peripheral assets

stress test	return on peripheral assets	benefit	drawback
zeroed-out	zero return	implementation is quite easy	ignoring co-movements is unrealistic
predictive	expected return based on correlation	idiosyncratic errors are averaged out	correlation is calculated in the normal situation
historical	actual return of the specific historical data	the stress situation is easily incorporated	idiosyncratic errors of the historical event cannot be removed

Source: Kim, Finger (2000)

Kim, Finger (2000) concludes that version with zeroed-out peripheral assets underestimate portfolio loss, while broken arrow approach, because of higher estimated conditional correlations for turbulent periods than unconditional correlation for all data in this example, gives higher estimate of loss for all quantiles than if we use unconditional correlations.

Here, the resulting loss depend heavily on the correlations used, while correlations among factors or assets significantly differ for calm and turbulent times and the combination of two multivariate distributions based on conditional correlations fits real data better than unconditional distributions. At least in this case we can conclude that for better prediction of loss distribution during turbulent time, conditional correlations should be used.

4.3.3 Worst-case scenarios

As already mentioned the stress tests should use scenarios describing rare but plausible situation, by which the portfolio value can be significantly hit. Both these conditions are highlighted by Breuer, Krenn (2001), who introduce a theoretical methodology, how to search for a scenario that would be plausible, cause significant losses to the portfolio and have economic sense.

The portfolio special worst-case scenario according to this paper can be defined based on an assumption on the distribution of risk factors` changes, on their correlations and on the level of scenario plausibility the institution wants to include into their stress testing program. As mentioned, the managers should always define which plausibility level is relevant for stress testing, in other words, up to how rare market events the institution wants to run stress tests, or decide case by case if a given scenario should be included into stress testing procedure, otherwise the results of a stress test using more improbable scenario than those accepted by management will not give credible result that could motivate them to decide about changes in the portfolio structure or other risk-lowering corrective measures.

A possible measure of plausibility of a scenario introduced here is a distance of factors´ values under the actual market conditions to those in a scenario situation (respecting given correlation structure of factors). Under the assumption of normal distribution of risk factor changes and given plausibility level p, we can find a multidimensional domain, where all such a plausible scenarios are located[39].

Let's assume that n risk factors $r = (r_1, r_2, \ldots, r_n)$ *have a significant impact on the portfolio. The difference of scenario from the current market situation can be expressed as:*

$$\Delta r = \frac{r - r_{CM}}{r_{CM}}$$

And correlation among risk factors is given by variance-covariance function[40]:

$$S = \begin{pmatrix} s_1^2 & s_{12} & \cdots & s_{1n} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ s_{n1} & \cdots & \cdots & s_n^2 \end{pmatrix}$$

Then under the assumption of normal distribution of risk factor changes, all scenarios of risk factors r with given plausibility level p form an ellipsoid[41]:

---

[39] For example if an institution decides that it wants to include in its stress testing 99 % of market situations, then it will search along the ellipse, outside which lie 1 % of less probable market situations, for the combination of factors with the worst impact on institution.

[40] The correlation among factors as mentioned in Breuer, Krenn (2000) can be ether historical or stress correlation as introduced by Kim, Finger (2000).

[41] For definitions of domains for other distributions od risk factors like t-distribution with thicker tails than under normal distribution see Breuer and Krenn (2001).

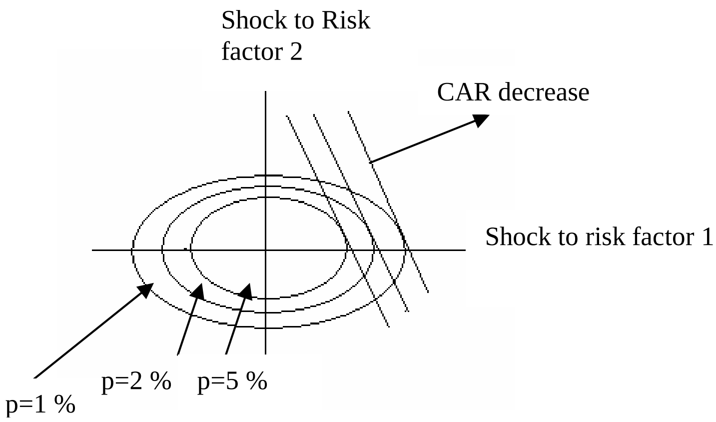
$$\Delta r_{stress}{}^{T} * S^{-1} * \Delta r_{stress} = k^2$$

$$F_{\chi_n^2}(k^2) = \frac{1}{2^{n/2}\Gamma(n/2)} \int_0^{k^2} x^{\frac{n}{2}-1} \exp(-x/2)dx = p$$

Where admissibility domain can be defined as all scenarios inside and on this ellipsoid and k denotes size of the ellipsoid, because its axes are proportional to k.

For a special case of only two risk factors as in Čihák (2007) we can depict the scenarios with the same plausibility as an ellipse, which shape depends on the correlation between these two factors (see Figure 4.8). The larger the ellipse the less plausible the scenario is, intuitively because the larger change in factors form current market situation is needed to reach this scenario. In the Figure 4.8 we can further find lines that connect market situations with the same impact on the institution (for example on its capital adequacy ratio (CAR), profitability or other important measures), these lines do not need to be linear[42]. The further the line from the origin the more adverse is the effect on the institution.

Figure 4.8: Searching for extremes under worst-case and threshold approach



Source: Čihák (2007)

According to this Figure we can define two possible approaches how to deal with the optimization problem for searching for a portfolio specific worst case scenario[43]. The

[42] depending on the combinations of the factors´ changes that will cause the same adverse impact on the institution.

[43] As highlighted by Breuer, Krenn (2001), even though we work with worst case scenario, we do not search for a scenario with worst impact on the institution, because results of such a stress test would not be useful

worst-case approach searches for a scenario that has the most adverse impact on the portfolio value or institution Figures for a given level of plausibility (shift along given ellipse). On the other hand threshold approach selects the given adverse impact and looks for the most plausible scenario (for an ellipse with the highest plausibility that touches the line with the given adverse impact).

More concrete facts on general methods for systemic search for worst case scenarios can be found in Breuer, Krenn (2000). The easiest method, so called factor push, use n-dimensional cuboids, which are inscribed into ellipsoids described above, but enable to find worst case scenarios just in corners of these cuboids. Further more general maximization algorithms like multidimensional simplex method and simultaneous annealing are described. However these methods can find extremes with much higher probability than factor push method, their algorithm is much more complicated.

---

for management of the institution, and such a scenario often can not be found as some portfolios do not have limited downside loss, but we always work with some plausibility level.

5. Stress testing techniques – examples from the literature

Once the scenario is defined, it must be translated into the credit risk model as denoted by the first arrow on the Figure 4.2 and 4.4. This is usually done by an additional model that conditions the development or setting of some of the variables from credit risk model on the scenario, to be able to incorporate changes in the risk factors into the credit model.

The most intuitive example of scenario describing changes in macroeconomic risk factors or shocks to some special parts of portfolio can be implemented into the model through a regression based model that will estimate sensitivities of the credit risk model input variables on the cycle. Further some assumptions of the credit risk model itself can be eased to make it more realistic and enable it to react to the scenario. In that case an assumption of some input variable being constant or developing in with some trend is changed to make it dependent on the scenario risk factors, as for example in a case of linking the transition matrices to the cycle.

In general we can divide scenarios according to the Consultative Paper on Credit Stress Testing (2002) to those describing change in:

- counterparty's behavior like deterioration in overall PDs, LGDs, spreads
- environmental changes like changes in economic or regulatory conditions, specific industry or region downturn, political changes
- analytics and model assumptions such as change in correlations, transition matrices and other assumptions of the credit risk models

The methods that are commonly used for implementation of these stress scenarios into the credit risk models are explained further.

5.1 Approaches to stressing the PDs and asset correlations

The PDs are inputs into the Basel II minimum capital requirement formulas as well as to credit risk models like CreditMetrics or CreditRisk+. In stress testing scenarios they are mostly made dependent on a macroeconomic development expected and on changes in obligor specific characteristics[44]. This is usually done as in Rösch, Scheule (2004) by

[44] Sometimes a additional model that links the PDs or also the asset correlations to the scenario conditions is not used as found in some of the Czech banks, in which the stress testing program is not so developed yet. In

application of a one factor model or as in Hamerle et al. (2004) by using a multifactor version, which enables better differentiation of obligors and to assign them different values of PDs. In both these papers the historical time series of PDs as a dependent variable is approximated by some similar variable like a chargeoff rate or insolvency rate respectively, because there are usually not enough data available on the original series.

The factor models are typical threshold models (see also Merton-type credit risk models in the chapter 3) that see default as a situation, when value of assets decrease under some threshold (default point), which can depend on the business cycle in more advanced versions of this model[45]. According to Hamerle et al. (2004) the multifactor model with a sTable default point can be described as follows:

If a logarithmic return on an obligor's assets r_{it} *fall below the threshold* c_{it} *a default occurs (the indicator of default* $y_{it} = 1$*; 0 otherwise):*

$$r_{it} \leq c_{it} \Leftrightarrow y_{it} = 1$$

where the logarithmic return on assets is dependent on a time lagged obligor specific risk drivers $X_{i,t-1}$[46]*, systematic risk factor* z_{t-1} *and other risk factor explaining risk factors* $f_t \sim N(0,1)$ *not captured by the model*[47]*. It is assumed that obligors in one risk segment (such as in the same industry) have the same risk factors and the same exposures to them.*

$$r_{it} = \beta_0 + \beta' X_{i,t-1} + \gamma' z_{t-1} + bf_t + \omega u_{it}$$

Then conditional probabilities of default under the assumption that the obligor did not default before and given the realization of f_t *can be expressed as:*

$$\lambda\,(X_{i,t-1}, z_{t-1}, f_t) = P(y_{it} = 1 \mid X_{i,t-1}, z_{t-1}, f_t) = P(r_{it} \leq c_{it} \mid X_{i,t-1}, z_{t-1}, f_t) =$$
$$P\left(\frac{c_{it} - \beta_0 - \beta' X_{i,t-1} - \gamma' z_{t-1} - bf_t}{\omega} \,\Big|\, X_{i,t-1}, z_{t-1}, f_t \right) = F(\tilde{\beta}_0 + \tilde{\beta}' X_{it-1} + \tilde{\gamma}' z_{t-1} + \tilde{b} f_t)$$

these cases the shifts in PDs are set arbitrarily or the changes in PDs prepared within the CNB stress testing program for macroeconomic scenario predictions for the next year are used in some way.

[45] In this case the threshold c_{it} would depend on macroeconomic variables and vary through the cycle as in Rösch, Scheule (2004).

[46] Here a one year lags are used, but sometimes other lags like multiples of quarters of the years etc. can be used according to frequency of the data that are used for PD approximation as in Jakubík, Schmieder (2008).

[47] The one factor model uses just one specific (idiosyncratic) and one systematic (common) risk factor.

where $\tilde{\beta}_0 = \dfrac{c_{it} - \beta_0}{\omega}$ and $\tilde{\beta} = -\dfrac{\beta}{\omega}$ etc.

The conditional PDs for logit and probit model respectively are dependent on the risk factors included in the model:

$$\lambda\ (X_{i,t-1}, z_{t-1}, f_t) = \exp(\tilde{\beta}_0 + \tilde{\beta}' X_{it-1} + \tilde{\gamma}' z_{t-1} + \tilde{b} f_t) / (1 + \exp(\tilde{\beta}_0 + \tilde{\beta}' X_{it-1} + \tilde{\gamma}' z_{t-1} + \tilde{b} f_t))$$

$$\lambda\ (X_{i,t-1}, z_{t-1}, f_t) = \Phi(\tilde{\beta}_0 + \tilde{\beta}' X_{it-1} + \tilde{\gamma}' z_{t-1} + \tilde{b} f_t)$$

Φ denotes standard normal distribution function. Further a correlation for one risk industry and for two risk industries m and l can be derived as well as default correlation among obligors[48]. All parameters can be estimated according to the maximum likelihood method. In some papers the computation of default and asset correlation is not needed thanks to the Monte-Carlo simulation[49].

A classical application of this model in Rodriguez, Trucharte (2007) on Spanish mortgage portfolio of banks shows a logistic regression model for PD as dependent on macroeconomic variables and loan specific characteristics. In this paper after an estimation of the regression parameters a Monte Carlo simulation is used to find the loss distribution of the portfolio under normal conditions. Further a stress scenario is built, in this case based on historical volatilities or values of parameters during the worst year in the sample.

---

[48] According to Hamerle et al. (2004) the asset correlations for one risk segment and different risk segments (industries) can be derived as:

$$\rho(r_{it}, r_{jt}) = \ = \frac{b^2}{b^2 + \varpi^2 Var(u_{it})}$$

$$\rho(r_{it}^{(l)}, r_{jt}^{(m)}) = \frac{\tilde{b}^{(l)} \tilde{b}^{(m)} Cov(f_t^{(l)}, f_t^{(m)})}{\sqrt{\tilde{b}^{(l)2} + \pi^2/3}\sqrt{\tilde{b}^{(m)2} + \pi^2/3}}$$

And default correlation dependent on the same parameters can be expressed as:

$$\rho(y_{it}, y_{jt}) = \frac{\lambda(x_{it-1}, x_{jt-1}, z_{t-1}) - \lambda(x_{it-1}, z_{t-1})\lambda(x_{jt-1}, z_{t-1})}{\sqrt{\lambda(x_{it-1}, z_{t-1})(1 - \lambda(x_{it-1}, z_{t-1}))}\sqrt{\lambda(x_{jt-1}, z_{t-1})(1 - \lambda(x_{jt-1}, z_{t-1}))}},$$

$$\lambda(x_{it-1}, x_{jt-1}, z_{t-1}) = \int_{-\infty}^{\infty} F(\tilde{\beta}_0 + \tilde{\boldsymbol{\beta}}' x_{it-1} + \tilde{\boldsymbol{\gamma}}' z_{t-1} + \tilde{b} f_t) F(\tilde{\beta}_0 + \tilde{\boldsymbol{\beta}}' x_{jt-1} + \tilde{\boldsymbol{\gamma}}' z_{t-1} + \tilde{b} f_t) \varphi(f_t) df_t .$$

$$\varphi(f_t) = (1/\sqrt{2\pi}) \exp(-0.5 f_t^2)$$

[49] In some papers like Mager, Schmieder (2008) an alternative scenario is prepared, where the link between PDs and asset correlations is interrupted to see how much their dependency influence the stress test results.

Relevant values of explanatory variables are used in regression and by repeating the same method a conditional loss distribution is found[50].

Further the model can be used for more advanced analysis as in Rösch, Scheule (2007) and Rösch, Scheule (2004), where a point-in-time and through-the-cycle setting of PDs[51] and asset correlations is defined and the impact of different settings of the parameter on the result for loss distribution is highlighted. In general it is concluded that point-in-time setting leads to lower estimates of asset correlation and higher standard error for intercept from the regression, which means higher VaR and broader loss distribution.

Comparison of the four versions of the model, which take into account different number of factors, based on data on insolvency rates from Germany is done by Hamerle et al. (2004). A most simple model, where all obligors are seen as from one segment, their PDs are driven only by firm specific factors and asset correlation are the same for all obligors, is found to be significantly less accurate than a model including also the systematic risk drivers. When two similar models including also the sector specification of the obligors are compared, the model with systematic factor included is again significantly better. From comparison of models with and without segmentation, it can not be decided which one is better. The inclusion of macroeconomic variables in the model for PDs and asset correlations decreases the variance of the forecasts.

