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DIPLOMOVÁ PRÁCE

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Optimalizace marže vkladových produktů banky

Katedra pravděpodobnosti a matematické statistiky

Vedoucí diplomové práce: Mgr. Martin Hanuš PhD.

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Svému vedoucímu práce Mgr. Martinu Hanušovi PhD. děkuji za poskytnuté konzultace a cenné rady při zpracování mé diplomové práce. Své rodině děkuji za podporu.

Prohlašuji, že jsem svou diplomovou práci napsala samostatně a výhradně s použitím citovaných pramenů. Souhlasím se zapůjčováním práce a jejím zveřejňováním.

V Praze dne 4. srpna 2009 Bc. Barbora Hejmová

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Abstrakt: Předložená diplomová práce se zabývá stanovením marží depozitních produktů na českém depozitním trhu. Finanční krize, která zasáhla celý finanční systém se přirozeně nevyhnula ani českým bankám. Tato situace měla za následek výrazné snížení obchodu na mezibankovním trhu a banky začali hromadit likviditu zejména z primárních depozit.

Tato práce se v jejím úvodu zabývá původem a následnému rozšíření likviditní krize. Následující kapitoly rozdělením primárních depozit a jejich vlastností na českém bankovním trhu. Důraz je kladen především na termínované vklady a stanovení jejich sazeb a marží v závislosti na různých faktorech ovlivňující trh. Podstatnou částí této práce je model stanovení optimální marže zejména v závislosti na poptávce a marži konkurenčních bank. Zde je popsán i dopad likviditní krize na navrhovaný model. Výsledky jsou popsány na datech termínovaných vkladů z období 2006-2008 jedné z velkých bank působící na českém trhu.

Klíčová slova: depozitní produkty, termínovaný vklad, finanční krize, marže

Title: The Optimization of the Deposit Products Margin of a Bank
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Abstract: In the presented work we study fixing of term deposit margins on Czech deposit market. Financial crisis which influenced almost each financial system hit also Czech banks. It resulted in almost no trading on interbank market and banks started to hoard liquidity from primary deposits.

At the beginning of the thesis we introduce origin and expansion of liquidity crisis. Next we introduce term deposits and their basic properties on Czech deposit market. Then work is focused on term deposits and on fixing their margins in dependence on several factors. Next part introduce model for optimal margin in dependence especially on demand for term deposits and competitive margin. There is also description of crisis impacts on suggested model. Results are built on term deposit of one of the largest banks on Czech deposit market. Data cover period from 2006 to 2008.

Keywords: deposit products, term deposit, financial crisis, margin

Chapter 1

Introduction

Economic conditions in the pre-crisis period were characterized by dynamic and continuous economic growth in developed countries. Relatively stable inflation and low interest rates resulted in an explosion of credits further encouraged by low long term interest rates in the US and global money surplus from China, Japan and oil exporting countries. Low volatility and small differences among assets contributed to the development of new financial products.

Pricing bubble and quick development of innovative products (especially securitisation and credit derivatives) left economic system in a fragile state. The credit crunch followed by a global crisis began with the defaulted subprime mortgage market. Consequently were forced into a liquidity hoarding using primary deposits.

This work is divided to six chapters. The first chapter describes actual situation on financial market. It deals especially with development of liquidity crisis not only in the world but also on Czech deposit market. The second chapter introduces primary deposit products and their main characteristics. At the end of chapter this is described impact of liquidity crisis on Czech deposit products. The third chapter describes severals models how

to fix term deposit interest rates in dependence on referential rates. First we assume Pribor rate and then interest rates of leader on deposit market. There is also shown position of banks according to interest rates. The next chapter introduces model of optimal margin and profit in dependence on several parameters which are introduced in following sections. The fifth chapter summarizes conclusions and the last chapter describes data which were used in practical part.

1.1 Source of Financial Crisis

Changes in the institutional environment and massive development of above mentioned structured products contributed to the start of the financial crisis. Credit standards and conditions for granting of mortgage loans were set up. It was especially important for *sub prime* segment i.e. segment of household with higher risk of default. In addition, The Federal Reserve System (FED) decided not to oversee new agencies which offer mortgages. Moreover, in 2004 the exemption from the rule about a capital requirement was given to a investment companies with a capital higher than 5mld USD[1]. In particular, the important investment companies such as Goldman Sach, Merrill Lynch, Lehman Brothers, Bear Stearns and Morgan Stanley.

As mentioned before, in a period before crisis investment banks, government sponsored companies, hedge funds and other involved institutions operated with a high debt-to-equity ratios under low and stable interest rate. This resulted in a rising profit euphoria on financial market.

[1]Basel II. - release debt to net capital ratio from original 12:1 to 30:1 how is mentioned in Monitoring centrálních bank [7]

1.2 Outbreak of Crisis and Interventions of Central Banks

Property market began to tumble in the first half of 2007 in the US as a result of an increasing amount of residential mortgage default. Although problems were initially confined to the sub prime mortgage markets, further deterioration of credit quality and increasing rates resulted in an expansion of the crisis to the other products and markets. Non-isolated sub prime mortgage problem spread to securities which were not hold only by financial institution in US, but even in Europe and other countries in the world. These events led to the liquidity crisis, decreasing confidence among subjects on financial markets and causing a shortage of reliable information. The loss of confidence created an uncertainty among rating agencies as to which institutions held problematic securitisated financial products and in what quantity. This resulted in a higher resistance by banks to provide a liquidity to other banks, especially for maturities longer than a few days. Reasons for liquidity hoarding were twofold:

- banks protection against potential liquidity needs
- uncertainty about the risk of counter party

Problems became more acute in the middle of 2008 after series of investment bank bankruptcies (Bear Stearns, Lehman Brothers) and massive take-overs of important investment and insurance companies by rivals (Merrill Lynch, Wachovia, Fortis, Dexia, AIG and Island banks). Major event that launched the international financial crisis was a bankruptcy of the investment bank Lehman Brothers[2].

Central banks and governments reacted differently to ensure a protection of their financial systems. The main efforts in the US, Eurozone and in Great

[2] 15th of September 2008

Britain concentrated on the following:

- to provide liquidity to interbank market
- to decrease interest rate
- to inject equity to troubled institutions

1.3 Situation in the Czech Republic

The financial system of the Czech Republic is partially isolated from global problems. Local financial institutions have not been massively exposed to problematic sub prime mortgages. The growth of local mortgage market past few years was fueled by a strong economic situation. In contrast to US, credit standards are rather high and the credit profile of loan portfolios is good.

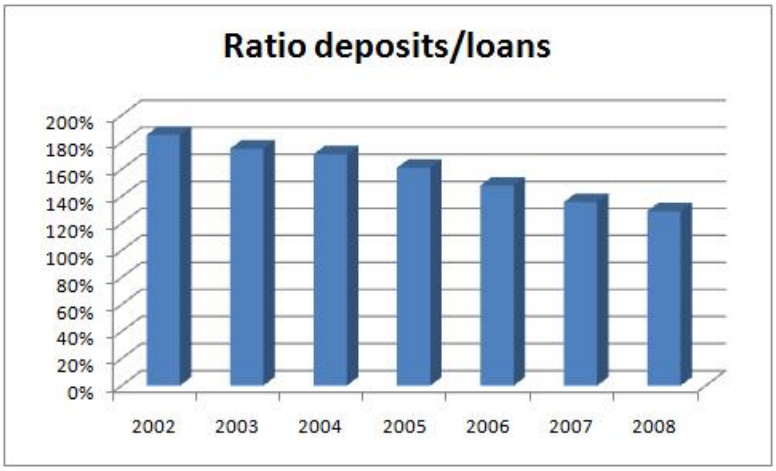

Figure 1.1: Deposit/Loan Ratio

Figure 1.1 shows the *Deposit-loan ratio*[3] in the Czech Republic. The ratio balances about 1.4 meaning that there is more deposits than loans in banks. In general, the ratio is decreasing, but not so wildly. Relatively high volume of primary deposits protects Czech financial market against early shortage of liquidity on the financial market. Nevertheless, the interbank market is dominated by nervousness and limited liquidity for a long term maturity. To get a liquidity on the interbank market is difficult due to the uncertainty on financial market and therefore banks look for other options. Usually, they try to build a liquidity pillow to protect themselves from the risk of the liquidity shortage come back. Since the financial system is paralyzed, the central bank funds can be drawn just against collateral. The best liquidity source seems to be primary deposits.

[3]all deposits from clients and receive credit divided by all client's credit - short, middle and long term

Chapter 2

Primary Deposits

Bank's clients can choose how to deposit their free financial sources from many bank's products. These deposits are most likely the cheapest source of liquidity for further financing.

This fact made competition among banks in the field of deposits even more aggressive. Furthermore new clients were easy to cross sell. It appears from this that banks should follow client's requirements, watch situation on market more carefully and innovate their own products more often. Banks can obtain capital from clients by deposit products which can be sort to following four categories

- current accounts
- savings accounts
- term deposits (TD)
- securities

All banks with license in the Czech Republic have to lead away some percent of their deposits to *The Fund of Guarantee - Fond pojištění vkladů*. This fund is used to return the investments to clients in case of bankruptcy.

According to Law of Banks[1] the following types of deposits (in all currencies) are insured:

- deposits of households and companies in bank
- current accounts and their interests
- deposits books
- deposits confirmed by certificate of deposits
- term deposits
- saving accounts
- accounts for building

For securities there is so called *Garanční fond burzy* which is the union of financial sources a risk covering from trades on Securities Exchange in Prague.

The income tax of capital equity is also related to deposit income[2]. For a physical entity it is 15% from incomes and different percentage for legal entity.

2.1 Current Accounts

Current account is one of the basic bank products. It is an instrument used for saving the temporary free financial resources of clients and especially for non cash payments. Deposits current accounts have usually small interest rate, due to the necessity to be able to dispose the money anytime. Almost all banks in the Czech Republic offer current accounts in foreign currencies. Dominant currency is the American dollar (USD) and Euro (EUR).

[1] no. 21/1992 Coll.

[2] Law of Income Tax no. 586/1992 Coll

For banks deposits on current accounts mean very important source of funds with the following properties as mentioned in Dvořák (2005) [3]:

- interest load is relatively low, with a high volume of payment transactions
- deposits on current account increase reserve requirements
- interest rates are almost stable (not linked to any market interest rates)
- advantage for banks comes from the *sediment*. Level of money on accounts change every day, but certain part of these deposits is still available for bank. Income and outcome payments are partially equal (under normal circumstances)

2.2 Savings Accounts

In comparison with current accounts they are instruments used to save deposits which client does not use on daily basis, but will use in the future. For banks these deposits are connected with lower transaction loads (resulting from a higher volume and the fact that some volume might be untouchable for a certain period). Reserve requirements can be lower for savings accounts than current accounts in some countries, but in the Czech Republic it is the same level. The *Sediment* with the same property occurs also for saving accounts. Interest rates are usually a slightly higher because clients deposit higher amounts and therefore these accounts are more resistant against inflation.

The features of saving accounts are different from bank to bank. They can be combined with other deposits products resulting in a wide variety of types. Also like current accounts, they are involved in guarantee of deposits.

2.3 Term Deposits

Term deposits offer the possibility of saving higher amounts with high interest rates. For the duration of the agreed upon period clients can not manipulate with money in the account. According to the withdrawal conditions the deposits are divided into the following:

- deposits with fixed maturity - money on deposits for an agreed upon period
- deposits with cancellation - disposition of money is agreed upon in a contract

Deposits with fixed maturity can be further divided into the following types:

- short term - maturity to 1Y (for example 1W,2W,1M,2M,3M,6M)
- middle term - maturity from 1Y to 5Y (for example 1Y,2Y,3Y)
- long term - maturity longer than 5Y (for example 5Y)

Term deposits are available for all clients however banks supply different interest rates depending on a client. The higher the volume and the longer the maturity of term deposits, the higher the interest rate is set. There can be one of the following types of interest rates:

- fixed - interest rate is fixed for entire period
- flexible - interest rate is bounded to another interest rate (basic bank's interest rate, discount rate of central bank, interbank interest rate, etc.)
- linked to some other financial characteristics with embedded option

Term deposits are an important source of deposits for bank. In comparison with current accounts, they are not used for payment transactions[3]. Moreover, the total volume of deposits is usually higher. As shown in Figure 2.1, short term deposits are dominant on the Czech market. There is relatively low demand for long term deposits, even so recently there has been a moderate increase. High inflation and low interest rates are two main reasons why the interest does not cover the inflation.

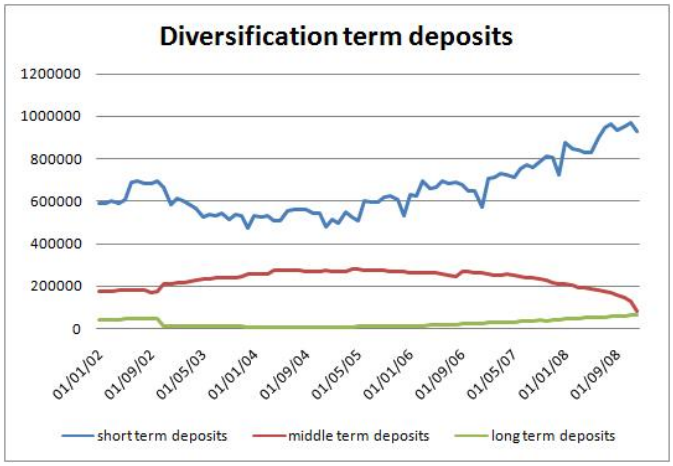

Figure 2.1: Short, middle, long term deposit, Source: ČNB - ARAD

2.4 Securities

Other common deposit product is the issuing of securities. Various types are differ in several products characteristics. In comparison with deposits they have one specific property - they are not covered by the mandatory insurance - guarantee deposits. This means higher risk for clients. The bank does not pay contribution to guarantee fund. Bonds are defined in Law of

[3]it is possible withdraw money before maturity but banks apply charge for it

Bonds[4]. They can be issued in paper certificates or book-entry bond and then traded on interbank market. Bank as an emitter can issued bonds, when *Komise pro cenné papíry*[5] certifies the emission conditions of bonds. There are laws and obligations for an emitter and holder of bonds and other details about bonds, an emit and market in conditions. Maturity can be fixed or flexible with several payment.

