This thesis is concerned with aerodynamic optimization of airfoils and wings. It focuses at using very fast solvers based on hybrid potential methods to evaluate the aerodynamic performance of the airfoils and wings. Compared to more widely used CFD solvers which are more computationally demanding, use of fast solvers brings different possibilities and different problems to tackle, some of which are analyzed in this work. A state-of-the-art fast solver for airfoils is resented, and a fast solver for slender wings is developed, and some of its aspects are discussed. An innovative evolutionary optimization algorithm is presented and both solvers are hen utilized to solve real-life airfoil and wing optimization problems.