5.2 Simulation of recovery rates and their correlation with default probability

Not just the PDs, but also loss given default (LGD) should according to empirical evidence depend on systematic risk factor and vary during the cycle[52]. As explained in Frye (2000) the recession can have impact on both default frequency and recovery rates that should be, based on a basic intuition, negatively correlated. Economic recession will lead to an increase in PDs and financial distress of many obligors. Banks will try to sell the assets that were used as collaterals on the market. The supply will become higher than demand for these assets and their value of further decrease; the LGD will increase. In Frye

---

[50] In most of the papers the LGD is not stressed, but kept constant also under stress according on some of Basel II prescribed values. The model described here could be according to Rösch, Scheule (2007) adjusted to a model of cycle dependent LGD. More on recovery rates simulation will be covered in the section 5.2.

[51] Under point-in-time approach the parameters are set according to current economic situation and may vary through the cycle. The through-the-cycle approach sets the parameters equal to their average or worst value (Saunders, Allen (2003)) during the cycle.

[52] Because both of the variables depend on the systematic risk factor they should be correlated.

(2000) there is offered an evidence that the lower is the LGD of a loan the higher is its sensitivity to the systematic risk factor and the higher is the loss in economic downturns. The same holds for PDs of the obligors.

As Altman, Resti and Sironi (2001) highlight the most common credit pricing models do not take into account changes in the LGD during the cycle, but assume that it is a constant parameter or a stochastic variable that is always independent of PD as analyzed on the Table 5.1. They introduce an empirical analysis for bond portfolio to compare estimated credit risk under three possibilities of the parameter setting. Within the first two models the recovery rates are assumed to be deterministic and stochastic, without any correlation with PDs. In the third model the most realistic setting is used; the recovery rates are simulated as stochastic and partially correlated with defaults.

Table 5.1: Treatment of LGD within credit risk models

Model	Treatment of LGD	Relationship between recovery rates (RR) and PD
CreditMetrics	Stochastic variable (beta distr.)	RR independent from PD
CreditRisk+	Constant RR	RR independent from PD
KMV	Stochastic variable	RR independent from PD

Source: Altman, Resti and Sironi (2001)

In both papers mentioned above it is concluded that the assumption of constant or stochastic LGD without any correlation with PDs leads to a significant underestimation of expected and unexpected loss. Linking both of these variables with the systematic factors would lead to higher procyclical fluctuations in the capital needs estimated due to positive correlation between PD and LGD that would better fit the real development of banks` portfolio loss during the cycle.

5.3 Stress tests incorporating portfolio concentration risk

According to Mager, Schmieder (2008) the stress test scenarios can be divided into those with the effect on the whole portfolio such as severe recession and other macroeconomic changes, to country or industry specific downturns and name-by-name deteriorations in credit quality. The latter two specific tests are usually connected with the goal to analyze high exposures to certain clients, few highly correlated industries or regions in the context of concentrations in the portfolio. The concentration on one side can be very logical as specialization on particular industry or product can increase bank's profitability. On the other side if the portfolio is not enough diversified and the industries or countries with high concentration are hit by the crisis the bank can experience severe loss. Once the crisis spreads the bank can start to change the portfolio structure and shift of exposure from low to higher rated clients, which is usually observed during the stress periods, can also lead to higher concentrations in the portfolio.

Both single name and sector concentrations in the portfolio and its impact on loss distribution are analyzed in Rodriguez, Trucharte (2008). Here at first the classical factor model is used for estimation of individual Basel II parameters under the stress conditions. Further, during the simulation of loss distribution by Monte Carlo method, particular restrictions are imposed on the portfolio structure. The tests are run for single name concentration in the portfolio as well as for sector concentration[53], in both cases the rest of the portfolio exposure is either randomly or proportionally distributed among other obligors.

As highlighted in the paper it is important to test an increase in concentration in the portfolio together with an adverse macroeconomic scenario, to see the effect of the crisis on the clients that the bank could be most exposed to. Because portfolio with concentration in few industries that will appear to be negatively correlated in the stress time or are low sensitive to changes in the economic environment do not need to lead to a higher credit risk for a bank.

Alternative specification how concentration risk as well as economic stress scenario can be incorporated into the stress testing is offered in Bonti et al. (2005). The joint distribution of systematic risk factors is at first found for different industries or regions. Further a joint distribution of factors is simulated based on the changes in the particular industry according to the scenario, while correlations between industries are fully respected. Finally the portfolio loss is estimated by Monte Carlo simulation considering just those simulations satisfying the specified factor changes.

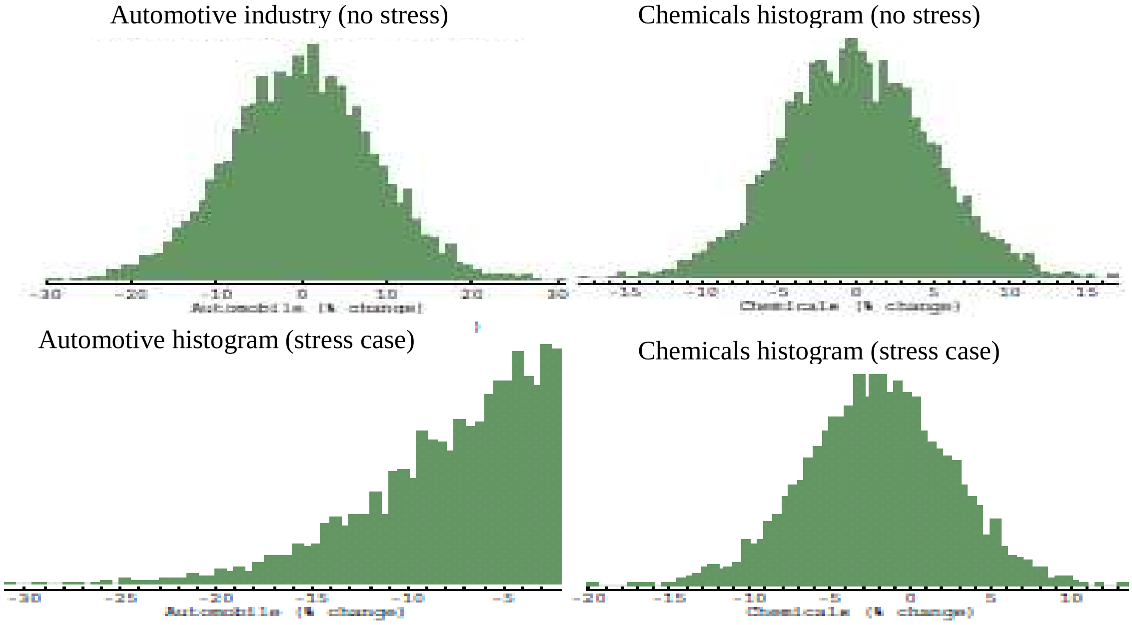
More concrete, Bonti et al. (2005) assumes a link between economic variables stressed and systematic risk drivers for each industry; in the paper the systematic risk drivers for each industry are taken as its stock indexes. For an example of scenario of a decline in automotive industry of at least two percent the rest of the distribution is cut off and the impact on other industries is simulated according to their correlations as depicted

[53] In the model for sector name concentration the name concentration must also be specified to see the effect of both of them on the portfolio loss.

on the Figure 5.2. Based on the joint distribution of factors the portfolio losses are estimated by Monte Carlo simulation.

Finally a more dynamic approach is introduced that takes into account changes in the portfolio structure during the stress situation and tries to incorporate the concentration risk that can arise due to default of certain part of the obligors into the loss estimation.

Figure 5.2: Histogram for automotive and chemical stock index for non-stress and stress cases



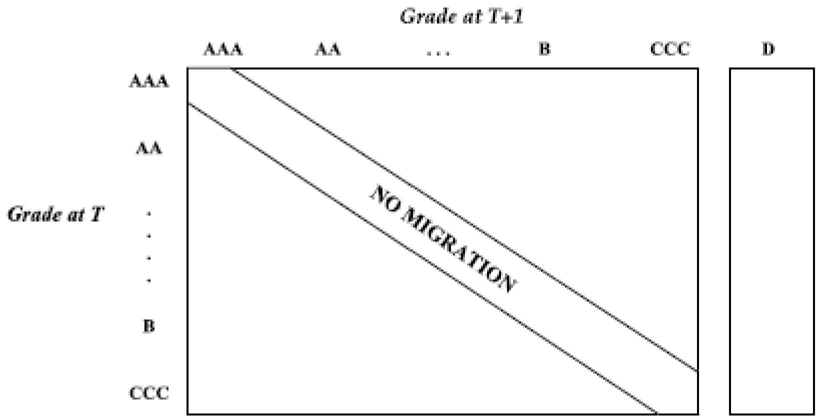
Source: Bonti et al. (2005)

5.4 *Migration matrices*

Migration matrices containing probabilities of transition from the initial to the final rating are one of the inputs into the mark-to-market types of credit risk models like CreditMetrics. They are an alternative to the concept of probability of default in default mode models that take into account just two possible stages of the world, default and non-default. On the other hand in mark-to-market models all possibilities of rating migration are considered as part of credit risk.

In general we can see the migration matrix as more general concept in comparison the probability of default. The data in migration matrix for a period T provide an information about the probability with which an obligor with given initial rating at the beginning of the period will end up with some final rating at time T. According to Bangia (2002) a one year migration matrix, which is mostly used for credit portfolio models, contains in rows data on probability of being at particular grade at time T+1 given the initial rating at time T. The sum of probabilities in each row of migration matrix must sum up to one to give a full description of rating distribution for all firms.

Figure 5.3: Structure of a migration matrix



Source: Bangia et al. (2002)

Further there can be found typical characteristics of the migration matrices such as that the lower is the initial grade the higher is the probability of shift to default grade, that "no migration" is the most probable final rating for all initial ratings and second probable is a shift to some rating next to the diagonal. The further the combination of grades from diagonal the lower is the probability of occurrence[54].

Technically, one can assign a rating to an obligor based on the return on its assets and threshold values for each rating grade as described in the chapter 3.3.2, the joint migration matrices can be derived from multivariate normal distribution of asset returns and correlations between each two assets. Servigny, Renault (2002) offers formulas for computing all probabilities of transition in individual migration matrix based on real data observed as well as joint probabilities and shows, based on the joint empirical migration matrix, how an implied asset correlation can be derived[55].

---

[54] For further typical characteristics of migration matrices see Bangia et al. (2002) and CreditMetrics model in chapter 3.

[55] Further an approximation of asset correlations by equity correlations is criticized. This concept is commonly used in CreditMetrics model, but according to Servigny, Renault (2002) they are not a sufficient proxy and neither a broader theory introduced by Zeng, Zhang (2002) that asset correlation should depend on equity correlations as well as on a volatility of risk-less interest rate give according to their paper better results.

Bangia et al. (2002) offers evidence that migration matrices do not stay the same during the extreme events[56]. First they estimate unconditional rating transition matrices and two conditional ones for economic expansion and contraction. Then it is shown that matrix for expansion significantly differs from the one for contraction, when the probability of default or downgrading is much higher. These two conditional matrices fit better the real transitions than the unconditional one. Finally these findings are used for stress testing within CreditMetrics model.

Under the expectation that next year will be a contraction and relevant transition matrix is used, Bangia et al. (2002) find a significantly different future distribution of portfolio value that has longer left tail and more probable values under mean. This means that using sTable unconditional migration matrices can lead to underestimation of losses under extreme events. As the relevant quantiles of the distribution of future value of the portfolio will be much further from the mean, the loss for given significance level will be underestimated under the unconditional transition matrix.

Similar analysis is run by Servigny, Renault (2002), who accepts the result of Bangia et al. (2002) and tries to separate the effect of change in univariate default probability and change in correlations with the cycle on the portfolio loss. It concludes that both of these components cause changes in conditional migration matrix and are relevant for portfolio loss distribution. The change in correlations due to recession influences the right tail of this distribution while the change in default probabilities affects the center of the distribution.

5.5 Regression-based transition probabilities

Except of conditioning the migration matrices on the recession and expansion, a classical assumption of stability of this matrix over time and types of borrowers in CreditMetrics model can be changed by linking the transitions probabilities directly to the changes in macroeconomic variables. This can be seen as an alternative to the model described in the section 5.1 for probabilities of default of obligors.

As described in Saunders, Allen (2002) the same probit or logit model can be developed for the element of the transition matrix describing the probability of shift from rating C to rating D, or alternatively for shift from any initial rating grade to default grade.

[56] The same is found by Nickell et al. (2001) who use a probit model to estimate the dependence of transition probabilities on the industry, country of the obligor and stage of the business cycle. The latter turns out to be most significant for changes in transition probabilities and will be discussed in the section 5.5.

The given probability depends on a set of systematic risk factors X_{it-j} (or macroeconomic factors) and two random values that describe shock to macroeconomic conditions ε_{it} and random shocks to economic system V_t.

$$p_{CD} = f(X_{it-j}, V_t, \varepsilon_{it}) \ldots\ldots where V_t, \varepsilon_{it} \sim N(0,\sigma)$$
$$X_{it} = h(X_{it-1}, X_{it-2} \ldots \varepsilon_{it})$$

In this model at first significant macroeconomic variables and their historical time series are found and model parameters are estimated. Further the series evolution is determined and based on the simulation of random variables the future default probabilities for each industry and initial rating can be estimated. Such simulated PDs can be compared to the unconditional sTable historical alternatives. If the simulated probability is higher than the unconditional probability of default the classical VaR using the latter would be underestimate the credit risk of the portfolio, there is a recession expected and the migration matrix should be adjusted so that the shifts to lower ratings will be more probable[57]. A migration adjustment ratio defined as a ratio of simulated and unconditional PD is defined for each period, being larger than one if there should be according to next year simulated scenario a shift of probabilities in the transition matrix to the right (downgrades should be more likely than in unconditional matrix) and lower otherwise.

Finally the unconditional matrix elements that describe probability of shift to default grade can be adjusted by multiplication by this adjustment ratio; other elements being adjusted according to the diffusion parameters. As soon as a transition matrix different for each year according to the simulated effects is obtained a cycle-sensitive VaR for each year can be estimated by CreditMetrics method described in 3.3.2.

Stress test can be run by using an extreme recession during the estimation of PDs for some or all industries/countries that would cause a significant increase in the elements in the right part of the migration matrix and influence the VaR estimated for that year.

[57] Because there are not any more just PDs but also probabilities of shifts to other rating grades in the migration matrix and sum of elements in rows of each matrix must be equal to one a special diffusion parameter dependent on the adjustment ratio is defined in this model to adjust the other row elements in the migration matrix.