One of the very important characteristics for a client is bond's liquidity, i.e how quickly and under what conditions it can be changed for money. The yield of a bond can be set as:

- fixed interest rate
- difference between nominal value and emission rate
- premium
- flexible interest rate bounded to other market interest rate

There are various types of securities issued by banks:

- bank bonds
- mortgage bonds - In a default situation, mortgage bondholders have a claim to the underlying property and can sell it off to compensate for the default as mentioned in Investopedia [8]
- covered bonds bonds
- Bills of Exchange - regulated by Bills of Exchange and Cheques Act[6]

[4]Bonds Act of the Czech Republic (No. 530/1990 Coll.) which is special regulation of Law of securities no. 591/1992 Coll.

[5]member of Committee of European Securities Regulators

[6]No. 191/1950 Coll.

2.5 Deposit Products During Crisis

In crisis the investors change their expectations. Before they prefer higher yields from their investment, now they are satisfied with lower or any returns. They are not aggressive, changed their strategies to defensive position and invest in safety products. The uncertainty of a financial market during a crisis forces people to save more money.

The situation before the financial crisis was as follows: deposits were guaranteed up to 90% of exposure below 25 000EUR in the Czech Republic. Then the crisis hit and almost all Europaen goverments began to rise the limits for deposits guarantee. Unfortunaltely due to the legislation procedures of the individual conutries, this has not occured uniformly across the Europe. In the Czech Republic, clients of czech banks began to withdraw money and to transfer them to others banks with higher insurance (Guarantees in other Europien countries are shown on Figure 2.2). Czech government reacted relatively quickly and approved 100% gurantee of all deposits and maximal bound on 50 000 EUR (or about 1,4 mil CZK[7]). The president of the Czech Republic signed the amendment of the law regarding deposit guareantee in December 2008 and subsequently this amendment was published in Collection of Laws.

Client with a significant amount of deposits reacted by diversification of their risk to several bank accounts. During crisis banks experience a reduction of the actual cash amount on accounts of their current clients and an influx of new clients wishing to open an account.

To put money on term deposits is profitable nowadays, especially even to the lower inflation and banks desire for more liquidity. This makes banks to create products just for clients with more convenient conditions.

Considering several alternatives how to deposit money in a bank, investors tend to choose safe options. Bank securities may bring higher yield

[7] Actual exchange rate 19.2.09 28,6CZK/EUR

Country	Guarantee
Denmark	all deposits
Germany	all deposits
Portugal	all deposits
Austria	all deposits
Slovakia	all deposits
Great Britain	to 65 000 EUR
Litva	to 100 000 EUR
Poland	to 50 000 EUR
Greece	to 100 000 EUR
Spain	to 100 000 EUR
Sweden	to 52 000 EUR
France	to 70 000 EUR
Italy	to 103 291 EUR
Hungarian	to 24 000 EUR
Netherlands	to 40 000 EUR

Figure 2.2: Comparison of limit in other countries

yet they also come witch the uncertainty. Given a current risk aversion on a financial market and non guarantee of securities, these investments are less sought after.

This behavior of investors is also shown on demand for term deposits. Clients are willing to accept worse conditions imposed by a of bank if the deposits are safe.

Chapter 3

Pricing Decisions in a Bank

3.1 Introduction

Pricing decisions are different in each banks. Some banks linked their term deposit (TD) interest rates directly to the reference rate or calculated functions depending on reference rate. Other follow situation on TD market and position of competitive banks. This chapter describes making decisions approaches based on referential rates and approach based on position on deposit market. And give ideas about Czech deposit market how banks fix TD interest rates and margins.

Before describing methods, we introduce Pribor like a referential rate and check whether it is right candidate. Pribor is reference rate fixed by the Czech Central Bank. This rate is considered as the interbank interest rates for CZK. Banks used to lend for this rate to each other on interbank market in several maturities 1D, 1W, 2W, 1M, 2M, 3M, 6M and 1Y. Now Pribor is used for fixing interest derivate rates and for reference rate for contract with large clients. For purpose of this paper about Czech deposit market use Pribor (Prague Inter Bank Offered Rate) as a referential rate. Other interbank markets use their own rates Bribor (Bratislava), Euribor

(Euro), Fibor (Frankfurt), Libor (London), Nibor (New Your) etc. The rate is published every work day (except state holidays) and it is calculated from *interest rates quoted by referential banks*[1] by algorithm based on arithmetical average.

Common behavior in deposit management of banks is profit from small clients. It means that they adjust long maturities and high volumes deposit rates to referential rates more often than to small clients (short maturities and small volumes deposit rates) thus, rates are set in stricter rules.

Banks are mainly interested in profit. They are finding optimal level between margin and volume of deposit. Margin of deposits is difference between referential rate and interest rate of examine deposit products. Since term deposit is parametrized product (margin, sector, time) introduce margin in general

$$m_{M,Bi}(t, a_j, a_{j+1}) = PRIBOR_M(t) - r_{M,Bi}(t, a_j, a_{j+1}) \tag{3.1}$$

where

- (a_j, a_{j+1}) - sector of volume, $j = 1, 2...n$, where $n - 1$ is number of sectors
- M - maturity
- t - time
- Bi - i-th bank on deposit market
- m - margin
- r - TD interest rate
- $PRIBOR$ - referential interest rate. For the Czech banks it is Pribor.

[1]Interest rate of referential bank is rate which bank are willing sell deposit to other ref. bank

To show if there is some dependence we can apply correlation method which in our case determined linear dependence between variables. Figure 3.1 shows this coefficient between 6M Pribor and TD interest rate in sector (500 000 - 1 000 000) for banks. According to correlation coefficient there is obvious linear dependence in all banks, therefor we can use it for next analyze.

Bank	correlation
Česká spořitelna	0.911
ČSOB	0.938
Komerční banka	0.893
Volksbank	0.913
Raiffeisenbank	0.912
CitiBank	0.959
GeMoney Bank	0.888
UniCredit	0.952

Figure 3.1: Correlation coefficient of 6M Pribor and corresponding TD interest rates of single banks for sector 500ths - 1mil

3.2 Delay model

Banks react on Pribor rate usually with delay. It means that published interest rate in time t is reaction on Pribor couple of days ago. Process starts by changes on deposit market (Pribor changes), then analyst offers suggestion as a reaction on changing. Pricing Committee make decision and then bank publishes their deposit rates. Last step is register their rates for the future by external agency. All interest rates (all sectors and maturities which banks offer) are processed in the same time.

In this section let's assume following hypothesis: Banks keep constant margins for all sectors and maturities. Margin is calculated according to

Equation 3.1 from referential rate. Second hypotheses: there is reaction delay which is created by approving or administration process in bank. Last consideration is that banks change deposit rates in the same time, therefor reaction delay is same for all maturities and sectors. In this section we use data described in Chapter 6 - Data. We test whether our hypothesis are satisfied.

Pricing Committee, where decisions about change rates are made, meet with different frequency in single banks. It depends on mechanism of individual management (once a week, twice a week, ...) and therefor reaction delay can be different for banks. Other options is that they give a rule: automatic changing rates every day according to calculation, but still there is some delay in administration process. Different rules in different banks are obvious on Figure 3.2. Citibank changes rates in high frequency and react immediately on Pribor rate while Raiffeisenbank changes rates not so often.

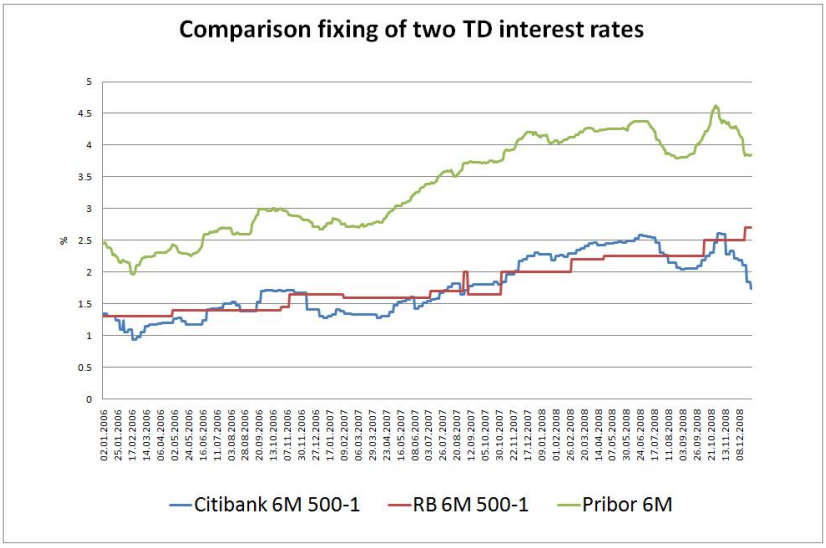

Figure 3.2: Comparison fixing of two interest rates

We named this approach based on delay hypothesis as a *Delay model*. For better explanation of hypothesis of the Delay model there is shows on

Figure 3.3 how a bank fixed TD interest rates for each sector and maturity. Aim of this part is to present how to search delay corresponding to respective bank processes.

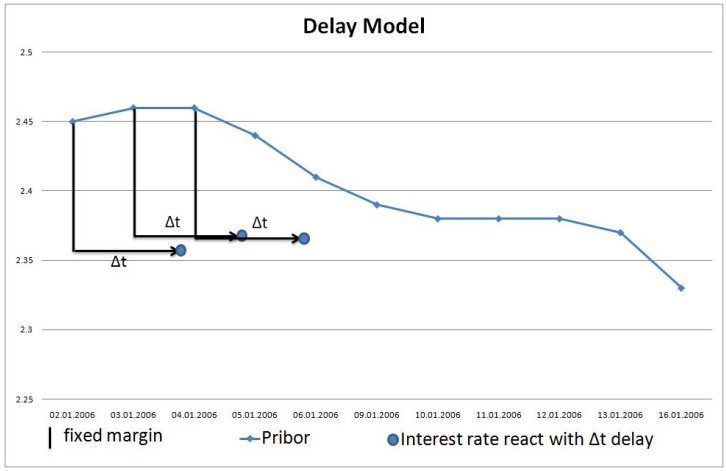

Figure 3.3: How bank fix TD interest rate in Delay Model

Now we define some variables that we use in next part about Delay Model. Denote

$S(t)$ - TD interest rate in some sector in time t
$P(t)$ - corresponding Pribor in time t
$\Delta(t)$ - reaction delay
$m(t, \Delta(t))$ - maturity in time t with delay $\Delta(t)$

Margin is defined in terms of a delay $\Delta(t)$ according to Equation 3.1 and time t as following

$$m(t, \Delta(t)) = P(t - \Delta t) - S(t)$$

Defined new TD interest rate $S'(t)$ as a interest rates with constant margin. The simplest method for calculating constant margin is an arithmetical average. Denote $\overline{m}(\Delta(t))$ and recall that it is also in term of $\Delta(t)$.

$$\overline{m}(\Delta(t)) = \sum_{t=1}^{n} m(t, \Delta(t))$$

where n is length of margin time series. Note that same length are Pribor and TD interest rate are same length. Now we can define New TD interest rate

$$S'(t, \Delta(t)) = P(t - \Delta t) - \overline{m}(\Delta(t))$$

Let us denote $\Delta(M)$ as standard deviation of difference between TD interest rate and the New TD interest rate

$\Delta(M) = \sqrt{Var(S(t) - S'(t))}$

$\Delta(M) = \sqrt{Var(P(t - \Delta t) - \overline{m} - P(t - \Delta t) + m(t))}$

$\Delta(M) = \sqrt{Var(m(t, \Delta(t)) - \overline{m}(\Delta(t)))}$

Aim of this model is to find such a $\Delta(t)$ to minimize $\Delta(M)$

$$\min_{\Delta t}(\Delta(M)) \tag{3.2}$$

Under assumption that decision making is uniform in bank, we are looking for such a $\Delta(t)$ which is equal for all maturities and sectors.

In banks which respond to Pribor in short frequency we assume clear result than in banks with long respond. Bank's management concentrate more on sectors with higher volume and longer maturities (half year and more). These clients bring bank higher utility than clients with small deposit volume and only for couple of weeks. Bank can better manage with longer and higher money.

Citibank is only one bank on Czech deposit market which react on Pribor with short frequency and also only Citibank get corresponding results. Figure 3.4 shows Pribor rate, New interest rate (shift by delay 4 working days) and corresponding margin. From chart is obvious that Citibank keep constant margin. An example is shown sector 1 000 000 - 3 000 000, 6Maturity. It is

result from Delay Model and is equal for all sectors and maturities, which corresponds to our hypothesis.

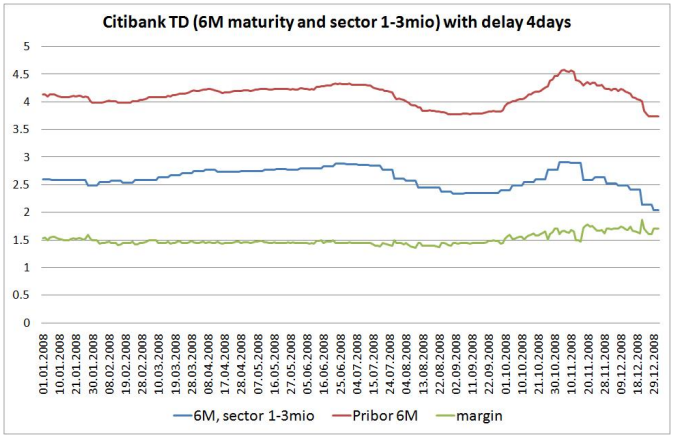

Figure 3.4: New TD interest rate and Pribor, 6M maturity and 1mil-3mil volume sector

These results are shown on following several charts.

