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Essays in Applied Microeconomics: School Admission Mechanisms and Corporate Bankruptcy

Ondřej Knot

Dissertation

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Abstract

In this dissertation, I address two topics in applied microeconomics. First two chapters deal with the functioning of school admission mechanisms and their effects on student school choice behavior. Third chapter deals with the question of optimal bankruptcy law design.

Matching mechanisms play a critical role in the schooling system. They affect the behavior of students and – through the information they convey – also the behavior of the schools and the authorities responsible for education policy. In the first chapter (joint with Daniel Münich) we use a computational simulation model to analyze the functioning of an admission scheme used in the Czech Republic, which can be seen as a prototype of decentralized, ability-based admission schemes widely used in the world to assign pupils to upper-secondary schools. Our findings show large incidence of strategic misrepresentation of school preferences among applicants, large differences between revealed and trued demand, and large incidence of justified envy in the resulting matching. We point out several implication for the functioning of schooling systems.

In the second chapter, I empirically study the behavior of students under the Czech pupil-school matching mechanism. Using district-level data on demand for public gymnasia, I find significant evidence that students do not apply to their most preferred schools, opting rather for a less-preferred, but safer option. Furthermore, using data on individual student school choices, I also find that students with weak socioeconomic background misrepresent their preferences more often than other students.

In the third chapter (joint with Ondrej Vychodil), we develop a model of a debt contracting problem under bankruptcy regimes differing by a degree of softness. In the model, the degree of softness is associated with the extent to which the absolute priority rule can be violated. We show that when the degree of softness can be set individually for each project, then the debtor's tendency to excessive risk-taking can be eliminated and the first best solution can be attained. When it is given exogenously by a bankruptcy law, then a completely tough law results in a lower distortion from the first best than a soft law with a moderate degree of softness.

Abstrakt

Tato dizertace se zabývá dvěma tématy v aplikované mikroekonomii. První dvě kapitoly se věnují fungování mechanismů pro párovaní žáků a škol a dopadům těchto mechanismů na chování studentů při výběru školy. Třetí kapitola se zabývá problematikou bankrotu podniku.

Mechanismy párování žáků a škol hrají důležitou úlohu v systému školství. Ovlivňují chování studentů a – prostřednictvím informací, které generují – také jednání škol a úřadů zodpovědných za politiku vzdělávání. V první kapitole (společná práce s Danielem Münichem) aplikujeme simulační model na analýzu fungování přijímacího systému využívaného v České republice. Tento systém lze považovat za prototyp decentralizovaného a na schopnostech studentů založeného mechanismu, které jsou v široké míre používány ve světě k párovǎni záků a škol při přechodu na střední školu. Naše výsledky ukazují vysokou míru strategického zkreslování preferencí, velké rozdíly mezi skutečnou a projevenou poptávkou po školách a vysokou míru "oprávněné závisti" (angl. justified envy) ve výsledném spárování. V této kapitole poukazujeme rovněž na několik dopadů, které by toto mohlo mít na systém vzdělávǎni.

Ve druhé kapitole empiricky studuji chování studentů v českém mechanismu párovǎni studentů a středních škol. S využitím dat na úrovni okresů o nabídce a poptávce po veřejných gymnáziích zjišťuji, že se studenti nehlásí na jejich preferované školy a vybírají si raději méně preferované, avšak jistější varianty. S využitím dat o volbě školy invidiualními studenty dále zjišťuji, že studenti s horším socio-ekonomickým zázemím zkreslují své preference více než ostatní studenti.

Ve třetí kapitole (společná práce s Ondřejem Vychodilem), představujeme model, který analyzuje uzavírání dlužnických kontraktů za přítomnosti hazardního boje o záchranu (angl. gambling on resurrection) v režimech úpadkového práva lišících se ve stupni měkkosti. V modelu je stupeň měkkosti svázan s mírou, do jaké může být porušeno pravidlo absolutní priority. Ukazujeme, že pokud lze stupeň měkkosti stanovit individuálně pro každý projekt, lze dlužníkovu tendenci k přílisnému riskovǎni zcela eliminovat a dosáhnout společensky optimálního výsledku. Pokud je ale stupeň měkkosti určen exogenně úpadkovým právem, zcela tvrdý zákon muže vést k menší odchylce od optimálního stavu, než mekčí zákon se středním stupněm měkkosti.

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Introduction

In this dissertation, I address two important in applied microeconomics. The first two chapters deal with the functioning of school admission mechanisms and their effects on student school choice behavior. The third chapter deals with the question of optimal bankruptcy law design.

Pupil-school matching mechanisms play a critical role in the schooling system. They affect the behavior of students and – through the information they convey – also the behavior of the schools and the authorities responsible for education policy.

I study a decentralized, ability-based matching mechanism,[1] different versions of which are used in many countries, especially in continental Europe.[2] The particular version I focus on is the mechanism used in the Czech Republic to match 9th grade students to upper-secondary schools. Its key feature is that in each round of the admission procedure, each student only has "one shot" (can choose a single school to apply to) and if she misses (ranks below the line of the admitted students), she risks that other schools she likes have filled their capacity with other students, leaving her only with choices she does not like.

In the first chapter,[3] we use a computational simulation model to study the school application strategies of rational agents under a decentralized, ability-based mechanism. Our simulations show that there is a significant degree of strategic preference misrepresentation among applicants. For a reasonable range of parameters, applying to a school other than the most preferred one is the optimal strategy for 30% – 50% of students. As a result, *revealed* demand for the best school (the number of students who actually apply) is about 50% higher than its capacity, while *true* demand (the number of students who prefer the school most) exceeds its capacity by 300%. The situation is the opposite for schools of lower quality, where revealed demand exceeds schools' capacities, while true demand is lower than capacity.

In the second chapter, I ask whether under the pupil-school matching mechanism used in the Czech Republic, students actually misrepresent their preferences. Using district-level data on demand for public gymnasia – the most over-subscribed type of school – I find that the revealed demand is co-determined by factors, which according to my model do not affect the true demand. In particular, I find that there is a significant positive relationship between the capacity of public gymnasia and the revealed demand for them,

[1] Individual schools administer the admission procedure themselves (= decentralized) and students' abilities play a role in the admission decision (= ability-based)

[2] In the UK and the US, to the contrary, the mechanisms used are typically centralized and ability-blind.

[3] Co-authored by Daniel Münich, CERGE-EI

even when controlling for potential endogeneity, supporting the preference misrepresentation hypothesis. Furthermore, using data on individual student school choices, I also find that students with weak socio-economic backgrounds misrepresent their preferences more often than other students. The most likely explanation for this is that the students from better-off families have a chance, if rejected by their most preferred school, to opt-out from the public school system and do not face the threat of ending up in a particularly low-quality school.

Several policy implications follow from the findings in chapters 1 and 2. From the ex-ante point of view, the mechanism induces significant psychic hardships on students and their parents related to the school application process. Because applying to one's most preferred school is often not the best strategy, students are forced to strategize and a wrong decision or bad luck can have severe consequences. From the ex-post point of view, a phenomenon called *justified envy* is widespread.[4] According to our results in chapter 1, justified envy affects approximately 10% – 20% of students, depending on parameter values.

A substantial flaw of the mechanism is that it distorts the school demand information. It creates an illusory balance between supply and demand for schools and does not provide reliable signals to school managers and to government officials, who make decisions concerning the overall school system, school capacities, and school funding. There are practically applicable mechanisms, which could be used instead of the mechanism currently used in the Czech Republic and similar mechanisms used in other countries. The two best-known examples are the Gale-Shaply matching mechanism[5] and the Top Trading Cycles mechanism.

In the third chapter,[6] we deal with the problem of an optimal design of a bankruptcy law, with particular regard to what should be the degree of softness / toughness of the law. Most bankruptcy laws in theory adhere to the concept of Absolute Priority Rule (APR), which states that shareholders should not get any value from the bankrupt firm unless creditors are paid in full. In practice, however, many bankruptcy systems enable violations of the APR, the best-known example being Chapter 11 of the U.S. bankruptcy code. One stream of research claims that a certain degree of softness (APR violations)

[4]This means that there is at least one student who ends up in a school which he ranks lower than some other school, while this other school would admit him over some other student which it actually admitted.

[5]This mechanism was used, for example, to replace the well-known Boston mechanism in 2006; see Abdulkadiroglu et al. (2005)

[6]Co-authored by Ondřej Vychodil

may, indeed, be beneficial ex-ante for all parties, since it can mitigate the gambling on resurrection problem. This problem, in short, means that when the financial situation of the debtor starts to deteriorate, she has a tendency to engage in risky endeavors with some small probability to safe the firm, but with a high probability to just burn cash and destroy the remaining value.

