

CHARLES UNIVERSITY IN PRAGUE  
FACULTY OF SOCIAL SCIENCE  
INSTITUTE OF ECONOMIC STUDIES

## BACHELOR THESIS

**Fish Wars: Dynamic Externality in Fishing**

**Author:** Tomáš Fiala

**Supervisor:** PhDr. Martin Gregor, Ph.D.

**Academic Year:** 2009/2010

## Declaration

I declare that I wrote this thesis myself and used only the literature listed in References.

In Prague, date \_\_\_\_\_

\_\_\_\_\_  
signature

## **Acknowledgments**

I would like to thank PhDr. Martin Gregor, Ph.D. for his support and valuable comments.

## Abstract

The dramatic state of world fish stock is often attributed to the open-access nature of fishing grounds. In this thesis we investigate the consequences of unrestricted access to fisheries by adopting game theoretic framework. We describe the situation of fish appropriation by dynamic model and find some of its Nash equilibria. We show that one of the possible results of the nonexclusive nature of fisheries is overexploitation. Moreover, we find that other outcomes are possible as well. The tragedy of commons is, thus, not inevitable.

**Title:** Fish Wars: Dynamic Externality in Fishing  
**Author:** Tomáš Fiala  
**Supervisor:** PhDr. Martin Gregor, Ph.D.  
**Academic year:** 2009/2010

## Abstrakt

Dramatický stav světové populace ryb je často připisován volnému přístupu k oblastem rybolovu a neomezenému právu v nich lovit. V této práci se zabýváme důsledkem tohoto neomezeného přístupu z hlediska teorie her. Popisujeme tuto situaci pomocí dynamického modelu a najdeme některé Nashovy rovnováhy tohoto modelu. Zjistíme, že jedním z možných důsledků volného rybolovu je přílišný lov. Zároveň ale popíšeme další Nashovy rovnováhy, které vedou k jiným následkům a ukážeme tak, že zničení zdroje není nevyhnutelné.

**Název práce:** Dynamická externalita při rybolovu  
**Autor:** Tomáš Fiala  
**Vedoucí práce:** PhDr. Martin Gregor, Ph.D.  
**Akademický rok:** 2009/2010

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Dynamic Games and Solution Concepts</b>	<b>3</b>
<b>3</b>	<b>Dynamic Resource Extraction Model</b>	<b>7</b>
3.1	The Model . . . . .	7
3.1.1	The General Framework . . . . .	7
3.1.2	The Example of Levhari & Mirman (1980) . . . . .	8
3.2	Cooperative Solution . . . . .	9
3.2.1	Single Person Dynamic Optimization . . . . .	9
3.2.2	Symmetric Pareto Optimal Solution . . . . .	10
3.3	Nash Equilibria in Open-loop Strategies . . . . .	12
3.3.1	Open Loop Strategies . . . . .	12
3.3.2	Derivation of Open-loop Solution . . . . .	13
3.4	Nash Equilibria in Feedback Strategies . . . . .	16
3.4.1	Feedback Strategies . . . . .	16
3.4.2	Nash Equilibrium in Linear Feedback Strategies . . . . .	17
3.4.3	Discontinuous Equilibrium . . . . .	21
3.5	Equilibria in History-dependent Strategies . . . . .	24
3.6	Resource Depletion . . . . .	26
3.6.1	Linear Growth Function . . . . .	26
3.7	Summary . . . . .	27
<b>4</b>	<b>Empirics and Experiments</b>	<b>30</b>
4.1	Evidence from International Fishing . . . . .	31
4.2	Experiments . . . . .	32
4.3	Local Shared Fisheries . . . . .	34

4.4 Summary . . . . .	37
<b>5 Conclusion</b>	<b>39</b>
<b>References</b>	<b>42</b>

# Chapter 1

## Introduction

Gradual destruction of the world fish resources is a well-documented problem. Recent effort to stop fishing of Bluefin Tuna <sup>1</sup> illustrates the severity of the problem. One of the causes, that Bluefin Tuna is nearly depleted, is that it is fished on high seas, where any fisher is free to fish it. Unrestricted access is common feature of many fishing grounds. Even if the fisheries are situated in an exclusive economic zone or inshore, they are often shared by a large number of fishers or open for all, because the exclusion is difficult to enforce.

It is widely accepted fact in economic theory that in the absence of clear-cut property rights assigned to individual decision-makers, the property tends to be used inefficiently. Indeed, this is the case of fisheries and the impossibility to assign such property rights might be a significant factor in deterioration of the stock.

Hardin (1968) identified a common cause of the inefficient use of shared property. He uses a parable of herders and a shared pasture. In the parable, the herders overuse the pasture, because they do not properly internalize the costs that they cause by their actions, and the improper internalization of the externalities associated with using the resource leads in turn to the tragedy of commons, deterioration of the pasture. He argues that there are several examples of commons, including fisheries.

The nature of fisheries, however, differs from the Hardin's pasture. While the pasture does not renew itself in time, the stock of fish does. The essential property of renewable resource is that it grows in time. Hence, we are concerned, whether we can expect the same result in this dynamic setting. To answer the question we adopt the framework of

---

<sup>1</sup>More about the devastation of Bluefin Tuna can be found, for example, in *The Economist* article *Managed to Death*, from October 30th 2008, available at: [http://www.economist.com/science-technology/displaystory.cfm?story\\_id=12502783&source=login\\_payBarrier](http://www.economist.com/science-technology/displaystory.cfm?story_id=12502783&source=login_payBarrier).

dynamic game theory and investigate a model of fishing first introduced by Levhari & Mirman (1980).

The model has two basic features. As we stated, the first feature is the presence of underlying state variable, which represents the stock of fish. The stock of fish grows with decreasing marginal growth. The second one is the strategic aspect. As there are more players and all extract from the stock, each appropriator must account for the actions of others.

We consider  $N$ -player version of the original 2 player model of Levhari & Mirman (1980). We identify several expected outcomes, Nash equilibria, for different assumptions on how much information players possess. We find, that the tragedy of commons is one of the possible outcomes, but the underlying state variable and repeated encounter of the players might produce other outcomes. Especially if the players observe other players' actions and use threats they might secure themselves better outcome. Moreover, if the players have the minimum information (only the pre-extraction level of the stock), the outcome is Pareto optimal. In addition, the model is quite general and could describe the problem of exploitation of other renewable common-pool resources as well.

Given the devastating effects that the tragedy of commons outcome has, we investigate, whether overfishing happens in real world. We found empirical and experimental evidence that the tragedy of commons in the dynamic setting of fishery takes place, although efficient outcomes are documented as well.

The thesis is organized as follows. In the second chapter we define basic concepts from dynamic game theory. In the third chapter we analyze the model and find several Nash equilibria. Fourth chapter is concerned with empirical evidence and the results of experimental testing. The last chapter concludes.

# Chapter 2

## Dynamic Games and Solution Concepts

In this chapter we define basic concepts, which we will use later. This chapter is based on Basar & Olsder (1995), all the definitions are taken from the book and modified for infinite-horizon case. First, we define a dynamic game suitable for our purposes.

**Definition 2.1 (Dynamic Game):** An N-person discrete-time, deterministic game with infinite horizon and stage-additive payoff function involves:

- An index set  $\mathbf{I} = \{1, \dots, N\}$  called the player's set.
- A set  $X \subset \mathbf{R}$ , called the state set of the game, to which the state  $(x_t)$  of the game belongs for all  $t \in \mathbf{N}$ .
- A set  $C_t^i \subset \mathbf{R}$ , defined for each  $t \in \mathbf{N}$  and each  $i \in \mathbf{I}$ , which is called action set of player  $i$  at stage  $t$ . Its elements are the permissible actions  $c_k^i$  of player  $i$  at stage  $t$ .
- A function  $f : X \times C_t^1 \times \dots \times C_t^N \mapsto X$ , so that  $x_{t+1} = f(x_t, c_t^1, \dots, c_t^N)$ ,  $t \in \mathbf{N}$  for some  $x_1 \in X$  which is called the initial state of the game. This difference equation is called the state equation of the dynamic game, describing the evolution of the underlying decision process.
- A finite set  $\eta_t^i \subset N_t^i$ , defined for each  $t \in \mathbf{N}$  and  $i \in \mathbf{I}$  as a subset of  $\{x_1, \dots, x_t, c_1^1, \dots, c_{t-1}^1, \dots, c_1^N, \dots, c_{t-1}^N\}$ , which determines the information gained and recalled by player  $i$  at stage  $t$  of the game (the information set). Specification of  $\eta_t^i$  for all  $t \in \mathbf{N}$  characterizes the information structure (pattern) of player  $i$ ,

and the collection (over  $i \in \mathbf{I}$ ) of these information structures is the information structure of the game.

- A prespecified class  $\Gamma_t^i$  of functions  $g_t^i : N_t^i \mapsto C_t^i$ , which are the permissible strategies of player  $i$  at stage  $t$ . The aggregate mapping  $g_i = \{g_t^i\}_{t=1}^\infty$  is a strategy for player  $i$  in the game, and the class  $\Gamma^i$  of all such functions so that  $g_t^i \in \Gamma_t^i, t \in \mathbf{N}$  is the strategy space of player  $i$ .
- An instantaneous payoff function  $u_t^i : X \times C_t^1 \times \dots \times C_t^N \mapsto \mathbf{R}$  defined for each  $t \in \mathbf{N}$  and each  $i \in \mathbf{I}$ , and discount factor  $\delta \in (0, 1)$ . Then the total stage-additive payoff of player  $i$  with players with strategies  $\{g_1, \dots, g_N\}$  is given by a functional  $W_i(g^1, \dots, g^N) = \sum_{t=1}^\infty \delta^{t-1} u_t^i(x_t, g_t^1(\eta_t^1), \dots, g_t^N(\eta_t^N), x_{t+1})$ .

Later we will refer to it as dynamic game. Below, we define information structures.

**Definition 2.2:** In a dynamic game we will say that player's  $i$  information structure is :

- an open-loop pattern if  $\eta_t^i = \{x_1\}$ ,
- a closed-loop feedback information pattern if  $\eta_t^i = \{x_t\}$ ,
- a closed-loop information pattern if  $\eta_t^i = \{x_1, \dots, x_t\}$ ,
- a full information pattern if  $\eta_t^i = \{x_1, \dots, x_t, c_1^1, \dots, c_{t-1}^1, c_1^N, \dots, c_{t-1}^N\}$ .

Below, we will define best response. It is the strategy, with which player  $i$  can secure himself the biggest payoff, when he takes his opponents strategies as given.

**Definition 2.3 (Best Response):** The strategy  $g_i^* \in \Gamma^i$  is best response of player  $i$  to other player's strategies  $g_j \in \Gamma^j, j \neq i$  if for all  $g_i \in \Gamma^i$  and all initial states  $x_1 \in X$  holds:

$$W_i(g_1, \dots, g_i^*, \dots, g_N) \geq W_i(g_1, \dots, g_i, \dots, g_N) \quad \blacktriangleright$$

In the following chapter we will be concerned mostly with describing the Nash equilibria. Below, we give the definition and then briefly discuss the meaning.