Table 5.4: Summary of stress testing approaches

Paper/authors	area of survey	message of the paper
Rösch, Scheule (2004)	effect of PDs and asset correlation on the loss distribution, models with different level of segmentation among sectors for: •constant PDs and Basel II correlations •constant PDs and estimated correlations •time-varying PDs and estimated correlations	asset correlations higher under Basel II than observed one, inclusion of variables related to the cycle decrease uncertainty of estimates and improves estimates of loss distribution and economic capital, correlations depend on same factors as PDs and should be modeled simultaneously
Hamerle et al. (2004)	models of PDs driven only by firm specific risk factors or also by systematic risk factors for versions with or without sector segmentations and asset correlations included	From comparison of models with and without segmentation otherwise same, it is not clear which one is better. Inclusion of macroeconomic variables in the model for PDs and asset correlations decreases the variance of the forecasts.
Rodriguez, Trucharte (2007)	logistic regression for PDs based on macroeconomic variables and loan characteristics, stress test for worst-case scenario	identification of systematic and idiosyncratic variables significant for mortgage portfolio PDs and stress testing exercise
Rösch, Scheule (2007)	stress tests for scenarios based on PDs trough the cycle and point in time setting and stress of regression coefficients according to their standard errors	larger standard errors for point in time regression coefficients, cause more severe stress, higher VaR and expected loss
Frye (2000)	makes the LGD cycle dependent and analyze impact for final loss distribution	theories why LGD change with the cycle and analysis of correlation between LGD and PD and their impact on loss distribution
Altman, Resti and Sironi (2001)	analysis of approaches to LGD modeling by different credit risk models, compares model with LGD constant or stochastic without correlation with PD and model with cycle dependent LGD and correlated with PD	The first two models significantly underestimate expected and unexpected loss. Cycle dependent parameters lead to higher procyclical fluctuations in the capital needs estimated.
Bangia (2002)	credit migration matrix analysis, comparison of unconditional and conditional migration matrix and example of its usage in CreditMetrics	conditional and unconditional migration matrices are significantly different and conditional fits better real development of losses. During contraction the distribution of future value under conditional matrix has longer left tail and more probable values under mean
Servigny, Renault (2002)	method of estimation of elements of migration matrix and asset correlation based on observed migrations, isolation of effect of change of univariate default probability and correlation with the cycle	change in correlation influence right tail of loss distribution, while the latter influence the center of the distribution
Nickell et al. (2001)	a probit model to estimate the dependence of transition probabilities on the industry, country of the obligor and stage of the business cycle	business cycle is most significant for changes in transition probabilities
Saunders, Allen (2002)	model of cycle dependency of elements of migration matrix	method how matrix elements under stress can be determined and how to adjust the migration matrix

Source: by author

6. Stress testing practice at mayor Czech Banks - with a focus on a loan book credit risk stress testing

In the following part I will introduce results of my research in five banks in the Czech Republic, which was focused on the loan book credit risk. During the survey I visited a credit risk management department in each of these banks and asked for facts on the two main topics. Firstly, information about credit rating systems used and main characteristics of regulatory and economic capital models built in the banks were collected; secondly, stress testing methods for loan book credit risk and sophistication of scenarios applied in these institutions were surveyed.

There was a great trade-off between receiving interesting and exceptional details about methods used and risk that I will cover some information that are crucial for the banks´ know-how that would lead to a loss of interview, so the questions covered were primarily prepared in such a way that they mostly ask for the facts that do not need to be kept in secret, but are contributive for the research, leaving up to the interviewed expert, how detailed his answer will be. (The Tables 6.1 and 6.2 compare answers to the questions covered during each interview which can be found in the appendix 1).

The main goal of the survey is to cover following three research questions:

- Are Czech banks influenced in their credit risk stress testing framework by their parents or the supervisory institution, beyond the regulatory rules?
- Do the credit risk stress testing techniques and scenarios vary across the Czech banking sector considerably and if they do, how and why?
- Did the credit risk stress testing techniques/scenarios/usage change considerably before or since the crisis started?

The answers heavily depend on the information received during the interviews, which I have to assume to be true, because mostly there are no public sources available for their verification. Just part of the facts and basic information about the institutions can be checked in the annual reports.

6.1 Characteristics of the banks in the sample

In the surveyed sample there are three out of four banks that are according to the Czech National Bank measure included in a group of large banks; Česká spořitelna further denoted as CS, Komerční banka denoted as KB and the UniCredit Bank. Further in the sample there is one medium-sized bank that wanted to stay anonymous (lets call it Bank A) and one branch of an international bank (the ING).

6.2 Key aspects of the credit risk quantification

In all of the surveyed banks the ratings assigned to a client from corporate sector depend on quantitative as well as qualitative factors. As the quantitative aspects client's financial statement Figures (like assets turnover or other economic ratios, indebtedness, profitability, etc.) are analyzed, while in the qualitative part soft factors like actual market position, future opportunities and threats, market share, regulation in the client's industry are considered.

The factors included and the weights in the rating function differ among the banks, but the basic methodology stays the same; given factors´ values are plugged into the rating function and according to the points received the calibrated PD values are assigned to each client. The historical series used for the calibration of the PDs differs from 3 years in the UniCredit Bank to 7 years in the KB[58]. The regulatory limit according to CNB decree 123/2007 and 282/2008 is 5 years for application for the IRB foundation approach (or 3 years in special cases).

The quantitative part is crucial for the final client's rating, while the qualitative one is used for an adjustment to get a final rating that will according to the experts in the bank fit more precisely the credit risk of a particular client[59]. The ratio of these two parts according to the received answers differs from 9:1 in the Bank A to less than 3:1 in the UniCredit Bank.

The number of parameters estimated for each client corresponds to the rating method used. The UniCredit Bank uses for the corporate clients the Standardized method with IRB foundation approach for some parts of this segment. The CS and the Bank A applies the IRB foundation[60] for all corporate clients and in the KB all clients, either retail

---

[58] In the KB this length of period is kept constant; each time the data from last seven years is used to keep the calibrated values as actual as possible.

[59] As the decree CNB 123/2007 and 282/2008 desires all clients must be divided into rating classes according to well-defined rating criteria for each class to be treated equally. But in some special situations it allows to use an expert judgment about the client rather then the pure rating system inputs.

[60] Even though the CS uses the IRB foundation they estimate for the corporate clients more parameters, which are used just for their internal models.

or corporate, are assessed according to the IRB advanced approach as well as in the ING, which specializes only on corporate clients[61].

The retail portfolio clients get ratings according to the weighted values of few characteristics, usually considering socio-demographic and economic facts (like age, education, disposable income, other loans taken etc.). The valuation is more quantitative and more automated than for clients from corporate sector, where banks use more case by case treatment. The UniCredit Bank, as the only one from the sample, do not assign PDs to the retail clients[62], the Bank A reported to use the Standardized method and other banks in the sample use IRB advanced.

In those institutions, where IRB foundation for retail or IRB advanced are used, also other Basel II parameters for capital requirement formulas must be estimated by internal system[63]. The LGD is according to the answers estimated in the CS, the KB and the ING as the CNB decree requires based on the product and the transaction specific characteristics.

For the credit risk measurement it is important not just to assign a rating when a credit to the client is approved, but also follow changes in the clients characteristics that are key for his ability to repay the debt. In the Czech Republic clients` rating is usually automatically changed, when he stops to meet his obligations; otherwise corporate clients are checked at least once a year according to their annual reports or more frequently in the case of some suspicion[64].

[61] Because the ING has only a branch in the Czech Republic and as such it has a strong interconnection with the parent bank, the same credit rating system was developed centrally and is applied on a global basis.

[62] According to information received during the interview, the retail clients are divided into three groups; those to whom a loan is approved, not approved and those that should be checked further. This type of system was called by the UniCredit Bank expert a traffic light rating system.

[63] CNB decree 123/2007 and 282/2008

[64] The rating checking is usually done by a special department in the bank manually at least once a year as the decree 123/2007 and 282/2008 calls for to keep the integrity of clients´ credit rating system. Only in the Bank A there was received information about special software that makes the rating checking for corporate clients automatically and warns in cases of some significant adverse changes.

Table 6.1: Summary of answers from respondents about credit risk rating system

	information on credit rating system								
	rating method used		factors considered for rating (PD)		factors considered for rating (LGD)	length of data series for calibration	Back testing	monitoring of client for possible changes in rating	
	corporate	retail	corporate	retail				PD	LGD
CS	IRB foundation*	IRB advanced	according to Figures from financial statements and expert qualitative part	demographic and economic factors	characteristics connected with particular product	***	***	monthly for retail and once a year for corporate (or more often in case of suspicion of downgrading)	***
KB	IRB advanced**	IRB advanced**	according to Figures from financial statements and expert qualitative part	behavioural + scoring	connected with particular product and historical experience with enforcement of repayment	min.5 years (max.7 years)	once a year (if differences detected possible to change the calibration)	at least once a year (in case of suspicion more often)	each month
UniCredit Bank	standardized (+IRB foundation for some segments of clients)	traffic light approach (no PD)	external ratings or quantitative and qualitative part if IRB foundation applied	do not estimate PD	do not estimate LGD	3 years (for part of portfolio where PD is estimated)	once a year	monthly	do not estimate LGD
Bank A	IRB foundation	Standardized	according to Figures from financial statements and expert qualitative part	behavioural + scoring	do not estimate LGD	5 years or more	once a year	at least once a year (but has a special automated software for more often clients´ monitoring)	***
ING	IRB advanced	no retail	quantitative and qualitative part	no retail	characteristics connected with particular product	5 years or more	once a year	once a year	once a year

*but LGD, CCF and effective maturity are estimated for internal purposes

**subsidiaries have IRB foundation

***information was not provided

Source: by author based on answers of interviewed representatives

6.3 *Stress testing at the mayor Czech banks*

According to the decree of CNB 123/2007 and 282/2008, which applies the Basel II directive into the Czech law, each bank[65] is obliged to identify possible future economic development that could have an adverse effect on the credit exposure and should access the capability to withstand such changes. Each institution should run stress tests that are meaningful, sufficiently conservative, take into account scenarios for at least a mild recession and covers the majority part of the portfolio. Within this quite broad definition the institution itself can decide about its stress testing procedure and scenarios, while the appropriateness must be finally approved by the regulator.

The scenarios used in the KB, the UniCredit Bank and the CS are of the same type. All these banks responded that they generally use three types of scenarios; those developed by the experts in a special department of the bank, those prescribed by the Czech National Bank[66] and those that were recommended or borrowed from their parent banks. While the KB and the UniCredit Bank sent the data on the Czech clients needed for stress tests based on a parent company's scenario to their parent bank, the CS tries to apply them on the its data by itself. This fact could give an evidence of a slight difference in the complexity of the stress testing models.

None of these banks had any problem to apply the CNB scenarios and offer the proper result of the stress test to the CNB for aggregation. Often the banks were even influenced in their expectations and scenario creation, as the UniCredit Bank representative mentioned. In the ING in the Czech Republic no stress testing takes place; all the data are regularly forwarded to the parent bank and the results of stress tests are then sent back to inform about the actual situation[67]. In the local branch all analysis are made just on the level of clients and based on the changes in the risks connected with individual transactions.

In the UniCredit Bank, where the CNB scenarios were accepted as reference scenarios, the stress testing is highly dependent on the parent company, to which all data

---

[65] which is obliged to follow this decree according to the part two § 3-6

[66]The stress tests run by the CNB and its cooperation with individual banks will be discussed in the following part.

[67] Because the ING has in the Czech Republic only a branch, the stress tests are run on the global basis by parent company, even if it searches for the effect of the scenarios on local players. ING can serve as a good comparison for a complexity of stress test methods of the whole international group with those run by local banks, but can not be any more included into the survey of the Czech stress testing methods.

about clients and scenarios are regularly provided. Parent bank estimates individual stressed PDs and in the Czech Republic the stress test is finished. This procedure is needed because of the fundamental type of rating system applied for the retail clients, but on the other hand it enables according to the UniCredit Bank respondent to pay attention to the sensitivity analysis and concentrate more on issues that are often not so properly tested. Thanks to cooperation with the parent company the UniCredit Bank can take the concentration and the diversification effect within the parts of the portfolio better into account and sensitivity analysis for different changes in macroeconomic conditions can be run faster and more often.

6.3.1 Characteristics of stress tests and scenarios

The complexity of the scenarios used by the Czech banks in the sample differs a lot. While the Bank A uses only sensitivity analysis and according to its representative it nowadays works on a more advanced models that could be used for running also the scenario analysis, in the CS scenario analysis amounts for about 50 % of all scenarios covered and in the KB only multi-factor analysis are applied. This statistics perfectly fits the general findings by the BIS _CGFS (2005) that larger institutions run mostly hypothetical and more advanced tests, while smaller institutions mostly apply sensitivity analysis.

The most complicated is the stress testing process for the UniCredit Bank, where because of the type of rating system, the parent bank itself applies all the scenarios on the clients´ data and sends back the appropriate parameter values; those can be further used for estimation of future losses and some basic sensitivity analysis in the Czech Republic.

All banks in the sample use only hypothetical scenarios and concluded that they do not consider historical scenarios in their stress testing process. The reasons were that the banks do not consider them as relevant for the possible future development and that in the Czech economy there was no situation that could be used as a historical stress event because of past favor development. Only the last years of economic downturn they could consider as a stress event situation, so these data are registered and could be used in the future as a scenario of historical crisis.

In the actual situation the banks see as the most important hypothetical, forward looking scenarios that try to forecast future development, while the knowledge of historical stress events can influence their experts in expectation and in the scenario creation[68].

One of the surveyed banks, the CS, runs the stress tests also for the mayor decisions on the future development of the bank like an introduction of a new product, a change in the orientation of the bank or a shift to different distribution channels. In the KB the stress tests for such a kind of scenarios are replaced by a less demanding analysis, while in the case of the UniCredit Bank all these are done by the headquarters in Italy.

Macroeconomic variables changes are the factors considered in the scenarios by all the banks in the sample. In the KB no other risk factors except of macroeconomic variables changes were identified as they consider their portfolio as enough diversified to be endangered for example by concentration risk and downturn or downgrade in some important industries[69]. Other banks in the sample reported that they run tests covering changes in an external conditions (like an important industry or region downturn), changes in portfolio characteristics (like in the transition matrices, concentration in portfolio) and in a behavior of clients (willingness to repay, increase in PDs etc.).

Except for such general scenarios that are often applied to the whole loan portfolio[70] (considering the difference in the effect of scenario on different products or clients groups) a special (sometimes an ad hoc, sometimes repeated) scenario can be created for some key products of the portfolio based on the most important risk factors. This is for example the case of the CS, which responded to search for an effect of a scenario based on the real estate prices decrease on their mortgage portfolio losses in the last year[71].

---

[68] The stress tests are nowadays often run also on an ad hoc basis; as reported during the interviews scenarios are sometimes created dynamically according to the actual situation and threats expected by experts.

[69] but on the other hand the KB includes in its stress test the future expected changes in the portfolio structure if the scenario materialize, which makes their stress tests more dynamic than if the assumption of no changes in the portfolio structure is accepted.

[70] Most common factors of this type are changes in interest rates, GDP growth, inflation, employment; among factors applied on special parts of the loan portfolio can be FX rates, real estate prices and other factors that have a significant influence just on some kind of product.

[71] Similar procedure is applied also in the Bank A, where special scenarios are created just for parts of the portfolio or the types of clients on the ad hoc basis, when some suspicion about the future development appears. In this bank, as their representative said, the scenario is created each time only for some important

The magnitude of the changes in the risk factors is usually based on an expert judgment and macroeconomic model used by a specialized department of a bank, however it can be in some way influenced by a historical experience and knowledge of past stress events the experts have or by the scenarios received from the Czech National Bank.