We investigate the problem using a theoretical model of contracting between a creditor and a debtor under asymmetric information. At the time of contracting, both parties have the same information; the asymmetry arises during the life of the financed project when the debtor learns a signal about the quality of the project, which the creditors do not receive. The debtor then decides whether to continue the project or to abandon it. Continuation is optimal when the signal is good, while liquidation is optimal when the signal is bad. The debtor, however, may decide to continue the project even when the signal is bad and the creditors, because they do not see (or cannot verify) the signal, cannot stop him. The softness / toughness of the bankruptcy law is a critical parameter in determining what strategy the debtor will follow.

We show that a sufficiently soft bankruptcy law may indeed eliminate the gambling on resurrection problem, but – under a law that is insufficiently soft – this problem gets even worse than under a completely tough law. We also show that the possibility of verification can as well eliminate the problem, either partly or fully, depending on its cost.

We see two practical issues with the soft law. First, although the sufficiently soft law outperforms the completely tough one, it can be very difficult to find the optimal degree of softness for a given economy – there is no one-size-fits-all solution both in terms of different projects and different creditor types.[7] Hence it might seem reasonable for the policy maker to fully preserve APR, rather than trying to find the optimal degree of APR violation. Moreover, could the optimal degree be found, it might still be impossible to reach it in practice. Even the best-known example of a clearly soft law, Chapter 11, is empirically documented to be substantially tougher than the optimal degree of softness found in our paper.[8]

[7] In the paper, we assume for simplicity only one creditor. In practice, however, there are typically multiple creditors with different levels of seniority and, thus, different optimal degrees of softness for each of them.

[8] In our paper, we find that to have the desirable effects on debtor behavior, the law has to enable the debtor to retain between 40% and 50% of the bankrupt firm value. Empirical studies of Chapter 11 of the U.S. bankruptcy code show that in reality this value ranges between 0% and 26%.

Chapter 1

The impact of decentralized, ability-based school admission mechanisms on efficiency, equity, and quality of educational outcomes

Co-authored by ***Daniel Münich***

Abstract

Pupil-school matching mechanisms play a critical role in the schooling system. They affect the behavior of students and – through the information they convey – also the behavior of the schools and the authorities responsible for education policy. In this paper, using a computational simulation model, we analyze the functioning of an admission scheme used in the Czech Republic as a prototype of decentralized, ability-based admission schemes widely used in the world to assign pupils to upper-secondary schools. Our findings show a large incidence of strategic misrepresentation of school preferences among applicants, large differences between revealed and true demand, and large incidence of justified envy in the resulting matching. We point out several implications this could have for the functioning of schooling systems.

Keywords: pupil-school matching, matching mechanisms
JEL classification codes: C78, D60, I20

1.1 Introduction

School admission mechanisms are indispensable components of each schooling system. Such mechanisms represent sets of more or less formal rules matching applicants to schools based on applicants' *preferences* for individual schools and schools' *priorities* over applicants. Resulting pupil-school matching outcomes are taken as given in numerous strands of the existing literature. In particular, the last decade saw great progress in describing skills production processes,[1] understanding the role played by the selectivity of schooling systems in the transmission of socio-economic inequalities,[2] and estimating the impact of schools on the labor market[3] and other outcomes.[4]

There is rich empirical evidence that educational outcomes are co-determined by pupils' and schools' characteristics.[5] However, the actual functioning of the admission mechanism itself has not been given much attention yet. But as we argue, the actual setup of a school admission scheme can have serious equity and efficiency implications. Moreover, admission schemes generate valuable information about the educational demand/supply gap. As desirable functioning of school-choice based schemes is conditional on well informed agents – parents and their children – imperfect or biased information generated by ill-shaped admission mechanisms could hamper their functionality.[6] Moreover, on the supply side, biased information about demand can lead to suboptimal managerial decisions of individual school and governmental schooling administrators. Finally, complex admission schemes requiring sophisticated strategizing can disadvantage applicants with weak family support.

A great deal of empirical evidence on the functioning of school admission mechanisms comes from the US and the UK. Most common there are *centralized-ability-blind* (CABl) admission mechanisms. Centralized administration consists of centralized collection of information on applicants' preferences for schools and of centrally set priority rules deter-

[1] See Krueger (1999), Cabrales, Calvo-Armengol, and Pavoni (2008), Zax and Rees (2002), McMillan (2004)

[2] See Hanushek and Luque (2003), Fleurbaey and Maniquet (2005), Fernandez and Rogerson (2001), Nechyba (2006)

[3] See Bowles, Gintis, and Osborne (2001)

[4] See Glewwe (2002), Bils and Klenow (2000)

[5] See great ongoing debate on the role of class-size and peer effects (Hanushek vs. Krueger), literature on school tracking and transmission of intergenerational inequalities (e.g., Krueger (2003), Rivkin, Hanushek, and Kain (2005), Hanushek, Kain, and Rivkin (2004), Cullen, Jacob, and Levitt (2005)).

[6] Recent studies on this are by Epple and Romano (1998), Rouse (1998), Nechyba (2000), Adnett and Davies (2000), Machin and Stevens (2004), Davies, Adnett, and Mangan (2002), Hoxby (2000), Clowes (2008), Greene and Kang (2004), Grosskopf et al. (2001), Hsieh and Urquiola (2003), Merryfield (2008) and Hastings and Weinstein (2008).

mining the priorities of applicants for individual schools. Since priority rules commonly preclude inclusion of a pupil's personal traits like study aptitude and other skills into the admission criteria, such mechanisms are called ability-blind. Our work contributes to the existing literature analyzing *decentralized*, *ability-based* (DABa) matching mechanisms which are widespread within continental Europe.[7] In this family of mechanisms, there is little or no inter-school coordination, and an applicant's personal traits are an important component of priorities. Inspired by a prototype of DABa mechanisms employed in the Czech Republic, we explore its key properties. While various implementations of DABa mechanisms can be found throughout Europe, we call the particular version which we analyze the *Czech mechanism*.[8]

In the Czech mechanism, admission priorities are set by individual schools commonly based on an applicant's performance in the school's own test. The exact procedure is that students first select a school to apply to in the first round and then sit down for a test on a centrally specified date. The schools then decide, based on the test results, which students to admit / reject. The rejected students proceed to the second round where the procedure should be repeated, though in practice the second round is already less rigorous. Having incomplete and imperfect information about their own admission chances – since admission testing happens only after applications are submitted – applicants are restricted in the number of schools they can apply to. By building a behavioral model of applicants and using quantitative computational simulations, we identify properties of the Czech mechanism and compare the matching outcomes it generates to those generated by other stylized mechanisms.

Our simulations of the Czech mechanism show that there is a significant degree of strategic preference misrepresentation among applicants. For a reasonable range of parameters, applying to a school other than the most preferred one is the optimal strategy for 30% – 50% of students. Due to this, revealed excess demand for the best schools is about 50% while true demand exceeds capacity by 300%. The situation is the opposite for schools of lowest quality, where revealed demand exceeds schools' capacities, while true demand is lower than capacity. We document that the degree of strategic misrepresentation depends on the degree of uncertainty, which depends on the degree of perception

[7] Ability-based mechanisms are also used in the US college system.

[8] The Czech mechanism considered here as a prototype of DABa mechanisms assigns almost a hundred thousand fifteen-year old applicants to thousands of high schools in the Czech Republic each year. Examples of other countries that use a similar mechanism are Austria, Croatia, Hungary, Poland and Slovakia.

error of one's own qualifications. Finally, we document that different parameters and choice of admission mechanism can have sizeable efficiency and equity implications.

We put the issue of matching into a broader perspective and review the related literature in section 1.2, describe our simulation model in section 1.3, and present our findings in section 1.4. Broader implications of our findings, possible setbacks, and policy issues are discussed in section 1.5.

1.2 Background and framework

The pupil-school matching problem is a special case of two-sided matching between heterogeneous agents.[9] Different cases of two-sided matchings have been studied in various fields. Spontaneous matching between workers and jobs is one important determinant in models explaining employment and wage patterns.[10] Another strand of literature employs the concept of a *matching function* to account for labor market frictions, i.e. interaction and simultaneous existence of unemployed and vacant jobs.[11] There, the matching function - a concept similar to the notion of production function in the theory of firm - is a tool capturing the extraordinary complex spontaneous interaction processes. Closely related to our topic is the matching of school graduates to specific jobs (professions). Thoroughly studied was the matching of medical interns to hospitals in the UK and US[12]

Another rich strand of literature explores the principles of matching on marriage markets[13] or in the house allocation problem where houses represent indivisible goods rationed among agents.[14] Many other examples of two-sided matching can be found.[15]

1.2.1 Properties of pupil-school matching mechanisms

Matching of pupils to schools - admission - is experienced by everyone several times in life; first as a student, and later as a parent. The process is frequently perceived as a quest for a desirable school or college. In their seminal study, Gale and Shapley (1962) (GS) described the matching of applicants to colleges as a mechanism design problem.