**Definition 2.4 (Nash Equilibrium):** A Nash equilibrium is a  $N$ -tuple of strategies  $(g_1^*, \dots, g_N^*)$ , such that for all  $i \in \mathbf{I}$  strategy  $g_i^*$  is a best response to other player's strategies  $g_j \in \Gamma^j, j \neq i$ , i. e. if for all  $i \in \mathbf{N}$ , all  $g_i \in \Gamma^i$  and all initial states  $x_1 \in X$  holds:

$$W_i(g_1^*, \dots, g_i^*, \dots, g_N^*) \geq W_i(g_1^*, \dots, g_i, \dots, g_N^*) \quad \blacktriangleright$$

If the strategies are in equilibrium, each player maximizes his payoff while taking the strategies of other players as given. When in equilibrium, no player has an incentive change his action, because he cannot raise his payoff by unilateral deviation from the equilibrium. Then, if the assumptions of our model are correct, players behave rationally and have perfect information about the structure of the game, we should not expect the outcome of the game that is not Nash equilibrium. Such an outcome would not be stable and would change quickly, as we would always find a player that would benefit from changing his action.

The concept of Nash equilibrium can be refined by adding additional requirements. The concept below reflects the idea, that if players believe that the other players' potential actions that would come after deviation from equilibrium are not credible, then the Nash equilibrium might collapse. A player would not believe the other players' action ,if they would would reduce their payoffs by playing it. Putting it differently, player cannot be deterred by threats, that would harm the threatening players as well. Such deterrence would not work, since the player would not consider it credible. Nash equilibrium with such threats could not be sustained. Thus, we come with refinement of Nash equilibrium that allows for credible threats only. We call it time consistency, since it requires, that the player would not wish to change his strategy during the game, later in the time.

We first introduce notation. Let  $D(\Gamma)$  be a dynamic game as defined in 2.1, where  $\Gamma$  is the product of strategy spaces  $\Gamma^i$ . Furthermore, let

$$g_{\{s,\dots,r\}} \in \Gamma_{\{s,\dots,r\}} \quad g_{\{s,\dots,r\}}^i \in \Gamma_{\{s,\dots,r\}}^i$$

denote the truncations of  $\Gamma$  and  $\Gamma^i$  to the case  $t \in \{s, \dots, r\}$ . Then, let:

$$D_{\{s,\dots,r\}}^\beta \equiv D(\{g \in \Gamma : g_{\{1,\dots,s-1\}} = \beta_{\{1,\dots,s-1\}}, g_{\{r+1,\dots\}} = \beta_{\{r+1,\dots\}}, g_{\{s,\dots,r\}} \in \Gamma_{\{s,\dots,r\}}\})$$

denote a version of  $D(\Gamma)$  where strategies for stages  $t \in \{1, \dots, s-1\}$  and  $t \in \{r+1, \dots\}$  are fixed as  $\beta_{\{1,\dots,s-1\}}$  and  $\beta_{\{r+1,\dots\}}$ . Now we can define weakly and strongly time consistent Nash equilibrium:

**Definition 2.5 (Weakly Time Consistent Nash Equilibrium):** A Nash equilibrium  $g^* \in \Gamma$  is weakly time consistent if its truncation  $g_{\{s,\dots\}}^*$  is Nash equilibrium in truncated game  $D_{\{s,\dots\}}^{g^*}$  for all  $s \in \mathbf{N}$ . If the equilibrium is not weakly consistent, then it is inconsistent. ▶

**Definition 2.6 (Strongly Time Consistent Nash Equilibrium):** A Nash equilibrium  $g^* \in \Gamma$  is strongly time consistent if its truncation  $g_{\{s,\dots\}}^*$  is Nash equilibrium in truncated game  $D_{\{s,\dots\}}^\beta$  for all  $\beta_{\{1,\dots,s-1\}} \in \Gamma_{\{1,\dots,s-1\}}$  and all  $s \in \mathbf{N}$ . ▶

Strong time consistency implies weak time consistency. The strong time consistency is different in that the optimal strategy must solve the truncated game, which starts in states that can be reached by any strategy, while the weak time consistency requires only that the Nash equilibrium must solve the truncated game that starts only from states lying on equilibrium path, i.e. is reached by equilibrium strategies. Lastly, we define several classes of strategies.

**Definition 2.7:** We say that strategy for player  $i$  is

- feedback if it is for all  $t$  a function  $g : X \mapsto U_t^i$ , so that the action in stage  $t$  depends only on current state  $x_t$ ,
- open-loop if it is given by functions  $g_t : X \mapsto U_t^i$ , so that the actions in all stages depends only on initial state  $x_1$ ,
- history-dependent if it is given by function  $g : X^t \times U_1^1 \times \dots \times U_{t-1}^1 \times U_1^N \times \dots \times U_{t-1}^N \mapsto U_t^i$ , so that the action in stage  $t$  depends on  $\{x_1, \dots, x_t, c_1^1, \dots, c_{t-1}^1, c_1^N, \dots, c_{t-1}^N\}$ .

In next chapter I will use these concepts to present a dynamic game that describes the use of renewable natural resources and characterize some Nash equilibria.

# Chapter 3

## Dynamic Resource Extraction Model

In the following chapter we present a model of exploitation of renewable natural resources (e.g. fisheries, forests). This model was first introduced by Levhari & Mirman (1980), who found a Nash equilibrium in feedback strategies and cooperative solution. Their solution finds that noncooperative use of renewable natural resource leads to the tragedy of commons, the resource is overexploited. There exist, however, other Nash equilibria, that lead to different outcome. We present the results concerning the original model of Levhari and Mirman, comment on them and compare them with respect to the information available to the players. Similar task was done in Amir & Nannerup (2004), but for less general case of two players. Moreover, we focus on the intuitive explanation of the results. By using slightly different approach, one can explain more intuitively why the inefficiencies occur when players do not cooperate.

### 3.1 The Model

#### 3.1.1 The General Framework

We present the general version of the model, similar to the one studied in Dutta & Sundaram (1993). There are  $N$  players and a single resource, which players either consume or invest in next periods. At the beginning of period  $t$  there is a stock of the resource available, denote it  $x_t$ . There is a given initial state  $x_1$  in the first period. Players decide how much they consume each period, the rest is invested in future consumption. Denote consumption of player  $i$  at period  $t$  by  $c_{it}$ . There is a growth function  $f : \mathbf{R}_+ \mapsto \mathbf{R}_+$ , which describes the replication of the resource. After players consume part

of the stock, the rest is transformed into the next period stock  $t + 1$  by the following relation  $x_{t+1} = f(x_t - \sum_{i=1}^N c_{it})$ . Then, the players decide on their consumption again with stock  $x_{t+1}$  and the process is repeated at every stage. If the consumption is infeasible ( $\sum_{i=1}^N c_{it} > x_t$ ), we assume that the players receive equal share of the stock. We will be concerned mostly with interior solutions, however. Moreover we assume that function  $f$  is continuously differentiable, increasing and strictly concave,  $f(0) = 0$ ,  $\lim_{x \rightarrow 0} f'(x) = \infty$  and  $\lim_{x \rightarrow \infty} f'(x) = \infty$ .

The instantaneous utility of each player  $i$  is given by utility function  $u(c_t) : \mathbf{R}_+ \mapsto \mathbf{R}$  and total utility is given by discounted sum  $\sum_{t=1}^{\infty} \delta^t u(c_t)$ , where  $\delta$  is discount factor ( $0 < \delta < 1$ ). Each player wishes to maximize his total utility given the actions of others. We assume that function  $u(c)$  is continuously differentiable, increasing, strictly concave and  $\lim_{c \rightarrow 0} u'(c) = \infty$ . The assumptions ensure that there is a interior Pareto optimal solution and positive steady state, if the growth is uninterrupted (Stokey et al., 2004, sect. 6.1).

### 3.1.2 The Example of Levhari & Mirman (1980)

Levhari and Mirman studied an example of the model outlined above. In this chapter we will be mostly concerned with the  $N$ -player version of this example. They consider exponential growth function:

$$f(x) = x^\alpha, 0 < \alpha < 1 \quad (3.1)$$

The growth is strictly concave function. If we take fishes as an example of the resource, we can justify the shape of the function by intuition. With small population of the fish, the overall stock of fish is growing, because the more fishes there are, the more offspring they have. The rate of growth is, however, decreasing, because the more there are fishes in the sea, the harder it is for them to find food. Eventually there could be a point where the effect from overpopulation offsets the effect of reproduction, and the stock begins to decrease. At this point the uninterrupted growth would lead to steady state, in the case of our exponential growth the steady state is  $x_{t+1} = x_t = 1$  (we normalize the steady state to 1).

We consider discount factor  $\delta$  and symmetric logarithmic instantaneous utility func-

tion. The total utility of each player to be maximized is given as:

$$\sum_{t=1}^{\infty} \delta^t \log(c_{it}), \quad 0 < \delta < 1 \quad (3.2)$$

Since logarithmic function is not defined in 0, we cannot have corner solutions. Players cannot consume the whole stock, since then in the next period the game would not be defined. The logarithmic function tends to  $-\infty$  as  $t \rightarrow \infty$ . Therefore, there is always a consumption level that produces arbitrarily low utility. This property ensures that the solution will be interior, since players would not want to come near to zero consumption.

This is clearly not entirely realistic, since the model does not allow for resource depletion, although it has occurred several times throughout history. In section 3.6 we will slightly modify our assumptions and examine a case with linear growth function. We will see that the modification can lead to case in which the stock would converge to 0.

## 3.2 Cooperative Solution

### 3.2.1 Single Person Dynamic Optimization

First we derive necessary condition for optimum in the general resource exploitation game for just one player. Later we will use the result to derive other type of equilibria. We will follow section 4.5 in Stokey et al. (2004). We want to solve:

$$\begin{aligned} & \arg \max_{\{c_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \delta^t u(c_t) \\ \text{s.t.} \quad & x_t > c_t > 0 \\ & x_{t+1} = f_t(x_t - c_t) \end{aligned} \quad (3.3)$$

Since  $c_t = x_t - f^{-1}(x_{t+1})$ , we can reformulate the problem, so we can use the results in Stokey et al. (2004). We have equivalent problem, in which we look for:

$$\begin{aligned} & \arg \max_{\{x_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \delta^t u(x_t - f_t^{-1}(x_{t+1})) \\ \text{s.t.} \quad & x_t - f^{-1}(x_{t+1}) > 0 \\ & x_t > 0 \end{aligned} \quad (3.4)$$

If function  $u$  and for all  $t$  the function  $f_t$  are strictly concave, increasing, and continuously differentiable on  $\mathbf{R}_+$ , then function  $u(x_t - f_t^{-1}(x_{t+1}))$  is strictly concave, increasing,

and continuously differentiable in both arguments as well. Then, if  $x_{t+1}^*$  is the interior solution the first order conditions must hold for all  $t$ :

$$\delta^t u'(x_t^* - f_t^{-1}(x_{t+1}^*)) \frac{1}{f_t'(f_t^{-1}(x_{t+1}^*))} - \delta^{t+1} u'(x_{t+1}^* - f_t^{-1}(x_{t+2}^*)) = 0 \quad (3.5)$$

Rearranging and substituting  $x_t^* - f_t^{-1}(x_{t+1}^*) = c_t^*$  we arrive at Euler equation:

$$u'(c_t^*) = \delta u'(c_{t+1}^*) f_t'(x_t^* - c_t^*) \quad (3.6)$$

The equation 3.6 gives us the necessary condition for optimum, moreover if the transversality condition  $\lim_{t \rightarrow \infty} \delta^t u'(c_t^*) x_t^* = 0$  is satisfied, then by the theorem 4.15 in Stokey et al. (2004) the conditions are sufficient as well.<sup>1</sup>

One can find intuitive interpretation for the expression above. It shows that in the interior optimum the marginal increase in utility from raising consumption is equal to the marginal effect on discounted utility in next period, caused by increased consumption in previous period. The left hand side of the equation is something like the marginal present value of opportunity cost in the next period, caused by increased consumption this period. If the left hand side of the equation was higher (lower) in any period, it would be always better action to consume more (less) in that period. In the subsections 3.2.2 and 3.3.2 we use the Euler equation 3.6 to derive the solutions for our model. We use unified approach, which is simpler than the approach used in the original articles, to derive several types of Nash equilibria for  $N$ -player version of the Levhari & Mirman (1980) model.