Some banks from the sample included into their stress testing program more scenarios with the same factors covered that differ only in the severity of the changes in the variables. These tests were applied on the whole portfolio in order to see how possible future loan book losses depend on the severity of the future development. Finally in all such banks it was found that the number of scenarios should rather not be too high and many of them were left, keeping mostly just the most and less extreme scenarios. The KB credit risk management department tried to run stress tests for even up to 10 scenarios with different severity and settings. It soon detected that such a high number of scenarios makes the stress testing excessively demanding and the interpretation of results was often confusing without bringing much additional information.

Finally the number of scenarios was reduced to two or three (baseline and most sever, sometimes accompanied by some middle scenario) to make the computation and presentation of results to management easier and more clear[72]. In the KB they believe that no important information was lost as the results for omitted scenario were between those left and find this number of the whole-portfolio scenarios for the periodical tests as optimal.

In the KB and CS a special model is developed for transmission of changes in macroeconomic variables into PDs. These are very important part of stress testing program as any scenario of future economic development can be by this model easily translated into the Basel II parameters to be used in regulatory formulas. The prediction of possible change in macroeconomic variables is usually based on some internal macroeconomic model, which ensures that such a combination of factor changes is realistic to occur. In other banks in the sample the transmission model was not identified.

---

part of its portfolio according to the most actual forecast of the development in this product and the relevant risk factors; the results are aggregated for the whole portfolio just in case when scenarios are similar.

[72] Two scenarios were applied also under the CNB joint stress testing program and can be probably seen as sufficient number of scenarios applied to the whole portfolio, if they such scenarios that fulfill all the characteristics of a good scenario, which are required in Basel II, by CNB or recommended in consultative papers.

6.3.2 Changes in scenarios, methods and importance of stress testing since the beginning of the crisis

All the representatives were asked if and how their stress testing processes changed before or after the current financial crisis started. In all the banks it was reported that stress testing changed considerably. In the Bank A an initiative from the managers to develop better stress testing procedure is observed by the credit risk management department; the stress tests are run more often and with higher precision[73].

CS started to be applied more variants of the similar scenarios with a different severity of risk factor changes since the beginning of the year 2009.Its expert adds that thanks to the crisis new factors that could have an adverse effect on their portfolio were identified and can be included in the scenarios.

Furthermore the CS representative noted that before the recession their stress tests were mostly based on the changes in the portfolio structure, while later their attention was shifted mostly just to the scenarios considering the market variable changes, as they started to be more volatile during the crisis in comparison with a quite sTable and positive macroeconomic development earlier. In all banks in the sample more severe changes in risk factors are incorporated into the scenarios since the crisis started.

On the other hand the KB representative highlighted that in the Czech Republic the Basel II concept came fully into force in 2008[74] and a preparation of new methods and processes was needed by banks to be able to fulfill the new obligations, so a more progressive development and motivation from the inside of the banks in the last years can according to their opinion partly be a result of a new regulatory requirements introduced. Even though also in this bank the experts concluded that in the last year more severe scenarios are considered thanks to more volatile market conditions.

[73] This finding is supported also by the trend depicted in the international survey BIS_CGFS (2005) as well as in the report about practice in German banks, Stress tests at German banks (2004), where overall increase in the number of stress tests (considering all risks) run by banks (however higher in scenario tests than for sensitivity analysis) and increase in the number of credit risk stress tests and its sophistication is described.

[74] According to the Financial Stability Report (2007) some banks introduced the Basel II system already since 1.7.2007, but the new rules were generally applied by all banks since January 2008. As mentioned the CNB decree 123/2007coll. amended by decree 282/2008 came into force on May 15 2007 and August 15 2008 respectively and takes effect since 1.7.2007, accepting a possibility that banks can apply this regulation since the beginning of 2008.

6.3.3 The frequency of stress testing and scenario changes

The frequency of periodical tests differs among the Czech banks, but in each case fits the Basel II limit that asks for at least annual stress tests. In the CS periodical tests are repeated each quarter based on a scenario according to the actual prognosis of the future development in the macroeconomic variables. More frequent changes are possible in case if external conditions are extremely volatile.

In the KB the frequency is semiannual with other characteristics the same as in the CS. The Bank A follows the minimal regulatory frequency and the UniCredit Bank ran the periodical tests each nine month or once a year, but during the last year it started to repeat the tests with a semiannual frequency and also in the future it wants to keep this trend. Moreover in the KB, CS and the Bank A the ad hoc scenarios are run very often; each time when they find some relevant scenario for a part of their portfolio. Results of such ad hoc tests are usually analyzed immediately and serve as an early warning against a possible extreme loss.

The time needed for one stress test, from defining the scenario till the final report with results takes from few weeks in Bank A, where only sensitivity analyses are run, up to 2 or 3 months in the UniCredit Bank and in the KB respectively. Here we can see the trade-off between obtaining the stress test result fast to react to possible adverse future development in the economic variables and the encompassment of all important factors and portfolio parts[75].

In the KB the scenario test can give much more precise results than if only sensitivity analysis is applied like in other banks, but in the case of the UniCredit Bank the length of the process is caused by the dependence on the parent company in the setting of clients´ parameters values needed for the stress test.

6.3.4 The reports on stress testing and possible corrective measures

The stress testing process as a part of an institution's risk management can be according to the Basel II rules in CEBS (2006) delegated by the management body to specific risk committees or to the senior management, but the role of management body stays crucial for approving the overall institutions´ stress testing framework and design of

[75] In the most cases it can not be generally said that the faster or the less complicated method the better or the worse, because the omitted effect on the other factors in the sensitivity analysis can be quite high and the too complicated tests can take a long time. The UniCredit Bank could however in the next years increase their possibilities in stress testing if at least IRB foundation was introduced for the whole portfolio.

scenarios.[76] The results of the main stress test must be according to this directive reported to the senior management and the management body clearly and with appropriate frequency, which should enable them to approve necessary steps to forego large losses.

The remedial measures applied depend on the circumstances and can include according to CEBS (2006, $41, p.12) for example reduction of exposures or business in some specific region, sector or portfolio, reviewing capital adequacy, reconsidering the funding policy or implementation of contingency plans.

In the Czech Republic the reports on results are prepared for the management each four month in the CS, each quarter in the UniCredit Bank and semiannually in the KB, but except for these periodical reports some stress test results can be reported immediately, as mentioned for the ad hoc tests that are a common practice in the Bank A.

In all the surveyed banks the results of the stress tests are used for a risk management purposes; the results (if adverse) can lead according to all the interviewed representatives to changes in the portfolio structure, bank's lending policy, underwriting criteria, limits on the amount offered to some clients and in better hedging.

[76] The definition of management body can be found under point 40 p.11 of this document.

Table 6.2: Summary of answers to questions about stress testing

	source of scenarios													
	own experts	parent company	CNB	risk factors covered	coverage of tests	sensitivity/scenario tests	historical/ hypothetical	magnitude of change in risk factors and settings of correlations among them	changes since the beginning of the crisis	frequency of scenario changes	duration of average test	frequency of regular stress tests	frequency of reporting	possible corrective measures based on reports
CS	x	x	x	mostly macro and market variables´ changes, also key decision of management, new products and distribution channels.	depends on the stress scenario, for whole portfolio or just parts	50% to 50%	hypothetical (because we have no data on severe stress events available from history)	according to experts´ expectations, macro models,	new risk factors detected, tests for more adverse changes, started to test more variants of one scenario with different severity of changes in factors, started to test market variables´ changes, before the crisis mainly scenarios of changes in portfolio	quarterly or more often	*	quarterly	at least three times a year + reporting of ad hoc tests	changes in portfolio and credit policy, limits, hedging
KB	x	x	x	mostly only changes in macro variables	mostly the whole portfolio covered, but effects of scenario on PD and LGD for different parts of portfolio differs	scen.	hyp.	set by colleagues from special department, based on macro model	increase in importance of stress testing in pillar II and with implementation of Basel II, more severe changes in risk factors	semiannually	3 months	once a year, this year and further ahead plans semiannually and more often on management request	semiannually + reporting of ad hoc tests	changes in portfolio structure, changes in granted amounts, changes in underwriting criteria
UCB	x	x	x	mostly changes in PDs, initial PDs for expected future development based on macro model are provided by parent company	the whole portfolio covered	scen. + sens.	hyp.	CNB scenario as a reference for changes in variables	no big changes, because earlier scenarios were very conservative, (just input parameters are now more adverse)	semiannually	2 months	each nine month or once a year, the year 2009 and further ahead semiannually	quarterly	shift in business strategy, change in limits or changes in credit policy
Bank A	x	just sends data	just sends data	macro and market variable changes, tests for key management decisions etc.	for parts of portfolio, if needed the results are aggregated	sens. only (intends to prepare a model for scenario tests)	hyp.	according to experts´ expectations	increase in importance of stress testing and motivation from inside the bank, increase in frequency and precision, more severe changes in risk factors	*	few weeks and more, depends on complexity	regular tests once a year, accompanied by ad hoc tests	all tests reported immediately	changes in underwriting criteria, limits and plans in case of materialization of negative events
ING (answers provided by parent company)	*	*	*	macro, downgrades of some clients, industries, regions.	the whole portfolio or just parts according to asset classes or regions	*	hyp.	according to time series modeling	increased focus on stress testing, more ad hoc test are requested, more severe changes in risk factors	*	*	once a year or semiannually + ad hoc tests	*	*

*no information provided

Source: by author

6.4 Concluding remarks to the research questions

During the survey four Czech banks and one branch were visited to collect information about their credit rating systems and loan book credit risk stress testing. The questions were the same for all banks in the sample. Based on the facts collected three research questions formulated at the beginning of this chapter can be answered.

All the surveyed banks are in some way influenced by their parent companies. In the ING a common rating system is shared, which was developed in assistance of experts from more banks in the financial group including Czech employees. But all the banks in the sample are influenced in the scenarios they use. As mentioned in the CS some scenarios from Vienna can be borrowed or recommended for applying in the local bank for stress testing.

In general it can be concluded that banks with more advanced rating methods that are able to run scenario stress tests by themselves (like the CS and the KB) are more willing to apply scenarios from parent companies by themselves or at least discuss in details the stress testing methods applied on their data, which can improve their own stress testing procedure. While in the other two cases, where the UniCredit Bank is highly dependent on the parent setting of parameter values for stress testing and in both banks just sensitivity analysis can be run, such sharing of experience with their parents was not mentioned.

Most of the banks were in some way influenced by the CNB new project and scenarios received, because in the most cases they were considered to be a valuable forecast of a possible future economic development. In the UniCredit Bank these were taken as reference scenarios, while in the KB the similarity of the results received from their own periodical stress tests proved, according to its representative, appropriateness of the scenarios prepared by the KB´s experts.

The types of scenarios also differ among the surveyed banks. In general larger Czech banks with more advanced rating systems run in comparison more scenario analysis, which corresponds to the experience from abroad, as showed in BIS_CGFS (2005) and Deutsche Bundesbank (2004).

Further in the Bank A, the CS and the KB, ad hoc or regular tests specialized on specific parts of portfolio are undertaken. Although in the Bank A ad hoc tests are according to its representative the only stress tests applied[77], in the CS these tests are part of stress testing program, which specializes on the most important parts of its portfolio, to ensure better credit risk management. In the CS and the KB also analysis based on the key decision of management about the future policies are covered. In both of these banks the stress tests are run with higher frequency, if we do not consider the new trend in the UniCredit Bank to run tests semiannually.

The time needed for one stress test depends according to this study on types of scenarios used by the institution, the coverage of the test and on a type of the rating system used, (as it enables the bank to use its own estimated parameters for stress testing). Corrective measures that are possible to take based on results of stress testing do not differ significantly.

The answer to the last research question appeared to be more complicated, because an experienced more dynamic development in stress testing application and methods that could have been motivated by the crisis is probably mixed with the impact of the implementation of the new regulatory rules. Basel II was introduced by Czech banks in January 2008 (in some cases earlier), so by this time Czech banks needed to improve their stress testing procedures to fulfill the CNB decree requirements. It is hard to differentiate what caused the observed progress and increase in the importance of stress testing, because the effects of both events were mixed.

[77] The representative wanted to cover during the interview only ad hoc tests. He reported that they are the only tests applied in the bank A, but he also highlighted that their stress testing program meets the Basel II minimum requirements.

7. Estimation of the capital requirement for an institution's portfolio – a bias that arises from low segmentation

7.1 Top-down vs. bottom-up approach to stress testing

According to the Quagliariello (2009), there are two main approaches to stress tests for the whole financial systems. The top-down approach uses data on the institution level, not seeing the segmentation to rating classes. Under this approach a uniform model is applied to transfer the scenario on the institution's figures. On the other hand, the bottom-up approach works with data on each contract and obligor. The tests are usually run by the financial institutions themselves, while each institution can use different internal models to link the shift in risk factors to PDs. The scenarios are in both cases determined by the supervisory body, which afterwards collects and aggregates the results to judge an overall impact on the financial stability and on individual institutions.

Based on these definitions it can be deduced that bottom-up approach can give more accurate results of estimated capital requirement, because the data used are more segmented and detailed and the portfolio loss is estimated according to each client's sensitivity to the scenario. In the bottom-up stress tests institutions use all the data available on clients and scenarios as inputs into internal models calibrated according to their historical experience.

On the other hand, because of the difference in settings of the models and assumptions used, the results from these tests are usually hard to aggregate or compare among institution. The complexity of scenario that can be applied in these stress tests highly depend on the models developed and rating methods used by the institutions.

According to Čihák (2007), a big advantage of the bottom-up method is that it can easily capture concentration in the portfolios, which can be overlooked under the top-down approach. But once the supervisory body decides to conduct a stress test for the whole financial system using a bottom-up approach, it will face problems to use more complex scenarios, coordinate the process and aggregate the results.

In fact, the supervisory bodies usually try to combine these methods to get advantages of both. For example Austrian stress testing program called Systemic Risk Monitor (SRM)[78] according to Boss et al. (2009) starts with the analysis of scenario impact

[78] Like most of the macro stress testing programs, the SRM covers more risk types. It is a model that can capture credit as well as market risk as two most important risks of a bank.

on individual institutions` portfolios on the level of contracts and obligors, which is further enlarged by an inter-bank model that can capture contagion effect. According to Laviola et. al (2009) the stress tests run under the FSAP program in Italy were also of two kinds. The supervisory body ran the top-down macro stress tests based on the flow of new bad debts[79]. The bottom–up stress tests were conducted by the institutions themselves according to the macroeconomic scenarios provided by the national bank.

The Deutsche Bundesbank uses the above mentioned bottom-up approach as well as a top-down stress testing. The latter contains two main parts, as in Deutsche Bundesbank (2007). At first, within the macro module, a scenario is prepared using a system of econometric linkages and models for macroeconomic variables and macro stress tests are run. The results are further used in a bank module, where institutions that can get in problems under given scenario are identified.

7.2 Stress testing in the Czech National Bank[80]

Stress tests for the whole Czech banking sector are one of the measures used by the Czech National Bank to monitor the financial stability. In fact there are two types of programs in the CNB that use stress tests. Firstly the top-down stress tests, which results are each year published in the Financial Stability Reports. The second type of stress tests is bottom-up, which is based on the information collected from banks. All the banks included in the sample for the survey described in chapter 6 (of course except for ING) reported that they took part in this program.