[9] See Roth and Sotomayor (1990)

[10] See Jovanovic (1984), Postel-Vinay and Robin (2004), Moscarini (2005).

[11] See Mortensen and Pissarides (1994), Albrecht and Vroman (2002), Petrongolo and Pissarides (2001) and Galuscak and Munich (2007).

[12] See Roth (1984a), Roth and Xing (1994), Roth (2002), Roth (1990) and Roth (1991)

[13] Roth (1985)

[14] See Abdulkadiroglu and Sonmez (1999) or Ergin (2000)

[15] Kidney exchange studied by Roth et al. (2004), Housing market by Shapley and Scarf (1974).

Various theoretical investigations followed. We provide a brief overview of the established terminology we use while details can be found in Abdulkadiroglu and Sönmez (2003) and Chen and Sonmez (2006).[16]

In the following, each student has *preferences* over schools and students have *priorities* within each school. *Student assignment mechanism* is a procedure leading to a particular matching for each school choice problem, where *matching* is an assignment of students to school seats such that each student is assigned just one seat and the capacity of no school is exceeded. Two types of mechanisms are distinguished: *school-choice* and *school-admission* mechanism. In the former one, students optimize choosing schools based on their own preferences, while the priorities of students within schools are externally set and not subject to optimization. In the latter case, priorities are also the outcomes of schools' optimizing behavior. In the latter case, too the welfare of both sides matters. Under certain conditions, the difference between school-choice and school-admission mechanisms has no impact on the essence of the matching problem.

Justified envy in matching appears if student i prefers school s to her actual assignment while student j assigned to school s has a lower priority at this school than does student i.[17] A mechanism eliminates justified envy if it always leads to matching which eliminates justified envy. Elimination of all cases of justified envy in the final matching corresponds to *pairwise stability property* in school-admission problem.

A matching is *Pareto efficient* if there is no alternative matching which would assign each student to a weakly better school and at least one student to a strictly better school. In other words, matching is Pareto efficient if no student can be made better off without making some other student worse off. Note that Pareto efficiency is defined from the perspective of students' welfare only. A mechanism is Pareto efficient if it always selects a Pareto efficient matching. Note that while Pareto efficiency relates possible gains between subsets of applicants, justified envy takes into account both priorities and preferences. As we point out later, there could be a trade-off between Pareto efficiency and the elimination of justified envy.

The matching mechanism is *strategy-proof* if no student can benefit by unilateral misrepresentation of preferences (i.e. dominant strategy is incentive compatible). Mis-

[16] For a literature on pupil-school matching, see also McVitie and Wilson (1970), Roth (1982), Roth (1984b), Roth (1989), Roth (2008), Roth and Sotomayor (1990), Roth and Vande Vate (1990), Roth and Vande Vate (1991), Roth and Rothblum (1999)

[17] The notion of justified envy corresponds to the concept of stability in school-admission models. Interpreting school preferences as school priorities, the notion of stability translates into the notion of justified envy.

representation of preferences appears if at least one applicant can benefit from unilateral misrepresentation of her own school preferences.

As we clarify in detail, another distinguishing feature of matching mechanisms is whether or not applicants' personal traits, like study aptitude or qualifications, co-determine preferences and priorities.[18] Mechanisms where applicants' qualifications determine priorities – *ability-based* mechanisms – tend to increase economic and social inequalities due to sorting of students into schools of heterogeneous quality and due to the peer effect, if present (see Nechyba (2006)). In some countries such schemes are viewed as merit-based and desirable. In other countries, the prevailing view is that applicants' study aptitudes are, to a great extent determined by social and economic environment rather than individual merit. In the latter case, on equity grounds, governments impose legal restrictions on the use of personal traits in admission, the freedom of individual schools to set own admission criteria is quite limited, and admission criteria are set by local or central authorities.

Ability-blind schemes are widespread in the US and the UK. The negative side effect of such arrangements is that they can have an adverse impact on competition among schools, which some consider an important disciplining device (see McMillan (2004)). This is the intention of proponents of school-choice programs – schemes based on per-pupil public funding, where schools have strong incentives to attract pupils by providing educational services of higher quality.[19] Furthermore, there can be a trade-off between equity and productive efficiency, as addressed by Adnett, Bougheas, and Davies (2002) and Machin and Stevens (2004).

1.2.2 Existing pupil-school matching mechanisms

Different admission schemes are used at different schooling levels and by different school types.[20] Most existing schemes have developed spontaneously or were designed with no account of mechanism design theory.

Research on existing admission mechanisms is still rare. Existing studies focus on a few cases, mostly centralized and ability-blind mechanisms in the US and the UK.

[18]One can also find mixed arrangements which allow schools to select a proportion of applicants based on ability or aptitude and requiring them to admit the rest by criteria other than those based on the cognitive skills of applicants.

[19]Survey studies like Wilie (1998) or Ladd and Fiske (2003) conclude that competitive pros are small while equity losses are large.

[20]Extraordinary high variation in admission schemes is documented in the UK by West, Pennell, and Noden (1998), West, Hind, and Pennell (), and White et al. (2001).

There, a mechanism used in the Boston region till 2005 was thoroughly examined.[21] The Boston mechanism is a centrally processed, ability-blind school assignment where school priorities of pupils are based on factors like walking distance, school attendance by senior siblings, special medical or social needs of the student, etc.[22] Ergin and Sönmez (2005) pointed out serious flaws of the Boston mechanism: (i) it does not eliminate justified envy[23], (ii) it is not Pareto efficient[24], (iii) it is not strategy proof.[25] These flaws have serious consequences: (a) the justified envy of applicants and their parents creates grounds for legal disputes. This is particularly so in public schools as it casts doubts on the realm of justice. (b) Built-in incentives to misrepresent school preferences together with incomplete and imperfect information carry serious psychic costs given that actual school assignment could have life-long implications. (c) Strategic misrepresentation of applicants' preferences generates biased information about demand at the level of individual schools, school types and regions. This can lead to inefficient allocation of public resources and poor managerial decisions at the school and central levels. (d) The need for strategic application can harm some demographic groups of applicants.

In 2005, based on critical arguments (i)-(iii), the matching mechanism in the Boston area was replaced with a *student-proposing deferred acceptance mechanism*, the principles of which were originally proposed by Gale and Shapley (1962) and used for decades

[21] See Abdulkadiroglu et al. (2005), Abdulkadiroglu et al. (2006) and Ergin and Sonmez (2006). For a study of the mechanism used in the New York City see Abdulkadiroglu, Pathak, and Roth (2009).

[22] In the Boston mechanism, applicants are assigned to public schools by centrally controlled procedure. Each applicant submits own school *preference* (ranking) list. Then, students are ordered by *priority rules* defined by several criteria. Assignments are done in a series of rounds. In the first round, only schools on the top of preference lists are considered and matched according to priority rules. Assignments to particular school are terminated when its capacity is filled. In the second round, second-top choices of still unassigned applicants are considered. This process is repeated until all applicants are matched. In the Boston mechanism, skills of applicants and economic and social family background translate into matching only indirectly, through residential mobility. See seminal theoretical work on this issue by Tiebout (1956). Further theoretical developments can be found e.g. in Epple and Romano (1998), Benabou (1996) and empirical examinations by Rosenthal (2003) and Gibbons and Machin (2006)

[23] The origin of these flaws in the Boston mechanism can be presented by an intuitive example. Assume that applicants do not misrepresent their school preferences. Consider applicant i who had put her most preferred school on the top of her preference list and was not admitted to this school in the 1st round due to capacity limit. If all slots in her 2nd most preferred school j were filled by applicants already in the 1st round (by applicants listing school j on top of their lists), it could be the case that some applicants admitted to school j have lower priority in this school than applicant i not admitted to this school (justified envy). The flaw of the Boston mechanism is that assignments realized in early rounds are not temporary but terminal. Incidence of justified envy creates incentives for applicants to misrepresent their school preferences, placing less preferred schools higher on their preference lists.

[24] There exists another matching in which the utilities of all students are weakly higher and the utility of at least one student is strictly higher.

[25] The optimal strategy of the students is not to reveal their true preferences. If students were, somehow, made to reveal their true preferences, the Boston mechanism would be Pareto efficient.

to match medical interns to hospitals in the US.[26] Using detailed records on actual students' choices and assignments, Abdulkadiroglu et al. (2006) provide empirical evidence of the incidence of sophisticated strategic behavior among parents under the old Boston mechanism. They convincingly show that the new mechanism substantially simplified the strategic choices of parents and improved matching outcomes.