### 3.2.2 Symmetric Pareto Optimal Solution

The symmetric Pareto optimal solution was found found by Levhari & Mirman (1980) for two-player case and later by Okuguchi (1981) for  $N$ -player case. Okuguchi used the technique of dynamic programming, he derived first the optimal consumption for finite horizon problem and then found the solution to infinite-horizon problem by letting the horizon tend to infinity. Here I use different approach and employ the Euler equation.

Cooperating players maximize the sum of their utilities. We will look for symmetric

---

<sup>1</sup>The theorem in Stokey et al. (2004) involves a case with only  $f$ , which is not dependent on time. The proof, however, is unaffected by this fact, hence, we can use it in our case as well.

solution:

$$\begin{aligned} & \arg \max_{\{c_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \delta^t N u \left( \frac{1}{N} c_t \right) \\ \text{s.t. } & x_t > c_t > 0 \\ & x_{t+1} = f_t(x_t - c_t) \end{aligned} \quad (3.7)$$

Then for the interior solution it follows from 3.6 that for all  $t$ :

$$\frac{1}{N} u' \left( \frac{1}{N} c_t^* \right) = \frac{1}{N} \delta u' \left( \frac{1}{N} c_{t+1}^* \right) f'(x_t^* - c_t^*) \quad (3.8)$$

We solve it for the example of section 3.6, for logarithmic utility and exponential growth we get:

$$\frac{N}{c_t^*} = \delta \alpha \frac{N}{c_{t+1}^*} (x_t^* - c_t^*)^{\alpha-1} \quad (3.9)$$

We then assume that optimal consumption is stationary and linear in the current stock (i.e. that  $c_t^* = \lambda x_t^*$ ). Then the consumption in next period is given by  $c_{t+1}^* = \lambda((1-\lambda)x_t^*)^\alpha$ . We plug it into 3.9 and find that the optimal consumption is indeed linear and stationary and that  $\lambda = 1 - \alpha\delta$ . For given  $x_1$  the optimal cooperative consumption for each player  $i$  is given by the following recursive relation:

$$\begin{aligned} c_{it}^* &= \frac{1 - \alpha\delta}{N} x_t^* \\ x_{t+1}^* &= (x_t^* - N c_{it}^*)^\alpha \end{aligned} \quad (3.10)$$

The stock of fish in optimum evolves according to difference equation  $x_{t+1}^* = ((1 - \alpha\delta)x_t^*)^\alpha$  and converges to steady state  $x_{PO}$ :

$$x_{PO} = \lim_{t \rightarrow \infty} (\alpha\delta)^{\sum_{j=1}^t \alpha^j} x_1^{\alpha^t} = (\alpha\delta)^{\frac{\alpha}{1-\alpha}} \quad (3.11)$$

Because the optimal stock converges, satisfies the transversality condition and is positive for any initial state  $x_1$ , our solution satisfies the sufficient conditions and, thus, solves the problem 3.7. We showed, that cooperating players would aggregately consume  $(1 - \alpha\delta)x_t$  each period and the stock would eventually approach a steady state. The more myopic (the lower is the discount factor) the players are, the more they consume and the lower is the steady state. The steady state and the consumption is decreasing in  $\alpha$  as well. The Pareto optimal consumption and steady state serves as a normative benchmark, to which we can compare the noncooperative outcomes, and draw conclusions about their optimality.

In a more general case, it can be shown, that under our assumptions the Pareto optimal solutions exists and that the cooperatively exploited stock converges monotonically to  $x^*$ , where  $x^* = f'^{-1}(\frac{1}{\beta})$ .<sup>2</sup>

### 3.3 Nash Equilibria in Open-loop Strategies

#### 3.3.1 Open Loop Strategies

In this section we will describe the noncooperative solution for situations, in which players use only open-loop strategies. The player using open-loop strategy decides the entire plan of his consumption throughout game after observing the initial state at the beginning of the game. The player specifies an infinite sequence  $\{c_{i1}, c_{i2}, \dots\}$ , which gives his consumption at each stage and depends solely on the initial state. It is the only class of strategies admissible in the open-loop information pattern, i.e. in a situation, in which the information set of each player consists each period solely of the initial state. Players may use, however, the open-loop strategies even, when there is more information available.

In single person deterministic optimization open-loop strategies are equivalent to other types of strategies (i.e. feedback, close-loop, history-dependent). In a deterministic system, single information (such as the initial state) already gives the maximum amount of information needed for maximization. It does not matter, which type of strategy player uses as long as the values of the realization of the strategy maximize the problem (Bertsekas, 2001). The situation with strategic interaction is different, however. Availability of more information allows players to condition their actions on more factors. Then, the dynamics of the state does not depend only on the maximizing player and he has to take into account the actions of others as well. Under various information patterns, a player can react differently to the actions of others, and hence it can produce various Nash equilibria.

In spite of that, the restriction to open-loop strategies is not very limiting. The open-loop strategies may be used under more sophisticated information patterns as well. Moreover, the Nash equilibrium in open-loop strategies will remain to be Nash equilibrium, even when other types of strategies are permitted. With the strategies of other players given, a player faces deterministic maximization and the amount of information is irrel-

---

<sup>2</sup>It is shown on p. 133 - 136 in Stokey et al. (2004). Their model describes optimal growth, but it is identical to our model of resource extraction.

evant for the actions chosen. The best response could be either open-loop or any other type, they are equivalent as long as they produce the optimum consumption at each stage. Hence the best response to open-loop strategy could be again open-loop. Therefore the open-loop strategies could form Nash equilibria even under more sophisticated information patterns (Basar & Olsder, 1995).

The major drawback of open-loop Nash equilibria is that they are generally not time consistent. Although they still form Nash equilibrium in a setting with information patterns that include more than initial state, the increased amount of information might produce possibility to change the prespecified consumption pattern during the course of the game. Hence, the issue of time consistency arises. Time consistency is desired property, as the time consistent Nash equilibrium is more realistic predictor of outcome. We will see later, that in the case of our example, the open-loop strategy is weakly time consistent.

### 3.3.2 Derivation of Open-loop Solution

We follow closely Amir & Nannerup (2004) in the derivation, but we show wider scope of equilibria (not just the symmetric one) and examine the  $N$ -player case. We will look for interior solutions. In order to derive the equilibrium we find a best response of player  $i$  given the strategies of other players. First we fix the open-loop strategies of other players  $\{c_{j1}, c_{j2}, \dots\}, j \neq i$ . Now we solve maximization problem:

$$\begin{aligned} & \arg \max_{\{c_{it}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \delta^t u(c_t) \\ \text{s.t.} \quad & x_t - \sum_{j \neq i} c_{jt} > c_t > 0 \\ & x_{t+1} = f(x_t - c_{it} - \sum_{j \neq i} c_{jt}) \end{aligned} \quad (3.12)$$

Using the result of section 3.2.1 after substituting  $f_t(x) = f(x - \sum_{j \neq i} c_{jt})$  and plugging into Euler equation 3.6 we get the necessary condition for the best response.

$$u'(c_{it}^*) = \delta u'(c_{i(t+1)}^*) f'(x_t^* - c_{it}^* - \sum_{j \neq i} c_{jt}) \quad (3.13)$$

We proceed similarly to the previous section. First, we suppose that all players' strate-

gies are linear in the current stock (i.e.  $c_{jt} = \lambda_j x_t^*$ ). Then, we solve for our example:

$$\frac{1}{\lambda_i x_t^*} = \frac{\delta \alpha}{\lambda_t \left( (1 - \sum_{j=1}^N \lambda_j) x_t^* \right)^\alpha} \left( (1 - \sum_{j=1}^N \lambda_j) x_t^* \right)^{\alpha-1} \quad (3.14)$$

After solving the equation 3.14 for  $\lambda_i$ , we find that  $\lambda_i = 1 - \alpha\delta - \sum_{j \neq i} \lambda_j$ . Then the best response  $c_{it}^*$  of player  $i$  against given other players' consumptions, that are linear in stock is:

$$c_{it}^* = (1 - \alpha\delta) x_t^* - \sum_{j \neq i} c_{jt} \quad (3.15)$$

Now, we look for such strategies that are linear in stock and simultaneously satisfy equation 3.15 for all players. This holds for the open-loop strategies, for which the consumption pattern is given by recursive relation:

$$\begin{aligned} c_{it}^* &= \lambda_i x_t^* \\ x_{t+1}^* &= (x_t^* - \sum_{j=1}^N c_{jt}^*)^\alpha \end{aligned} \quad (3.16)$$

with the initial state  $x_1$  given  
where  $\sum_{j=1}^N \lambda_j = 1 - \alpha\delta$ , and for all  $j \in \mathbf{I} : \lambda_j > 0$

When all players play according to strategies defined in 3.16 the game attains its Nash equilibrium. Thus, we have found infinitely many such Nash equilibria for all the combination of  $\lambda_j$  defined by the last row in expression 3.16. The aggregate consumption is in any stage  $(1 - \alpha\delta)$  times the current stock, which coincides exactly to the aggregate Pareto optimal consumption.

Because the aggregate consumption stays the same, the dynamics follows the same pattern as in the cooperative solution. The stock develops in monotonous way and converges to steady state  $x_{OL} = x_{PO}$ . That implies that transversality condition holds and the solution is indeed interior, hence the necessary condition for maximization 3.12 are sufficient.

Moreover, the identity of symmetric Pareto optimal solution and Open-loop Nash equilibrium holds partly even for more general case. The symmetric Pareto optimal consumption pattern must solve 3.8, hence it solves the 3.13 for all  $i$  as well. The state dynamics follows the same pattern, because the aggregate consumption is the same. Hence the transversality condition in the open-loop case is satisfied if and only if it is satisfied for the pareto-optimal case. Therefore, if there is symmetric Pareto-optimal solution, then there is also identical symmetric open-loop solution. The fact, that the stock is exploited

optimally even in the asymmetric case is due to the particular functional form and does not have to be valid in more general setting.

No inefficiency arises when players use open-loop strategies. This result is surprising and rather unintuitive. Since one might expect that exploitation of a common pool resource shared by more than one user would produce the so called tragedy of commons. The inefficiency in our model stems only from its dynamic nature. In this model, the players do not have direct effect on each other's utilities, but interact indirectly through influencing the level of stock. By restricting the possibility to react directly on the state after each period, we suppress the dynamic strategic incentives. I will comment more on it in subsection 3.4.2 after finding the equilibrium that leads to overexploitation.