Figure 3.5 and 3.6 show that delay 4 days gives the smallest standard deviation in all follow sectors 50 - 100 000, 100 000 - 300 000, 300 000 - 500 000, 500 000 - 1mil, 1-3 mil, 3-5mil, 5-7.5mil and 7.5-10 mil. Difference between new TD interest rate with constant margin and original interest rate is minimum for 4 working day delay. Minimum standard deviation reach approximately 0.3 % in average.

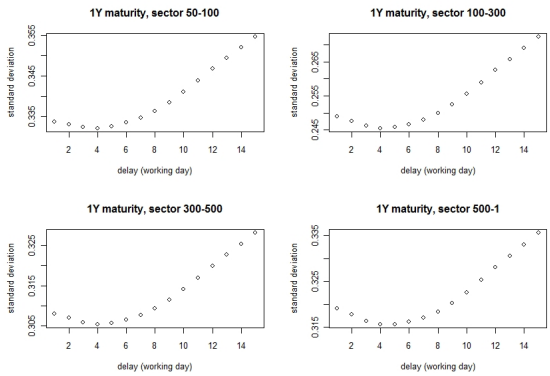

Figure 3.5: 1Y maturity, sector 50-100, 100-300, 300-500, 500-1mil

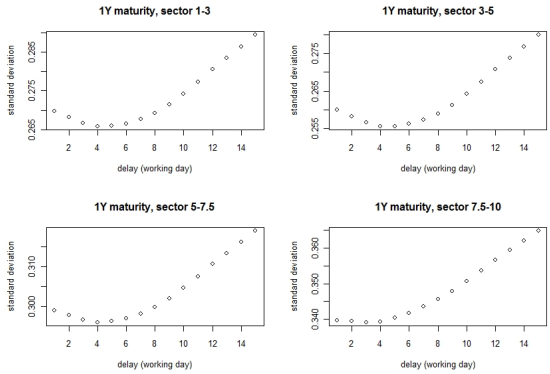

Figure 3.6: 1Y maturity, sector 1-3mil, 3-5mil, 5-7.5mil, 7.5-10mil

Figure 3.7 and 3.8 show 6M maturity in all sectors which Citibank offer. Also this maturity reach 4 working days delay on Pribor. Except sector 7.5 - 10 mil. Bank usually approach to client with high maturity and volume individuality. Account rates are linked on Pribor as well, but not with strict rules. Minimum standard deviation reach approximately 0.3 % in average.

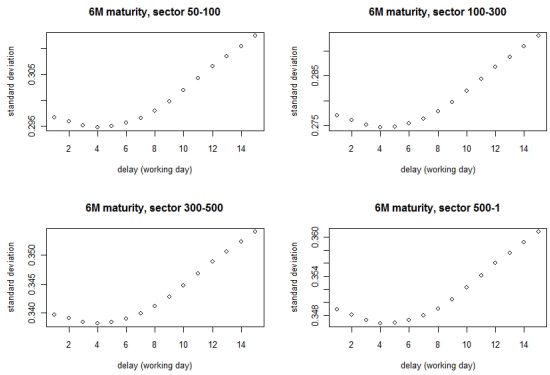

Figure 3.7: 6M maturity, sector 50-100, 100-300, 300-500, 500-1mil

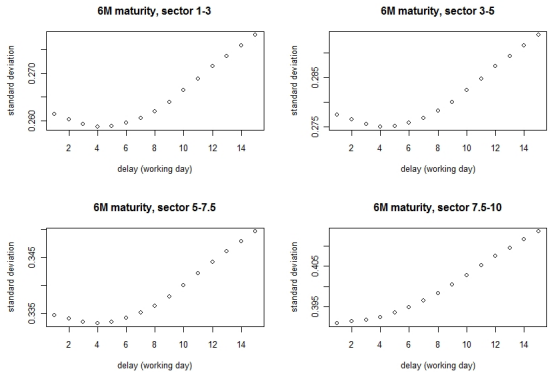

Figure 3.8: 6M maturity, sector 1-3mil, 3-5mil, 5-7.5mil, 7.5-10mil

On Figure 3.9 is shown that Delay model not suits for all rates and all banks. As an example is shown on ČSOB, 6M maturity in four sectors 0-150, 150-500, 500-1mil and over 1mil. There is no delay - no minimum for standard deviation.

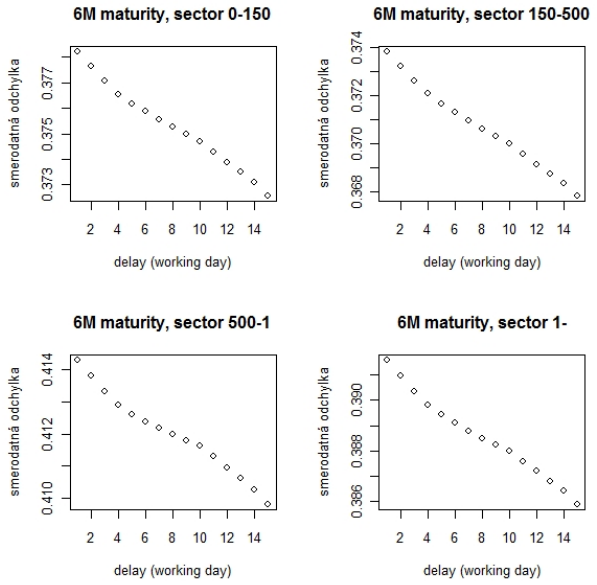

Figure 3.9: CSOB, 6M maturity, sector 0-150, 150-500, 500-1, 1-

Model assume frequent changes of interest rates and therefore other banks (with change "once a while") do not get satisfactory results. This model does not satisfy hypothesis for all banks. In some banks margin is not constant. It could be partially constant function with trend which depend on size of liability. Margin is function of more factors and addition more parameters can improve model and bring better results.

3.3 Treshold Model

How it was already mentioned in part about correlation coefficient, it is obvious that there is linear dependence between TD rates and Pribor. Where Pribor represents referential rate. This section shows different approach in making decision about TD rates. Our model is based on following three hypothesis: There exist a function of TD rate and Pribor. Banks fix TD interest rates according to development of this function Equation 3.3

$$\varphi(t) = f(S(t), P(t)) \tag{3.3}$$

where $S(t)$ is TD interest rate of given sector in time t and $P(t)$ is corresponding Pribor in time t. Last consideration is, that there exist a mean, lower and upper treshold of this function and bank is following these tresholds. We named this approach based on treshold hypothesis as a *Treshold model*. Let's define this function from Equation 3.3 as a ratio between margin defined according to Equation 3.1 and Pribor:

$$\varphi(t) = \frac{P(t) - S(t)}{P(t)} \tag{3.4}$$

This variable is interesting especially in time when bank changes TD rate. On one hand when increase rate and on the other when decrease rate. Let consider three tresholds of this variable. Next consider that calculates this ratio every day and when variable reach upper or lower treshold then change TD interest rate. Set new interest rate to calculated treshold. Denote these tresholds:

φ_L - lower treshold
φ_U - upper treshold
φ_M - mean treshold

Consider that decision making is based on following algorithm:

1. calculate $\varphi(t)$
2. if $\varphi(t) \geq \varphi_U$ or $\varphi(t) \leq \varphi_L$ then $S(t) = P(t)\,(1 - \varphi_M)^2$
3. otherwise $S(t) = S(t-1)$

for $t = 1...T$ where T is length of examined time series.

Calculate these tresholds as an arithmetical average:

$\varphi_L = \sum_{t=1}^{T} \varphi(t)$ where $t : S(t) > S(t+1)$
$\varphi_U = \sum_{t=1}^{T} \varphi(t)$ where $t : S(t) < S(t+1)$
$\varphi_M = \sum_{t=1}^{T} \varphi(t)$ where $t : S(t-1) < S(t)$ or $S(t-1) > S(t)$

Now all hypothesis were done and we can analyze data which are described in Chapter 6 - Data whether satisfy our hypothesis. Some banks did not decrease their DP rates in examined period therefore we calculate upper and mean tresholds.

Let's first estimate these tresholds and then estimate so call *new interest rate* which keeps tresholds hypothesis. Finally compare original TD rate and *new interest rate*.

Estimation of *new interest rate* is done from algorithm where tresholds are calculated from arithmetical average.

Model reached similar results for whole Czech deposit market. Let's introduce result of Treshold model on Ge Money Bank, 1M maturity, 500ths - 1mil sector. First let's describe four charts on Figure 3.10:

Up left chart: Dash line is 1M Pribor (%) and continuos line is TD rate 1M maturity

[2] it comes from Equation 3.4

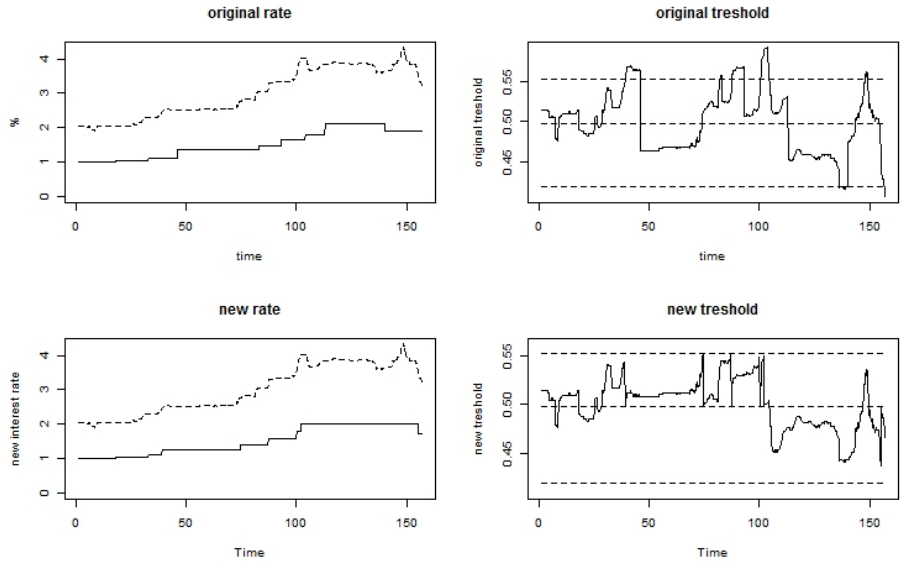

Figure 3.10: GeMoney Bank, 1M maturity, sector 500ths-1mil

Up right chart: Continuous line is ratio $\varphi(t)$. Upper dashed line is φ_U, lower dashed line is φ_L and medium dashed line is φ_M estimate from the ratio

Down left chart: Continuous line is New TD interest rate modeled according to algorithm mentioned and dash line is 1M Pribor

Down right chart: Ratio $\varphi(t)$ for new interest rate

If comparing two original rates (continuous line on up left chart) and *new TD rate* (continuous line on down left chart) is evident that this approach of fixing TD rates do not exactly satisfy our hypothesis on Czech Deposit market. Setting another function which banks follow or addition another parameter can improve approach and bring better conclusions.

3.4 Compare Delay Model and Treshold Model

In previous section we introduced two approaches (models) for fixing TD interest rates which depend on referential rate - Pribor. In this section is summarized comparison between Delay model and Treshold model.

Both Models are based on static hypothesis. Delay model assumes that there is some delay of reaction and banks fix rates on constant margin. Treshold model gives higher spread rate for changing rates in comparison to Delay model. Rules in Treshold models give to banks option to let constant TD rates in some period. This spread is different for all banks and maturities. But still there is static hypothesis based on ratio and it does not reflect other variations. Other option to improve this fact is develop model in bigger detail or combine both models. But it is overreach this work.

3.5 Dependence on Other Reference Rate

First part of this chapter considered Pribor as a referential rate. Now assume that banks fix TD interest rates based on deposit rates of other bank. Name this bank as a *leader of deposit market*. Assume that other banks are changing their TD rates, when they observe rate change in *leader bank*.

First we find *leader of deposit market*. Each bank take some position on deposit market in terms of retail deposit in their balance. Assume that position is determined by share of retail deposits on Czech deposit market and *leader* is bank with the highest volume of deposit. This share is changing in time and on Figure 3.11 is shown progress in share. Česká spořitelna reaches the highest value in last two years and second highest in 2006 and therefore assume this bank as *leader of term deposit market*.

Since banks published several TD rates (sectors, maturities) select sectors and maturities in examined period 2006-2008 which are same for all banks. In reality, it is only one sector from 500 000 to 1 000 000 CZK and 1M, 3M,

Share of retail deposits			
bank	31.12.2006	31.12.2007	30.9.2008
Komerční Banka	22%	27%	22%
ČSOB	31%	25%	25%
Ge Money Bank	3%	3%	4%
Česká spořitelna	26%	29%	30%
Citibank	4%	4%	4%
UniCredit	9%	8%	8%
Raiffeisenbank	4%	4%	6%
Volksbank	1%	1%	2%

Figure 3.11: Share of retail deposit on Czech TD market
Source: Annual reports

6M and 1Y maturities.

Let's analyze dependence on *leader*. Correlation coefficient give us linear dependence. Figure 3.12 shows linear dependence on Česká spořitelna in all maturities. But we can not note from correlation method that all banks react on Česká spořitelna. This dependence not shows us this property.

Correlation of Česká spořitelna							
maturities	Citi	CSOB	GeMB	KB	RB	Volks	UniCredit
1M	0.894	0.977	0.955	0.912	0.902	0.925	0.960
3M	0.893	0.973	0.962	0.935	0.913	0.833	0.960
6M	0.882	0.910	0.867	0.886	0.884	0.862	0.942
1Y	0.874	0.857	0.890	0.888	0.853	0.876	0.958

Figure 3.12: Correlation between Česká spořitelna and single banks, sector 500 000 - 1 000 000 in 1M,3M,6M and 1Y maturities

Trash model and Delay model are introduced in first part of this chapter and they are based on Pribor rate as a referential rate. In this part we assume another referential rate which has almost same properties as Pribor.

One difference is that leader rate does not change so frequently. But it is not problem for applying these models and hypothesis on Česká spořitelna rates.