The common flaw is that these mechanisms create incentives such that strategic decisions are superior to the revelation of true school preferences. Rare non-US empirical case is described by Balinsky and Sönmez (1999) studying centralized college admission based on standardized testing across Turkey. Identifying serious deficiencies of this mechanism, they propose an alternative mechanism with superior properties: strategy proofness and fairness.

The theoretical literature offers another CABl strategy-proof mechanism called *top trading cycles mechanism (TTC)*. It assigns applicants in the order of their priorities but allows them to trade the schools for which they have highest preferences if mutually beneficial (Pareto improving) trade is possible.[27] Contrary to the GS mechanism, the TTC mechanism is Pareto efficient but does not eliminate justified envy. Which of these two mechanisms is more appealing depends on whether policy makers prefer Pareto efficiency or the absence of justified envy.

If priority orderings of students are the same across schools or do not exist, the TTC mechanism reduces to a *serial dictatorship mechanism (SD)*. In the SD mechanism, students are ranked in random order (*random serial dictatorship*) or in the order of test-scores (*deterministic serial dictatorship*[28]) and choose their favorite school among schools with remaining slots. The SD mechanism produces Pareto efficient matching with respect to the order in which applicants choose a school. When priorities are given by test scores, the deterministic SD mechanism also eliminates justify envy but random SD does not.[29]

[26] This mechanisms also belongs to the family of centralized ability blind mechanisms - CABl

[27] The TTC mechanism was employed to assign over 400 students in the after-market in New York City's High School match. See Abdulkadiroglu and Sönmez (2003) for a detailed outline of the TTC mechanism designed for school-choice purpose. Pais and Pintér (2008) provide experimental results in which the TTC mechanism slightly outperforms the GS mechanism in terms of both efficiency and percentage of truthful revelations.

[28] Serial dictatorship mechanism is currently used to sort pupils into high schools in Turkey. Students are assigned to available slots in their most preferred schools in the order given by test scores (nation-wide, standardized).

[29] Random lotteries are a component of numerous mechanisms in the US. Motivated by the New York City High School supplementary matching where large fraction of high schools employ a lottery to order students instead of setting priorities, Pathak (2006) compares random serial dictatorship and TTC mechanism with random priority and shows that a random serial dictatorship is equivalent to top trading cycles with random priority.

Real implementation of admission schemes confronts peculiarities deviating more or less from their pure theoretical counterparts described above. A comprehensive typology of pupil-school matching mechanisms has not yet been developed, and there is no systematic survey of real mechanisms used.

1.2.3 Ability-based decentralized matching

The mechanism we investigate assigns almost one hundred thousand fifteen-year-old students to upper-secondary schools in the Czech Republic each year.[30] Pupils can freely apply to any school they wish and inter-district school choices are not restricted. Each school administers its own admission test and results determine the students' priorities within the school.[31] The fact that student priorities are determined by admission tests means that they are not known at the time of school choice decision, creating a risk for the students. Admissions proceed in a number of rounds and timing is coordinated by the central government. In the first round, the applicant is allowed to submit an application to just one school of her choice.[32]

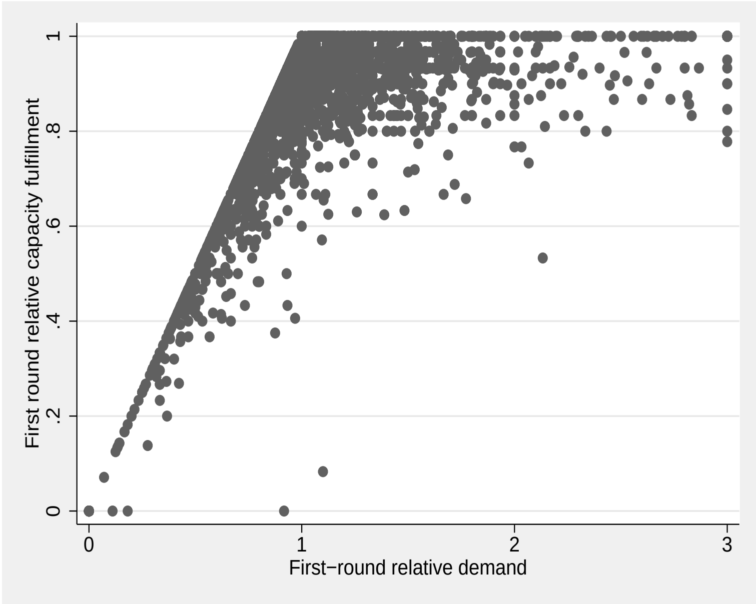
Unfilled slots remain at schools which experience a demand below their capacity in the first round. The schools which face excess demand typically fill most of their seats. One might expect that, given the properties of the mechanism, schools could behave strategically and keep certain amount of seats unfilled for the second round, in which they could win high-ability students who were rejected by their first-round schools. This, however, is not the case. The schools appear to be highly risk-averse and rather than running the risk of not being able to fill the capacity, they tend to accept everybody who exceeds some minimum threshold. This behavior of the schools is depicted in figure 1.1. The horizontal axis shows the ratio of first-round applicants to capacity; value above 1 means excess demand. The vertical axis then shows the ratio of students admitted in the first round to capacity. It is clear from the chart that those schools which have enough applicants typically fill most of their capacity in the first round. Thus, most seats

[30] It corresponds to the transition from junior high school to high school in the US system.

[31] In the following, we use term *priorities* for schools and *preferences* for applicants.

[32] The actual mechanism used in the Czech Republic has subsequently been amended and from the year 2009 students are allowed to submit 3 applications in the first round. However, strong restriction on the number of schools applicants can apply for is still very common. Slovakia has a very similar system. In Croatia applicants apply to one school but admissions are not based on test scores. In the US, restriction to three schools applies in Columbus Student Assignment mechanism. There, priorities for schools among applicants are determined by random lottery. The random element resembles the imperfect knowledge of applicants about their own study aptitude in the Czech mechanism. For details see Abdulkadiroglu and Sönmez (2003).

Figure 1.1: Students admitted in the first round vs. applicants in the first round



available in the second round are from schools, which faced lower demand in the first round.

The admission procedure is effectively finished in the second round. All applicants are served because the total capacity of schools exceeds the number of applicants.

The Czech mechanism belongs to the family of *decentralized-ability-based* (DABa) mechanisms. It differs from the thoroughly studied *centralized-ability-blind* (CABl) mechanisms, such as the well known Boston mechanism, in four ways. (1) In CABl mechanisms, pupils' intellectual qualifications do not play any direct role because they cannot be incorporated into priority rules.[33] In the Czech mechanism, on the contrary, applicants' qualifications are a key determinant of priorities of applicants within individual schools. (2) In CABl mechanisms, schools do not have the discretion to set their own priority rules; these are set externally by the schooling administration. In contrast to this, the Czech mechanism gives individual schools large autonomy to adopt their own priority rules. (3) In CABl mechanisms, assignments are processed centrally. In the Czech scheme, the matching procedure is decentralized to the level of individual schools and the central government coordinates only the timing of admission rounds. (4) In the Czech mechanism, admission test scores are not nation-wide standardized but school specific.[34]

[33] There, racial and ethnic quotas are imposed on priority rules. See Chen and Sonmez (2006) for a detailed analysis of controlled choice models.

[34] There are legal restrictions requiring equal treatment of applicants in the sense that an applicant

Features (3) and (4) of the Czech mechanism imply that applicants and their parents choose the school to apply to with imperfect and incomplete information. At the moment of submitting an application, applicants have only imperfect information about their true qualifications. Applicants perceive their own qualifications mainly through grades obtained at the preceding lower-secondary schooling level and through comparison to peers in a class. Comparability of grades as a qualification measure across schools is limited. Moreover, when submitting an application to a particular school, applicants have imperfect knowledge about the total number of applicants for that school and of their qualifications. Applicants form expectations based mainly on past years' evidence.[35]

Contrary to CABl mechanisms, DABa mechanisms have been given little attention in the literature. But the general perception is that the Czech system is highly selective on ability as well as social (and in some regions also ethnic) background of pupils.[36] At the same time, compared to the UK and to the US, social segregation is mitigated by the extraordinarily low residential mobility and notably lower socio-economic inequalities in the society. Residential migration motivated by school quality is rare. The authors' own experience and frequent commentaries in the media supply convincing anecdotal evidence that the Czech mechanism is not strategy proof. Opinions like the following appear frequently in the media when the annual application deadline approaches: *"The rule of one application in the first round is stressful especially for parents. The major problem is that parents are not sure which schools will still have open slots in the second round in the case their child would fail in the first round, ... "*[37]

Not being strategy proof, the Czech mechanism is neither Pareto efficient, nor does it

with better admission test scores has priority in admission. Most schools use written exams and some also add an oral examination or interviews.