We cannot, however, generalize this finding to asymmetric case. In this case with asymmetric discount rates (i.e. for a case in which:  $\delta_i \neq \delta_j$  for some players  $i \neq j$ ) the solution of the game would become more complicated, possibly it would not exist. We show that by arguing similarly as above we find that best response of player  $i$ :

$$c_{it}^* = (1 - \alpha\delta_i)x_t^* - \sum_{j \neq i} c_{jt} \quad (3.17)$$

Each player reacts to the others consumption in such way, that he consumes up to a point, where the total consumption of all players reaches  $(1 - \alpha\delta_i)x_t^*$ . Since the optimal total consumption is different for some players, all individual consumptions of players cannot be best responses at the same time. Hence, there is no equilibrium in pure open-loop strategies, that are linear in the optimal stock and stationary. Finding whether there are other equilibria in open-loop strategies would be more complicated than in the previous case, because we could not use the simplifying assumptions that allowed us to find the results presented above.

Even though the equilibria in open-loop strategies are generally not time consistent, the previously found equilibrium for the example is weakly time consistent. The truncated equilibrium strategies that start in stage  $k$  are given as in 3.16 only with the difference that the initial state is  $x_k^*$ . Hence, if the truncated starts in stage  $k$  from the point on optimal path  $x_k^*$ , the truncated strategy is also Nash equilibrium and the Nash equilibrium is weakly time consistent. If, however, the strategy starts from arbitrary point  $x_k$  off the equilibrium path, than the Nash equilibrium forms  $N$ -tuple of strategies given by 3.16 with initial state in  $x_k$ , which is not the truncated Nash equilibrium of the original game, therefore, the Nash equilibrium in open-loop strategies is not strongly time consistent. Thus, this equilibrium lacks the desired property of time consistency and cannot be

expected as an outcome. It serves, however, as a benchmark to show that inefficiency arises only if players can react on the development of the stock.

## 3.4 Nash Equilibria in Feedback Strategies

### 3.4.1 Feedback Strategies

The feedback strategy is a function, which determines the consumption in dependence on the current state. Under the feedback strategy, the player's  $i$  consumption at time  $t$  is given by  $c_{it} = g_i(x_t)$ . Feedback strategy is stationary, i.e. the function  $g_i$  stays the same during the whole duration of the game and is independent on time.<sup>3</sup> Each player consumes always the same amount, given the same stock, regardless of the time. In the game each player chooses his feedback strategy at the beginning and keeps it for the whole duration of the game and reacts just on the current stock.

The feedback strategy is the only admissible strategy in a game with the feedback information pattern, i.e. when the information set at time  $t$  for each player  $i$  consists solely of the current stock  $x_t$ . They can be used, however, even under an information patterns that consist of more inclusive information sets.

Again, there is no loss of generality in restricting ourselves just to feedback strategies (as in the Open-loop case in section 3.3.1). When player is finding his best response against given strategies, he faces deterministic dynamic optimization problem. The problem can be maximized by any type of the strategy, as all the optimal strategies lead to the same values of consumption. The best response strategy to feedback strategies is feedback as well. Therefore, the Nash equilibrium feedback strategies still form Nash equilibrium even if we allow the use of more sophisticated strategies (Basar & Olsder, 1995).

Contrary to the Nash equilibrium in open-loop strategies, the equilibrium in feedback strategies is always strongly time-consistent. Because the strategies are stationary and they react directly on the current stock, whenever the game starts again in arbitrary point in any stage, they still maximize payoff in the new game facing other player's strategies. Hence even in the new truncated game, they are all simultaneously best responses against each other and form Nash equilibrium even in the new game.

---

<sup>3</sup>The feedback strategy can be also referred to as Markovian or Markov-stationary as in Dutta & Sundaram (1993).

### 3.4.2 Nash Equilibrium in Linear Feedback Strategies

Levhari & Mirman (1980) published the results for 2 player case. Later Okuguchi (1981) generalized the result for  $N$  players. Levhari & Mirman (1980) and Okuguchi (1981) derived the equilibrium for a case with asymmetric interest rates. We will present their findings with symmetric discount rate, but derive it differently using the Euler equation as in the previous sections.<sup>4</sup> Levhari, Mirman and Okuguchi solve their model by method of dynamic programming. They first consider only one-period horizon and assume that players will divide the fish equally in the last period. Thus, the player  $i$  solves problem:

$$\arg \max_{0 < c_i < x} \log c_i + \delta \log \left( \frac{x - \sum_{j=1}^N c_j}{N} \right) \quad (3.18)$$

Then, by simultaneously solving the previous problem for all  $i$ , we arrive to equilibrium strategy for all players  $i$ . In one-period horizon the Nash equilibrium consumption is given as:

$$c_i^* = \frac{1}{N + \alpha\delta} x \quad (3.19)$$

By iterating this step  $m$  times, they found the optimal consumption of player  $i$  in the first stage is given by:

$$c_i^{(m)*} = \frac{1}{\alpha\delta \frac{(1-(\alpha\delta)^m)}{1-\alpha\delta} + N} x \quad (3.20)$$

Then, the solution of the problem with infinite horizon is a limit of  $c_i^{(m)*}$ .

$$c_i^* = \frac{1}{\frac{\alpha\delta}{1-\alpha\delta} + N} x = \frac{1 - \alpha\delta}{N - (N - 1)\alpha\delta} x \quad (3.21)$$

Instead of using dynamic programming we can follow the approach that we have used in previous sections. We proceed as in the section 3.2.1 we transform the maximization problem as:

$$\begin{aligned} & \arg \max_{\{x_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \delta^t u(x_t - \sum_{j \neq i} g_j(x_t) - f_t^{-1}(x_{t+1})) \\ \text{s.t.} \quad & x_t - \sum_{j \neq i} g_j(x_t) - f_t^{-1}(x_{t+1}) > 0 \\ & x_t > 0 \end{aligned} \quad (3.22)$$

---

<sup>4</sup>This method was also suggested in the original paper Levhari & Mirman (1980) as well.

Solving of first order conditions, provided the strategies of other player's are differentiable, we get necessary conditions. For all  $t$  must hold:

$$u'(c_t^*) = \delta u'(c_{t+1}^*) f' \left( x_t^* - \sum_{j \neq i} g_j(x_t^*) - c_t^* \right) \left( 1 - \sum_{j \neq i} g'_j(x_{t+1}^*) \right) \quad (3.23)$$

We solve it for the case of example of Levhari & Mirman (1980). We proceed identically as in the previous cases. First we plug in the concrete functions and obtain:

$$\frac{1}{c_t^*} = \alpha \delta \frac{1}{c_{t+1}^*} \left( x_t^* - \sum_{j \neq i} g_j(x_t^*) - c_t^* \right)^{\alpha-1} \left( 1 - \sum_{j \neq i} g'_j(x_{t+1}^*) \right) \quad (3.24)$$

Similarly to the previous sections, we suppose that all players adopt linear strategies  $g_j(x_t) = \lambda_j x_t$ . The consumption in period  $t + 1$  is, then, given by  $\lambda_j ((1 - \sum_{k=1}^{\infty} \lambda_k) x_t)^\alpha$ . We can simplify the equation to:

$$\frac{1}{\lambda_i x_t} = \alpha \delta \frac{1}{\lambda_i \left( (1 - \sum_{j=1}^{\infty} \lambda_j) x_t \right)} \left( 1 - \sum_{j \neq i} \lambda_j \right) \quad (3.25)$$

And solve for  $\lambda_i$ :

$$\lambda_i = (1 - \alpha \delta) \left( 1 - \sum_{j \neq i} \lambda_j \right) \quad (3.26)$$

Then,  $g_i^*(x_t) = \lambda_i x_t$  gives us the best response of a player  $i$  against other player's linear strategies given by  $g_j(x_t) = \lambda_j x_t$ . Solving simountaneously  $\lambda_i$  for all players. We obtain symmetric Nash equilibrium. The equilibrium strategies are given by 3.21. <sup>5</sup>

Our approach has the advantage that it is easier to compute than the original dynamic programing approach in Levhari & Mirman (1980) and Okuguchi (1981). Amir & Nannerup (2004) used the Bellman's equation to arrive to 3.24. Cave (1987) found the equilibria by directly supposing that the strategies are linear. He summed the utilities over infinite horizon and used calculus to find the maximum. This approach has a major drawback, because it can be used only if we know that the strategies in the equilibrium are linear. Otherwise it would provide only the best linear response. With our approach we can be sure that best response to linear strategies is indeed linear.

Moreover, our approach allows intuitive interpretation. By looking at the necessary conditions, we can explain why inefficiency occurs. Under cooperation each player con-

---

<sup>5</sup>The optimal best responses to the equilibrium strategies satisfies the sufficient conditions, it follows from applying theorem 4.15 in Stokey et al. (2004). The reasoning is more detailed in subsection 3.2.1.

sumes  $\frac{1-\alpha\delta}{N}$  share of the stock, while under the noncooperative equilibrium each player consumes  $\frac{1-\alpha\delta}{N-\alpha\delta(N-1)}$ . The latter consumption is clearly higher. If the interior Nash equilibrium exists, we can make following observations. We can compare the conditions for the noncooperative case:

$$u'(c_t^*) = \delta u'(c_{t+1}^*) f'(x_t^* - \sum_{j \neq i} g_j(x_t^*) - c_t^*) (1 - \sum_{j \neq i} g'_j(x_{t+1}^*)) \quad (3.27)$$

and the conditions in Pareto optimal case:

$$u'(c_t^*) = \delta u'(c_{t+1}^*) f'(x_t^* - Nc_t^*) \quad (3.28)$$

We see, that in optimum a marginal utility from consumption  $c_t^*$  in one period equals the marginal effect, which a change of the current consumption  $c_t^*$  has on the utility in next period. In optimum the marginal profit, the utility in the current period, equals the marginal "costs", the present value of marginal reduction in next period utility caused by current consumption. If the left hand side of the equation 3.27 is bigger (smaller) than the right hand side, then the player  $i$  raises (lowers) his consumption until he reaches the maximum, i.e. until the equalities 3.27 hold for all  $t$ . Any marginal decrease in current consumption raises the stock available for consumption in next period by  $f'(x_t - \sum_{j \neq i} g_j(x_t) - c_t)(1 - \sum_{j \neq i} g'_j(x_{t+1}))$ , while in Pareto optimal case the positive effect of postponing consumption is  $f'(x_t - Nc_t)$ . If  $x_t$  and  $x_{t+1}$  is the same in both cases, the former is  $(1 - \sum_{j \neq i} g'_j(x_{t+1}))$  times the latter. Hence, the investment in the noncooperative case is less profitable, because the consumption of other players will reduce the marginal effectivity of the investment to the  $(1 - \sum_{j \neq i} g'_j(x_{t+1}))$  times the original value. The opportunity cost of investing is borne solely by one player, while the profit from increased stock is shared by all. The incentive to invest by postponing consumption is, thus, weakened. The inefficiency occurs because players are aware that others will react on their decisions. Any effort to increase future consumption by reducing current consumption will be partly offset by increased consumption of other players. The investment is not as effective, when there are other players reacting on the stock. Hence, there arises a situation similar to the Hardin's pasture. There is also an externality, that causes the overusage, but the externality has a dynamic nature and, thus, reflects the appropriation in fishing.