Delay model

Delay model is describing in previous section, so we show results again on Figures. Assume maximal delay 60 working day which is almost 3 months. Calculations over all banks in sector (500 000 - 1 000 000) and maturities 1M,3M,6M and 1Y bring similar results. There is no equivalence in all maturities for one bank. Show it for example on two maturities 1M and 1Y.

Figure 3.13 shows 1M maturity where is obvious minimum standard deviation for Raiffeisenbank (delay 46 working days), Volksbank (delay 50 working days) and Ge Money Bank (delay 5 working days). Other banks do not reach minimum for data in examined period 2006-2008.

From Figures 3.13 and 3.14 is shown conclusions of model in two maturities 1M and 1Y and 500ths-1mil sector. For 1Y maturity ČSOB and Komerční Banka have 26 working days delay, Raiffeisenbank 25 and Ge Money Bank 15days. Other banks do not reach any minimum. For 1M maturity only Volksbank reach minimum 50 days and Ge Money Bank 5days. Thus, they do not follow our hypothesis (banks change all TD interest rates in the same time). Also high delay is unreal and these delays are influenced by other factors. Conclusion of Delay Model is that each bank and each rate react on Česká Spořitelna rates not according to our assumptions.. Only Citibank does not reach any minimum for all maturities which confirms results from previous model that Citibank react on other referential rate.[3]

[3]It is shown in section Delay Model

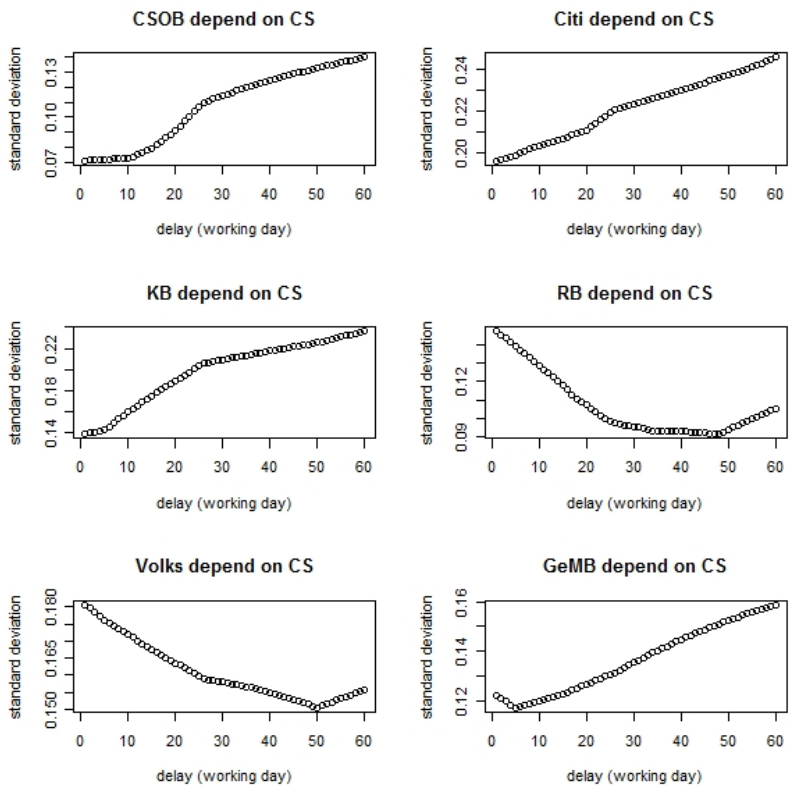

Figure 3.13: Delay Model with referential bank Česká spořitelna TD rate, 1M maturity, sector 500ths-1mil

Treshold Model

Also for Treshold model let's shown results for other referential rate than Pribor. First let's look if replacing Pribor rate by ČS deposit rate gives sense in Treshold model. Ratio $\varphi(t)$ is defined according to Equation 3.4

$$\varphi(t) = \frac{S_{CS}(t) - S_B(t)}{S_{CS}(t)}$$

where $S_{CS}(t)$ is TD interest rate for Česká spořitelna in time t and $S_B(t)$ is corresponding (in sector and maturity) interest rate of competitive bank.

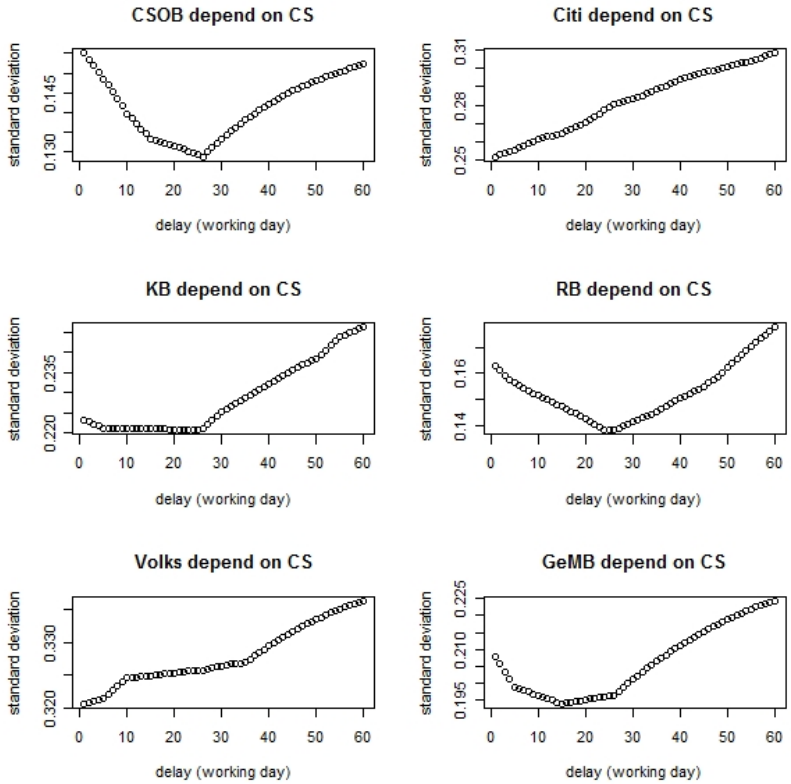

Figure 3.14: Delay Model with referential bank Česká spořitelna TD rate, 1Y maturity, sector 500ths-1mil

All rates are defined and ratio $\varphi(t)$ make sense.

Calculate three tresholds as an arithmetical average how it is mentioned in section 3.3 Treshold Model. Spread between upper φ_U and lower φ_L is not higher than maximal. Then let's estimate new interest rate according to mentioned algorithm and compare them. Since referential (Česká spořitelna) rate is partial constant therefor also ratio $\varphi(t)$ is constant and reach mean treshold and new rate are not changing. When referential rate is increased or decreased also *New interest rate* is changed because spread of change is not high and then calculated treshold exceeds upper or lower treshold.

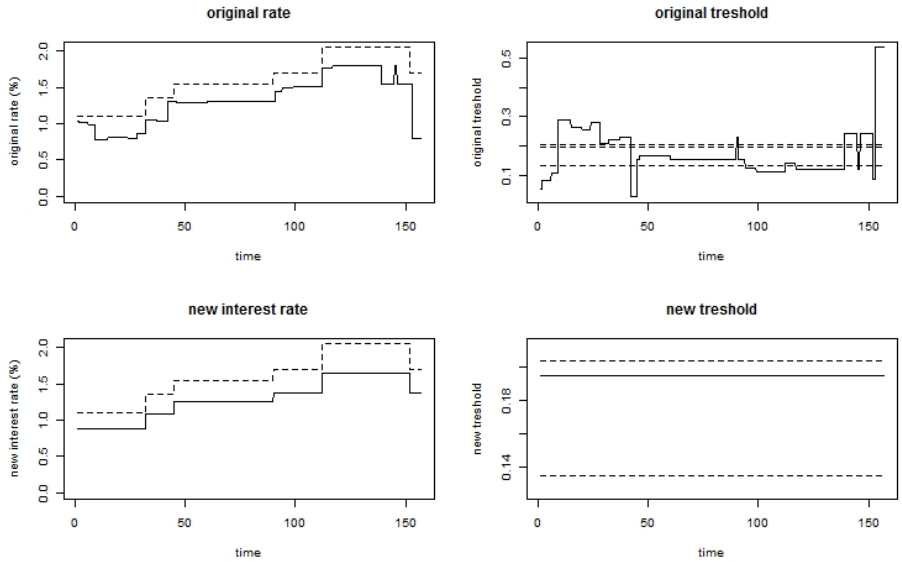

Figure 3.15: Komerční banka, 1M maturity, sector 500ths-1mil

All tested rates for all banks give similar results. Let's see it in bigger detail on Komerční Banka example. Behavior of rates is shown on Figure 3.15 where is evident parallel process in referential rate and new interest rate (down left chart). We compare original rate (continuous line, left up chart) with new interest rate (continuous line, left down chart) and we note different development of rates in this model. Treshold model does not bring good results for finding dependence between referential rate and bank term deposit rate and do not satisfy Treshold model hypothesis.

Other possibility is that, Česká spořitelna as a leader on term deposit market does not satisfy leader assumption. Finding another leader of market or spreading models for more parameters how was described in part about models can improve results given by these models.

3.6 Position of Bank on the Deposit Market

This section deals with behavior of banks according to its position on deposit market. Banks follow TD interest rates of other banks and keep some position. One of the possible positions for banks are: have the highest rates, around average of TD rates or have the lowest rates on market. In this part we test these possibilities and make some conclusions about Czech deposit market.

Position is given by value of TD interest rates. When one bank decide to change its TD rates, the automatically positions on market change and when other banks want keep their position, they also have to change rates. For our purposes we are looking for banks which does not change rates so often (Figure 3.2). On Czech deposit market is only CitiBank which satisfy these properties therefor we except this bank as a good sample for following explorations.

Sector 500 000 - 1 000 000 is again only one sector which can be analyze as a sector with unbiased rates. We examine which position banks kept the most often in period 2006-2008. In this period some banks had equal interest rates to other and it is not possible to set unique position. Therefor some banks can get 4.5 position on market.

Komerční banka is bank which kept the lowest position, which means they try to have the lowest TD interest rates. On Figure 3.16 is shown histogram where rank 1 is the position with the lowest interest rate and 7 with the highest interest rate. There are four charts with 1M, 3M, 6M and 1Y maturities made from 780 observations. KB keeps almost 500 times the lowest position in 6M and 1Y maturities and 500 times the two lowest positions in 1M and 3M maturities.

This analyze shows approach how bank can fix their TD interest rates. Some banks keep position on deposit market and according it fix term deposit rates.

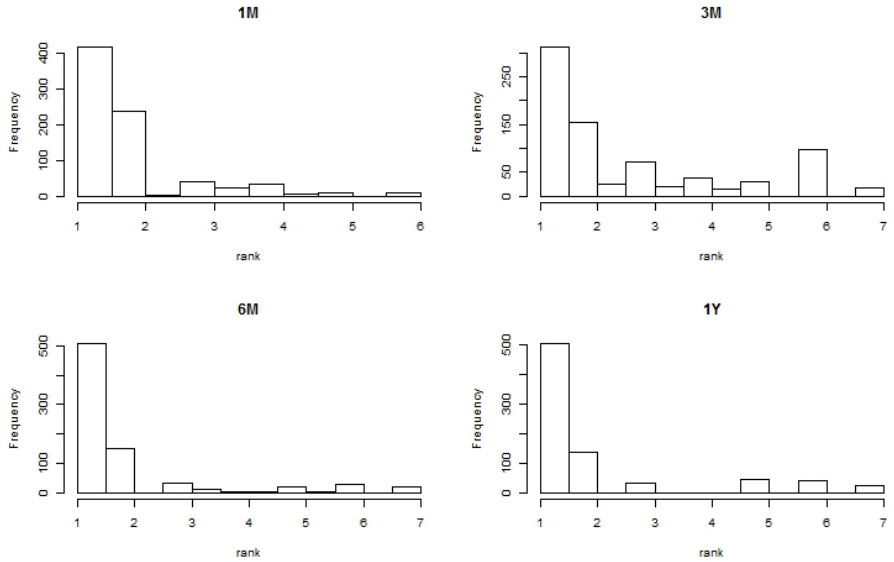

Figure 3.16: Histogram of Komercní banka position on Czech TD market

3.7 Behavior Change in Examined Period

Banks change their policy of deposit management also according other influence factors as well. Unexpected events and changes on interbank market can result in situation that Pribor nor another referential rate or position on market is not dominant for bank decision making. Change of deposit interest rate is influenced by unexpected factors. One of them is financial crisis mentioned in introduction. Lack of liquidity made banks to perform steps which increase deposit income.

Effect from lowering central bank interest rates is lowering deposit interest rates on Czech deposit banks. But some of banks react by holding rates or some of them by increasing TD rates to attract liquidity. They also prepare packages and special offers to attract clients for deposit free financial sources to their bank. Most common special offers are: setting fees for deposit accounts to zero, advanced interest payment or just temporary

increase of interest rates. Always it means decrease profitability for bank and acquiring clients from competitive banks. Keeping high interest rates in time when Czech Central Bank decreased rates on its historical minimum is impossible. High deposit rates are profitable for bank only and only if they attract many clients and then again decrease rates.

Static models do not satisfy assumptions in long period and do not reflect fluctuation on bank market. It is needed to modify this strong considerations.

We analyzed banks and their term deposit interest rate and margins on Czech deposit market in this chapter. Conclusions give us some ideas how they fix TD rates or which position keep on market or which bank keep constant margin. These informations can help to bank fix some parameters in following chapter.

Chapter 4

Optimal margin

This chapter describes model for setting optimal margin in dependence on several parameters which depend on TD. First part is describing profit and its development in time. In next section analysis is extend by competitive parameter. Next parameter which is described in third section is demand parameter. There is also described model of optimal profit and margin. The third section end with discussion about values of individual parameters and their impacts on optimal values. Fourth section introduced changes of optimal values in connection with liquidity crisis. The last section described calculation of competitive interest rates for TD sectors which do not correspond to all of banks on the deposit market.