[35]See Hastings and Weinstein (2008) on the role of available information in school choice.

[36]The Czech schooling system sorts fifteen-year old pupils to schools of three very different types. There are apprentice schools, vocational schools and gymnasia schools enrolling 40%, 40%, and 20% of the 15-year old age cohort, respectively. Apprentice schools offer a profession-specific curriculum with a large share being manual training. Vocational schools offer a curriculum for specific professions (technical, business administration, social services) but the component of manual training is minor. Vocational schools differ a lot by fields, quality, and the degree of oversubscription. Gymnasia schools offer a general curriculum (standard components of the curriculum at gymnasium are mathematics, foreign languages, IT, fields of natural sciences, and arts) which is rather homogeneous across schools. Most gymnasia schools have been oversubscribed for decades and are perceived as a preparatory stage for future studies at a college. Most schools are public, tuition-less, jointly financed by central and regional governments (non-public schools, private and church, enroll less than 10 percent of pupils). Information about individual schools is available at relatively low cost through a centrally managed internet-based information system. Information is provided on school facilities, admission requirements, characteristics of students enrolled, college admission rate, over-subscriptions in previous years, etc.

[37]MF Dnes daily newspaper, November 28, 2005 from the article "Strašák jedné přihlášky na SŠ zůstal i letos" [The nightmare of one application for upper-secondary school remains].

eliminate justified envy. Despite this, schooling ministry officials do not recognize the high incidence of preference misrepresentation and argue in favor of the existing system: *"Most children who had self-esteem and applied to schools they wanted to made a good choice. Nine out of ten children got admitted to the school they have applied to. According to the analysis of the ministry, admission chances of children are not lowered by the mechanism and the ministry is not preparing any reform."*[38]

We show that the Czech mechanism creates strong incentives for strategic misrepresentation of preferences which precludes Pareto efficiency and elimination of justified envy. We explore the scope of strategic behavior for a wide range of uncertainty in the admission process and heterogeneity among applicants. We also explore equity and efficiency implications of the admission scheme and compare matching outcomes to those hypothetically achieved by a *deferred-acceptance mechanism*, which is strategy-proof and eliminates justified envy.

1.3 Computational model

While the existing literature offers a good account of the properties of stylized matching mechanisms, these properties do not necessarily capture peculiar features of real mechanisms. The extraordinary complexity of spontaneous matching processes and involvement of a high number of independently acting agents makes it impossible to find analytical solutions when investigatingthe mechanism's properties and matching outcomes. There are two alternative methodological approaches: laboratory experiments and numerical simulations. Experiments are run in laboratory settings with real agents mimicking real situations.[39] While the advantage of experiments is that they provide evidence based on the decisions of real agents, it is difficult to simulate comprehensively real conditions in artificial laboratory settings. Moreover, the number of agents involved in experiments is very small and the marginal costs of experimental rounds are high.

The alternative approach we rely on are numerical simulations. To our knowledge, this is a novel approach to school-choice problems, not applied in the literature so far. Specifying the optimizing behavior of agents, the advantage of computer simulations is that they enable us to model complicated matching process and determine matching

[38] MF Dnes daily newspaper, September 22, 2005 from the article "Změny v přijímačkách nebudou" [Admission scheme will not change].

[39] For an example of experimental approach see Chen and Sonmez (2006) or Pais and Pintér (2008).

outcomes that would not be possible to obtain using analytical methods. Another advantage of simulations, especially compared to experiments, is that the number of agents and number of runs for various setups is constrained only by computational time. The usual shortcoming of simulations is that the optimizing behavior of agents as defined by a researcher does not necessarily comply with the behavior of real agents.

Nevertheless, the simulation approach allows us to analyze various setups of admission mechanisms and vary the parameters to reflect uncertainty and heterogeneity among applicants. This provides insight on the impact on (i) equity in terms of allocation of heterogeneous talents to heterogeneous schools, (ii) efficiency in terms of aggregate production of skills, and (iii) biases in generated demand/supply gap indices.

As outlined in section 1.2, the Czech mechanism is a prototype of DABa mechanisms. We build our computational model to resemble a real situation. Some simplifications are made to secure clarity of exposition but they do not affect key patterns in the results obtained. The model allows us to simulate strategic application decisions of a large number of heterogeneous pupils who have imperfect information about their own qualifications and incomplete information about the application behavior of other applicants.

1.3.1 Heterogeneous applicants and schools

We consider set of n applicants $I = \{i_1, i_2, ..., i_n\}$ described by a vector of *true qualifications*, $A = \{A_1, A_2, ..., A_n\}$ distributed $iid \sim N(100, \sigma)$. Qualification represents an applicant's general aptitude for study, i.e. her capacity to acquire further skills in a school. Ex-ante, before her application is submitted and qualification is tested, the applicant knows her qualification imperfectly, *perceived qualification*, $\tilde{A}_i$ defined as

$$\tilde{A}_i = A_i + \epsilon_i \tag{1.1}$$

where ϵ_i is perception error $iid \sim N(0, \sigma_\epsilon)$. The actual value of the error and true qualification is revealed ex-post through an admission exam.[40] Ex-ante, $E(\epsilon_i|A) = 0$. The perception error accounts for imperfect information[41] and contributes to the uncertainty in the admission process. This uncertainty is greater in systems without nation-wide stan-

[40]We make a simplifying assumption that even though the test is school specific, it reveals the true qualification of the student. A centralized test would be better in this respect, but at least the school-specific test compares the pupils from different elementary schools and is objective in this sense.

[41]The perception error also captures uncertainties in passing admission exams.

dardized skills testing such as the Czech one.[42] We assume that distribution parameters of A_i, its mean and variance σ, are known to applicants ex-ante, such that $\sigma_\epsilon \leq \sigma$.

There is set of schools $S = \{s_1, s_2, ..., s_m\}$ characterized by quality, $Q = \{Q_1, Q_2, ..., Q_m\}$, and capacity, $C = \{C_1, C_2, ..., C_m\}$, vectors with non-negative elements $Q_s > 0$, $C_s > 0$ $\forall s$. We assume that school capacities are equal or greater than the number of applicants

$$\sum_{j=1}^{m} C_j \geq n \tag{1.2}$$

School quality represents the production capacity of educational technology employed by the school to upgrade a student's true qualification into productive skills, Π, i.e. labor market rental price of skill productivity. In the following we call Π *individual productivity*. Defined in this way, Π also captures pupil-school match specific features like heterogeneous skills, school curricula, school commuting preferences of applicants, heterogeneous skills demanded by local labor markets etc. What we abstract from in the explicit modelling, however, are the effects of peer quality.[43] Although incorporating peer effects into the model might be of interest, this would create a much more complicated strategic game with multiple equilibria, for the analysis of which our simulations approach would not be applicable.[44]

Following the standard approach in the literature, we assume that applicants making their application decisions maximize the expected value of Π.[45] Applicants form their true school preferences according to individual productivities obtained from individual schools defined in additive form[46] as

$$\Pi_{ij} \equiv Q_j A_i + \omega_{ij} = Q_j(\tilde{A}_i - \epsilon_i) + \omega_{ij} \tag{1.3}$$

[42] This is the case of the Czech Republic not having nation-wide standardized examinations.

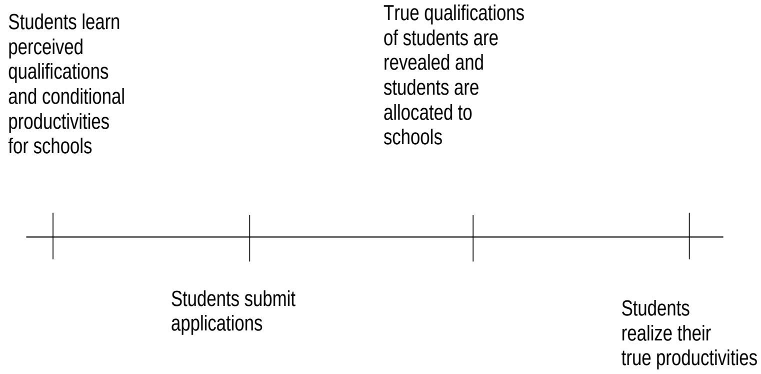
[43] For discussion of peer effects on students' performance is see, for example, Markman et al. (2003).

[44] In a stable environment, accounting for peer effects would most likely not change the results substantially. Accounting for peer effects might be especially useful when there are some external shocks to the system. In this case, a critical mass of students might change their school preferences, which could lead to a different equilibrium.

