Report on Dissertation by Libor Křižka "Moduli spaces of Lie algebroid connections"

The thesis presented by Mgr. L. Křižka is devoted to some very interesting and important problems related to the advanced subject of Lie algebroids, their connections and related moduli spaces. In general, moduli spaces have many applications in mathematics and physics, since they allow construct invariants of manifolds such as, for instance, Gromow-Witten or Seiberg-Witten invarinats. Also, moduli spaces are closely related to deformation theory. For instance, moduli spaces of holomorphic structures, moduli spaces of stable holomorphic bundles, moduli spaces of flat connections or moduli spaces of Higgs bundles play an important role in gauge theory, low-dimensional topology, theory of integrable systems and string theory.

L. Křižka's motivation for the study of moduli spaces comes from the subject of holomorphic structures on complex vector bundles over a compact complex manifold and its relation to Hitchin-Kobayashi correspondence, and also from complex and generalized complex geometry and their relation to moduli spaces of A- and B-branes and hence to mirror symmetry and Langlands duality.

The main body of thesis consists of three chapters. The core of the thesis containing original results is in chapters 2 and 3.

First chapter is a very nicely written introduction to Lie algebroids and their differential geometry; in particular many examples of Lie algebroids are presented in detail. Also, concepts and notions important for the rest of the thesis like those of differential forms, vector fields and the de Rham differential on Lie algebroids are introduced and described. Finally, the Lie algebroids are put into a wider context of generalized complex geometry.

The second chapter is devoted to linear Lie algebroid connections. In this chapter, first, the definition of a linear Lie algebroid connection along with some well known results are summarized, .including the description of the gauge group action on the space of linear Lie algebroid connections. Next, the notion of the moduli space of linear Lie algebroid connections is introduced and its Sobolev completion is defined, which allows to equip the moduli space with a structure of a geometric space. It is an interesting and important original result of Mgr. L. Křižka that irreducible Lie algebroid connections form (a possibly non-Hausdorff) principal bundle with structure group being the reduced gauge group and that the corresponding moduli space is a locally Hausdorff Hilbert manifold (Theorem 10). Also, a further important original result is presented in this section. It is proved that the moduli space of smooth irreducible flat Lie algebroid connections is (near a smooth point) a smooth finite dimensional manifold of dimension equal to the dimension of the first Lie algebroid cohomology group (Theorem 11). Altogether, in this chapter highly nontrivial results are presented generalizing results described by Kobayashi in his seminal monograph on differential geometry of complex vector bundles and results of papers by Lübke, Okonek and Teleman.

The third chapter introduces the general concept of a principal Lie algebroid connection. Some concepts and results described in chapter 2 are generalized from the case of linear Lie algebroid connections to the principal Lie algebroid connections. Particularly, the covariant exterior derivative, the induced Lie algebroid connection on an associated vector bundle, the action of the gauge group and the parallel transport along a Lie algebroid path and holonomy are introduced. The central result of this chapter is the Theorem 18 establishing the isomorphism of the holonomy group and the isotropy group of a principal Lie algebroid connection.

Altogether, the thesis is excellent. It presents some very important and original contributions to the theory of moduli spaces of Lie algebroid connections, with potential applicability in both mathematics and theoretical physics. The results of the thesis also give rise to some very interesting questions for the future research. For instance, as conjectured by Mgr. L. Křižka in the "Conclusion" section a generalization of the Riemann-Hilbert correspondence should exist, relating the proper generalizations of the character variety and G-local system.

The thesis is concisely written and shows that Mgr. L. Křižka has a profound knowledge of the relevant parts of mathematics. There is a good reason to believe that the work presented in the thesis is of real interest to a wide community of mathematicians and mathematical physicists.

There is no doubt that Mgr. L. Křižka can work successfully independently on his own original research projects and that he will continue to produce relevant contributions to the mathematics in the future.

Mgr. L. Křižka certainly deserves to be awarded the PhD title.