Within this new project (also called "joint" stress testing), which was introduced in September 2009, Czech banks[81] were provided with two scenarios, a baseline and an adverse scenario, defined in terms of future development of macroeconomic variables as well as in a percentage change in PDs for each stressed segment of the portfolio. Relevant parameters for these two scenarios were estimated by each bank, using their own models.

[79] Such a type of data is a commonly used PD proxy, because of its availability in national credit registers.

[80] General information about the joint stress testing project was received during the interview in the banks as well as a consultation in the CNB.

[81] Up till now not all Czech banks were asked to take part in this project, because of different levels of stress testing methods developed in the banks. For the purpose of project the IRB foundation or at least capability to use this method (considering institutions that already applied to use this approach) was necessary for this stress test.

Finally the results for capital requirement and other important measures under these scenarios were sent back for aggregation.

Only some of the banks reported during the survey that they are able to link a macroeconomic scenario to the PDs, so they could probably estimate their own PDs relevant for the CNB's scenarios. The rest of the banks that took part in the joint stress testing program probably used the percentage change in PDs estimated by the CNB for an average portfolio.

For the top-down approach the CNB publishes each year one baseline and two alternative scenarios describing possible future economic development[82]. The results are interpreted in the following two ways; how much the capital adequacy ratio (CAR) would decrease and how much money would need to be invested into the financial sector to keep all institutions above the CAR required level.

7.3 Comparison of capital requirements estimated under models for different level of segmentation

In this section I will introduce a unique model for a comparison of the capital requirement that is estimated under different levels of portfolio segmentation. Because under the top-down approach the micro data, that would enable the regulator to estimate for example the models of PDs for each rating class of an institution, are not available, the estimates can turn out to be less accurate. On the other hand the bottom-up stress testing models use data with higher segmentation, which should lead to more precise results.

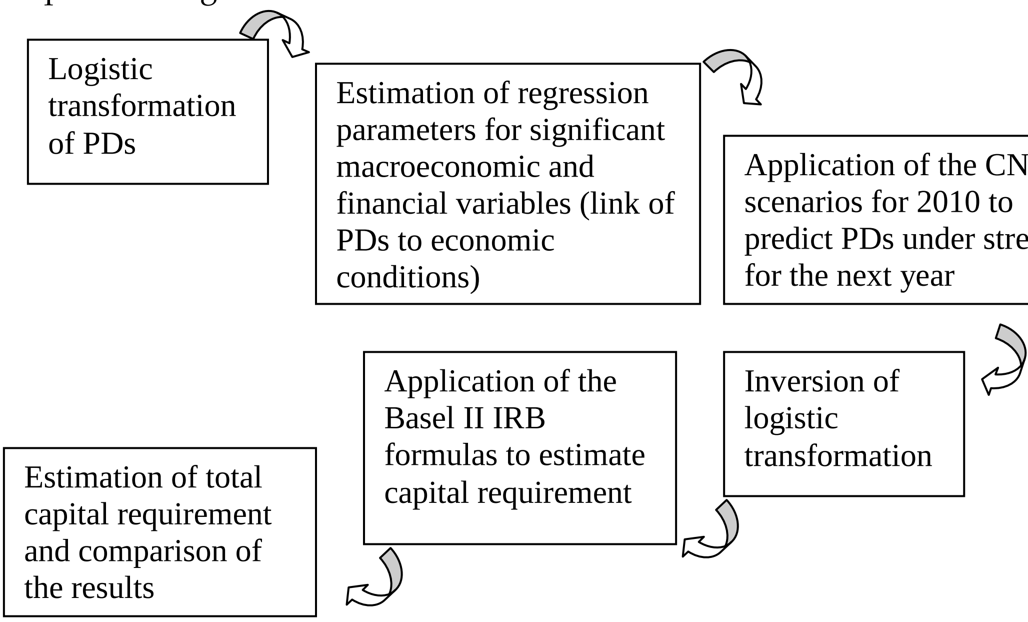
Most of the empirical papers that contain models of stress testing, use aggregated data for the whole banking sector and try to find significant explanatory variables for some PD proxy or compare them among more countries. In this chapter I will compare results of stress tests under a model with segmentation of the portfolio according to the credit quality to the ones without segmentation. Basically, I will try to find out, whether a segmentation of the data on portfolio can lead to a significantly different estimated capital requirement

[82] For the year 2008 CNB prepared three scenarios–safe heaven, property market crisis and loss of confidence. The real economy finally followed the safe heaven scenario for more than half a year and than shifted toward the property market crisis. For the year 2009 three different scenarios of Europe in recession, market nervousness and economic depression were prepared (see Financial stability report 2008/2009, CNB for more details about macroeconomic variable setting for each scenario) out of which the market nervousness would according to the CNB results hit the most banks, but would not endanger the stability of the financial market under the assumption of one year duration of the shock.

for the same stress scenario. The estimates of the capital requirement under the tests run by the institution are usually based on data of each client or contract and i.e. should be the most accurate. On the other hand, if we use as inputs into the stress test the average data (without any segmentation) that characterize the whole portfolio, the estimates can be biased, which can lead to an inaccurate conclusion about the institutions` stability. The model developed further will show, if there is some significant difference in estimates due to segmentation.

The inputs into the model are real data on PDs provided for purpose of this thesis by one Czech bank. Three sets of historical time series with different level of segmentation were used. The most general data is the time series of PDs for the whole corporate segment. Further data on PDs for its investment (including rating classes 1 to 6[83]) and sub-investment part (including lower quality classes 7 to 13) were available. The data for model with the highest segmentation are represented by a set of series on PDs for each rating class. All data are available quarterly since June 2005 till December 2009; for the next year a stress scenario will be implemented. On the Figure 7.1 the logic of the model is depicted.

Figure 7.1: Model for the estimation of a capital requirement under different levels of input data segmentation



Source: by author

[83] The financial institution differentiates its corporate clients into 14 rating classes according to their credit quality; the lower the number of the classes, the higher the quality of the client. The rating class 14 contains only clients in default.

7.3.1 Description of the regression model

At first a regression model is needed to link the PDs on different levels of segmentation to the key economic variables. Because the data on PDs are between or equal to 0 and 1 the logistic regression model would be most suiTable[84]. However it is also possible to use an equivalent method, where a linear regression is applied on a logistic transformation of the PDs[85].

The most common explanatory variables that determine corporate default rates according to Jukivuolle, Virolainen, Vähämaa (2008) and Jakubík, Schmieder (2008) are measures of indebtness of corporate sector, interest rates, GDP and in the latter as well unemployment rate, exchange rates, inflation and GDP of the main trading partners. I covered all of these categories and found as significant the following four variables.

Table 7.2: Descriptive statistics of explanatory variables [86]

variable description	abbreviation	mean	median	standard deviation	C.V.	skewness	ex.kurtosis
real interest rate 3M	Real 3M	-0,0019	0,00078	0,012	6,458	-0,83	-0,016
ratio of value of loans to corporate sector to GDP	Loans/GDP	0,8037	0,81042	0,086	0,107	-0,09	-1,133
price of industrial products	Ind. prices	102,33	102,85	2,76	0,027	-0,655	-0,353
exchange rate – EUR/CZK	EUR/CZK	0,0369	0,03672	0,002	0,061	0,414	-0,761

Source: by author

The regression equation for the model of PDs for the whole corporate segment as dependent on the explanatory variables can be expressed as follows:

[84] For model for individual PDs with similar structure see the section 5.1.

[85] The logistic transformation changes the domain of definition to all real numbers, so the transformed PDs can directly enter a linear regression model as dependent variable.

[86] The source of the first three variables is a statistic database of the CNB (ARAD), the last variable is from www.oanda.com. Real interest rates were derived based on the Fisher equation from inflation rate and nominal PRIBOR 3M (three month Prague Interbank Offer Rate). The ratio of loans to GDP was calculated as a total value of loans to corporate sector in local and foreign currencies provided by Czech banks to the absolute GDP in current prices. The exchange rate EUR/CZK is defined as amount of EUR one can get for one Czech koruna. Index of industrial prices is expressed as annual moving average in constant prices of 2005.

$$\ln\left(\frac{PD_t}{1-PD_t}\right) = \beta_0 + \beta_1 * real\_3M_t + \beta_2 * loans / GDP_{t-4} + \beta_3 * ind\_prices_t + \beta_4 * EUR / CZK_{t-4} + \varepsilon_t$$

The model for more segmented data describing the PDs of investment and sub-investment parts of the portfolio was estimated using two separate equations of the same structure as above. The PDs for each rating class were considered as a panel data. The estimated regression parameters and their significance is shown in the following Table 7.3.

Table 7.3: Regression parameters for models with different level of input data segmentation and their significance

Coeff.	β0	β1	β2	β3	β4	adjusted R-squared	unadjusted R-squared
variab.	Const.	Real 3M	Loans/GDP	Ind.prices	EUR/CZK		
model for PD-for the whole corporate sector							
	-18,6073	25,4027	6,93398	0,137738	-117,729	0,80825	0,85938
	(6,86E-05)***	(0,0012)***	(6,98E-05)***	(0,0011)***	(0,0131)**		
model for investment part of portfolio							
	-8,34907	16,6605	6,58874	-	-60,4344	0,94853	0,95883
	(9,08E-09)***	(3,72E-05)***	(1,11E-07)***		(0,0247)**		
model for PDs for sub-investment part of portfolio							
	-17,8624	24,8311	6,48495	0,133622	-110,326	0,7989	0,85253
	(7,30E-05)***	(0,0011)***	(9,31E-05)***	(0,0011)***	(0,0154)**		

Source: by author, values in brackets are p-values of the estimated parameters; ** means that the estimate is significant at 5% level and *** at 1% level.

Because of a logistic transformation of the PDs, it is not possible to interpret the coefficients as elasticities. Although, because the natural logarithm (ln x) is an increasing function of its argument (x), we can still comment the signs of the estimated coefficients and conclude if the variable influences the PDs in a positive or negative way.

For the results above, an interpretation of the coefficient's signs is quite intuitive. An increase in a real three month interest rate will cause an increase in a probability of default as a rate on a corporate debt increases and the financial resources of the firms become more expensive; it will be more difficult to meet their obligation from credits. The higher is the ratio of loans to firms to GDP and the higher the prices of industrial products the more corporate client will stop to repay their debts. In the first case the corporate sector will become too indebted relatively to the GDP and many firms will not have enough financial resources from production to repay the credits. In the latter the expenses on inputs and production in most the firms will increase.

The last significant coefficient for exchange rate is negative, which is exactly opposite to the empirical findings of CNB (2006) that an appreciation leads to a decrease in profitability measured as return on equity[87]. For this particular portfolio, the effect is exactly opposite. This may be a case, when most of the clients use as inputs imported materials and resources that become more expensive when Czech koruna depreciates. This will increase their production costs as well as their probability of default.

The coefficient of determination is sufficiently high in all regression models; in all cases the explanatory variables can explain at least 80% of movements of the dependent variable. All the assumptions for the OLS method were checked[88]. According to the collinearity test using variance inflation factors, there is no multicolinearity present among the data series. The null hypothesis of no heteroskedasticity in the White test can not be refused for none of the models. The Durbin-Watson statistics is sufficiently close to 2 in both models for aggregated data and sub-investment PDs, so we can not refuse that there is no autocorrelation in disturbances. For the model for investment part of portfolio a more general Breusch-Godfrey test for autocorrelation (tested for autocorrelation up to order 4) as well as Ljung-Box test was further checked and both these tests showed that their hypothesis of no autocorrelation in disturbances can not be refused. The normality test did not refuse that the distribution of residual is normal.

Similar model was applied on the data series on PDs for each rating class. For an estimation of panel data regression by OLS method the errors terms must be uncorrelated with each other and have equal variance. The Breusch-Pagan test showed that the variance covariance matrix of residuals is not diagonal and error terms are correlated. For this reason a weighted least square method was used instead of OLS. The estimated of coefficients are listed on the following Table 7.4 and results of tests for assumptions needed for weighted least square method can be found in Appendix 2.

[87] The Czech Republic is one of export-oriented countries, so the stronger is the appreciation of Czech koruna the higher will be prices of the goods of Czech exporters on the foreign markets and the more difficult it will be to sell the products (this may lead to higher probability of default).

[88] Results of all tests, their test statistics and p-values can be found in appendix 2.

Table 7.4: Coefficients and their significance in model for segmented data on corporate loans

variable	constant	real 3M	Loans/GDP (t-4)	ind_prices	EUR/CZK (t-4)
coefficient	β0	β1	β2	β3	β4
corp1	28,6747	-	-	-0,43015	-
	*0,0004****			*6,35E-06****	
corp2	40,2686	-115,104	-	-0,544601	-
	*0,0248***	*0,0026****		*0,0040****	
corp3	37,2372	-	-	-0,582748	305,983
	*0,0060****			*3,55E-05****	*0,0175***
corp4	33,7924	-	-	-0,606517	484,022
	*1,77E-05****			*2,89E-09****	*3,21E-07****
corp5	-0,148912	41,5501	9,61276	-0,158023	-
	0,9804	*0,0009****	*6,18E-06****	*0,0118***	
corp6	-0,215094	25,0003	9,34077	-0,146874	-
	0,9396	*0,0001****	*2,10E-09****	*7,35E-05****	
corp7	-8,93056	27,8108	10,5724	-0,0619968	-
	*0,0053****	*2,59E-05****	*2,63E-010****	*0,0220***	
corp8	-10,815	26,2588	6,67183	-	-
	*8,84E-010****	*0,0004****	*3,63E-06****		
corp9	-21,7964	39,8532	10,7348	0,130025	-126,02
	*6,59E-05****	*0,0001****	*5,18E-06****	*0,0046****	*0,0202***
copr10	-19,095	14,7936	8,13092	0,0877137	-
	*6,43E-06****	*0,0033****	*3,30E-09****	*0,0022****	
corp11	-20,9685	25,0528	2,24667	0,160223	-
	*8,92E-05****	*0,0011****	*0,0131***	*0,0004****	
corp12	-25,2162	26,6036	3,88669	0,234942	-91,7544
	*5,34E-06****	*0,0010****	*0,0062****	*1,56E-05****	*0,0463***
corp13	-12,2683	18,3239	3,3206	0,13199	-115,234
	*0,0049****	*0,0212***	*0,0275***	*0,0041****	*0,0303***

Source: by author, values under the estimated values for the coefficients are p-values; ** means that the estimate is significant at 5% level and *** at 1% level.