[45] A more general specification would consider applicants as productivity maximizers and present value of the stream of future earnings would enter as an argument. Our approach is advantageous in the sense that productivities have monetary expression and can be summed into aggregate productivity measures.

[46] An additive specification is commonly used in the literature. For example, Chen and Sonmez (2006) use additive utility function composed from three components accounting for school quality, proximity, and school random factor. Our specification does not consider school quality as a separable factor but a factor entering through interaction with study aptitude.

Figure 1.2: Timing of the application process



where ω_{ij} represents *school preference heterogeneity* component.[47] Specification 1.3 determines the truly preferred ranking of all schools by each pupil. Note that in the absence of school preference heterogeneity, applicants would rank schools merely according to school qualities and rankings of all pupils would be the same, depending only on school quality Q. The presence of school preference heterogeneity allows for equally qualified pupils having different true school preference lists.[48] We assume that ω_{ij} is $iid \sim uniform[-int^u, int^u]$[49] with actual values known by applicants ex-ante. Note that school preference heterogeneity is more important determinant of true school preferences among applicants with lower perceived qualification.

The ex-ante expected productivity to be obtained at school j by student i is

$$E(\Pi_{ij}|\tilde{A}_i) = Q_j\tilde{A}_i + \omega_{ij} \tag{1.4}$$

Pupils make application decission based on a complete set of m values $\{E(\Pi_{ij}), j = 1, .., m\}$ and corresponding admission probabilities. The timing of the application process is depicted in figure 1.2.

[47] School quality also accounts for rigor of the school curriculum, determining education value added in interaction with student's qualification. Defined in this way, equally qualified students enrolled by two schools of different quality will obtain different value added. On the other hand, low-qualified pupil in lower quality school can achieve value added higher than identically qualified pupil in high quality school. This specification allows for optimizing matching between pupils' qualifications and schools' qualities.

[48] Note that in a $\Pi - A$ space, ω and Q represent intercept and slope of the $\Pi - A$ locus. Let us consider an individual choosing from two schools 1 and 2 with heterogeneous school preferences $\omega_2 < \omega_1$, and schools such that $Q_1 < Q_2$ (school 2 has higher quality). Under these conditions, pupil's preferred school depends on her perceived qualification $\tilde{A}$. For low values of $\tilde{A}$, she will prefer school 1 over 2, but for sufficiently high $\tilde{A}$ she will prefer school 2 over 1. In other words, the gain of low qualified pupil from attending high quality school 2 is not sufficient to compensate her for relatively low school preference heterogeneity ω_2.

[49] Note that iid assumption implies $\omega \perp A$.

1.3.2 School choice

In the Czech system, an applicant submits an application to just one school in the first round.[50] Applicants not matched in the first round are matched in the second round to schools with remaining open slots. These slots are due to short demand in the first round. Let $\Psi = \{\psi_1, \psi_2, \dots\dots, \psi_n\}$ describe a *strategy profile* where $\psi_i = j$ means that student i applies to school j. The actual admission outcome is determined by the true qualifications of applicants which are revealed by an admission exam.[51] Given school's j capacity C_j and access demand $N_j > C_j$, the ex-ante admission probability of applicant i at school j is given by the probability that her true qualification A_i exceeds the true qualification of a marginally admitted applicant – the pupil with the lowest qualification (i.e. $\Pr(A_i > A_{C_j})$ where A_{C_j} is the qualification of a marginally admitted applicant to school j). Note that in case of under-subscription, $N_j < C_j$, all applicants to school j are admitted with certainty. Applicants determine probabilities of admission to individual schools conditional on particular strategy profile Ψ. Conditional admission probabilities can be expressed as

$$p_{ij}(\Psi) = \Pr[A_i > A_{C_j}(\Psi)] = 1 - \Phi[\frac{A_{C_j}(\Psi) - \tilde{A}_i}{\sigma_\epsilon}] \tag{1.5}$$

where $\Phi(.)$ is standard normal c.d.f. Note that admission probability depends not only on a student's perceived qualification $\tilde{A}_i$, but also on the application decisions of other pupils accounted for by Ψ. Assuming risk neutrality, ex-ante productivity expected from school j by applicant i, given profile Ψ, is

$$\pi_{ij} = p_{ij}(\Psi)E(\Pi_{ij}) + [1 - p_{ij}(\Psi)]E(\Pi_{im}) \tag{1.6}$$

The additive term on the right represents fall-back productivity obtained in case of rejection. If rejected, the applicant obtains productivity $E(\Pi_{im})$ in the least demanded school.

Summing up, applicants apply to schools offering them the highest ex-ante expected productivity so that $\psi_i = \arg\max_j(\pi_{ij})$. This in turn, according to (1.6), depends on (i) applicants' ex-ante productivities $E(\Pi_{ij})$ and arguments of Π_{ij}, (ii) the probability of

[50] The legal limit of just one application only corresponds to the case of infinite costs of a 2nd application. Chade, Lewis, and Smith (2006) study a school admission model incorporating positive but finite marginal application costs.

[51] We assume that schools seek to fill their capacity with the very best students.

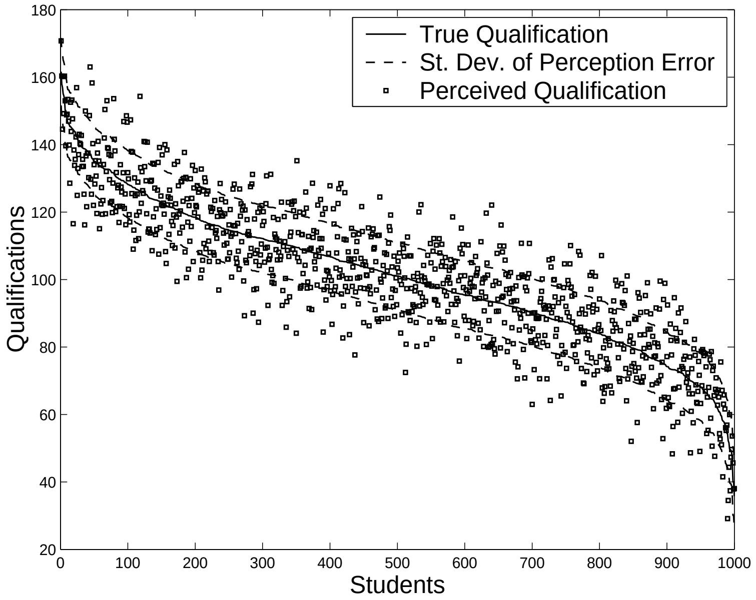
passing the admission line $p_{ij}(\Psi)$, and (iii) ex-ante fall-back productivity obtained in the least demanded school, $E(\Pi_{im})$.

In line with reality, we assume that priority rules give higher priority to applicants with higher qualifications. This implies homogeneous priorities across schools. To find the resulting matching of this school choice game, we are looking for a *Nash equilibrium strategy profile* (NESP). The NESP is a strategy profile, Ψ^N, such that ex-ante all applicants prefer their current choices given the choices of all others so that $\psi_i \in \Psi^N$. Even if an analytical solution to NESP exists, finding it would be extraordinarily cumbersome due to the high number of heterogeneous applicants. Instead, we find the NESP by an iterative computational approach. This enables us to explore the sensitivity of matching outcomes to various parameterizations of the model.

Let us conclude this section with a note on two simplifying assumptions we have made to facilitate the modelling and to focus on the key issues. The first concerns the automatic fall-back option for students rejected in the first round, which we have assumed to be the school of type 4. We believe this is a plausible simplification of reality, since the good schools typically fill most of their seats in the first round. As shown in table **??**, in the school year 2004 / 2005, the percentage of students admitted in the first round was 90% for Gymnasia, 74% for Vocational schools and 39% for Apprentice schools. Since the Vocational schools category includes both good-quality and low-quality schools, the average may be misleading. It is therefore interesting to note that for 56% of these schools, the percentage of students admitted in the first round was 90% or above.

The other simplifying assumption is that of risk-neutral students. Introducing a common level of risk aversion for all the students would not change the overall picture. It would only increase the the mismatch between the true and the revealed demand and would, thus, make our case even stronger. Our analysis, therefore, under-estimates the impact of the Czech mechanism on students' behavior. An interesting question is how different levels of risk aversion among students would affect their behavior and the outcomes of the admission procedure. Although we do not include risk aversion in our model, we can get an insight regarding the effect of risk aversion by looking at the perceived vs. true qualification. Low risk aversion would make a student bid more aggressively for his most preferred school; high perceived relative to true qualification has the same effect (see figure 1.12 in section 1.4).