The dynamic inefficiency occurs in the feedback case contrary to the solution in open-loop strategies, because open-loop strategies do not allow to condition the consumption directly on the stock. A player's investment, then, is not wasted on increased consumption

of others and the incentive of each player to invest in future yields is stronger. The player do not affect each other directly. The only source of interaction is the dynamics of stock. The players affect each other only through influencing the state variable. If players have to specify their consumption pattern at the beginning of the game, the players cannot take advantage of somebody's reduction in consumption. A player, who is aware that other players are not able to react on his decision to invest more, maximize as in the Pareto-optimal case without the inefficiency of investment being present. The requirement for each player to specify the whole consumption pattern ahead suppresses the dynamic aspect of the strategic interactions. Players will invest efficiently and the optimal aggregate extraction through time will be maintained.

At the end we evaluate the stock dynamics. Every period  $N \frac{1-\alpha\delta}{N-\alpha\delta(N-1)} x_t$  is consumed. Therefore, the stock develops according to equation:

$$x_{t+1} = \frac{\alpha\delta}{N - \alpha\delta(N - 1)} x_t \quad (3.29)$$

And converges in infinity to steady state:

$$x_{CL} = \lim_{t \rightarrow \infty} \left( \frac{\alpha\delta}{N - \alpha\delta(N - 1)} \right)^{\sum_{j=1}^t \alpha^j} x_1^{\alpha^t} = \left( \frac{\alpha\delta}{N - \alpha\delta(N - 1)} \right)^{\frac{\alpha}{1-\alpha}} \quad (3.30)$$

As one would expect, due to higher consumption the steady state for this equilibrium is lower than the Pareto optimal steady state. With increased number of players the overconsumption worsens and the steady state decrease. Therefore, the more players there are, the more severe is the inefficiency. The steady state is still greater than zero, thus in this setting of the model, the depletion of the resource does not take place, but as  $N$  grows to infinity the steady state tends to zero. Hence in a situation with many players, situation approximately close to depletion might happen.

We demonstrated directly that the Nash equilibrium in closed-loop strategies is inefficient for the example of Levhari & Mirman (1980). For the more general case, we reasoned intuitively, that equilibria in feedback and differentiable strategies would lead to overexploitation, because of the reduced incentive to invest. The result was proven formally in Dutta & Sundaram (1993). They proved for 2-player case that the feedback equilibria, if they exist, are always inefficient. Moreover, they proved that under feedback equilibrium the stock develops in monotonous way. In the case, when the strategies are differentiable, the stock converges to lower than Pareto optimal steady state; in other words, the resource is overexploited. In the next subsection, I present a different equilib-

rium in feedback strategies that shows that overexploitation does not have to be always the case.

### 3.4.3 Discontinuous Equilibrium

Hardin (1968) argued that the exploitation of natural resource by several appropriators leads overexploitation by arguments similar to the reasoning above. The equilibrium, that we have found in previous section, indeed confirm that the tragedy of commons is possible outcome. Dutta & Sundaram (1993) showed that there exists a different Nash equilibrium in feedback strategies that results in underexploitation. This outcome might occur, because of the presence of state variable in the game.

To motivate for this equilibrium, we first compare this dynamic game to a repeated game. In the repeated games that have unique Nash equilibrium of the stage game, unless players can condition their strategies on observed actions of other players, the only time-consistent Nash equilibrium of the whole game consists of the stage game equilibria. For example, in repeated prisoner's dilemma the only time-consistent Nash equilibrium is to deviate at every stage. Here, we have also a similar structure to the one in prisoner's dilemma. The players choose between consuming higher amount and overconsumption or cooperation by consuming less, which would lead to higher overall payoff. Our game differs, however, in one important aspect, the players do not face the same game every stage. The game develops in time according the actions the players choose. The development is described by the state variable.

Normally in the repeated games, there exist other equilibria, whenever players can observe directly the actions of others. Knowing the past actions of others, players may use threats to force them to cooperate. This „enforced“ equilibrium secures higher payoff to the players. This is the idea of trigger strategy, normally used in the repeated game (I will describe it more thoroughly in next section). One can use similar approach in the dynamic game, even without players having the knowledge of the history of the other players' actions. Because the state variable, at least limitedly, reflects the past action the state variable, it may serve as a proxy for the history of past actions. Hence, players might use the stock, as a indicator on which to condition their threats.

Dutta & Sundaram (1993) developed the idea as follows. The players choose some point (level of stock) that cannot be reached by the noncooperative feedback strategies given by 3.21 (below, I will refer to it as noncooperative strategies). Then, if the level of the stock is above that point, the state indicates that cooperation might exist. Then the

players consume the exact amount to reach that point in one stage and then stay at the point. Any higher consumption is punished by returning to playing the noncooperative strategies. Hence, this way the trigger strategy, which is normally used to explain cooperation, can be imitated even in situation when the full history of the game is not available to players. Below I describe a way how to find this equilibrium more formally, it follows the result of Dutta & Sundaram (1993) for two player case. I use the same arguments to show that it is equally valid for  $N$  players.

First, let  $x'$  denote:

$$x' = \left( \frac{1}{N - (N-1)\alpha} \right)^{\frac{\alpha}{1-\alpha}} \quad (3.31)$$

And  $c'$  is one  $N$ th of consumption, that ensures the  $x'$  stays at the same level:

$$\begin{aligned} c' &= \frac{1}{N} (x' - (x')^{\frac{1}{\alpha}}) = \frac{1}{N} \left( \left( \frac{1}{N - (N-1)\alpha} \right)^{\frac{\alpha}{1-\alpha}} - \left( \frac{1}{N - (N-1)\alpha} \right)^{\frac{\alpha}{1-\alpha} + 1} \right) = \\ &= \frac{1}{N} \left( \frac{1}{N - (N-1)\alpha} \right)^{\frac{\alpha}{1-\alpha}} \frac{(1-\alpha)(N-1)}{N - (N-1)\alpha} \end{aligned} \quad (3.32)$$

Now, we define strategies  $g_i$ :

$$g_i(x) = \begin{cases} \frac{1 - \alpha\delta}{N - (N-1)\alpha\delta} x & , x \in [0, x'] \\ \frac{(x - x') + Nc'}{N} & , x \in [x', \infty) \end{cases} \quad (3.33)$$

We will confirm that strategies  $g_i$  indeed form Nash equilibrium. If the state is  $x \in [0, x')$ , a single player cannot raise the stock to level higher than  $x'$ , because even if he did not consume anything the stock would develop according to equation:

$$x_{t+1} = x^\alpha \left( \frac{1}{N - (N-1)\alpha} \right) \quad (3.34)$$

This converges monotonically to a steady state  $\left( \frac{1}{N - (N-1)\alpha} \right)^{\frac{\alpha}{1-\alpha}}$  which is strictly smaller than  $y'$ , because the discount factor is always smaller than 1. For initial states  $x \in [0, x')$ , the Nash equilibrium stays the same as in the previous section, because the strategies are the same for states smaller than  $x'$  and the modification does not affect the best responses.

If the initial state is  $x \in [x', \infty)$ , the consumption would be constantly  $c'$  after the first stage and the stock will be constantly in steady state  $x'$ . There would be no incentive

to change action from the one prescribed by  $g_i$  for sufficiently high discount factor. If he consumes more than  $c'$ , the stock in next will be under  $x'$  and all players start play noncooperative strategies.

The noncooperative consumption converges in infinity to:

$$\bar{c}_\delta = \frac{1 - \alpha\delta}{N - (N - 1)\alpha\delta} \left( \frac{\alpha\delta}{N - (N - 1)\alpha\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (3.35)$$

As  $\delta$  approaches 1,  $\bar{c}_\delta$  tend to limiting value, given by:

$$\bar{c} = \frac{1 - \alpha}{N - (N - 1)\alpha} \left( \frac{\alpha}{N - (N - 1)\alpha} \right)^{\frac{\alpha}{1-\alpha}} \quad (3.36)$$

Comparing equations 3.32 and 3.36, we find out that  $c' > \bar{c}$  if:

$$\frac{N - 1}{N} > \alpha^{\frac{\alpha}{1-\alpha}} \quad (3.37)$$

which holds for sufficiently high  $\alpha$ . Moreover, because  $c' > \bar{c}$ , for high  $\delta$  it also holds:  $c' > \bar{c}_\delta$ . If in the state  $x \in [x', \infty)$  a player increased his consumption over the strategy  $g_i(x)$ , he would fell under the threshold state  $x'$  and as the time would grow to infinity, his consumption would get strictly smaller than the consumption  $c'$ . For high  $\delta$ , all gains over first finite number of periods are swamped by strictly smaller consumption thereafter. Therefore, it is unprofitable the raise consumption above the strategy  $g_i(x)$ , when playing against strategies  $g_j(x)$  of other players. Moreover, the more players there are, the smaller can  $\alpha$  be, so that the inequality 3.37 still holds.

We have shown, that even if we admit only feedback strategies, overconsumption does not have to be the only equilibrium. We have found that, surprisingly, there could be another Nash equilibrium, which leads to underconsumption. The nature of dynamic game, the presence of state variable, allows for this equilibrium. Contrary to the previous game, the steady-state, to which the stock converges, depends on the initial state. The Nash equilibrium is different only if the initial state is higher than  $x'$ . The stock converges to  $x_{CL}$  if  $x_1 \in [0, x')$  and to  $x'$  if  $x_1 \in [x', \infty)$ . The long-term outcome  $x'$  is still inefficient, but it Pareto dominates the outcome from noncooperative strategies. In addition, the inefficiency stems from underconsumption rather than from overconsumption. This is because in order for the strategy to be in equilibrium, there must not be possibility to reach the threshold state  $x'$  unilaterally. Paradoxically, the result of exploitation by several players can be the exact opposite of the tragedy of commons. The players might switch to different source of inefficiency, in order to avoid the tragedy, which has much more devastating results.

### 3.5 Equilibria in History-dependent Strategies

Common way how to explain cooperation using the noncooperative game theory is to assume that players interact repeatedly for very large number of periods. In many games, like prisoner's dilemma, Nash equilibria are suboptimal. Although all players would be better off, if they cooperated and jointly maximized their utilities, this is not a Nash equilibrium. When the cooperation is not Nash equilibrium, the rational players will not be able to secure such outcome without enforcement from third party. Players cannot agree on cooperation, because there would always be an incentive to break the agreement and deviate to another action. In one-shot games there is no way how to enforce that players will stick to the cooperative actions, when these are not in Nash equilibrium.

Contrary to one-shot games, in infinitely repeated games, there is a possibility to punish suboptimal behaviour of other players if they are able to observe other player's past actions. One way to enforce an agreement that all players will play some prespecified action that would lead to outcome that Pareto dominates the Nash equilibrium, is to play trigger strategies. The player that adopts trigger strategy will play the prespecified action unless any player breaks the agreement, then he begins to play the Nash equilibrium action. If the discount factor is enough high, a player does not want to sacrifice the future yield from cooperation by deviation. Hence, if players observe the others' actions, new Nash equilibria can be sustained.

