We study countable embedding-universal and homomorphism-universal structures and unify results related to both of these notions. We show that many universal and ultrahomogeneous structures allow a concise description (called here a finite presentation). Extending classical work of Rado (for the random graph), we find a finite presentation for each of the following classes: homogeneous undirected graphs, homogeneous tournaments and homogeneous partially ordered sets. We also give a finite presentation of the rational Urysohn metric space and some homogeneous directed graphs. We survey well known structures that are finitely presented. We focus on structures endowed with natural partial orders and prove their universality. These partial orders include partial orders on sets of words, partial orders formed by geometric objects, grammars, polynomials and homomorphism orders for various combinatorial objects. We give a new combinatorial proof of the existence of embedding-universal objects for homomorphism-defined classes of structures. This relates countable embedding-universal structures to homomorphism dualities (finite homomorphism-universal structures) and Urysohn metric spaces. Our explicit construction also allows us to show several properties of these structures.