Figure 1.3: Students' true and perceived qualifications



1.3.3 Model description

Model parameters

In this section, we describe our computational model simulating the school-choice behavior of applicants. We generate a population of applicants who are characterized by their true and perceived qualifications, and school preference heterogeneity distributed orthogonally so that $A_i \perp \epsilon_i \perp \omega_{ij}$. Figure 1.3 on page 22 provides an example of such a population. We consider the supply side represented by four schools $j = 1, \ldots ..., 4$, each characterized by quality-capacity pair (Q_j, C_j). The relative school capacities approximately correspond to the actual overall capacities in the Czech Republic, with gymnasia (typically the best schools) accounting for approx. 15% of the capacity, vocational schools for 50% of the capacity and the apprentice schools for the rest. The school qualities are chosen without a corresponding empirical benchmark in such a way that the worst school does not add anything to the starting qualification of a student, while the best school doubles it. The other schools are in between these two values.[52] See table 1.1 for details.

To summarize, we generate:

- Population of $n = 1000$ applicants,

[52]The deliberate choice of school qualities can be considered a weakness, but the results would not change fundamentally if we assume different ratios; only the degree of misrepresentation (and other phenomena studied) would be different.

Table 1.1: Model structure of the secondary education market

School Type	Capacity	Quality
Gymnasium	150	2
Vocational A	250	1.6
Vocational B	250	1.3
Apprentice	350	1

- Each of the applicants has true qualification A_i, perception error ϵ_i, and school preference heterogeneity components ω_{ij}. In the basic setup of the model we assume ability A_i is normally distributed with mean 100 and standard deviation 20; perception error ϵ_i is normally distributed with mean 0 and standard deviation 20; and preference heterogeneity component ω_{ij} is normally distributed with mean 0 and standard deviation 50.

- Population of $j = 1, \ldots ..., 4$ schools,

- with capacities $C_j > 0$ so that $\sum_{j=1}^{m} C_j = n$ and qualities Q_j such that $Q_1 > Q_2 > Q_3 > Q_4 > 0$.

Equilibrium finding algorithm

Our algorithm finding the NESP, as defined in section 1.3.2, proceeds as follows:

- We choose initial strategy profile (SP) such that all applicants apply to the lowest quality school ($j = 4$).[53]

- For a given SP, we find the best response of the first applicant – school choice that yields her the highest expected ex-ante productivity in (1.6) given the choices of others – and we modify the corresponding element of the SP.

- We repeat the previous steps for all applicants.

- We repeat the previous two steps until no applicant modifies her school choice. The resulting SP fulfills the properties of the NESP.

[53]The choice of a particular initial strategy profile or of the order that the students make choices has no effect on the NESP found.

1.4 Results

In this section, we investigate the properties of the Czech pupil-school matching mechanism described in section 1.3.2 using the computational model described in section 1.3.3. We review key quantitative patterns of resulting matchings and compare them to those generated by the GS mechanism. We select the GS mechanism as the benchmark because of its desirable properties and because of its relative simplicity and applicability in practice. It should be noted that if priorities are homogeneous across schools – as is the case of the Czech system – matchings generated by the strategy-proof TTC and GS mechanisms are the same.

First, we explore the incidence of strategic preference misrepresentation, comparing matchings obtained by the Czech mechanism and the GS mechanism. Second, we explore distributional outcomes and efficiency and equity trade-offs. Examination of the effect of variance in the school preference heterogeneity component are relegated to the Appendix. Implications of our findings, including policy ones, and possible shortcomings of our approach are debated in section 1.5.

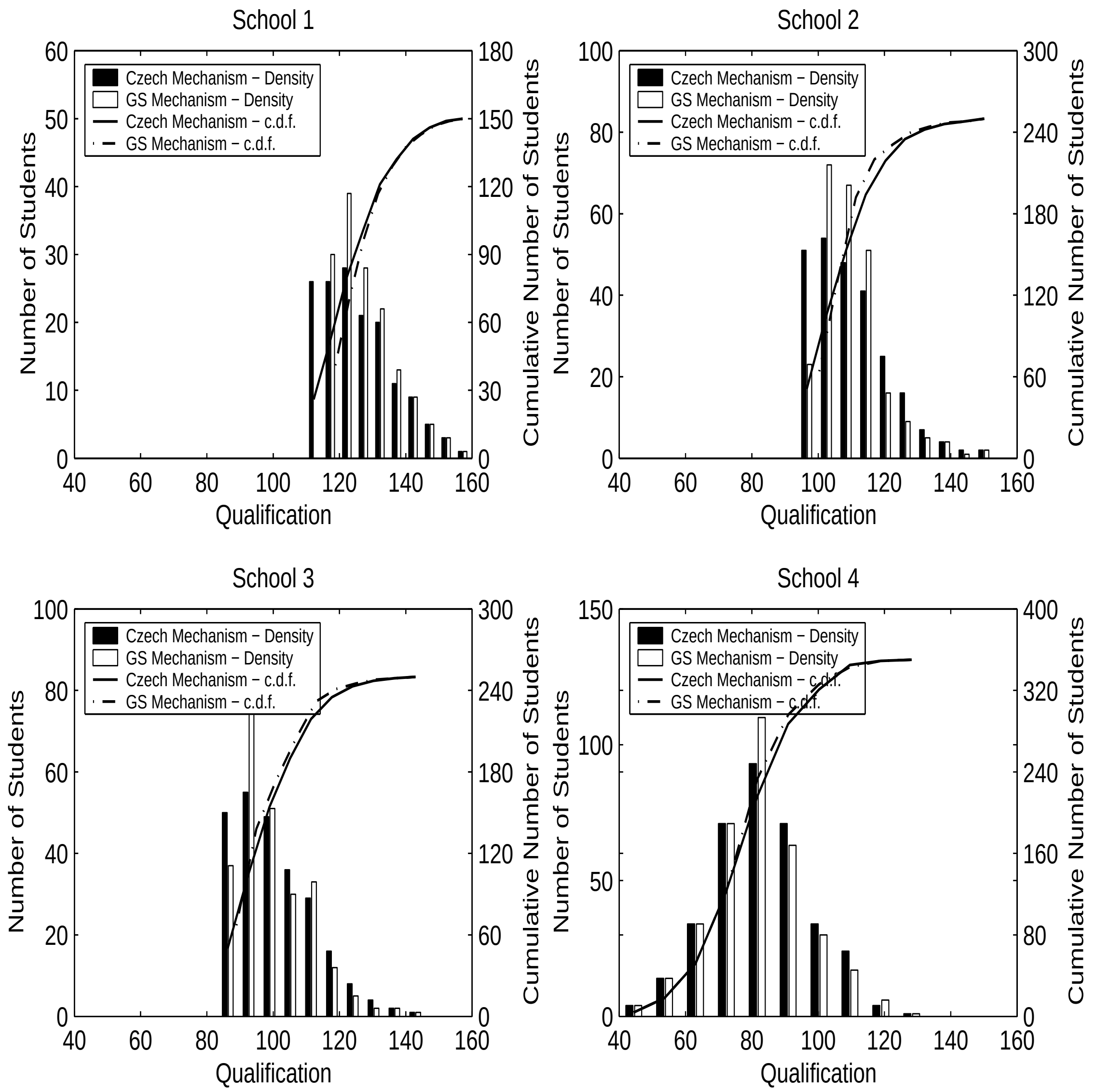
1.4.1 Sorting of applicants to schools

The analysis is performed for plausible values of $\sigma = 20$, $\sigma_\epsilon = 20$, and $\sigma_\omega = 50$.[54] Compared are matchings produced by the Czech mechanism and the GS mechanism.[55] Resulting distributions of students by their true qualifications within our four schools are depicted in the four panels of Figure 1.4. For each decile of true qualifications, bar pairs compare the number of students enrolled under two mechanisms. The Czech mechanism obviously leads to higher within-school heterogeneity of true qualifications. As can be seen in the top-left panel, in the Czech mechanism the best school ($j = 1$) admits a group of less qualified students who, under the GS mechanism, would be sorted to school $j = 2$. Correspondingly, the Czech mechanism sorts some relatively highly qualified pupils - those who would be sorted to school $j = 1$ by the GS mechanism - to lower quality school $j = 2$. This pattern of upward promotion of less qualified applicants and downgrading of more

[54]The particular choice of standard deviation values is not critical for the conclusions, which remain valid for any plausible values as we show later in the sensitivity analysis.

[55]When calculating the matching under the GS mechanism, we assume students make their choices based on true qualifications, e.g. after taking a common test. This is quite realistic, given that the GS mechanism needs to be centrally administered, so a common test would be a natural part of the process. However, assuming choices based on perceived qualifications would only change the results little, by changing preference rankings of some students.

Figure 1.4: Qualification of students by schools



qualified applicants appears also on the margin of schools $j = 2$ and $j = 3$. Admission outcomes of both mechanisms differ only slightly in school $j = 4$ (lowest quality school). Clearly, *promotion* and *downgrading* effects in the Czech mechanism contribute to higher within-school variance of true qualifications.