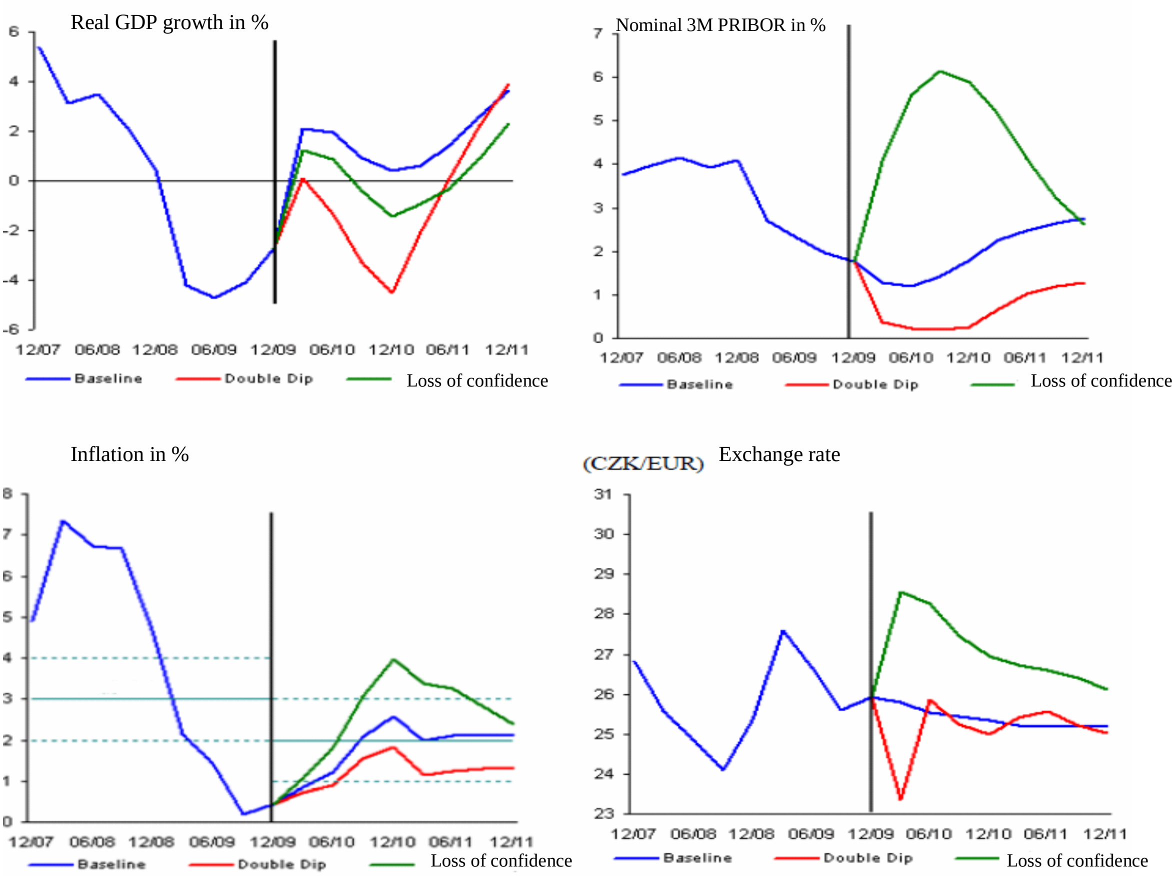
The Doornik-Hansen test for multivariate normality confirms that residuals in this system are normally distributed. Further there was no ARCH effect found in first four lags and according to Ljung-Box test in most of the equations none of the autocorrelation coefficients up to lag 4 is significant.

7.3.2 Stress scenarios used

As already mentioned, the Czech National Bank offers regularly scenarios that are used for stress testing the financial sector. Each year there is a baseline scenario that represents an expected development in economic variables and two or more alternative scenarios that are more adverse. Based on an estimation of an impact of these adverse scenarios on baking sector’s key variables, the resistance of the banking sector and its stability is assessed.

For purpose of this model I will use the most actual scenarios from CNB (2010), which predicts scenarios for next two years (2010 and 2011). The predictions on each individual factor in these scenarios are based on an internal macroeconomic model of CNB, which takes into account correlations among factors and ensures economic consistency of the scenario. The two adverse scenarios used are depicted on the Figure 7.5 as Double dip and Loss of confidence.

Figure 7.5: Scenarios by CNB used for stress testing



Source: CNB (2010)

Double dip scenario describes, according to CNB (2010), a recession characterized by GDP decrease of more than 4 % at the end of 2010. The exchange rate will appreciate to less than 24 CZK per EUR and interest rates as well as inflation will stay on low levels. On the other hand, Loss of confidence is a more adverse scenario that could materialize in a case, if a risk aversion to Czech Republic will increase due to increasing public deficits. The exchange rate will depreciate to nearly 28,5 CZK per EUR in the first quarter, which will lead to an increase in inflation (reaching nearly 4 % at the and of the year) and in interest rates. The nominal 3M PRIBOR will raise to 6% in the third quarter and the real GDP growth will reach the bottom at the end of the year at less than -1,5 %.

Considering the significant variables that were found as crucial for development of probabilities of default of this portfolio, the three month real interest rate can be derived from CNB forecasts based on Fisher equation. Because the ratio of corporate loans to GDP and exchange rate enter the regression with one year lag, it is not needed to forecast them to estimate the PDs during 2010. The only significant variable that is left to be forecasted is the index for a price of industrial products. This was predicted based on a side regression model that could sufficiently explain development of this variable as dependent on the exchange rate with a lag of 2 quarters and real GDP growth with a lag of 4 quarters, both of which are forecasted by the CNB. The dependence can be expressed as follows (more details on tests for assumptions needed for OLS are available in Appendix 2):

$$ind\_prices = 70{,}7351 + 720{,}667 * EUR/CZK_{t-2} + 89{,}8726 * real\_GDP\_growth_{t-4}$$

(2,26E-7)*** (0,0019)*** (8,55E-6)***

Adjusted R-squared=76,31%

Once having all significant explanatory variables forecasted for the above defined scenarios, the stressed PDs for 2010 can be derived based on the inversion of the initial logistic transformation.

$$\ln\left(\frac{\hat{PD}_t}{1-\hat{PD}_t}\right) = \hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 * real\_3M_t + \hat{\beta}_2 * loans/GDP_{t-4} + \hat{\beta}_3 * ind\_prices_t + \hat{\beta}_4 * EUR/CZK_{t-4}$$

$$\hat{PD}_t = \frac{e^{\hat{y}}}{(1+e^{\hat{y}})}$$

7.3.3 Capital requirement computation

Assuming that the portfolio structure meets all the requirements for using the Basel II formulas, the minimum capital requirement can be computed based on the formulas below. Because the data on LGD are not available, I will accept an assumption that the portfolio is homogeneous in LGD, which will not be stressed but set equal for all classes and levels of aggregation to 45 %[89]. This level should be assigned according to BCBS

[89] As a possible improvement of this model, the LGD (or RR) could be made dependent on systemic variables and stressed as explained in the section 5.2.

(2006, Article 281) and Czech regulation (decree 123/2007 and 282/2008) to senior claims on corporates not secured by a collateral. For correlation no adjustment according to firm size is used as I deal with data for rating classes (or even less segmented) and have no information on individual clients. The exposure (E) is assumed to be equal to a book value of a loan[90]. Maturity (M) is set to 2,5, which is according to Czech decree 123/2007 and 282/2008 a relevant value for corporate loans for an institution that does not want to estimate maturity on its own.

$$Correlation(R) = 0{,}12 * \frac{1-\exp(-50*PD)}{1-\exp(50)} + 0{,}24 * \left[1 - \frac{1-\exp(-50*PD)}{1-\exp(-50)}\right]$$

$$Maturity\_adjustment(b) = (0{,}11852 - 0{,}05478 * \ln(PD))^2$$

$$Risk\_weight(r) = \left(LGD * N\left[\frac{1}{\sqrt{1-R}} * G(PD) + \sqrt{\frac{R}{1-R}} * G(0{,}999)\right] - PD * LGD\right) *$$

$$* \frac{1}{1-1{,}5b} * (1 + (M - 2{,}5) * b) * 12{,}5 * 1{,}06$$

where N denotes cumulative distribution function for a standard normal distribution and G denotes the inverse cumulative function to this distribution.

Risk weighted exposition(RWE)= E · r (where r=0 if PD = 100 %)

*Capital requirement for credit portfolio= 0,08*ΣRWE*

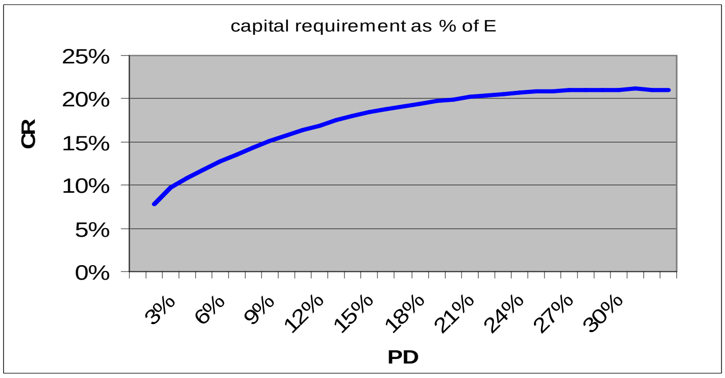
7.3.4 Estimated capital requirement under different levels of aggregation of input data[91]

The capital requirement in percent of exposition is a concave function of PD as depicted on the Figure 7.6

[90] When the capital requirement (CR) is expressed in absolute numbers, the total portfolio exposure (E) is considered to be 200 billion Czech Koruna. Because the data were provided by one of big-size banks, I consider the exposure of around 25 % of total corporate loans (according to CNB statistic in ARAD) as adequate. In most cases the CR will be expressed as percentage of E.

[91] Further I will comment only results for Loss of Confidence scenario, which turned out to be more adverse and mostly compare the value of variables at first quarter of 2010, when the scenario seems to hit the portfolio considerably. Further the portfolio PDs return to values close to values during 2009. So the capital requirement estimated should reach the peak in the first quarter of 2010. The whole analysis is made under the assumption that the portfolio structure does not change in time.

Figure 7.6: Capital requirement as percentage of exposition for different levels of PD



Source: by author

Based on the results from regression models it is obvious that lower PD rating classes (loans with higher credit quality) are more sensitive to the stress applied on the portfolio. While the PD in the model for non-segmented data increased between the fourth quarter of 2009 and the first quarter of 2010 by 70 %, all PDs for rating classes 1 to 9 increased by a significantly higher percentage. The same result was found for an investment and sub-investment parts of portfolio, which increased by 75 % and 65,5 % respectively (some of PDs values and changes are listed in the Table 7.7). This would lead to a general conclusion that low quality expositions are less risk sensitive than a high quality ones.

Table 7.7: Shift in PDs for different level of segmentation under stress scenario

	forecasted PD (1Q 2010)	real PD (4Q 2009)	absolute change (1Q2010 to 4Q2009)	relative change (1Q2010 to 4Q2009)
Portfolio – no segmentation	7,20%	4,23%	2,98%	70,50%
inv.	2,07%	1,18%	0,89%	75,23%
spec.	8,53%	5,19%	3,34%	64,42%
corp1	0,00%	0,00%	0,00%	261,71%
corp3	0,10%	0,04%	0,06%	146,81%
corp4	0,24%	0,14%	0,10%	73,14%
corp5	0,73%	0,18%	0,55%	306,04%
corp6	0,97%	0,34%	0,63%	186,37%
corp9	5,89%	2,39%	3,50%	146,32%
corp10	7,02%	4,60%	2,42%	52,56%
corp11	5,29%	4,53%	0,76%	16,76%
corp13	42,32%	35,10%	7,22%	20,58%

Source: by author

The weights of rating classes in the portfolio could not be estimated from information received from the bank. Because the PDs describe only the probability of direct default for given rating class, it is not possible to estimate relevant weights by weighting the PDs. As the correct weights to sum the results of capital requirements for segmented data are missing and can not be estimated, it turned out to be impossible to estimate an absolute bias that can arise from an omission of segmentation.

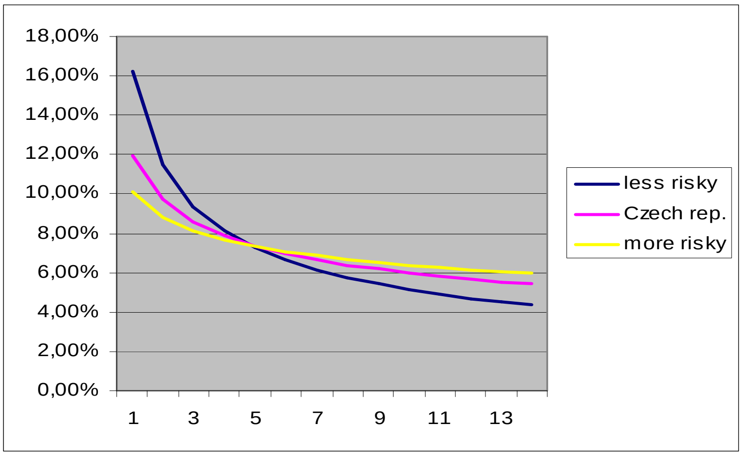
But based on the statistics of CNB about a total distribution of credits between standard and classified loans and a rule of thumb how a typical banks portfolio composition can look like, we can assume that classes belonging to the investment part of the portfolio have higher weights than the sub-investment classes. Under this assumption it can be concluded that, because of the higher sensitivity of low PD classes and their relatively higher weight in the portfolio, the capital requirement computed in the model with no segmentation will be lower than the one based on the data on each rating class. Because of no segmentation the capital requirement needed under the stress scenario will be underestimated.

Further, I have decided to broaden the analysis by applying the percentage shift estimated for the non-segmented data on the rating classes` PDs. The same was done by some banks within the joint stress tests. Those banks that have not yet developed the model to link PDs to economic variables used average percentage change offered by the CNB as the one consistent with the scenarios for changing their rating classes PDs. These banks took initial PDs increased by this average shift and computed in this way the level of capital requirement under stress, which was sent to the CNB.

For this analysis I needed to estimate weights that will be used for computation of the exposition in each rating class. The same weights were used in both models, in the one where PDs for classes are based on regression model and in the second where stressed PDs are set equal to the pre-stress values increased by an average shift of 70 percent estimated for whole portfolio. Even though the average shift was computed based on data on aggregated portfolio, using the same weights for aggregation in both models should help to get comparable results.

The final weights were based on the CNB statistics on a structure of a Czech portfolio and on an expert judgment how the portfolio of the bank that provided the data can look like. Furthermore I add other two portfolios, one more risky that can be typical for a bank in developing country in crisis, as the weights for lower rating classes are higher, and other that is less risky for a bank that have higher exposure in higher quality credits. The structure is described on Figure 7.8.

Figure 7.8: Assumed structures of a portfolio for capital requirement estimation

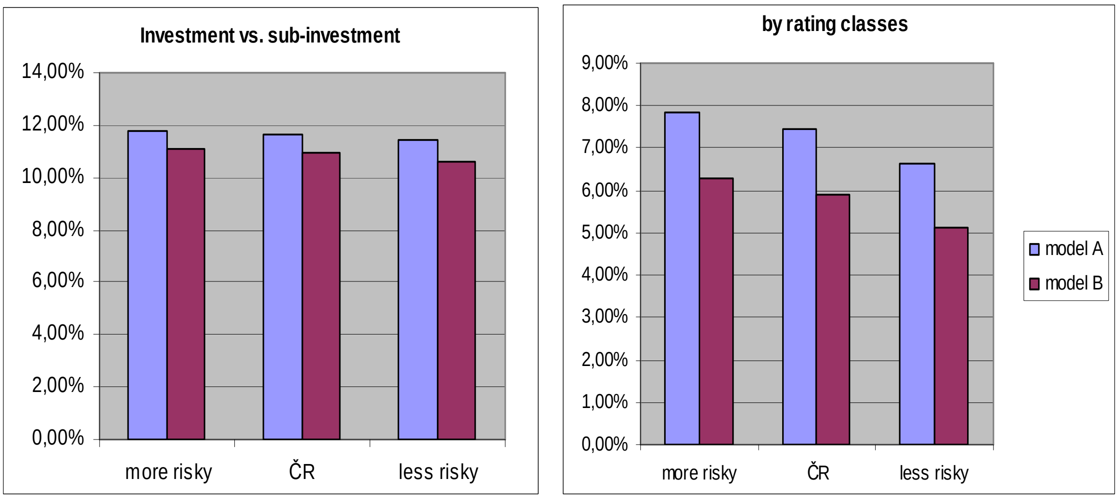


Source: by author – the portfolio called Czech Rep. is based on the ARAD statistics describing the percentage of credits for 5 rating categories and adjusted according to expectation on the structure of portfolio of this particular bank.