In such equilibria each player is locked in playing the cooperative action, because any other action would be punished by the reversion to suboptimal outcome. Although, in equilibrium, the threats are only hypothetical, they must be credible. Each player has to believe that if he deviated the punishment would indeed come. Hence, the punishment itself must be best possible action in the stages following deviation, because the punishing players must not punish themselves as well. Any threat must form a new Nash equilibrium in a game that begins one period after deviation. In other words, the equilibrium must be strongly time consistent, i.e. even if the state is reached by suboptimal strategies, there will still be no incentive to change the strategy. The worst Nash equilibrium of the stage game satisfies the requirement for credible punishment, therefore the trigger strategy uses it as a threat to secure new Pareto improving equilibria.

There are several so-called folk theorems that describe that trigger strategies can produce various Nash equilibria in any repeated game. Friedman's theorem (Gibbons, 1992) states that a Pareto optimal strategies form the Nash equilibrium provided that the discount factor is high enough. Moreover, there is a vast number of other Nash equilibria

that produce payoffs that are strictly higher than the payoffs from the equilibrium of original one-shot game. Thus, if the discount factor is sufficiently large, almost any payoff ranging from the lowest yield, given by the original equilibrium, to the highest Pareto optimal yield can be reached. Other variation of folk theorem (Abreu, 1988) has been concerned with specifying the set of all Nash equilibria given some discount factor.

Similarly, we are interested, whether such result hold in the case of dynamic games as well. As we pointed out in previous section, the setting of our model is different from the framework of repeated games. The state variable is a feature, which is not present in repeated games. However, in both cases the players interact repeatedly infinitely many times, hence, there is room to punish other players for not complying with some consumption pattern. If we admit history-dependent strategies, we want to find a set of all equilibria that can be supported by the trigger strategies. Cave (1987) derives a variation of such folk theorem for the example of Levhari & Mirman (1980) and describes the set of equilibria supported by trigger strategies. He assumes only stationary strategies that are linear in stock. Under the trigger strategy each player consumes  $\lambda_i$  share of stock, when cooperating and punishes deviation with the noncooperative strategy given by equation 3.21. The trigger strategy is defined as:

$$g_i(x_t, c_1^1, \dots, c_{t-1}^N) = \begin{cases} \lambda_i x_t & , \text{ if and only if } \{c_1^1, \dots, c_{t-1}^N\} = \{\lambda_1 x_1, \dots, \lambda_N x_{t-1}\} \\ \frac{1-\alpha\delta}{N-(N-1)\alpha\delta} x & , \text{ otherwise} \end{cases} \quad (3.38)$$

Cave found<sup>6</sup>, that for each  $\delta$  there exists a set of  $(\lambda_1, \dots, \lambda_N)$ , such that the associated trigger strategies form Nash equilibrium. Such set has nonempty interior, is convex, compact and symmetric. The Pareto optimal extraction rates might also be in the set, provided the discount rate is high enough. The shares of the stock consumed under noncooperative strategies are member of the set as well; moreover, they induce the highest overall consumption than any other point in the set. The minimum aggregate consumption vary with the discount rate. The aggregate consumption is, thus, always equals to or is reater than consumption under the noncooperative strategies. Since, the set is convex, every aggregate consumption between the lowest and the highest is possible. In infinity

---

<sup>6</sup>In the article, he presents the proof for 2-player case, but he claims that the results hold in a N-player case too.

the stock converges to steady state:

$$x_{TS} = \left(1 - \sum_{i=1}^N \lambda_i\right)^{\frac{\alpha}{1-\alpha}} \quad (3.39)$$

Because the overall stock is decreasing in the overall share consumed the steady-state of stock is always greater than or equal to the noncooperative steady-state. The payoff for all players is never smaller than in the noncooperative case. However, unless the discount rate is high, the Nash equilibria are still inefficient.

## 3.6 Resource Depletion

So far, we have been concerned with the interior solutions. All of the solutions yielded steady state that was strictly greater than zero. The exploitation of a natural resource often leads to the destruction of the resource. The example of Levhari & Mirman (1980) produced no such outcomes, therefore it is not a good framework to study the mechanism of resource depletion. Since the logarithmic utility function is not defined in zero, it is not possible to get corner solution.

In the general framework, however, and under the assumption that the players split the resource if the aggregate consumption is not feasible, the corner solutions are easy to construct. As Dutta & Sundaram (1993) pointed out, such solution might be, for example, the feedback strategy  $g_i(x) = x$  for all players. Similarly the open loop strategy, in which each player for each stage consumes the whole initial stock is equilibrium as well. This way we can get equilibria, in which the resource is depleted after first stage.

### 3.6.1 Linear Growth Function

There is another possibility to get solution, in which the stock converge to zero. We modify the model of Levhari & Mirman (1980) to allow the possibility of gradual depletion of the resource. The modification was done for 2-player in the original paper Levhari & Mirman (1980) and later for open-loop strategies in Amir & Nannerup (2004). We found the results for  $N$  players.

Instead of the growth function  $f(x) = x^\alpha$  we assume linear growth function in the form  $f(x) = ax$ , where  $a$  is some positive constant. The utility function is still logarithmic,

which means that the consumption can never be zero. We just state the results. They can be found by the procedure used in previous subsections 3.2.2 and 3.4.1.

The symmetric Pareto optimal strategy is:

$$g_i^{PO}(x) = \frac{1 - \delta}{N} x \quad (3.40)$$

The stock develops according to difference equation:

$$x_{t+1} = a\delta x_t \quad (3.41)$$

Then, as the  $t$  tends to  $\infty$  the stock converges to 0 if  $a\delta < 1$  or it converges to  $\infty$  if  $a\delta > 1$ . If  $a\delta = 1$  the initial stock is a steady state and stays constant over time.

There is a symmetric Nash equilibrium in feedback strategies. The equilibrium strategy is for each player:

$$g_i(x) = \frac{1 - \delta}{N - \delta(N - 1)} x \quad (3.42)$$

The equilibrium stock develops according to difference equation:

$$x_{t+1} = \frac{a\delta}{N - \delta(N - 1)} x_t \quad (3.43)$$

Similarly, the stock either converges to 0 or  $\infty$ . If  $\frac{a\delta}{N - \delta(N - 1)} < 1$ , the stock converges to 0. If  $\frac{a\delta}{N - \delta(N - 1)} > 1$  the stock converges to  $\infty$  and if  $\frac{a\delta}{N - \delta(N - 1)} = 1$  the initial stock is a steady state.

Because  $\frac{a\delta}{N - \delta(N - 1)} > a\delta$ , it is possible that  $\frac{a\delta}{N - \delta(N - 1)} > 1 > a\delta$ . In such situation the outcome on the dynamics of the resource under the cooperative and under the noncooperative regime radically differs. While the stock would be gradually depleted by noncooperating players, under the cooperation regime the stock would grow to infinity. In other cases, however, the outcome in the long run would be the same.

### 3.7 Summary

In this chapter, we introduced a model of the resource exploitation and show some, possibly not all, Nash equilibria. The model describes the dynamics of a renewable resource in time, when it is exploited by  $N$  players. The main feature of the model is that the decisions are made in discrete time. Then, it is assumed that both the growth and utility

functions are concave, both realistic and reasonable assumptions. Moreover, additional technical assumption has been made that ensure the existence of interior Pareto optimal solution, because it is reasonable to assume that Pareto efficient extraction does not deplete the resource.

To further simplify the case, we made further assumptions and adopted the functional forms of the example of Levhari & Mirman (1980), logarithmic utility and exponential growth. This allows to evaluate directly the equilibrium consumptions and steady states. We argued informally about the general model results, which seem to resemble the results in the particular example that we have adopted, although they cannot be directly evaluated. Hence, we suppose that the findings for the general model and for the example are quite similar.

We found different equilibria for various information patterns and a Pareto optimal solution, so that a judgment about efficiency can be done. We found that under the Nash equilibrium in open-loop strategies, the overall extraction of the resource is Pareto efficient, but the distribution of the consumption among players may be asymmetric and vary as long as it has positive value for each player. When the players have more information than just about the initial state, the equilibrium is not strongly time consistent and, thus, we do not expect it to be the possible outcome.

Under the feedback information pattern, we found the least efficient equilibrium. It is symmetric and leads to overconsumption. This equilibrium follows the logic of the tragedy of commons and yields the lowest steady state. We show that for sufficiently high discount factor and high ability of the resource to reproduce the tragedy is not inevitable even under the feedback information pattern and there might exist other equilibrium that leads to underconsumption. All equilibria in the feedback strategies are inefficient, however.

Lastly, we have stated a variation of folk theorem for our case. By it, the equilibrium outcome may vary and the consumptions can attain various values from the lowest given by the noncooperative equilibrium to the highest Pareto optimal if the discount rate is high enough.

As we have argued, if there is Nash equilibrium under one information pattern, it forms a Nash equilibrium even under more inclusive information pattern. Hence, if we admit full information, all of the strategies specified above form Nash equilibria, but the open-loop Nash equilibrium will not be time consistent. The Nash equilibrium only gives us the necessary condition for an outcome to be expectable. Hence, the results are rather ambiguous. All the outcomes ranging from the tragedy of commons to underconsumption

are possible. Thus, the model as a predictor is not very useful. We have found that unless the discount rate is high and the cooperative outcome is supported by trigger strategies, the strongly time consistent equilibria are Pareto inferior and we have described the mechanism of the inefficiency caused by strategic interaction that leads to the dynamic variant of Hardin's tragedy of commons in fisheries.

Because it is often argued that the appropriation contributes to overfishing and depletion of the world fisheries, we are particularly concerned with this worst possible outcome. In next chapter, we will survey empirical literature to find out, whether this outcome, indeed, takes place on larger scale.

## Chapter 4

# Empirics and Experiments

The overexploitation of world fish resources is a widely recognized problem. According to Food and Agriculture Organization of United Nations (2005), 24 percent of the world fish stock, for which there were available data, was overexploited or depleted. There is a well-documented decline of fish population caused by the exploitation of the fish. One of the causes of the overexploitation might be the dynamic externality problem which we have outlined in subsection 3.4.2. The intuition is compelling, yet, direct empirical evidence is lacking.

The setting of our model considered one isolated aspect of the common-pool resource use, that of dynamic externality. We hardly encounter such isolated setting in reality. Schlager in Ostrom et al. (1994, chap. 11) identifies several kinds of externalities in a shared fishery. Apart from the appropriation externality, with which we have been concerned in our model, there are the technological externality and problem of assignment of fishing spots. The technological externality reflects the negative effects which the appropriators have on each other by using diverse technology. In addition, Schlager particularly stresses the assignment problem, in which a considerable amount of resources is devoted to contest for the best fishing spots and thus leading to inefficient use of the resource. Wilson (1982), for example, considers the problem of appropriation as relatively small and unimportant, although he acknowledges it. Given the presence of other strategic effects it is difficult to separate one from another in empirical examination.

Moreover, considerable effort was already devoted to solving the externality problem. Ostrom (1990) and Ostrom et al. (1994) have argued that several types of institution have emerged in response to solve the commons problems. Similarly, the fisheries are under heavy state regulation (Ostrom, 1990). Even on the international level, the problem of fishing on high seas and cross-border externalities have been subject to various interna-

tional treaties (e.g. the United Nations Convention on the Law of the Sea). Therefore, it is hard to get data, from which we would be able to infer that the tragedy of commons effect indeed takes place in large scale.

