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Referee report on the doctoral thesis
Analysis in Banach spaces
by
Antonín Procházka

It is my pleasure to write a referee report on this thesis. The thesis deals with four areas of research:

1. The area of the use of topological games in the theory of Banach spaces with the Radon Nikodým property. Here, it is shown in particular, that a Banach space has this property if and only if there exists a mapping $F : X \to X^*$ such that a bounded sequence (x_n) is convergent whenever

$$\big(F(x_n), x_n\big) \leq \big(F(x_n), x_{n+1}\big)$$

for every n.

Let me note that this is an interesting new approach as related properties usually characterize spaces the dual of which has the Radon Nikodým property. New ideas in this area are needed as it is for example unknown if the space has the Radon Nikodým property whenever each bounded closed convex subset of it has an extreme point.

2. The area of the dentability and Szlenk indices of Banach spaces. This is an area used in the study of geometric properties of spaces as well in nonlinear analysis. Let me remark that it is still unknown if a Banach space is isomorphic to c_0 if it is uniformly nonlinearly homeomorphic to c_0.

From many results in this section of the thesis let me mention only one practically useful corollary that two separable $C(K)$ spaces are isomorphic if and only if they have the same dual dentability index.

3. The area of renormings by higher order differentiable norms. From this section let me mention the result in the thesis that if α is an ordinal, then $C([0, \alpha])$ admits an equivalent norm that is locally uniformly rotund, Fréchet differentiable and is a limit, uniform on bounded sets, of norms that are C^∞ smooth. This was an open problem since the pioneering paper of M. Talagrand in 1986.

The proofs here required a very fine subtle considerations. This is because the techniques for renormings by rotund norms and higher order differentiable

norms do not go well together: Note that any space with C^2 smooth norm that is at the same time locally uniformly rotund is necessarily superreflexive and there is no C^2 smooth rotund norm on $c_0(\Gamma)$ if Γ is uncountable. Also, real analytic norms on $C(K)$ spaces can exist only if they are separable. In the case of continuous functions on the ordinal segment, it was so far even open if it has a locally uniformly rotund norm that is C^1 smooth. The method of the proof of the results here uses a sofisticated version of the Kuiper method of constructiong norms on $C(K)$ spaces.
The problem if the C^1 smooth norms are dense in all equivalent norms on $C[0,\alpha]$ if α is uncountable remains open.

4. The section dealing with parametric variational principles contains valuable results that will sure be frequently used in analysis on Banach spaces. This section contains also many examples that are of independent interest and greatly help in understanding the subject.

As I mentioned above the thesis is an excellent thesis which can be accepted in the most prestigious universities.

The thesis contains new results and methods that show that the author of the thesis is capable of doing independent valuable research in this area.

The proofs of the results in the thesis required a deep and fine analysis of the principles and techniques as well.

The results in the thesis will certainly be used in solving further open problems in the area.

For this purpose, it is important that this thesis does formulate clearly the main principles and techniques.

The thesis is well written and its organization is excellent.
I did not find any mistakes, typos or anything else that would require a change or an improvement.

Let me claim here that out of, say, 30 PhD theses that I dealt with as a referee during my career in Mathematics, I would place this one in the group of the best three or four theses.

Conclusion:

The thesis of Mr. Procházka is an excellent, well written thesis that contains many new valuable results. It is a true contribution to this active area of analysis in Banach spaces. I like this thesis very much.

The thesis in its present form fully satisfies all the requirements for a PhD thesis in Mathematics and I strongly recommend it as a doctoral thesis for granting the degree of PhD in Mathematics.

Václav Zizler
Director of Research
Mathematical institute
Czech Academy of Sciences
Zitna 25, 11567 Prague 1
e-mail: zizler@math.cas.cz
Czech Republic
and
Professor of Mathematics, Emeritus
University of Alberta
Edmonton, Alberta, Canada