Deposits are sources for financing bank credit for banks. Therefor banks compete with each other and try to acquire clients. Total deposits on deposit market move between banks how is indicated on Figure 4.1. The main tool for this fight is interest rates. Higher interest rates, good marketing strategy and attractive deposit products can provide the higher share on deposit market.

While REPO rate decreased to 1,75%[1] during financial crisis also all

[1] 6.2.2009

banks decrease their deposit interest rate. Ge Money Bank took opposite strategy, offered new savings product with 3,6% p.a with minimal deposit 40 000CZK. This strategy resulted in higher market share on deposit market.

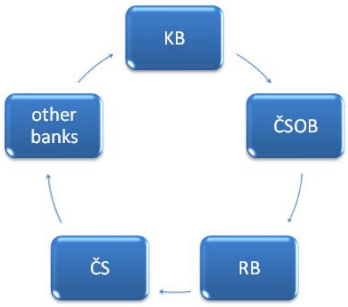

Figure 4.1: Circle of deposits on deposit market

Before analysis let's define profit, margin and other variables and explain their relations to each other. Term deposits are not identical in each bank. Because of competition among banks each bank is trying to offer to client the most attractive product and which is also profitable as much as possible from bank's prospective.

Let's call *analyzed bank* the bank, which we examine and its interest rate *analyzed interest rate*. Other important term is so called *competitive interest rate*. This rate reflect all competitive interest rates of banks on interbank market and corresponds to analyzed TD interest rate. For some banks it could be only one bank - *leader* on deposit market. For following model we assume that competition interest rate include information about more than one competitive interest rate.

In any case TD competitive interest rate must correspond in two parameters:

- maturity
- sector of volume

Nearly each large bank in Czech deposit market offers TD in 1W, 2W, 1M, 3M, 6M and 1Y maturity at least. Therefor following analyses involve only these maturities. The second parameter is sector of volume (resp. TD interest rate for this sector), which is different in each bank. On Czech deposit market exist one common sector for all large banks, it is sector from 500 000 to 1 000 000 CZK. We verify our assumptions and build the result from following models on this sector. How to analyze other sectors and calculate corresponding competitive interest rate is described at the end of this chapter.

Actual number of banks (foreign banks and national banks) in the Czech Republic is 38 [2], but not all of them offer deposits product and not all data of interest rates are available. For purpose of this model, we will consider only banks with term deposits products with known TD interest rates (see section Data).

Each bank keeps some position on financial market, which influence also its competitive interest rate. This size of bank can be included in rate. Its position can be qualify by several measurable. Show some of them:

- Equity
- Liabilities to clients
- Deposit loans ratio
- Bank rating

But for purpose of this paper, we use the simplest method - arithmetical average. It means that competitive interest rate $r_M(S)$ for maturity M and sector S is

$$r_M(S) = \frac{1}{m} \sum_{j=1}^{m} r_{M,Bj}(S) \tag{4.1}$$

[2] data to 31.9.2008, source:ČNB

where Bj, $j = 1, ..., m$ is a j-th bank.

4.1 Indifference Analysis of Profit

Profit is the most monitored element in each company. It is positive gain from business operation. Banks follow their profit from each business very carefully. Also term deposits bring cash and profit to banks. And now TDs are one of the cheapest way to get a cash. Aim of this section is analyze parameters of profit by indifference analysis and suggest how to set these parameters to reach planned profit.

Each deal as well as term deposit is contract between client and bank and it is characterized by several variables. Denote:
t_s - start time
t_e - end time
M - maturity (defined by t_s and t_e)
S - sector
V - volume of deal

Maturity is calculated as $M = t_e - t_s$. It has usually standardized intervals according to bank's product offer. Let's consider, that S and M are fixed to simplify following denotations.

We define total volume $V(t)$ as a volume of all deal in time t.

$$V(t) = \sum_{i=1}^{n_t} V_i(t) I_{<t_s,t_e>}(t)$$

$\forall t \in 1, ..., T$, where $V_i(t)$ is volume of i-th deal in time t, $i = 1, ..., n_t$ is number of deal in time t and T is length of time series. Indicator means that $V_i(t) = 0$ when $t \notin < t_s, t_e >$ because deal expire and bank do not dispose of this volume.

Also margin $m(t)$ is calculated from TD interest rate according to Equation 3.1. Now let's define profit for period $(0, T)$ in general:

$$Pr(0,T) = \int_0^T \frac{V(t)m(t)}{360} dt \tag{4.2}$$

Assume monthly volumes and margins, where t denote months and t_m number of working days in month. Then

$$V(t) = \sum_{i=1}^{t_m} V(i)$$

and

$$m(t) = \frac{1}{t_m} \sum_{i=1}^{t_m} m(i)$$

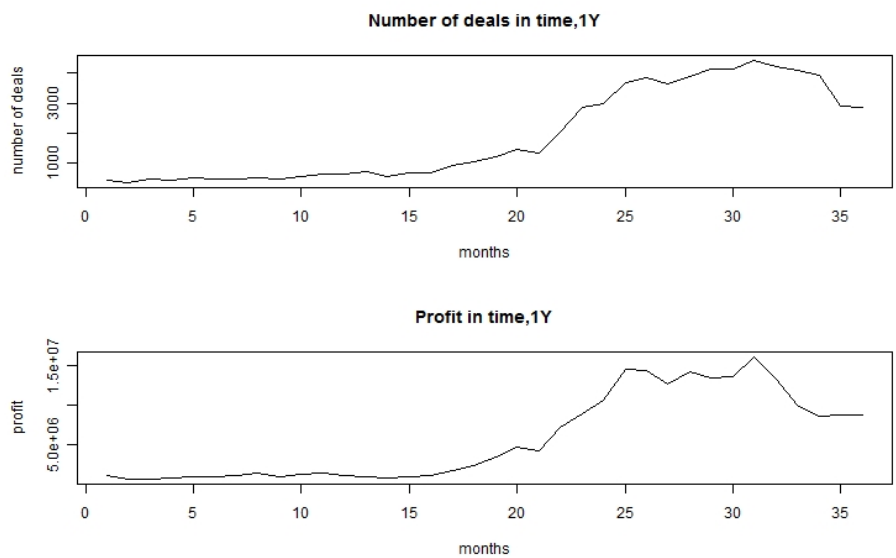

Figure 4.2: Number of deals and profit in time, sector 500ths - 1 mil, 1Y maturity

Profit is also influenced by number of deals. The more deals the higher profit. Upper chart of Figure 4.2 shows number of deals in months and

lower chart monthly profit in sector 500ths - 1mil and 1Y maturity. Each bank has several marketing operations in period hence number of deals is different in time. Successful marketing operations are result in increasing number of deals. This progress is obvious on upper chart on Figure 4.2 in 2008. Marketing operations can include all maturities or only some of them. Charts of profit and number of deals of other maturities see in Appendix A.

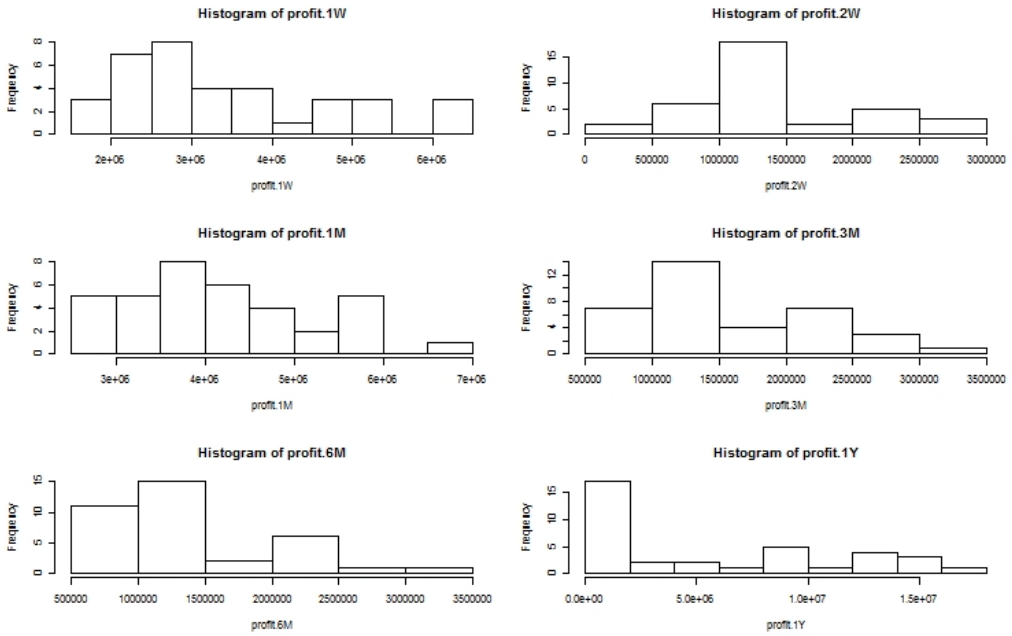

Figure 4.3: Histogram of profit, 1W, 2W, 1M, 3M, 6M, 1Y maturities, 500ths-1mil sector

First we analyze profit by histograms, which show the most frequently of profit (calculated by Equation 4.2) reached in sector 500ths-1mil and all maturities. From Figure 4.3 is obvious that the most reached frequency of value of profit is between 1mil - 1,5mil in almost each maturity. One option how to reach this profit is to set low margin (resp. higher interest rates) which will results in attracting new clients and their deposits. The second option to reach same profit is higher margin (lower interest rates) which

means that there is less clients with lower volume creating higher profit. These considerations also come from Equation 4.2.

Now let's explain term indifference analysis. So called indifference analysis based on indifference curve is *combination between two goods which brings same satisfaction* how is defined in Holman (2005)[11]. In a way margin and volume are two goods and profit presents *satisfaction*. Thus, same profit can brings not only one combination of volume and margin. Higher indifference curve means higher profit. But not all profit and its combinations are real and practicable. When bank specify their yearly profit and plan to months, indifference curve shows all combinations how to reach it.

Now let's analyze historical data of term deposits and find some indifference curves. Since histograms show us the most frequency value of monthly profits, let's concentrate on this maturities and values. Figure 4.4 shows all combinations how to reach 1,4,6 and 8 millions monthly profit. Is obvious that during three examined years bank reached profit around 1 million in many cases (red continue line). There is also combination when profit is higher than 8 millions which can reflect some marketing operations. Other several charts of indifference analysis in other maturities see in Appendix A.

Using indifference analysis is one method how to reach planned profit. This model contain only two parameters: margin and volume. Because volume is depend on margin (resp. interest rate) bank can set only margin parameter. As mentioned above some combinations of parameter are not suitable because bank is also member of deposit market and must follow competitive banks and market development. This method for reach profit do not reflect situation on term deposit market and TD rates of competitive banks. Hence in next section is introduced parameter of competition.

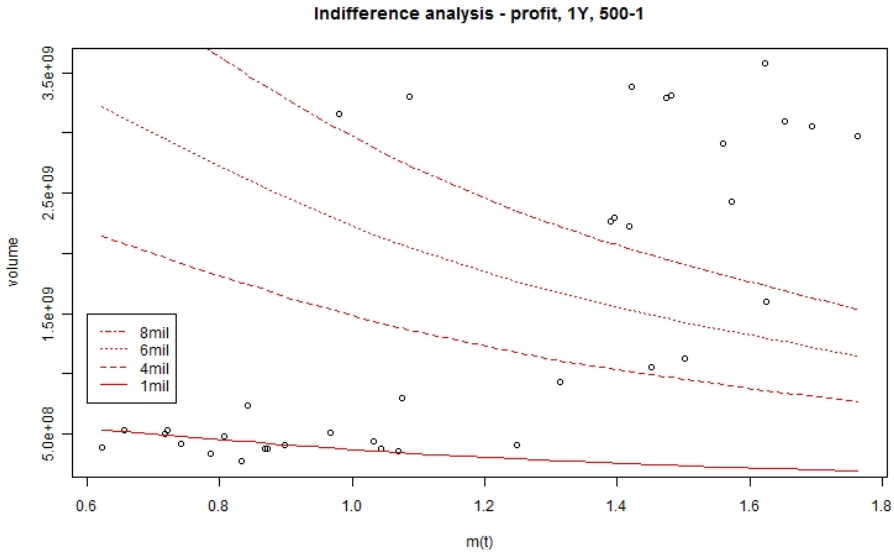

Figure 4.4: Indifference analysis of margin and volume, 1Y, sector 500ths-1mil

4.2 Parameter Competition

In this section we will imply competitive parameter to our analysis of profit. Volume and profit of term deposits are influenced by not only analyzed interest rate but also by other factors. One of the most important factors is competitive interest rate. Banks also analyze changes on financial market and especially changes of competition's products. Events on financial market often result in changes in deposit products. Competitive deposit can be represented by several products, for examples primary deposit (see in Chapter 2 - Primary Deposits). For simplicity and because this paper deals with term deposits, consider as competitive deposit to TD only term deposits of other banks.

Since we analyze profit which is function of margin and not interest rate, let's examine analyzed margin (AM) and deviation from competitive

margins (CM). Thus define margin $m(t)$ as

$$m(t) = m_c(t) + \Delta m \tag{4.3}$$

where $m(t)$ is analyzed margin, $m_c(t)$ is competitive margin calculated according to Equation 3.1 from competitive interest rate and Δm is margin deviation for time t.

Banks can fix their interest rates according to themselves thus, Δm can be any rational number. Now introduce some properties of Δm versus other variables:

1. $\Delta m < 0$
   AM is lower than CM $\Rightarrow$ analyzed TD interest rate is greater than comp. interest rate $\Rightarrow$ attractive interest rate $\Rightarrow$ increase demand $\Rightarrow$ higher volume, but lower margin $\Rightarrow$ profit is higher due high volume.

2. $\Delta m > 0$
   AM is greater than CM $\Rightarrow$ analyzed TD interest rate is lower than comp. interest rate $\Rightarrow$ higher demand for competitive TD products $\Rightarrow$ lower volume, but higher margin $\Rightarrow$ profit is higher due to margin.