In this chapter we present various types of empirical evidence which indicates that shared fisheries are prone to overexploitation due to the presence of dynamic externality. Some of the cases are compelling, but evidence that would directly link the dynamic externality and overfishing is missing.

## 4.1 Evidence from International Fishing

McWhinnie (2009) examines the effect of sharing econometrically. She estimates whether stocks shared by more countries tend to be overexploited. A number of players serves as an explanatory variable. The number was found by counting all exclusive economic zones in specified area, where certain species of fish was caught and dummy for high seas was included. To specify it further, I cite an example from McWhinnie (2009):

For clarity of how the number of countries is counted, consider the following example. In the NW Pacific, pink salmon is harvested in Japanese, North and South Korean, Russian, and American waters plus the high seas so it is classified as being shared by five countries (with the high seas dummy equal to one). In the NE Pacific, pink salmon is harvested in Canadian, Russian, and American waters but not the high seas so it is classified as being shared by three countries (with the high seas dummy equal to zero).

The explained variable is qualitative and indicates the state of exploitation of the resource. The indicator is taken from two *Reviews of World Marine Resources of Food and Agriculture Organization of United Nations* from years 1994 and 2002. Furthermore, she controls for other factors such as technical capability of each country, price of the fish, management ability, the ability of fish to grow, the costs of fishing and discount factor. Many of the factors are not measurable, hence she uses proxies for many of them (average index of political risk for management capability, average GDP per capita for technical capacity, average interest rates for discount factor and dummy variables for doubling time of the stock and climate, which indicate the ability of the stock to reproduce), she also includes dummy for a dataset from year 2002. To estimate the effects she uses ordered probit.

She finds that the number of countries is significant at 5 percent significance level. The estimated effect is that the more countries share a resource, the more is the resource prone to overexploitation. Moreover she found that:

These predicted proportions mean that if a fish stock is shared between two countries it is 9% more likely to be overfished and 19% more likely to be depleted than a stock fished by one country. If the stock is shared by five countries it is 36% more likely to be overfished and 82% more likely to be depleted. When the stock is shared by 10 countries it is 68% more likely to be overfished and 183% more likely to be depleted than a stock fished by just one country.

Indeed, the effect of the number of countries on stock depletion is strong. The other significant variables were a proxy for management capability, doubling time of fish stock and price of the fish. The results are robust to various modifications. Several different methods of determination of the number of countries, changes of the proxies and variations of the methods of estimation were considered and all these variants confirmed the significance of the number of players and the other findings.

The negative effect of increasing number of countries catching a species of fish on overexploitation is consistent with the feedback noncooperative equilibrium. Thus, the findings support the hypothesis that sharing of resource leads to tragedy of commons.

## 4.2 Experiments

I have already emphasized some problems of empirical testing of the model. One way to avoid these troubles is to test the motivation of the players directly by conducting an experiment. When conducting experiment, one has complete control over the setting of the decision making problem and can create environment that exactly matches the setting of the theoretical model. One can create institution-free environment, in which players interact in exactly one specified way. Hence, one can directly examine the predicted effect separately from other factors that are present in the real world. One can also easily determine the Pareto optimal value and find out, whether there is inefficiency.

There is a vast number of experimental literature on commons (e.g. Ostrom et al. (1994) and Ostrom et al. (1992)), which usually finds that in the absence of communication the outcome is overexploitation. The findings are consistent with the predictions

based on theory. The framework used for these games however is repeated game without underlying development of the stock. These findings cannot be generalized to the dynamic setting of our model. Herr et al. (1997) conducted experiments with both the dynamic interactions and the repeated static games, compared them, and found out that the tragedy of commons was a good predictor of actual behavior in experimental settings. The results for the repeated static games and the dynamic games, however, differed greatly. Although the nature of appropriation of renewable resources is dynamic, surprisingly few experimental studies accounted for it.

Walker & Gardner (1992) conducted an experiment, in which the players exploited a resource and the more they appropriated the resource the higher was the probability that the resource will be depleted next round. There were two designs of the experiment. In the first one there was only one point, where the probability of the destruction was zero and in the second one there was an interval, which allowed for positive extraction and still granted the zero probability of destruction if the aggregate consumption was below some level. The players exploited the resource by investing effort, which was limited for every period and could be invested in safe but lower yield alternative. The game had finite a horizon of twenty periods and the participants of the experiment were experienced students. There were eight players in each group in the experiment.

In this experiment, there is a dynamic interaction through the increased probability of destruction in the next round. The setting of our model is, however, very different in several important aspects. It is deterministic, has infinite horizon and players influence by their actions not only the next period but the whole remaining part of the game. The experiment also lacks the state variable, on which the players react and from which they infer about the progress of the game. The experiment still gives us some insight, whether the fact that future state of stock is negatively influenced by other players leads to overconsumption. The intuition gained by the noncooperative equilibrium solution imply that players will neglect the future yields in the presence of other players, because they are aware that the future benefits from investment (be it increased stock or increased probability of the survival of the stock) will be in large part consumed by others. We can look whether the same mechanism will be present in the experiments.

In the first design there was one subgame perfect equilibrium, which yielded the expected payoff that was 22% of the Pareto optimal expected outcome. The Pareto efficient strategy was to wait for the first 16 periods and collect the safe yields and then start to exploit the resource in the last four stages. The results from 5 different groups strongly favor the overconsumption hypothesis with actual earnings ranging from 11% to 36% percent

and average earning 21% of the expected Pareto optimal income. They systematically overconsumed as the average overall consumption of the resource was 2,8 times higher than the Pareto optimal. There was one group that participated in the game repeatedly and had the highest aggregate consumption exactly coinciding with the equilibria value, hence we might conjecture that with increasing experience and understanding of the game the outcome approaches that of perfect Nash equilibrium.

In the second design the results were similar. There were two subgame perfect equilibria: one identical to the previous equilibrium and a second one, which yields nearly efficient income, 97% of the optimal outcome. Although there was almost Pareto efficient perfect equilibrium, most of the experiments resulted in a much worse outcome. In five out of seven groups the result was significant overextraction, with average overall consumption 1.7 times higher than the optimal one, which is similar to the worst equilibrium outcome. The remaining two groups extracted values similar to the best equilibrium consumption. Again, there was one experienced group, which had the highest average extraction. Hence, even if there is better Nash equilibrium, the players are more likely to select the worse one.

We see that when players interact intertemporally, and the loss for the whole group in next period is related to profit for individual in the present period, the players tend adopt more myopic strategies leading to overall inefficiency. Also, when they could choose between efficient and inefficient equilibria, they chose the inefficient one on the majority of cases. The experiment was conducted without additional information that plays significant role in our model, i.e. the the players do not have information about the actions of other player neither there exists a underlying state variable. Although the setting of our model differs in several important aspects, it supports hypothesis that players, who have negative on each other through influencing the dynamics of the resource, will overuse the resource.

### 4.3 Local Shared Fisheries

As we have argued it is hard to get evidence that would link directly the overexploitation and the dynamic externality. There are several factors that influence the state of the exploitation. In local commons, i.e. when the property is owned by a relatively small community with a limited number of appropriators that have access to it, the significant

factor is the institutional environment (Ostrom, 1990). The institutions can have various forms, for example the following ones: binding and enforced contracts; internalized standards of behavior; informal norms, where the disobedience can be punished by social exclusion; regulatory bureau of government and many others.<sup>1</sup> To maintain the language of the game theory, we can think of institutions as of rules that change the setting of the game. The fishery game (i.g. the set of players, dynamics of the stocks, available strategies, information patterns) is determined by the physical world (the properties of fish stock, sea environment) as well as by the man-made constraints (intentional or unintentional). Contrary to the physical world, these constraint imposed by humans can be changed and are to some extent endogenous to the problem. This endogeneity can be hardly captured, however, by a simple game-theoretic model.

These institutions are more or less present in each problem of the common-pool resources. This, thus, disables direct empirical examination of the structure of the game that involves only the physical world and highly specific interaction among players. Below, I cite two case studies that indicate that a lack of institutions constraining the appropriation and high number of players make the resource more vulnerable to over-exploitation. The cases serve merely as illustration and motivation, as they are selected rather randomly.

Berkes (1986) presents four case studies of different fishing spots on southern coast of Turkey. In the first three studies, Berkes identifies a set of institutions, some of which have legal basis and are recognized by state, while the others are informal. These institutions consist of local cooperatives, in which the decisions how much to fish and how are made. Also, the entrance of new players is restricted, either on legal basis, or by the danger of threats. The physical properties of the fisheries are also an important factor that makes keeping the number of fishers restricted easy. Neither of these fisheries shows signs of overfishing.

Contrary to the previous cases, fishery in Bodrum (Turkey) lacks any arrangement that would ensure the restrictions on appropriation. Also the entry into the fishery is not restricted. Until 1970s, local fishermen operated in the area throughout the history without causing any sustainability problems. This small scale fishing collapsed in the

---

<sup>1</sup>Detailed discussion of institutions is beyond the scope of this thesis. They are, however, major factor affecting the efficient use of resource. Ostrom (1990) and Ostrom et al. (1994) provide more information about the role of institutions in successful government of the commons. In addition, Sethi & Somanathan (1996) create interesting model of evolutionary emergence of institution of punishment in the context of commons.

early 1970s. Meanwhile number of large appropriators has steadily increased from just one trawler in 1960s to eleven trawlers in 1976. By year 1976, the stock was completely destroyed. In the absence of effective restrictions on appropriation, the increase of players using a resource was accompanied by rapid depletion of the resource.

Acheson (1975) comes with another example. He observed lobster fishing communities in Maine (USA). He found out that the fisheries are divided in several areas, in which the residents of the area have exclusive fishing rights. Although the right to fish in certain area is not based on law, as the land in the sea is officially public property with open access, it is widely recognized. The fishing rights are based on custom and violators of this common law are generally punished. The punishment can vary from destruction of fishing gear to physical violence. The punishments can be very harsh for repeated offenders. Every area is used by a group of fishers, a „gang“, whose members have the exclusive right to fish and protect the area from strangers. Within the gang, there is no system of assigning any constraints how much to fish. The amount fished is determined by each fisher individually.

Acheson identifies two basic regimes of protecting the fishing area: perimeter-defended and nucleated. The latter is centered around a mouth of a port, the close proximity of the port is regarded as strictly exclusive area and is eagerly protected from trespassers. The more distant is a spot from the harbor the less is the exclusivity enforced. Many distant places are shared by two gangs at the same time.

Perimeter-defended area are vigorously protected on the whole area. The boundary is drawn precisely and anyone fishing in the area is punished, contrary to nucleated area, where the boundaries are not so strictly enforced. The perimeter-defended areas usually surround islands. The gang governing perimeter-defended area is more exclusive and the membership is usually hereditary. Meanwhile, the membership in a gang in the nucleated area is connected with residency in the harbor town, and any newcomer that lives in the town for short time can get it. As a result the gang of perimeter-defended area has less members and therefore there less fishing boats per square mile in the perimeter defended area than in the nucleated, concretely it is two thirds times less.

