7188/70/ 14 -06- 2010

Report on doctoral thesis of Jan Hubička

This thesis reports on structures which are universal for a given class, under either embedding or homomorphism (that is, every structure in the class should have an embedding in or homomorphism to the given structure).

For embedding, the best situation is where there exists a structure which is ultrahomogeneous (that is, isomorphisms between its finite substructures extend to automorphisms). Such structures are universal in the class of all structures younger than themselves, and satisfy a stronger version of universality (which implies, in particular, that embeddings can be found "on-line"). For some special classes (graphs, tournaments, digraphs, posets, certain finite metric spaces) the ultrahomogeneous structures have been classified. Understanding these is important, since they play a role in many areas of mathematics including Ramsey theory and topological dynamics. Hubička's main achievement in this area is the construction of explicit descriptions or "finite presentations" for many such structures. These are related to one another in surprising ways, and also to Conway's "surreal numbers".

In other cases where ultrahomogeneous structures do not exist, two lines of approach are possible. One is to give direct constructions of universal structures which are not ultranomogeneous. A wide variety of such constructions is given for posets. The other is to add extra relations to make the structure homogeneous; this is achieved by the recent method of "lifts and shadows", where Hubička gives an improvement and extension of known results.

For homomorphism embedding, the phenomenon of duality arises. The theory appears to be quite different but, as Hubička shows, there are surprisingly close links. The universality of oriented paths is given a new and cleaner proof.

It is a substantial thesis with some impressive new results, and in my opinion amply justifies the award of a doctorate.

The only quibbles I have concern a certain lack of clarity in some places. For example, the crucial concept of finite presentation of a relational structure is nowhere clearly defined, although examples make reasonably clear what is intended. I suspect that a more precise definition would be needed in order to prove that certain structures are not finitely presented. Similarly the statement "transitive closure [is] axiomatised by first order formulas" on p.47 is not sufficiently precise. These criticisms in no way disturb my overall judgment on the thesis.

Peter J. Cameron
June 9, 2010