3. $\Delta m = 0$ rates are equal to each other. Profit is same as profit of competition banks, because interest rates are same.

These facts are also obvious on Figure 4.5 where monthly profit is presented by 3M maturity and 500ths - 1mil sector. For better overview we smooth noisy observations. We use a cubic spline function which is represented by red line.

How is obvious from Figure 4.5 and from consideration of curve development, to maximize profit means to be on tails of curve

- to be on left tail of curve is result in higher interest rates and increasing volume.

- to be on right tail of curve is result in lower interest rate (higher margin) and in a few periods it also results to lower volume.

Edge value where profit is higher are unreal, because clients are not able to accept these conditions on the one side and banks do not fix so low interest rates on the other side. Thus we add next parameter - demand for term deposits.

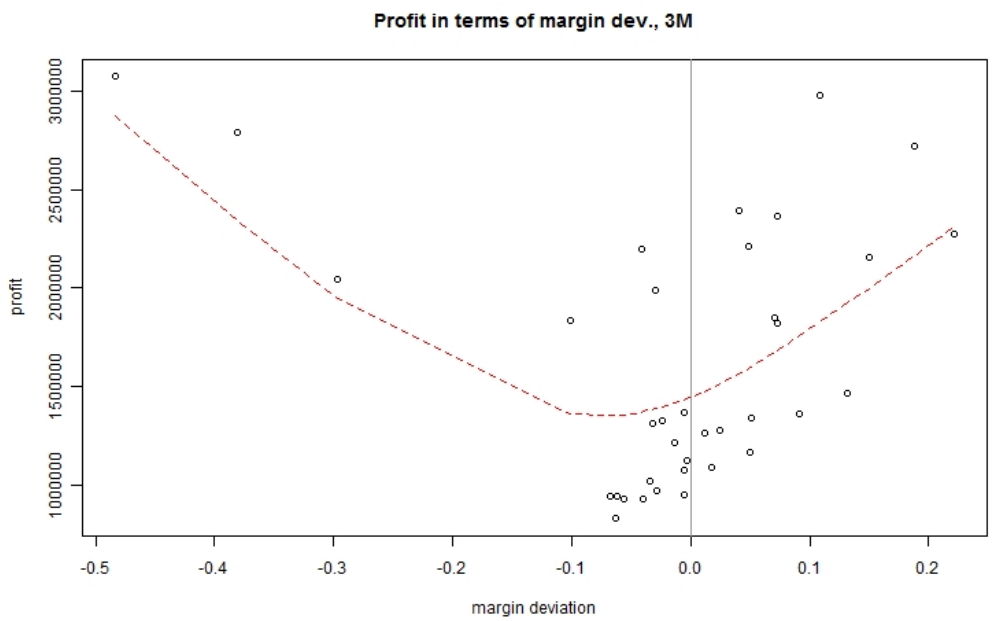

Figure 4.5: Smoothing profit in terms of margin deviation, 3M maturity, sector 500ths-1mil

4.3 Parameter Demand

Demand is very important parameter for fixing TD interest rates. In this section we describe model of optimal profit which depends on several variables.

Then different development of optimal profit in dependence on changes of these parameters is discussed.

Demand is function of demand amount of good in terms of several variables: price of given good, price of other goods (which are substituents or complements) and also of consumers income comes from Holman (2002)[12]. Demand for term deposit of bank can be characterized by two variables:

- volume
- number of deals

We study volume as a variable of demand for two reasons:

1. calculation of profit is function of volume
2. between number of deals and total volume exist direct proportion how is shown on Figure 4.6

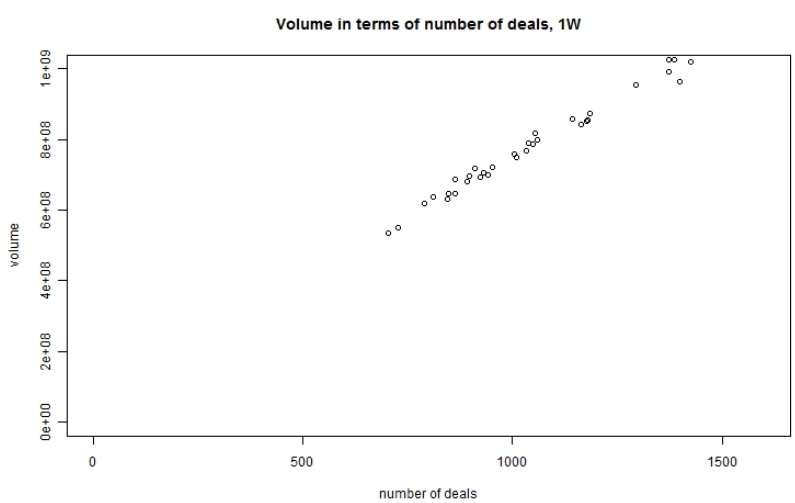

Figure 4.6: Direct proportion between volume and number of deals, 1W maturity, 500ths-1mil sector

Each demand is different in time, because it is influenced by other events in financial world. Therefor there are many different possible shapes of demand function.

In next part we introduce price sensitivity. There are two common measures: the slope and the elasticity defined in Lewis (2005)[10]. For purpose of this paper demand is represented by volume.

The first measure is *Slope*. It is equal to the change in demand divided by the difference in prices

$$\delta(p_1, p_2) = \frac{[D(p_2) - D(p_1)]}{(p_2 - p_1)} \tag{4.4}$$

It is common to specify the slope at a single price and it is computed as the limit of Equation 4.4 as p_2 goes to p_1. That is the derivative of demand function at p_1,

$$\delta(p_1) = D'(p_1)$$

Second is *Elasticity*. It is measure of sensitivity of demand to price. It is equal to percentage change in demand divided by percentage change in price. Formally, we can write

$$\epsilon(p_1, p_2) = \frac{[D(p_2) - D(p_1)]\, p_1}{(p_2 - p_1)/D(p_1)} \tag{4.5}$$

where $\epsilon(p_1, p_2)$ is the elasticity of margin change from p_1 to p_2.

Downward-sloping property guarantees that demand always changes in the opposite direction from price. The same way as slope, we define point elasticity at p by taking the limit of Equation 4.5 as p_2 goes to p_1

$$\epsilon(p_1) = D'(p_1)p_1/D(p_1)$$

If we take linear demand function $D(p) = ap + b$ then slope is $\delta(p) = D'(p) = a$ which is typically negative for demand function.

First let's consider period without big changes on financial market. This situation is typical by almost whole period 2006-2008. Even though, financial

crisis started in the middle of this period, impact on deposits market was recorded at the end of 2008. Development of profit in dependence on changes demand for TD is described in section Term Deposit Demand in Financial Crisis.

Now we can describe development of demand according to Δm by three following situations:

1. $\Delta m < 0$ - AM is lower than CM $\Rightarrow$ competitive TD interest rate is lower than analyzed interest rate and we can expect higher demand for deposit product of analyzed bank.

2. $\Delta m > 0$ - AM is greater than CM $\Rightarrow$ competitive TD interest rate is higher than analyzed interest rate and we can expect lower demand for deposit product of analyzed bank.

3. $\Delta m = 0$ - margins and also interest rates are equal to competition and we expect demand for TD product of analyzed bank same as demand for other TD product on deposit market.

From this we expect downward sloping function of demand. One of the simplest shape of curve is linear dependence, thus for estimation of demand V use linear regression model.

Definition of linear regression model

$$F(\Delta m) = \alpha \Delta m + \beta + \epsilon$$

where α and β are unknown parameters and ϵ is error term, which captures all other factors which influence the dependent variable other than the regressors.

For estimation use the simplest and very common estimator Ordinary Least Squares (OLS). The OLS method minimizes the sum of squared residuals:

$$min \sum_{i=1}^{n} (\alpha \Delta m_i + \beta - V_i)^2$$

where n is a number of observations. We used Statistical software R 2.7.1 for estimation of unknown parameters. Calculated result is shown on Figure 4.7 where red lines is fitted function from observations. It is shown on sector 500ths - 1mil and 3M maturity. Not all maturities of our examined sector comply with our expectation requirement of downward sloping function.

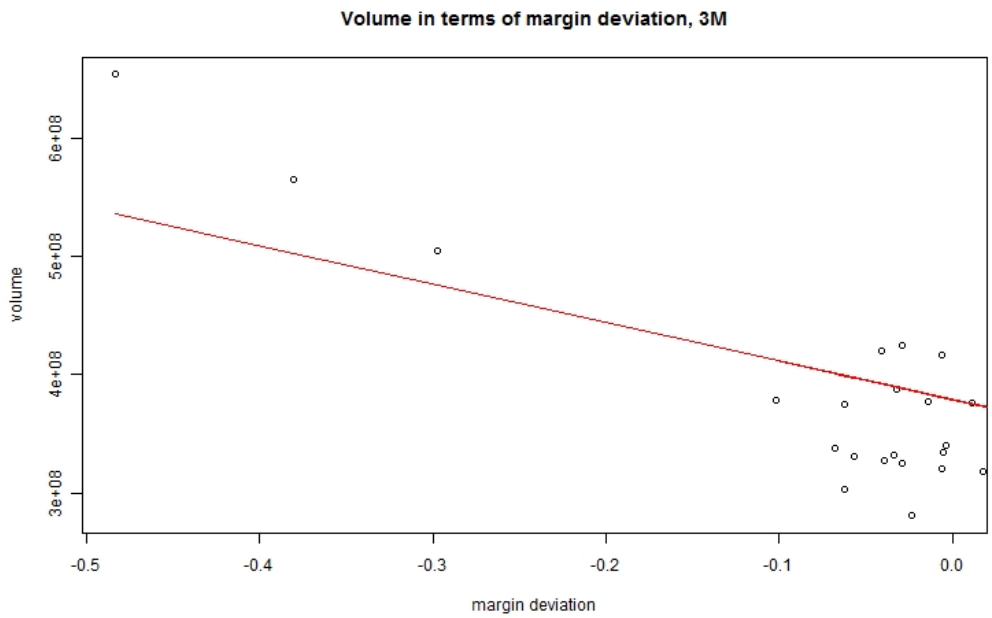

Figure 4.7: Fitted volume function, maturity 3M, sector 500ths-1mil

Because it is decreasing linear function, slope is given by negative parameter α. Now we can define demand function $V(t)$ in terms of margin deviation in time t. Denote
V_0 - initial volume of analyzed bank in time 0

Then according to assumption of linear function of volume we can define

$$V(t, V_0, \alpha, \Delta m) = V_0 + \alpha \Delta m t \tag{4.6}$$

where α is estimated slope by linear regression model.

This Equation 4.6 determine development of volume in time in dependence on margin deviation and initial level of volume.

Now we expressed volume as a function of margin deviation. Let's focus on profit in period $(0, T)$ according to Equation 4.2:

$$\begin{aligned} Pr(0,T) &= \frac{1}{12}\int_0^T [(m(t))(V_0 + \alpha\Delta mt)]dt \\ &= \frac{1}{12}\int_0^T [(m_c(t) + \Delta m)(V_0 + \alpha\Delta mt)]dt \end{aligned}$$

now solving only integer

$$\begin{aligned} I &= \int_0^T [(m_c(t) + \Delta m)(V_0 + \alpha\Delta mt)]dt \\ &= V_0\int_0^T m_c(t)dt + \alpha\Delta m\int_0^T tm_c(t)dt + \\ &\quad + \int_0^T \Delta mV_0dt + \int_0^T \alpha t\Delta m^2dt \\ &= V_0M_1 + \alpha\Delta mM_2 + \Delta mV_0T + 1/2\alpha\Delta m^2T^2 \end{aligned}$$

where $M_1 = \int_0^T m_c(t)dt$ and $M_2 = \int_0^T tm_c(t)dt$.

Using these results we have

$$Pr(0,T) = \frac{1}{12}[V_0M_1 + \alpha\Delta mM_2 + \Delta mV_0T + 1/2\alpha\Delta m^2T^2] \tag{4.7}$$

Our examined profit is term of several variables: initial volume V_0, parameter of demand α, competitive margin expressed by $M1$ and $M2$, length of examined period T and especially margin deviation Δm. Aim of this paper is to find such a margin deviation which maximize profit from Term deposits. Mathematically it is:

$$max_{\Delta m} Pr(0,T) \tag{4.8}$$

Derivative function $\frac{\delta}{\delta \Delta m} Pr(0,T)$ we have

$$\frac{1}{12}\left(\alpha M_2 + V_0 T + \alpha \Delta m T^2\right)$$

Setting this derivative to zero and replacing Δm with $\tilde{\Delta m}$ we have

$$\tilde{\Delta m} = \frac{-V_0}{\alpha T} - \frac{M_2}{T^2} \tag{4.9}$$

Now we check that our estimate of Δm to maximum by seeing whether the second derivative is negative when $\Delta m = \tilde{\Delta m}$. We have

$$\frac{\delta^2}{\delta \Delta m^2} Pr(0,T) = \alpha T^2$$

which is negative because α is negative and square is always positive number.

Optimal margin deviation consist of two terms. First positive term is reduced by term which contain competitive margin. It is very hard to predict competitive margin. Methods how to do it overreach scope of this paper. Previous Chapter 3 - Pricing Decisions in Bank monitors banks on Czech deposit market. There is shown how banks fix their interest rates resp. margins, if there is some leader of deposit market and also the most frequent positions of the banks on the market. This information give us some ideas about development of competitive interest rates and margins. Assume that in short period competitive margin is constant. For example Citibank satisfy this hypothesis on Czech deposit margin.

