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Report on the Doctoral Thesis

Stability and Approximations for Stochastic Programs

submitted by Mgr. **Michal Houda**

The thesis addresses the important topic of stability and approximation in stochastic optimization. The relevance of this subject is based on the fact that the theoretical probability measure underlying a stochastic optimization problem is not known in general and, even if it was known, it might be desirable for computational reasons to replace the original problem by a structurally simpler one (e.g., convex or linear). In any case one has to study the effect of approximation on the optimal value and the optimal solutions of the given problem.

The paper consists of 8 chapters. The first part serves as an introduction to stochastic programming problems and as an overview of probability metrics and stability theory in this context. This survey is written up very thoroughly and demonstrates the candidates good understanding of the mathematical background. The original contributions are contained in the second part of the thesis. Particular attention is paid there to problems involving chance constraints, a topic which has recently stimulated a lot of international research.

Chapter 4 is devoted to stability of stochastic programs. Theorem 4.4 provides a semi-continuity result for solution sets and a calmness result for optimal values w.r.t. to the Wasserstein metric which is later used in Theorem 4.6 to prove Hölder continuity of solution sets under a strong convexity assumption. Chapter 5 deals with empirical approximations in stochastic programming. Here, Theorem 5.3 shows how the independence assumption for empirical distribution functions can be relaxed to so-called weak independence. The results are illustrated in a numerical convergence study.

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Chapter 6 takes up the important topic of convexity for chance constraints. Here, the candidate develops a nice idea of approximating a chance constraint from above and below by convex sets. Using a result from the literature which guarantees under suitable assumptions the convexity of chance constraints for random vectors with independent components, he introduces a dependence coefficient which allows to sandwich a possibly nonconvex chance constraint in the correlated case between two convex chance constraints. If correlations are not too strong, this approach offers a methodology for approximating the optimal value of a minimization problem involving chance constraints by means of convex programming tools. Indeed, the candidate supports this idea by employing arguments from the stability theory of chance constraints.

Chapter 7 turns to another issue of much interest in chance constrained programming, namely its relation to some recently emerged robust approaches promising better computational tractability. More precisely, the robust sampling approach introduced by Calafiore and Campi is compared with the original chance constrained program and with its empirical approximation (sampling average). The latter is known from stability analysis to provide an improving (with sample size) approximation of the original problem. In contrast, robust sampling defines a model which on the one hand exhibits nice features (like convexity of the feasible set) but which on the other hand has no meaningful relation with the original feasible set. This fact is illustrated impressively in the numerical study of chapter 7, demonstrating that the optimal solution of the robust sampling approach actually becomes worse with respect to the true solution when the sample size increases.

Specific comments:

1)

I am not completely sure about the metric regularity estimate on top of p. 66 (below the figure): The left hand side in the definition of metric regularity represents the distance of a point to a set. I do not see why the norm on the left hand side of the mentioned estimate should correspond to such a point-to-set distance.

2)

The idea of inner and outer approximation of chance constraints by convex chance constraints - coming from random vectors with independent components - could be complemented by means of individual chance constraints: It is well known that individual chance constraints provide an outer approximation of the original joint chance constraint. Often it is easy to check convexity of individual chance constraints (for instance, in case of a stochastic coefficient matrix with Gaussian data, one may rely on the classical Van de Panne/Popp/Kataoka result). Then, the intersection of the individual chance consraints with the chance constraint providing the convex outer approximation under the assumption of independence would yield a smaller, still convex outer approximation of the original joint chance constraint. This could just be an idea for practical improvement at least in certain special cases.

Conclusion:

The thesis is well organized and clearly written. The literature is well reflected and the author proves his understanding of the mathematical background in stability theory of stochastic programs. His creativity becomes evident in particular in chapter 6. The thesis contains a lot of practically useful observations which are supported by numerical experiments. Sometimes, arguments would have allowed a deeper analysis.

In conclusion, the candidate has proved his ability to get interesting new results and definitely merits the title of a PhD.

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