On the Figure 7.9 the capital requirements for the two models are compared. The estimated capital is:

1. based on the stressed PDs for each rating class/sub-segment as they were forecasted by the regression analysis and relevant estimated coefficients (further denoted as model A)
2. based on PDs equal to their initial real values at the end of 2009 increased by the average shift of 70 %. (Model B)

Figure 7.9: Estimated capital requirement for PDs stressed according to a regression analysis and PDs increased by an average shift



Source: by author

As obvious from the picture, if the bank would use the average change in PDs it would underestimate its capital requirement under stress scenario under the assumption made on the weights in the portfolio. The bias is the higher, the more detailed data on the portfolio are used and the less risky the portfolio is. For the model, which is based on the data on rating classes, the properly stressed PDs leads to about 25 % higher estimated capital requirement relatively to the model with a percentage shift in PDs. In the model of investment and sub-investment parts of portfolio the estimate under the first is about 6,5 % higher relatively to the latter model. Precise values are summed in the Table 7.10.

Table 7.10: Estimated capital requirements for PDs stressed according to a regression analysis (model A) and PDs increased by an average shift (model B)

estimated capital requirement investment versus sub-investment	weights used	absolute CR (1Q 2010)	CR (as % of E)	relative difference
Model A	more risky	23 515 943,42	11,76%	5,96%
	ČR	23 321 523,36	11,66%	6,51%
	less risky	22 898 221,61	11,45%	7,71%
Model B	more risky	22 193 676,21	11,10%	
	ČR	21 895 242,06	10,95%	
	less risky	21 258 303,96	10,63%	
estimated capital requirement for rating classes	**weights used**	**absolute CR (1Q 2010)**	**CR (as % of E)**	**relative difference**
Model A	more risky	15 697 386,38	7,85%	24,48%
	ČR	14 905 811,72	7,45%	25,90%
	less risky	13 225 983,62	6,61%	28,99%
Model B	more risky	12 610 485,05	6,31%	
	ČR	11 839 598,72	5,92%	
	less risky	10 253 634,89	5,13%	

Source: by author

Within this chapter I at first proofed that the segmentation according to the credit quality can result in a more precise estimation of the capital requirement. It was shown that the low PD classes (containing high quality credits) are relatively more sensitive to the stress scenario. Under an assumption that the portfolio has the most common structure, the capital requirement estimated based on the non-segmented data will be lower that the more precise estimate considering different clients` credit quality.

Further, it was highlighted that when the banks use a percentage shift for stressing their PDs based on the estimate from the model for non-segmented data, instead of applying their own model that links PDs of each segment to the shifts in economic variables in the scenario, the capital requirement estimate will be biased.

It can be concluded that the segmentation according to the credit quality is an important parameter that should be taken into account during the stress tests to get more precise estimates of capital requirement and to better judge the financial stability of an institution. The implementation of segmentation into the stress tests for the whole financial sector is easier under the bottom-up approach. Based on the second part of this chapter, the banks however need to have in place well developed methods to stress test their portfolio based on their own internal models. The PDs for different parts of portfolio should be linked to the development of economic variables under the scenario to get precise estimate of capital requirement.

8. Conclusion

Stress testing in general, considering any type of risk, is defined as a term for describing the various techniques (quantitative or qualitative) used by institutions to gauge their vulnerability to exceptional but plausible events. It is an in-depth analysis of a potential impact of a critical event on the institution's portfolio, financial prosperity and capital needed to withstand such a negative development. Although the credit risk is the most important risk run by the bank, according to many international studies a methodology for its measurement and stress testing is significantly less developed than for other risks. This can be an effect of lower availability of data and special characteristics of the loss distribution for typical credit products.

A more dynamic development of credit risk measurement and stress testing came with the implementation of the new Basel II regulatory framework, which is more sensitive in capital requirement estimation and asks for regular stress testing. This regulation impose many quantitative and qualitative requirements on institution's internal processes and models that must be fulfilled, if the institution wants to apply for an implementation of more advanced methods, which lead to a lower capital requirement estimated.

The stress testing is required under both the first and the second pillar to test whether the minimum capital requirement increased by the additional buffer would be sufficient in adverse conditions. In this sense the stress testing is considered to be a complement to the classical credit risk models that assume estimate the maximum loss that will materialize with some probability over given time period. The stress testing forecast loss under extreme market conditions, which are not covered under the VaR. There already exist theoretical approaches such as extreme value theory that tries to incorporate these two frameworks into one model, in the reality these two methods stay separated.

Within the stress testing program the institution is obliged to at first identify all significant risk factors, build a scenario based on the shifts in these factors, which can be either a sensitivity analysis or a scenario stress test. The former consider movement in only one risk factor, is easier to be performed, but omit the effect of comovement in other risk factors. Under the latter in the hypothetical scenarios the correlations among factors are usually set according to some macroeconomic model or in historical scenarios according to some past crisis to ensure its plausibility. Under scenario analysis the loss distribution under stress can be estimated more precisely.

The shift in risk factors is further translated to the shift in the input parameters of the credit risk model. In this way the stress testing is closely connected to the credit risk model used for capital measurement. Under the first pillar the shifts in risk factors are linked to the shifts in Basel II parameters, but under the second pillar the assumptions on the input variables` settings are relaxed, modified or linked to some explanatory variables.

The most common scenarios consider changes in economic and other external variables, in a behavior of clients or assumptions of the model. The PDs can be for example linked to systematic economic variables through regression model, keeping the dependence of asset correlation on PDs or not. In few papers also the LGD is linked to risk factors as there is evidence that the recovery rates also fluctuates with economic cycle, then it will be logically possible that the PDs and LGDs are correlated if they depend on the development of the same economic variables.

Further the portfolio can be tested for a concentration risk applying the scenario of an increase in exposure to one sector, region or industry which will be downgraded. The migration matrices can be changed for stress testing in two ways, either they can be made conditional on the economic recession and expansion or their elements can be directly linked to explanatory variables and whole matrix can be estimated based on the forecasted economic development.

Once the scenario is translated into changes in the credit risk model input parameters the conditional loss on given scenario can be estimated. The whole process of stress testing ends with an analysis of results on the loss distribution and in case an implementation of corrective measures to forego materialization of such loss.

During the survey of the stress testing practice in the Czech Republic, it was found that the sophistication of stress tests vary considerably. Bigger banks in the sample mostly use hypothetical scenario stress tests, while smaller ones usually run sensitivity analysis, which corresponds to the findings of an international survey by BIS. The bigger banks also run the periodical tests with higher frequency even though the scenarios are more complicated. The time needed for one test depend on the sophistication of the scenario, the rating method used and the coverage of the test.

The stress test methodology seems to differ according to the type of rating method used. Just two banks in the sample that use more advanced methods responded that they use a model that can link the risk parameters to economic cycle. The banks with more advanced methods are also more willing to run stress tests for scenarios prescribed by parent banks by themselves or at least discuss the results and methods applied on the

Czech data in details. The banks with less advanced rating methods (the Foundation or Standardised) more often send data for these stress tests to the parent company. Most of the banks were in some way influenced by the CNB stress testing program, because the received scenarios were often considered to be a trustful forecast of possible future development.

In all the banks the stress testing methods developed considerably during the last years, but this dynamics can be not just an effect of the crisis but also of the implementation of the Basel II regulation. Because in the Czech Republic the Basel II became fully into force at the beginning of 2008 the changes observed in the institutions` stress testing programs can be result of both of these.

Comparing the stress tests run for different levels of portfolio segmentation, I proofed that the bottom-up approach stress tests, if precisely applied by the financial institutions, play a key role in stress testing the financial system.

Based on the real data on corporate segment with different levels of segmentation I at first found significant explanatory variables and then forecasted the PDs for the most actual stress scenarios offered by the CNB. I analyzed the sensitivity of PDs for different levels of segmentation and - assuming a portfolio structure similar to the CNB statistics - I concluded that omission of segmentation can lead to an underestimation of the capital requirement needed under stress.

The CNB offered to the banks an average increase of PDs, which was consistent with the scenarios used for joint stress testing project. This average increase was probably used for computation of capital requirement within this project by the banks that can not link the PDs to macroeconomic variables yet.

Based on this information, I decided to estimate the total minimum capital requirement for the portfolio based on the stressed PDs forecasted by the regression model and compare it to the same measure using as PDs under stress the values at the end of 2009 increased by an average change from the model for non-segmented data. To get comparable results, in both cases I assumed the same portfolio weights that were set very close to the CNB statistics.

In this case I found that application of an average shift in PDs lead to a lower estimate of capital requirement than for the PDs precisely forecasted according to the regression models for each rating class. The bias seems to be the higher the less aggregated the data on the portfolio are used and the less risky the portfolio is.

List of abbreviations

BCBS	Bank Committee on Banking Supervision
BSM	Black-Scholes-Merton
CaR (CVaR)	Credit (Value) at Risk
CAR	Capital adequacy ratio
CNB	Czech National Bank
CR	Capital requirement
DM	Default mode
E	Exposure
EAD	Exposure at default
EC	European Commission
EDF	expected default frequency
EL	Expected loss
EU	European Union
EVT	Extreme Value Theory
FSAP	Financial Stability Assessment Program
FXrates	Foreign exchange rates
GDP	Gross domestic product
IMF	International Monetary Fund
IRB approach	Internal rating based approach
LGD	Loss given default
M	Effective maturity
MTM	Mark-to-market
OLS	Ordinary least square
PD	Probability of default
PRIBOR	Prague Interbank Offer Rate
RR	Recovery rate
S&P	Standard&Poor`s
SRM	Systemic Risk Monitor
UL	Unexpected loss
VaR	Value at Risk
WB	World Bank
WLS	Weighted least square

References

1) Altman E., Resti A. and Sironi A. (2001): Analyzing and Explaining Default Recovery Rates, December 2001
http://www.isda.org/c_and_a/pdf/Analyzing_Recovery_rates_010702.pdf

2) Andersson H., Lindell A. (2008): Risk Tolerance Concepts and Scenario Analysis of Bank Capital, Stress Testing for Financial Institutions, Risk Books, Incisive Media 2008, ISBN 978-1-906348-11-3, pp.399-423

3) Bangia A., Diebold F.X., Kronimus A., Schagen Ch., Schuermann T. (2002): Ratings Migration and the Business Cycle, With Application to Credit Portfolio Stress Testing, Journal of Banking and Finance 26, 2002, pp. 445-474, Elsevier Science B.V.

4) Bank Committee on Banking Supervision (2006): International Convergence of Capital Measurement and Capital Standards, A Revised Framework, Comprehensive Version, June 2006
http://www.bis.org/publ/bcbs128.pdf

5) BIS_CGFS (2005): Stress Testing at Mayor Financial Institutions: Survey Results and Practice, Bank for International Settlement, Report by Working Group of the Committee on the Global Financial system, January 2005, ISBN 92-9197-675-X
http://www.bis.org/publ/cgfs24.pdf?noframes=1

6) BIC (2004): International Convergence of Capital Measurement and Capital Standards, A Revised Framework, Bank Committee on Banking Supervision

7) Bonti G., Kalkbrener M., Lotz Ch., Stahl G. (2005): Credit Risk Concentration under Stress, October 2005
http://www.bis.org/bcbs/events/crcp05bonti.pdf

8) Boss M., Krenn G., Puhr C., Summer M. (2009): An Integrated Approach to Stress-Testing: the Austrian Systemic Risk Monitor (SRM), Stress-testing the Banking System, edited by Mario Quagliariello, Cambridge University Press 2009, pp. 202-237

9) Breuer T., Krenn G. (2000): Identifying Stress Test Scenarios
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.27.118&rep=rep1&type=pdf

10) Breuer T., Krenn G. (2001): What is a Plausible Scenario?, L. Kuncheva et al. (eds.): Computational Intelligence: Methods and Applications, pp. 215-221, ICSC Academic Press

11) CEBS (2006): Technical Aspects of Stress Testing under the Supervisory Review Process – CP12, Committee of European Banking Supervisors, December 2006
http://www.c-ebs.org/getdoc/e68d361e-eb02-4e28-baf8-0e77efe5728e/GL03stresstesting.aspx

12) Cherubini U., Della Lunga G. (1999): Stress Testing Techniques and Value-at-Risk Measures : A unified approach, Milano, July 1999, C. Bovina
http://gloriamundibeta.org/UploadFile/2010-2/cher.pdf

13) Čihák M. (2007): Introduction to Applied Stress Testing, International Monetary Fund Working Paper 07/59, March 2007
http://www.imf.org/external/pubs/ft/wp/2007/wp0759.pdf

14) CNB: Decree No. 123/2007 Coll. as amended by Decree No. 282/2008 Coll.
http://www.cnb.cz/en/legislation/banking_supervision/prudential_rules/contents.html

15) CNB (2008): Financial Market Supervision Report, 2008
http://www.cnb.cz/m2export/sites/www.cnb.cz/en/supervision_financial_market/aggregate_information_financial_sector/financial_market_supervision_reports/download/fms_2008.pdf

16) CNB: Financial Stability Report 2008/2009
http://www.cnb.cz/m2export/sites/www.cnb.cz/en/financial_stability/fs_reports/fsr_2008-2009/FSR_2008-2009.pdf

17) CNB (2007): Financial Stability Report 2007
http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/en/financial_stability/fs_reports/fsr_2007/FSR_2007.pdf

18) CNB (2006): Financial Stability Report 2006
http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/cs/financni_stabilita/zpravy_fs/fs_2006/FS_2006.pdf

19) CNB (2010): Výsledky zátěžových testů bankovního sektoru ČR, February 2010
http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/cs/financni_stabilita/zatezove_testy/2010/zatezove_testy_vysledky_2009_4q.pdf

20) Consultative Paper on Credit Stress Testing, Singapore, Working paper of Monetary authority of Singapore, 2002
http://www.gloriamundi.org/Library_Journal_View.asp?Journal_id=5444

21) Credit Metrics Technical Document: J.P.Morgan, New York, April 1997
http://www.ma.hw.ac.uk/~mcneil/F79CR/CMTD1.pdf

22) Crosbie P., Bohn J. (2003): Modeling Default Risk, Moody's KMV Company, December 2003
http://www.business.uiuc.edu/gpennacc/MoodysKMV.pdf

23) Decree of European Parliament No. 2006/48/EC
http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2006:177:0001:0200:CS:PDF

24) Deutsche Bundesbank (2004): Stress tests at German banks – Methods and Results, Monthly Report, October 2004, Deutsche Bundesbank, pp. 75-84
http://www.bundesbank.de/download/volkswirtschaft/mba/2004/200410mba_en_stresstest.pdf