Let's defined period when competitive margin is constant by $(0,T)$ and denote this constant margin $m_c(T)$, where T denotes length of period. Define value of CM by value in time 0

$$m_c(T) = m_c(0)$$

Using these facts we get:

$$\mathrm{M}_1 = Tm_c(T) \quad and \quad \mathrm{M}_2 = 1/2T^2 m_c(T)$$

Replacing Equation 4.7 with $M1$ and $M2$ we have:

$$\begin{aligned} Pr(0,T) &= \frac{1}{12}[TV_0 m_c(T) + \alpha\Delta m 1/2T^2 m_c(T) + \Delta m V_0 T + 1/2\alpha\Delta m^2 T^2] \\ &= \frac{1}{12}[TV_0(m_c(T) + \Delta m) + 1/2\alpha\Delta m T^2(m_c(T) + \Delta m)] \end{aligned} \tag{4.10}$$

Then the first derivative is

$$\frac{\delta}{\delta\Delta m} Pr(0,T) = 1/12[1/2m_c(T)\alpha T^2 + V_0 T + \Delta m \alpha T^2] \tag{4.11}$$

Setting the derivative to zero and replacing Δm with $\tilde{\Delta m}$ we have

$$\tilde{\Delta m} = \frac{-V_0}{\alpha T} - \frac{m_c(T)}{2} \tag{4.12}$$

And we can define optimal margin $\tilde{m(T)}$ for the time T and optimal profit $Pr(\tilde{0}, T)$:

$$\tilde{m(T)} = m_c(T) + \tilde{\Delta m}$$

and

$$Pr(\tilde{0}, T) = \frac{1}{12}[TV_0 \tilde{m(T)} + 1/2\alpha\Delta m T^2 \tilde{m(T)}]$$

Optimal margin deviation depends on several variables. Now we introduce them properly and we show how is changing optimal values in dependence on changing variables.

As mentioned before T is length of period, where we expect constant competitive margin. T is also the same period where we maximize profit.

Constant margin depends on development of Pribor and also on development of TD interest rates on market. Thus, note that competitive margin $m_c(T)$ is in terms of length of period T.

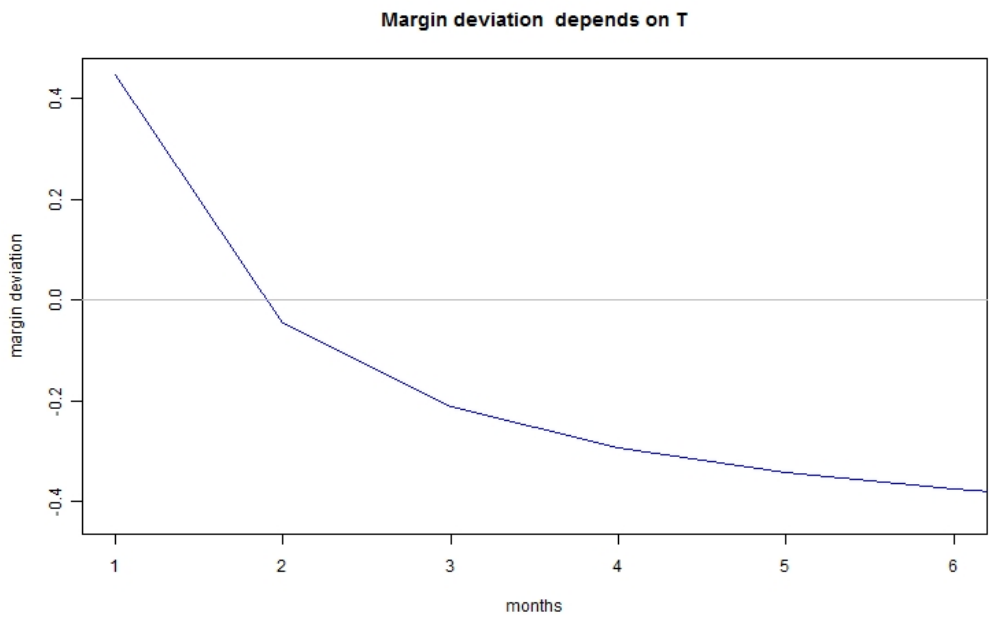

Figure 4.8: Development optimal margin deviation in time, 3M maturity, sector 500ths - 1mil

First we show development of optimal margin deviation in time. Assume that m_c is constant in whole examined period. It means, that all supposed banks have constant margin and therefor competitive margin is constant as well. Figure 4.8 shows optimal margin deviation in time for sector 500ths - 1mil, 3M maturity. This curve comes from Equation 4.12, where α is estimated parameter from demand curve, V_0 is initial volume (January 2006) and $m_c(T)$ is value of competitive margin in January 2006. It is obvious that optimal margin deviation is increasing with higher T.

Now we explain development of optimal profit. Figure 4.9 shows development of profit for specific T. From Equation 4.12 is obvious, that each T

gives another value of optimal deviation margin and therefor optimal profit as well. There are three developments of profit for three T in this period calculated from 4.10, where α, V_0 and $m_c(T)$ are parameters mentioned above.

- for $T = 3$ optimal margin deviation is $\tilde{\Delta m} = -0.2103$
- for $T = 4$ optimal margin deviation is $\tilde{\Delta m} = -0.2923$
- for $T = 5$ optimal margin deviation is $\tilde{\Delta m} = -0.3416$

It means that higher T results in the lower optimal margin deviation $\tilde{\Delta m}$ and the lower optimal profit.

Constant competitive margin in long period is unreal and does not correspond to situation on deposit market. Therefor, T should be fixed according to development of competitive rate.

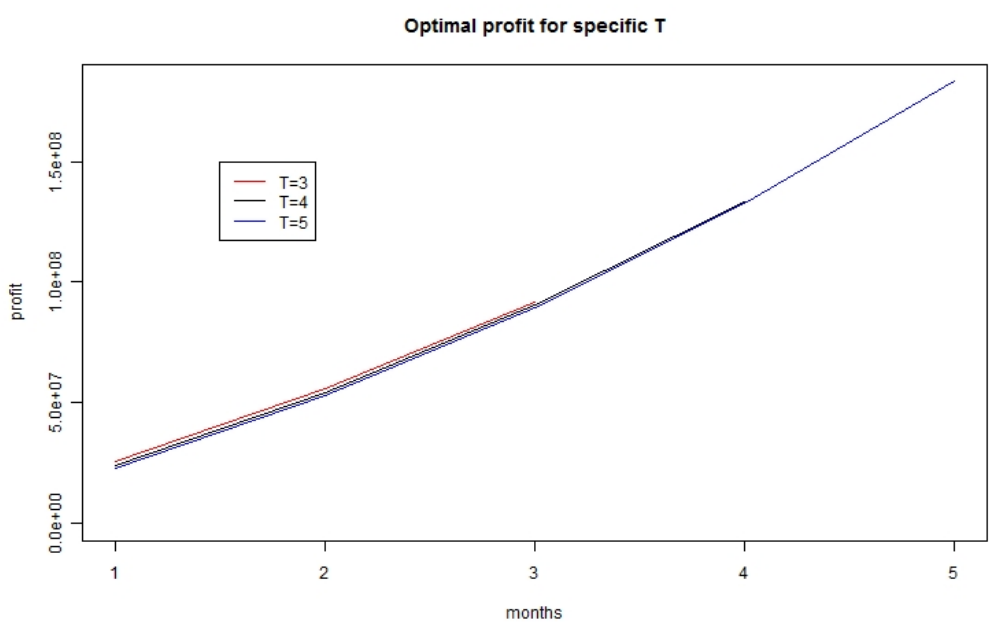

Figure 4.9: Optimal profit in dependence on time T

Model admit values of $\tilde{\Delta m}$ in real numbers. But in practice bank does not fix negative margin. Therefor we assume:

$$\begin{aligned} 0 &< \tilde{m(T)} \\ 0 &< m_c(T) + \tilde{\Delta m} \\ m_c(T) &> \frac{-V_0}{\alpha T} - \frac{m_c(T)}{2} \\ T &> -2/3 \frac{V_0}{\alpha m_c(T)} \end{aligned}$$

For our initial values[3] we have $T > 0.609$ months. This is additional condition for setting T.

These previous calculations were shown on period 2006-2008 on Czech deposit market. In this period liquidity crisis did not impact on deposit market so hard. In following section we introduce expected impacts of crisis on development of optimal values.

4.4 Term Deposit Demand in Financial Crisis

In previous section Deposit Products During Crisis is described impact of liquidity crisis on deposit products. These facts should also appear in our model and on value of optimal values. α is variable which is changed in crisis and influence our optimal values. This parameter gives to the model information about demand for TD of analyzed bank.

TD became more attractive due to 100% guarantee TD, clients were affraid of losing investments in other more risk products. Moreover, clients

[3] 3M maturity, sector 500ths-1mil, V_0 initial volume in January 2006, $m_c(T)$ margin in January 2006, estimated α from demand

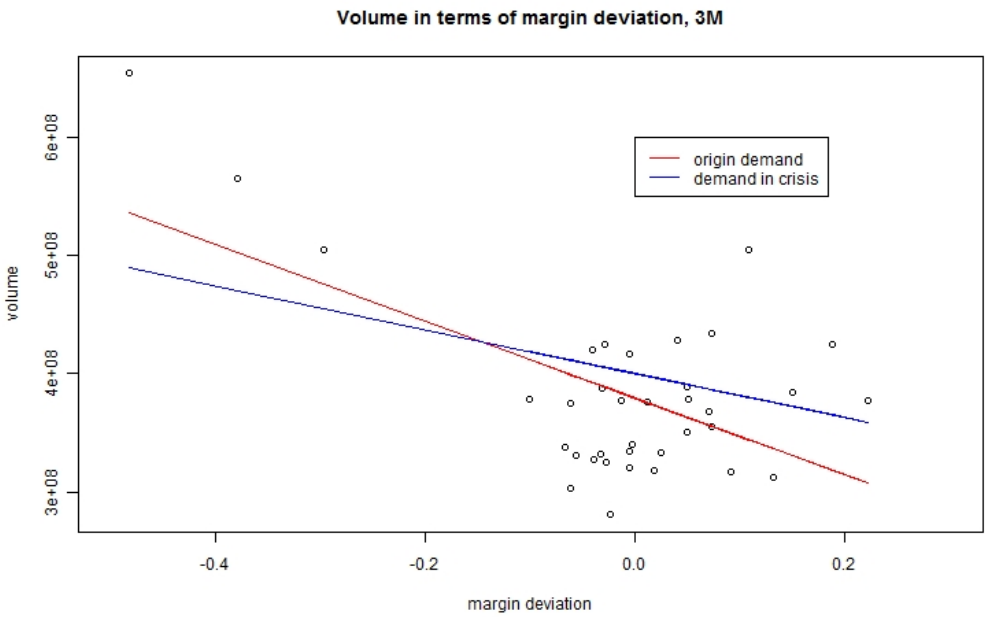

Figure 4.10: Expectation changes of demand for TD caused by financial crisis

are able to accept worse conditions than in normal situation. It menas that they trust to some banks and deposit free financial source even if they do not offer so profitable conditions (especially interest rates) than others.

Since TD sector of financial market was influenced at the end of our examined period we do not have enough data to prove our hypothesis. In next part we show development of profit and optimal margin deviation in dependent on change of demand.

We assume that clients are able to accept higher margin deviation. It means that TD interest rate are lower than before. There is red line as original estimated demand and blue line shows expectation change of demand curve on Figure 4.10. Due to liquidity crisis we expect higher demand which means higher α than in period before crisis.

Now we show impact of changes of α on optimal margin deviation. As example we show Figure 4.11 for 3M maturity and 500ths-1mil sectors. Also

from Equation 4.12 is obvious that higher α results in higher optimal deviation.

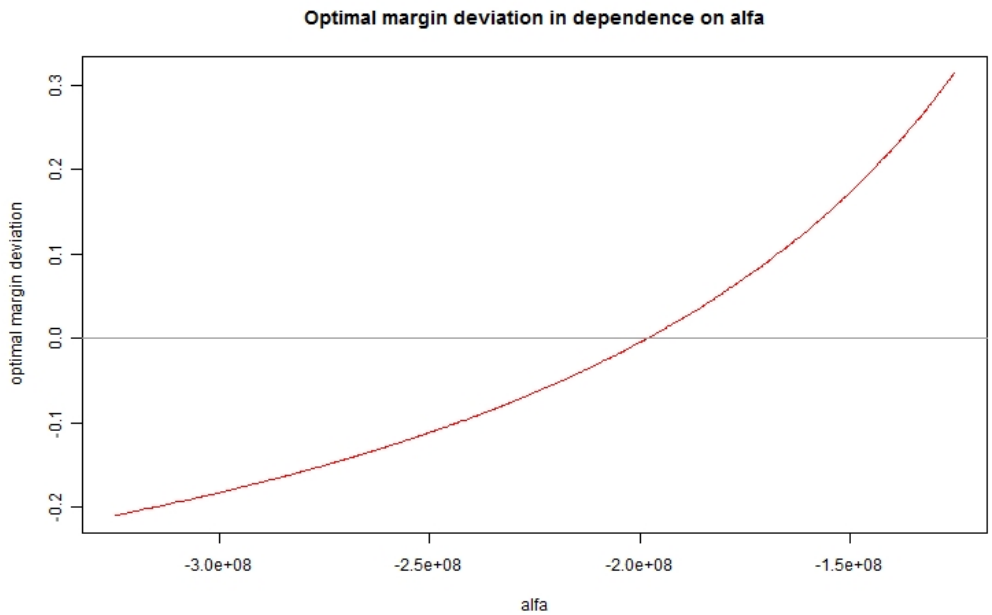

Figure 4.11: Deviation alfa

Now we show impacts of demand parameter changes on optimal profit. The higher demand for TD results in higher α. People are able to accept lower interest rates and it results in shift demand curve and change of α. These changes of moving estimated demand curve are shown on Figure 4.10.