Acheson (1975) investigated how the biological quality of the stock and the average yield differ in the two types of areas. Both types of the regimes are uniformly spread across the coast of Maine and fish the same species of lobster, thus, the productivity of the natural environment should be the same. The only thing that is different is the institutional environment and the number of appropriators it induces. He found that the density of the stock in perimeter-defended areas is on average twice as high as in the nucleated area. Moreover, the yield given by pounds of lobster caught per one man

and unit of time reaches on average twice as high levels as well. We can observe a close positive relation between the exclusivity and number of players on one side and the level of the stock and yield on the other. Although no tragedy occurs in either case, in the sense that the resource is not entirely depleted, the relation is again consistent with the noncooperative equilibrium.

Ostrom (1990) describes several cases of successful fisheries management that use no government enforcing or violence to restrict the consumption. In several commons that are communal property, coordination occurs, the community decide cooperatively on the rules concerning fishing and the rules are obeyed. Thus, welfare improving cooperation might emerge, even without enforcement. The possibility of cooperation was, also, confirmed by experiment (Ostrom et al., 1992). The experiment was a standard common-pool resource game, which had the structure of repeated prisoner's dilemma. The players observed the aggregate actions of others and could communicate after every round. In this experiment, the outcome was on average very close to Pareto optimal.

This closely resembles the trigger strategy equilibria of section 3.5. One way to interpret institutions that enhance cooperation through communication is to assume that these players play some kind of trigger strategy. The culture of mutual trust exists, which prevents the tragedy of commons. The trust is, however, supported by implicit threat, that if someone deviated, the whole system of appropriation restrictions would collapse.

## 4.4 Summary

The empirical literature that we have presented in this chapter suggests a strong link between the number of appropriators and efficiency. The more players using the resource, the worse is the condition of the resource.

First, we showed the link by evidence from international fisheries. The number of players was a significant explanatory variable in predicting, whether the species of fish was overexploited. The higher the number of exclusive economic zones that had access to a certain species of fish, the higher was the probability that the stock was overfished or depleted and the lower was the probability that the stock is used efficiently or under-exploited.

Similarly, we have chosen two case studies to illustrate the link between number of players and inefficiency. In the first case the inability to restrict access to fishery resulted

in increase of appropriators and this in turn caused depletion of the stock. In the second study, two regimes of restricting access were identified in otherwise identical fisheries. The research found out that the relative ineffectivity of one arrangement to keep the number of fishers low resulted in lower yields and worse condition of the stock.

The effect of number of players is consistent with the prediction of the noncooperative equilibrium that results in overfishing. While in the Pareto optimal case the overall extraction and the level of the fish stock is unaffected by change in number of appropriators; in the „tragic“ Nash equilibrium, increasing number of players worsens the overexploitation of the stock.

Also, an experiment supports the hypothesis that the dynamic appropriation externality, which is inherent in fishing, causes overexploitation of the fish stock. Indeed, the evidence supports that shared fisheries are inefficient; especially if the number of fishers sharing the resource is large. If it is impossible assign and enforces property rights to individuals, there is a room for government or community regulation to enhance efficiency. Our model suggests that the most efficient solution would be nationalization and heavy regulation. This policy suggested by Hardin (1968) would be wrong, however, as there is more to issue of governing commons.

We have briefly stated that several examples of successful management of fisheries are documented. One would have to investigate other issues more closely to identify all the aspects of the commons in fishery, which is beyond the scope of this thesis. Yet, we have suggested that some of these successful arrangements could be possibly interpreted as the trigger strategy equilibria.

# Chapter 5

## Conclusion

At the beginning, we raise a question, whether the dramatic state of world fisheries can be attributed to their common property nature. To answer this question, we adopt a dynamic game-theoretic model of fishing. The model has two basic features: the stock of fish grows in time, and there are several appropriators that influence the dynamics of the stock, thus, there is strategic interaction among the players.

We find that various outcomes are possible. When the players cannot react directly on the development of the stock, the outcome is efficient. When the players observe directly the extractions of others and the discount factor is sufficiently high, efficient outcome is also possible. Otherwise, the strategic interaction leads to inefficiency. Therefore, the efficiency arises only under rather limited conditions and we are more likely to expect Pareto inferior outcome.

We demonstrate from theoretical perspective, that common sharing of a resource leads, except few marginal cases mentioned above, to inefficiency. We particularly stress the outcome that leads to the well-known tragedy of commons. Under this equilibrium we can directly link the nonexclusive nature of fisheries and its tendency to be overused. We argue that tragedy of commons happens, because when more appropriators share a resource, they do not take in account the effect of their consumption on others. This equilibrium also predicts that with increasing number of players, the stock tends to depletion. The empirical and experimental results also indicate that there is a strong link between the number of fishers exploiting a resource and overexploitation. Hence, when the access is not restricted, high number of fishers bring the stock on the brink of depletion. However, several successful cases of management of shared local fisheries are documented as well. We conjecture that some of these could be interpreted as trigger strategy equilibria.

To prevent the inefficiency predicted by our model, the access to fish must be re-

stricted. The first best solution would be to assign exclusive property rights to fishing grounds. However, because of the nature of fisheries, this is practically impossible in reality. Our model suggests the nationalization and/or heavy state regulation, where possible, as the best solution. It is documented, however, that increased government involvement has led often to disastrous results (see for example Ostrom (1990)). There are many more factors influencing the efficiency of the resource exploitation and our discussion is by far not exhaustive. Other authors have stressed the importance of institutions (Ostrom, 1990), uncertainty and limited information (Wilson, 1982). It is clear that access to fish must be restricted to enhance efficiency and halt the destruction of fish, but to draw any more concrete policy recommendation, one must deal with the issue more thoroughly and take in the account all the factors affecting efficiency. Yet, the game theoretic approach, which we presented in this thesis, enhances our understanding of why the inefficiencies occur and might, thus, contribute to better policies.

# References

- Abreu, D. (1988). On the theory of infinitely repeated games with discounting. *Econometrica*, 56(2), 383–396.
- Acheson, J. M. (1975). The lobster fiefs: Economic and ecological effects of territoriality in the maine lobster industry. *Marine Policy*, 3(3), 183–207.
- Amir, R. & Nannerup, N. (2004). Information structure and the tragedy of the commons in resource extraction. CORE Discussion Papers 2004040, Université catholique de Louvain, Center for Operations Research and Econometrics (CORE).
- Basar, T. & Olsder, G. J. (1995). *Dynamic Noncooperative Game Theory*. New York: Academic Press.
- Berkes, F. (1986). Local-level management and the commons problem: A comparative study of turkish coastal fisheries. *Marine Policy*, 10(3), 215–229.
- Bertsekas, D. (2001). *Dynamic Programming and Optimal Control*. Belmont, Massachusetts: Athena Scientific.
- Cave, J. (1987). Long-term competition in a dynamic game: The cold fish war. *RAND Journal of Economics*, 18(4), 596–610.
- Dutta, P. K. & Sundaram, R. K. (1993). The tragedy of the commons? *Economic Theory*, 3(3), 413–26.
- Food and Agriculture Organization of United Nations (2005). Review of the state of world marine fishery resources. FAO Fisheries Technical Paper 457.
- Gibbons, R. (1992). *A Primer in Game Theory*. Boston: Twayne Publishers.
- Hardin, G. (1968). The tragedy of commons. *Science*, 162(3859), 1243–1248.

- Herr, A., Gardner, R., & Walker, J. M. (1997). An experimental study of time-independent and time-dependent externalities in the commons. *Games and Economic Behavior*, 19(1), 77 – 96.
- Levhari, D. & Mirman, L. J. (1980). The great fish war: An example using a dynamic cournot-nash solution. *Bell Journal of Economics*, 11(1), 322–334.
- McWhinnie, S. F. (2009). The tragedy of the commons in international fisheries: An empirical examination. *Journal of Environmental Economics and Management*, 57, 321–333.
- Okuguchi, K. (1981). A dynamic cournot-nash equilibrium in fishery: The effects of entry. *Decisions in Economics and Finance*, 4(2), 59–64.
- Ostrom, E. (1990). *Governing the Commons*. Boulder: Westview.
- Ostrom, E., Gardner, R., & Walker, J. (1994). *Rules, Games and Common-pool Resources*. Ann Arbor: The University of Michigan Press.
- Ostrom, E., Walker, J., & Gardner, R. (1992). Covenants with and without a sword: Self-governance is possible. *The American Political Science Review*, 86(2), 404–417.
- Sethi, R. & Somanathan, E. (1996). The evolution of social norms in common property resource use. *The American Economic Review*, 86(4), 766–788.
- Stokey, N. L., Lucas, R. E., & Prescott, E. C. (2004). *Recursive Methods in Economic Dynamics*. Cambridge, Massachusetts: Harvard University Press.
- Walker, J. M. & Gardner, R. (1992). Probabilistic destruction of common-pool resources: Experimental evidence. *The Economic Journal*, 102(414), 1149–1161.
- Wilson, J. A. (1982). The economical management of multispecies fisheries. *Land Economics*, 58(4), 417–434.

UNIVERSITAS CAROLINA PRAGENSIS  
založena 1348

Univerzita Karlova v Praze  
Fakulta sociálních věd  
Institut ekonomických studií



Opletalova 26  
110 00 Praha 1  
TEL: 222 112 330,305  
TEL/FAX: 222 112 304  
E-mail: [ies@mbox.fsv.cuni.cz](mailto:ies@mbox.fsv.cuni.cz)  
<http://ies.fsv.cuni.cz>

Akademický rok 2008/2009

## TEZE BAKALÁŘSKÉ PRÁCE

Student:	Tomáš Fiala
Obor:	Ekonomie
Konzultant:	Martin Gregor

Garant studijního programu Vám dle zákona č. 111/1998 Sb. o vysokých školách a Studijního a zkušebního řádu UK v Praze určuje následující bakalářskou práci

Předpokládaný název BP:

Fish Wars

Charakteristika tématu, současný stav poznání, případné zvláštní metody zpracování tématu:

This thesis examines the strategic problem of natural resources extraction. It will present several different game theoretic models of fish exploitation and compare the results of different approaches.

Struktura BP:

1. Introduction
2. Cooperative Solutions
3. Non-cooperative Solutions
4. Cooperative Solutions using Trigger Strategy
5. Biological Interaction between Species of Fish
6. Conclusion

Seznam základních pramenů a odborné literatury:

- Amir R., Nannerup N.: Information structure and the tragedy of the commons in resource extraction, Journal of Bioeconomics, 2006
- Cave, J.: Long-term competition in a dynamic game: the cold fish war, The RAND Journal of Economics, 1987
- Denisova, E., Garnae, A.: Fish Wars: Cooperative and Non-Cooperative Approaches, AUCO Czech Economic Review, 2008
- Fischer, R. D., Mirman L. J.: Strategic dynamic interaction: Fish Wars, Journal of Dynamics and Control, 1992

Levhari, D., Mirman L. J.: The great fish war: An example using dynamic Cournot-Nash solution, The Bell Journal of Economics, 1980
------------------------------------------------------------------------------------------------------------------------------------

Datum zadání:	5. června 2009
Termín odevzdání:	červen 2010

V Praze dne 5. června 2009

*S předloženou tezí souhlasím.*

*Martin Gregor, v.r. 5. června 2009*