25) Deutsche Bundesbank (2007): Stress Tests: Methods and Areas of Application. Deutsche Bundesbank Financial Stability Review 2007, pp. 97–112
http://www.bundesbank.de/download/volkswirtschaft/finanzstabilitaetsberichte/financialstabilityreview2007.pdf

26) Financial Service Authority (2005): Stress Testing, Discussion Paper, May 2005
http://www.fsa.gov.uk/pubs/discussion/dp05_02.pdf

27) Frye J. (2000): Collateral Damage, Risk Publications, Vol. 13, No. 14, April 2000
http://www.bis.org/bcbs/events/oslo/frye.pdf

28) Hamerle A., Liebig T., Scheule H. (2004): Forecasting Credit Portfolio Risk, Discussion Paper, Series 2: Banking and Financial Supervision, No 01/2004, Deutsche Bundesbank
http://www.bundesbank.de/download/bankenaufsicht/dkp/200401dkp_b.pdf

29) Jakubík P. (2005): Macroeconomic Credit Risk Model, Financial Stability Report, CNB, 2005, pp. 84-92
http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/en/financial_stability/fs_reports/fsr_2005/FSR_2005.pdf

30) Jakubík P., Schmieder Ch. (2008): Stress Testing Credit Risk, is the Czech Republic different from Germany? Working Paper Series 9, CNB, September 2008
http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/en/research/research_publications/cnb_wp/download/cnbwp_2008_09.pdf

31) Jokivuolle E., Virolainen K., Vähämaa O. (2008): Macro Model-Based Stress Testing of Basel II Capital Requirements, Stress Testing for Financial Institutions, Risk Books, Incisive Media 2008, ISBN 978-1-906348-11-3, pp. 379-399

32) Kim J., Finger C. (2000): A Stress Test to Incorporate Correlation Breakdown, CreditMetrics Journal Volume 1, May 2000, J.P.Morgan Reuters Ltd., pp. 61-80

33) Kafetzaki-Boulamatsis M., Tasche D. (2001): Combined Market and Credit Risk Stress Testing based on the Merton Model, June 2001
http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.21.8821

34) Kalkbrener M., Lotter H., Overbeck L. (2004): Sensible and Efficient Capital Allocation for Credit Portfolios
http://www.risklab.ch/workshops/computational_finance/lecturers/kalkbrenner.pdf

35) Laviola S., Marcucci J., Quagliariello M. (2009): Stress-testing Credit Risk: the Italian Experience, Stress-testing the Banking System, edited by Mario Quagliariello, Cambridge University Press 2009, pp.133-148

36) Longin F.M. (2000): From the Value at risk to Stress Testing the Extreme Value Approach, Journal of Banking & Finance, Volume 24, Issue 7, July 2000, pp. 1097-1130

37) Mager F., Schmieder Ch. (2008): Basel II-Type Stress Testing of Credit Portfolios, Stress Testing for Financial Institutions, Risk Books, Incisive Media 2008, ISBN 978-1-906348-11-3, pp. 423-443

38) Nickell P., Perraudin W., Varotto S. (2001): Stability of Ratings Transitions, Bank of England Working Paper, 2001
http://www3.imperial.ac.uk/pls/portallive/docs/1/43680.PDF

39) Quagliariello M.(2009): Macroeconomic Stress-testing: Definitions and Main Components, Stress-testing the Banking System, edited by Mario Quagliariello, Cambridge University Press 2009, pp. 18-35

40) Rodriguez A., Trucharte C. (2007): Loss Coverage and Stress Testing Mortgage Portfolios: A Non-Parametric Approach, 2007
http://www.bis.org/bcbs/events/rtf07rodriguez.pdf

41) Rösch D., Scheule H. (2004): Forecasting Retail Portfolio Credit Risk, Journal of Risk Finance, 5 (2), pp. 16-32
http://www.econbiz.de/archiv1/2009/97145_forecasting_credit_risk.pdf

42) Rösch D., Scheule H. (2007): Stress Testing Credit Risk Parameters – An Application to Retail Loan Portfolio, Journal of Risk Model Validation, Vol. 1, No. 1, (Spring 2007), pp. 55-75.
http://www.melbournecentre.com.au/workingpapers/Stress_Testing_Credit_Risk_Parameters_MCFS.pdf

43) Saunders A., Allen L. (2002): Credit Risk Measurement: New Approaches to Value at Risk and Other Paradigms, Second Edition, John Wiley & Sons, Inc., New York, 202, ISBN 0-471-21910-X

44) Saunders A., Allen L. (2003): A Survey of Cyclical Effects in Credit Risk Measurement, BIS Working paper 126, 2003
http://www.bis.org/publ/work126.pdf?noframes=1

45) Schachter B. (2001): How Well Can Stress Test Complement VaR, January 2001
http://www.gloriamundi.org/UploadFile/2010-2/stressandevt1.pdf

46) Servigny A., Renault O. (2002): Default Correlation: Empirical Evidence, October 2002

http://www3.imperial.ac.uk/portal/pls/portallive/docs/1/568015.PDF

47) Soberhart J. (2008): Uncertainty, Credit Migration, Stress Scenarios and Portfolio Losses, Stress Testing for Financial Institutions, Risk Books, Incisive Media 2008, ISBN 978-1-906348-11-3, pp.197-237

Internet sources

www.imf.org

http://www.imf.org/external/np/fsap/fsap.asp#C

www.bis.org

www.cnb.cz

www.oanda.com

Appendix 1: Questions to the survey on loan book credit risk measurement and stress testing in Czech banks

Questions considering an area of credit risk quantification:

1) How would you characterize your portfolio exposed to credit risk? (your loan book, trading book) Do you specialize on some specific clients/product etc.?

2) What kind of rating method do you use? (Quantitative/qualitative, what do you consider and how do you estimate rating of a client) Do you use Standardised, IRB foundation or IRB advanced method? Which risk parameters do you estimate by internal models?

3) Could you describe the link between credit ratings and risk parameters (LGD, PD, CF)? How long time series do you use for calibration of the parameter values and are they backtested? (How and how often?)

4) How are these parameters further used for minimum required capital estimation, economic capital estimation etc.?

5) What kind of model do you use for economic capital estimation?

Questions to credit risk stress testing:

1. What kind of scenarios do you use for stress testing credit risk and why?
   Are some of them prescribed by regulator, by parent company or are they based on your own analysis only?

2. Which risk factors do you consider important for your portfolio or its segments?
   Are your stress tests based on
   - economic variables changes (macro changes, industry/region downturn, changes in industry volatility)
   - changes in risk parameters (PD, LGD)
   - changes in portfolio (holding period, changes in correlations, transition matrices)

3. Do you stress test whole loan book or do you prefer to apply stress tests on separate parts of portfolio?

4. How many scenarios would you define as sensitivity analysis (change just in one risk factor) and how many are scenario tests (simultaneous change in more risk factors) in percentage? Do you use scenario tests based on historical events (and which historical stress events do you consider relevant)? Do you use hypothetical scenarios? Do you run tests for key decision of management on new products, policy changes etc.?

5. How do you decide about the magnitude of changes in risk factors for hypothetical scenarios? (based on historical changes, on experts' forecasts, on some macro models….)?

6. Did your methods/scenarios/models/importance of stress testing changed considerably since this financial crisis started?

7. What kind of link do you use between defined changes in risk factors in scenario and risk parameters to set their value under stress correctly?

8. How often do you run regular stress tests? How long does it take on average to run one test (from defining scenario till the report with results)? How often do you change (update) scenarios for stress testing?

9. How often do you present the results of stress tests to management and which actions could be taken based on the results?

Appendix 2: Complete results of tests for regression models from chapter 7

1) Test for model for aggregated data on PD for whole corporate

Durbin-Watson statistic = 2,38482
multicolinearity test

Values > 10.0 may indicate a collinearity problem
27) real_rate_3M 3,513
45) CZK_EURb_4 4,540
46) corp_loans__4 5,114
42) ind_prices 3,967

White's test for heteroskedasticity

Test statistic: TR^2 = 11,055375,
with p-value = P(Chi-square(14) > 11,055375) = 0,681680

normal residuals - Test for null hypothesis of normal distribution:

Chi-square(2) = 1,483 with p-value 0,47634

Breusch-Godfrey test for autocorrelation up to order 4

Test statistic: LMF = 1,834637,
with p-value = P(F(4,7) > 1,83464) = 0,227
Alternative statistic: TR^2 = 8,188887,
with p-value = P(Chi-square(4) > 8,18889) = 0,0849
Ljung-Box Q' = 2,94456 with p-value = P(Chi-square(4) > 2,94456) = 0,567

2) Tests for regression model for investment part of portfolio

Durbin-Watson statistic = 2,66126
colinearity

Values > 10.0 may indicate a collinearity problem
27) real_rate_3M 1,188
45) CZK_EURb_4 2,667
46) corp_loans__4 2,387

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present
Test statistic: LM = 13,2581
with p-value = P(Chi-Square(9) > 13,2581) = 0,151273

Test for normality of residual -

Null hypothesis: error is normally distributed
Test statistic: Chi-square(2) = 2,65089
with p-value = 0,265685

Breusch-Godfrey test for autocorrelation up to order 4

Test statistic: LMF = 2,594700,
with p-value = P(F(4,8) > 2,5947) = 0,117
Ljung-Box Q' = 5,92118 with p-value = P(Chi-square(4) > 5,92118) = 0,205

3) Tests for regression for sub-investment part of portfolio

Durbin-Watson statistic = 2,34718
collinearity

Values > 10.0 may indicate a collinearity problem
27) real_rate_3M 3,513
45) CZK_EURb_4 4,540
46) corp_loans__4 5,114
42) ind_prices 3,967

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present
Test statistic: LM = 11,2125
with p-value = P(Chi-Square(14) > 11,2125) = 0,669268

Test for normality of residual -

Null hypothesis: error is normally distributed
Test statistic: Chi-square(2) = 1,53255
with p-value = 0,464741

LM test for autocorrelation up to order 4 -

Null hypothesis: no autocorrelation
Test statistic: LMF = 1,8604
with p-value = P(F(4,7) > 1,8604) = 0,222275

4) Model for non-aggregated data – PDs for each rating class as panel data

Breusch-Pagan test for diagonal covariance matrix: (based on OLS estimate)

Chi-square(78) = 175,376 with p-value 0,0000

WLS method

autocorrelation test

Equation 1:
Ljung-Box Q' = 10,4874 with p-value = P(Chi-square(4) > 10,4874) = 0,033
Equation 2:
Ljung-Box Q' = 4,21425 with p-value = P(Chi-square(4) > 4,21425) = 0,378
Equation 3:
Ljung-Box Q' = 4,2367 with p-value = P(Chi-square(4) > 4,2367) = 0,375
Equation 4:
Ljung-Box Q' = 4,82708 with p-value = P(Chi-square(4) > 4,82708) = 0,306
Equation 5:
Ljung-Box Q' = 3,22715 with p-value = P(Chi-square(4) > 3,22715) = 0,521
Equation 6:
Ljung-Box Q' = 1,55863 with p-value = P(Chi-square(4) > 1,55863) = 0,816
Equation 7:
Ljung-Box Q' = 10,5157 with p-value = P(Chi-square(4) > 10,5157) = 0,0326
Equation 8:
Ljung-Box Q' = 4,88051 with p-value = P(Chi-square(4) > 4,88051) = 0,3
Equation 9:
Ljung-Box Q' = 5,14776 with p-value = P(Chi-square(4) > 5,14776) = 0,272
Equation 10:
Ljung-Box Q' = 6,90357 with p-value = P(Chi-square(4) > 6,90357) = 0,141
Equation 11:
Ljung-Box Q' = 2,33826 with p-value = P(Chi-square(4) > 2,33826) = 0,674
Equation 12:
Ljung-Box Q' = 10,3691 with p-value = P(Chi-square(4) > 10,3691) = 0,0346
Equation 13:
Ljung-Box Q' = 9,81969 with p-value = P(Chi-square(4) > 9,81969) = 0,0436

test for ARCH effects of order 4

Equation 1:
Null hypothesis: no ARCH effect is present
Test statistic: LM = 8,77165
with p-value = P(Chi-Square(4) > 8,77165) = 0,0670675
Equation 2:
Test statistic: LM = 0,24951
with p-value = P(Chi-Square(4) > 0,24951) = 0,992836
Equation 3:
Test statistic: LM = 0,504241
with p-value = P(Chi-Square(4) > 0,504241) = 0,973087
Equation 4:
Test statistic: LM = 0,336976
with p-value = P(Chi-Square(4) > 0,336976) = 0,987304
Equation 5:

Test statistic: LM = 2,58376
with p-value = P(Chi-Square(4) > 2,58376) = 0,629703
Equation 6:
Test statistic: LM = 2,37995
with p-value = P(Chi-Square(4) > 2,37995) = 0,666254
Equation 7:
Test statistic: LM = 3,8675
with p-value = P(Chi-Square(4) > 3,8675) = 0,424235
Equation 8:
Test statistic: LM = 9,27797
with p-value = P(Chi-Square(4) > 9,27797) = 0,0545149
Equation 9:
Test statistic: LM = 5,77588
with p-value = P(Chi-Square(4) > 5,77588) = 0,216523
Equation 10:
Test statistic: LM = 2,10442
with p-value = P(Chi-Square(4) > 2,10442) = 0,71656
Equation 11:
Test statistic: LM = 0,645542
with p-value = P(Chi-Square(4) > 0,645542) = 0,957871
Equation 12:
Test statistic: LM = 4,95361
with p-value = P(Chi-Square(4) > 4,95361) = 0,29209
Equation 13:
Test statistic: LM = 1,29733
with p-value = P(Chi-Square(4) > 1,29733) = 0,861829

Normality of residuals

Test for multivariate normality of residuals
Doornik-Hansen Chi-square(26) = 26,1473, with p-value = 0,455034

5) Model for prediction the ind_prices

OLS estimates Dependent variable: ind_prices

	coefficient	std. error	t-ratio	p-value	
const	70,7351	7,21531	9,803	2,26E-07	***
CZK_EURb_2	720,667	185,546	3,884	0,0019	***
real_GDP_gr_4	89,8726	12,7305	7,060	8,55E-06	***

Mean of dependent variable = 101,894
Standard deviation of dep. var. = 2,76971
Sum of squared residuals = 23,6159
Standard error of the regression = 1,34781
Unadjusted R-squared = 0,79477
Adjusted R-squared = 0,76319
F-statistic (2, 13) = 25,1716 (p-value = 3,39e-005)
Durbin-Watson statistic = 1,28933
First-order autocorrelation coeff. = 0,32233
Log-likelihood = -25,8177
Akaike information criterion (AIC) = 57,6353
Schwarz Bayesian criterion (BIC) = 59,9531
Hannan-Quinn criterion (HQC) = 57,754

Breusch-Pagan test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 0,841197
with p-value = P(Chi-Square(2) > 0,841197) = 0,656654

Test for normality of residual -

Null hypothesis: error is normally distributed
Test statistic: Chi-square(2) = 0,699044
with p-value = 0,705025

LM test for autocorrelation up to order 4 -

Null hypothesis: no autocorrelation
Test statistic: LMF = 1,18682
with p-value = P(F(4,9) > 1,18682) = 0,379603