We show development of optimal profit on Figure 4.12. There are three different lines. The black continuous line corresponds to estimated α demand from period 2006-2008 for term deposit (3M maturity, 500ths-1mil sector). Other two lines can present new expectation demand. Values of α and optimal margin deviation $\tilde{\Delta m}$ is shown in following lines:

- for $\alpha = -1e + 8$ optimal margin deviation is $\tilde{\Delta m} = 0.528$

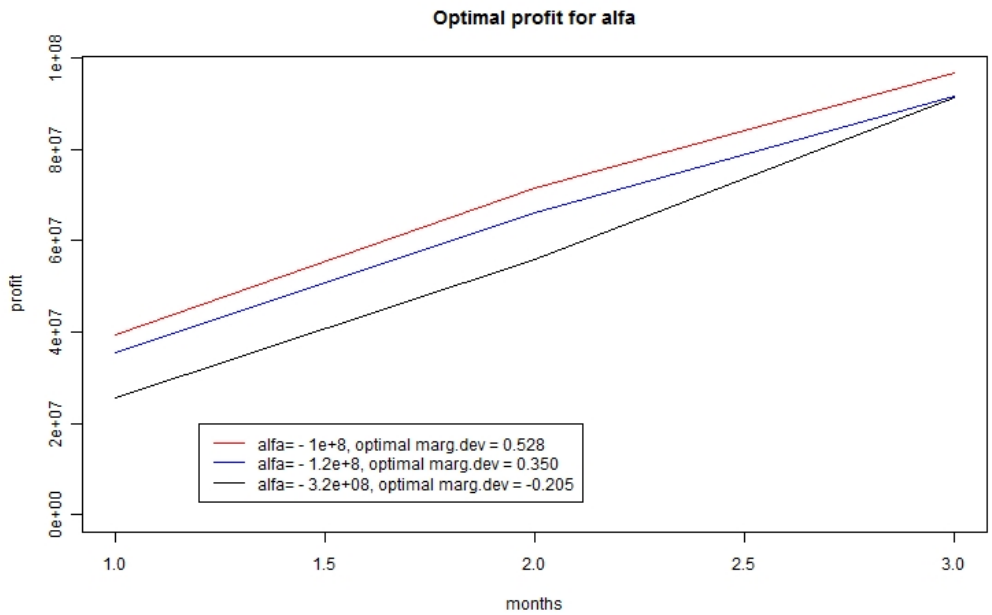

Figure 4.12: Optimal profit alfa for initial values 3M maturity, sector 500ths-1mil

- for $\alpha = -1.2e + 8$ optimal margin deviation is $\tilde{\Delta m} = 0.350$
- for $\alpha = -3.2e + 8$ optimal margin deviation is $\tilde{\Delta m} = -0.205$

The higher demand (the higher α) results in the higher profit which is caused by higher margin.

4.5 Competitive Margin for All Sectors

This chapter introduce how to calculate competitive margin for other volume sectors of TD. Since, TD products are often different by each bank, it is not easy to find corresponding interest rates of other bank to analyzed rate. The main difference is in sectors. Each bank fix interest rate to another sectors.

Let's introduce method to calculate competitive interest rate as it is defined at the beginning of Chapter 4 - Optimal Margin in general.

Let's assume that volume range of analyzed bank is divided to $\frac{n}{2}$ sectors. Denote them:

$$(a_1, a_2), (a_3, a_4), \ldots, (a_{n-1}, a_n)$$

where $a_i > 0$, $a_{i+1} > a_i, a_1$ define minimal volume and a_n maximal volume of deposit.

Next consider competitive bank with $\frac{m}{2}$ sectors and corresponding interest rates for maturity M

$$(b_1, b_2), (b_3, b_4), \ldots, (b_{m-1}, b_m)$$

$$r_M(b_1, b_2), r_M(b_3, b_4), \ldots, r_M(b_{m-1}, b_m)$$

where $b_i > 0$, $b_{i+1} > b_i, b_1$ define minimal volume and b_m maximal volume of deposit.

Assume that TD market include m competitive banks. Next denote for bank B_1 on deposit market corresponding competitive interest rate as $r_{M,B_1}(a_i, a_{i+1})$. Relation between sectors can be in two following options:

- analyzed sector is subset of competitive sector
- analyzed sector has intersection with several competitive sectors

Analyzed sector is subset of competitive sector

It means that analyzed sector (a_i, a_{i+1}) and competitive sector (b_j, b_{j+1}) with $r_M(b_j, b_{j+1})$ satisfy following inequality

$$b_j \leq a_i < a_{i+1} \leq b_{j+1}$$

In words it means that analyzed sector is subset of competitive sector. Then corresponding interest rate to sector (a_i, a_{i+1}) is

$$r_{M,B_1}(a_i, a_{i+1}) = r_M(b_j, b_{j+1})$$

Analyzed sector has intersection with several competitive sectors
Range of analyzed sector (a_i, a_{i+1}) interfere to consecutive sectors $(b_j, b_{j+1}), (b_{j+2}, b_{j+3})$ with $r_M(b_j, b_{j+1})$ and $r_M(b_{j+2}, b_{j+3})$. Interval bounds satisfy following inequality

$$b_j \leq a_i < b_{j+1} < b_{j+2} < a_{i+1} \leq b_{j+3}$$

then corresponding interest rate to sector (a_i, a_{i+1}) is weighted average

$$r_{M,B_1}(a_i, a_{i+1}) = (\frac{b_{j+1} - a_i}{a_{i+1} - a_i} * r_M(b_j, b_{j+1}) + \frac{a_{i+1} - b_{j+2}}{a_{i+1} - a_i} * r_M(b_{j+2}, b_{j+3}))$$

where weight is interference to several sector in dependence to length of analyzed sector.

We can calculate all corresponding interest rates of analyzed bank sectors by this algorithm. It is shown in following table:

$$\begin{array}{llll}
r_{M,B_1}(a_1, a_2) & r_{M,B_1}(a_3, a_4) & \dots & r_{M,B_1}(a_{n-1}, a_n) \\
r_{M,B_2}(a_1, a_2) & r_{M,B_2}(a_3, a_4) & & \vdots \\
\vdots & & \ddots & \\
r_{M,B_m}(a_1, a_2) & r_{M,B_m}(a_3, a_4) & \dots & r_{M,B_m}(a_{n-1}, a_n)
\end{array}$$

Now we can calculate competitive interest rate for each analyzed sectors according to Equation 4.1:

$$r_M(a_i, a_{i+1}) = \frac{1}{m} \sum\nolimits_{j=1}^{m} r_{M,Bj}(a_i, a_{i+1})$$

where sector is characterized by (a_i, a_{i+1}) and competitive margin is calculate according to Equation 3.1.

Chapter 5

Conclusion

Liquidity crisis influenced almost every financial sector and also hit Czech deposit market at the end of 2008. Banks started to hoard liquidity and one of the cheapest sources were deposit products. Volume of obtained liquidity is especially influenced by deposit interest rates in relation to competitive interest rates. Paper is focused on Czech deposit sector. We introduced several approaches for fixing term deposit interest rates and margins. Since there can be many factors influencing TD we introduced only some of them. This information gave us ideas how to fix parameters of model for Czech deposit market. The suggested model introduces how to fix optimal term deposit margin depending on several parameters. It is based on two main parameters: demand and competition parameters. There are shown results of model on term deposits values (from January 2006 - December 2008) of one Czech bank in practice part. Liquidity crisis influenced especially demand parameter and thereby changed optimal values.

Chapter 6

Data

Practice part of this paper was made on term deposits data which will be introduced in this chapter. If some modifications or aggregations of this data were used, we provide here arguments for these operations.

The practice part of paper was made on Czech bank market. TD interest rates of 8 large banks which offer term deposits on Czech deposit market were used. The mentioned banks are:

- Komerční banka
- Česká Spořitelna
- Raiffeisenbank
- ČSOB
- UniCredit
- GeMoneyBank
- Volksbank
- Citibank

We calculated interest rates for accounts with different sectors denominated in CZK. These time series of rates cover period from January 2006 - December 2008.

Pribor[1] in 1W, 2W, 1M, 3M, 6M, 9M and 1Y was used for calculations. Models used only accounts equal Pribor maturities although banks offer accounts also in other maturities.

In examined period some banks merged. For example HVB Bank and Živnostenská banka merged to UniCredit, then analyzed period is shorten and starts by merging day. But for models based on comparison rates we need the same length time series. In this case took interest rates of HVB and analyze UniCredit as bank with same length as other banks.

Available data of interest rates contained only one value for week so we used this value for the rest days of the week. Thus, this time series corresponds to Pribor time series which is fixed every working day.

For almost all practice part where we calculate competition interest rate we used section 500ths-1mil which contains unbiased interest rates. This sector is same for all mentioned banks on Czech deposit market.

The second part of this work (Chapter 4 - Optimal margin) studies the volume. For purpose of this paper one of the banks provided historical volume data from January 2006 - December 2008. These data contain individual deals of term deposits in time. Thus, we aggregated them from daily basis to monthly basis and used monthly values for calculations.

For all calculation was used Statistical software R 2.7.1 with package *stat*. Code of calculation is attached on CD.

[1] source ČNB

Appendix A

Charts

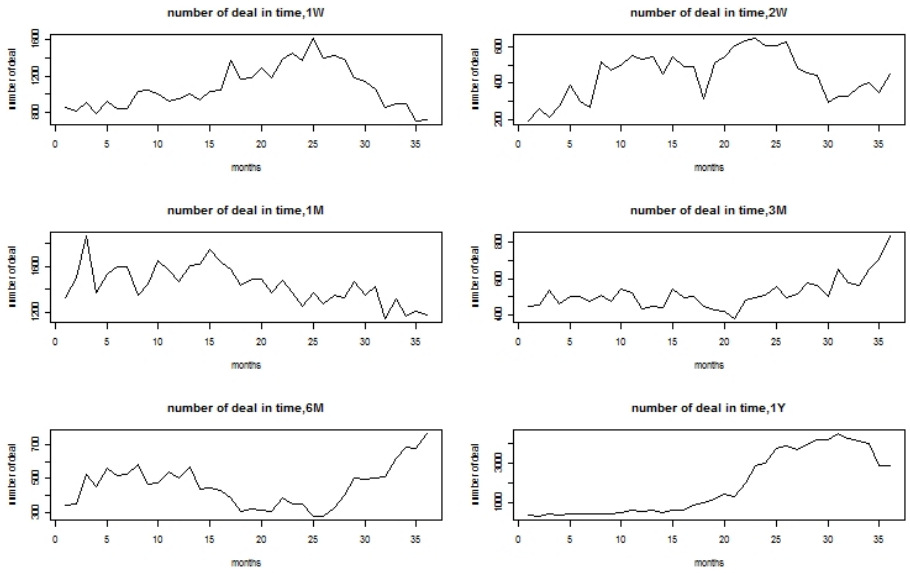

Figure A.1: Number of deals in time (months), sector 500ths - 1 mil

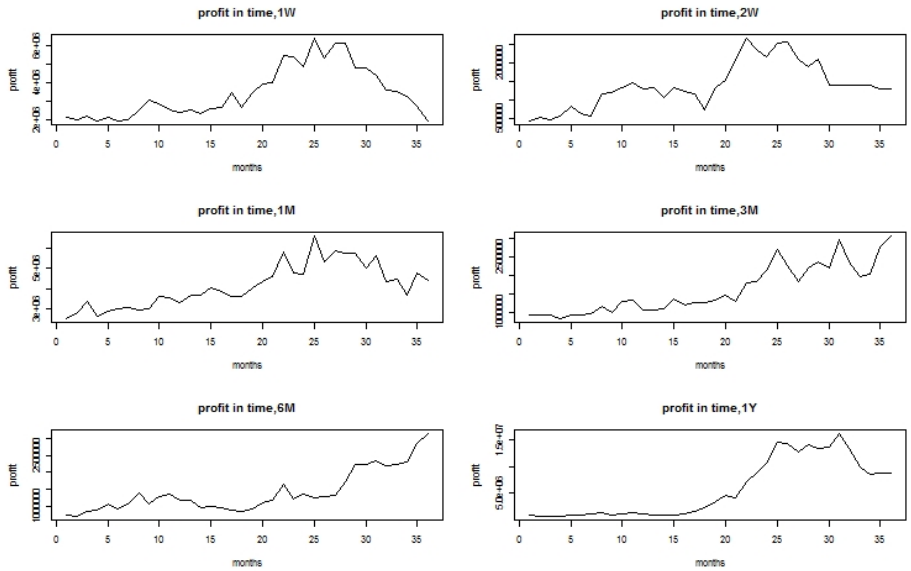

Figure A.2: Profit of deals in time (months), sector 500ths - 1 mil

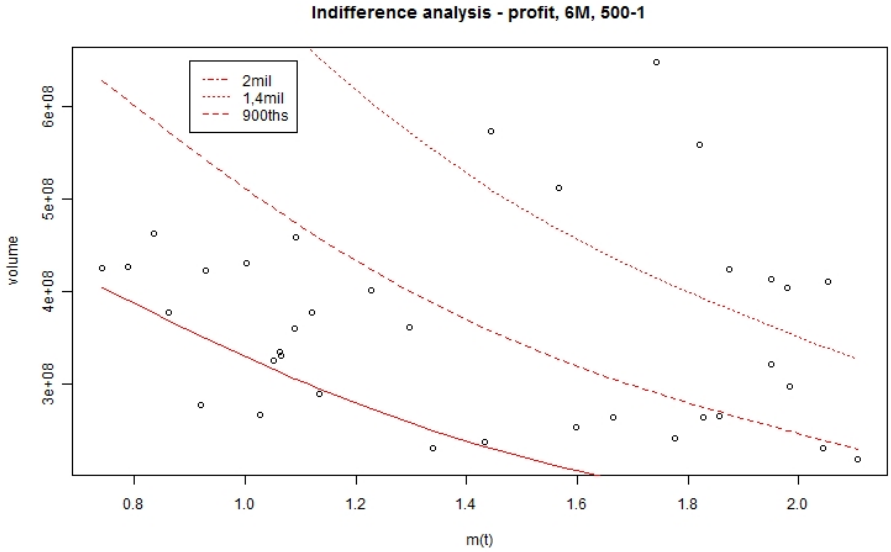

Figure A.3: Indifference analysis - profit. 6M maturity, sector 500ths-1mil

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