1.4.2 Misrepresentation of preferences

Strategic behavior of applicants leads to misrepresentation of school preferences. Applicants intentionally misrepresent their preferences if they apply to schools that are not at the top of their true preference list according to their perceived qualification and school preference heterogeneity component. Preference misrepresentation has important equity and efficiency implications, which we outlined in section 1.2 and discuss further in section 1.5.

The degree of preference misrepresentation in the NESP) of our model is documented in figures 1.5 and 1.6. Figure 1.5 shows the incidence of strategic choices comparing true and revealed preferences.[56] On the horizontal axis, we sort applicants into decile groups in ascending order according to their perceived qualifications. For each decile pairs of bars are presented: the left bar shows the average quality of schools applicants *prefer the most*, while the right bar shows the average quality of schools applicants in given decile *actually apply to*. Different heights of neighboring bars indicate the degree of preference misrepresentation among applicants.[57] Misrepresentation is negligible in the top decile and is higher among less qualified pupils. This is because applicants with lower perceived qualification face a higher probability of rejection at their most preferred school and are more cautious about applying to oversubscribed schools. Note that a rejection implies that the applicant ends-up in a fall-back school ($j = 4$).[58] The perceived risk of this outcome leads some applicants to misrepresent preferences and bid for other than their first best option. This secures them a high probability of admission to their second best school.

Figure 1.6 shows supply-demand gaps comparing revealed and true demand to capacities of individual schools. At the highest quality school ($j = 1$), both the revealed and true demand exceed the school capacity, but the true demand by significantly more so (the revealed by 60%, the true by more than 300%). In other words, while 63% of all applicants prefer the *highest* quality school ($j = 1$), only 24% actually *apply to* this school. At both schools of medium quality ($j = 2, 3$) revealed demands exceed school capacities which exceed true demands. The lowest quality school, ($j = 4$), faces both true and revealed demand notably below its capacity.[59]

[56] In our terminology, revealed preferences are those revealed in the application process by actual applications submitted. True preferences are those perceived by applicants based on simple comparison of $\Pi_j's$. neglecting admission probabilities.

[57] Recall that although there is a tendency to prefer the higher-quality schools, the individual preference orderings may differ due to ω_{ij}.

[58] This is because, in our model, all the other schools face excess demand in the first round. In reality, there will be more schools available after the first round, but not the high quality schools. Therefore, we believe this assumption is a plausible simplification of reality.

[59] The positive number of applications to the lowest quality school is due to positive school preference component ω in the case of some applicants - where it is high enough to compensate for low school quality Q. Also note that the revealed and true demands for this school are equal. This is because the school ranks at the top of the applicants' true preference list and admission to this school is certain.

Figure 1.5: Students' preferences over schools

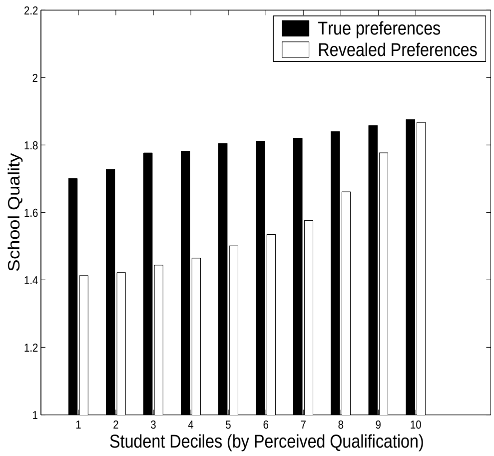
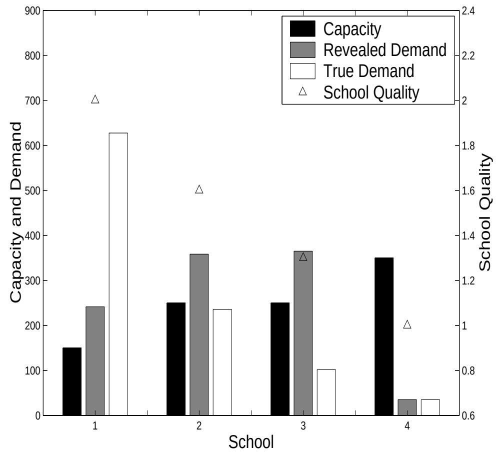


Figure 1.6: School demand



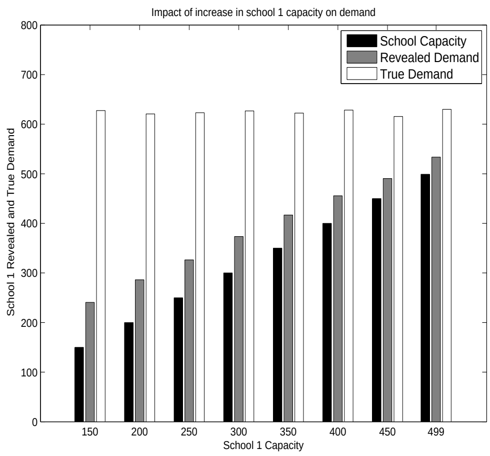
Discrepancies between true and revealed demand confirm a large degree of preference misrepresentation among applicants in the Czech mechanism. The incidence of strategic behavior of applicants leads to an illusory balance between supply and demand at the level of individual schools.[60]

An interesting question in this context is what happens if the school capacities change, in particular when the capacity of the most demanded school increases, while the mechanism to assign students to schools remains the same. We have examined a scenario in which the capacity of school 1 (the top quality school) gradually increases from 150 to 499, while the capacity of school 4 (the lowest quality school) decreases correspondingly.[61] The results are shown in figure 1.7. True demand (white bars) is stable, fluctuating only slightly due to random effects. Revealed demand (grey bars) increases along with capacity (black bars). This is intuitive: the higher capacity of school 1 encourages students, who prefer it most but who did not apply before due to the low probability of admission, to apply under the new circumstances. The revealed demand, however, increases at a lower rate than capacity, resulting in a decreasing relative (and also absolute) gap between those two. This is also intuitive, since the utility of the additional applicants is

[60] Misrepresentation of preferences has two components. The first is due to the strategic behavior of applicants, the second is due to the imperfect information of applicants about their own qualifications. To distinguish their relative role, we generated matching by the Czech mechanism but not allowing for strategic misrepresentation of preferences by applicants. As a result, applications for other than most preferred schools are due to imperfect information about their own qualification by individual applicants. In plausible range of σ_ϵ, the extent of non-strategic misrepresentation is small and results are available upon request.

[61] The total capacity of these two schools is 500, which we always maintain (we don't change the capacity of schools 2 and 3). For technical reasons, we were not able to reduce the capacity of school 4 to 0, but had to stop at 1, which is also why the maximum capacity of school 1 is 499.

Figure 1.7: Impact of increase in school 1 capacity



lower than the utility of those who already applied under lower capacity. For them to apply, the probability of admission has to increase, which means the relative gap between revealed demand and capacity has to decrease.

1.4.3 Perception error

As introduced in section 1.3.1, perception error ϵ_i reflects the imperfect information of the applicant about her own qualification. We replicated simulations for different variance of perception error measured by *perception error ratio*, $r = \sigma_\epsilon/\sigma$. The case of $r = 0$ corresponds to perfect certainty about qualification so that $\epsilon_i = 0$ (implying $\tilde{A}_i = A_i$), corresponding to the availability of nation-wide standardized testing of qualifications. In this case, misrepresentation of preferences is caused purely by strategizing behavior on the side of applicants and imperfect information about one's own qualifications plays no role. Growing r implies more uncertainty among applicants about their own qualifications. The case of $r = 1$ corresponds to very high variance in perception error compared with the variance of qualifications in the whole population, ($\sigma_\epsilon = \sigma$). We present results for r in the range of plausible values $[0, 1]$.

Figure 1.8 shows the impact of perception error ratio on the aggregate incidence of preference misrepresentation. In the left panel, the upper profile depicts the proportion of all applicants who are admitted in the 1st round. In the case of perfect certainty ($r = 0$), all applicants predict admission outcome perfectly, fully eliminate the risk of rejection and apply in such a way that all of them are admitted in the 1st round. Growing uncertainty decreases the proportion of applicants being admitted to about 70% for higher values of r. The bottom profile shows the share of applicants who apply to their truly most preferred school. This proportion is increasing with r since growing perception error causes some applicants to bid for their most preferred school where they get rejected afterwards. This incentive to overbid is the strongest for low-qualification students whose opportunity costs of rejection are lower. Secured admission at the lowest-quality school and non-zero probability of admission at a higher quality school creates incentives for some low-qualification applicants to consider a risky strategy.

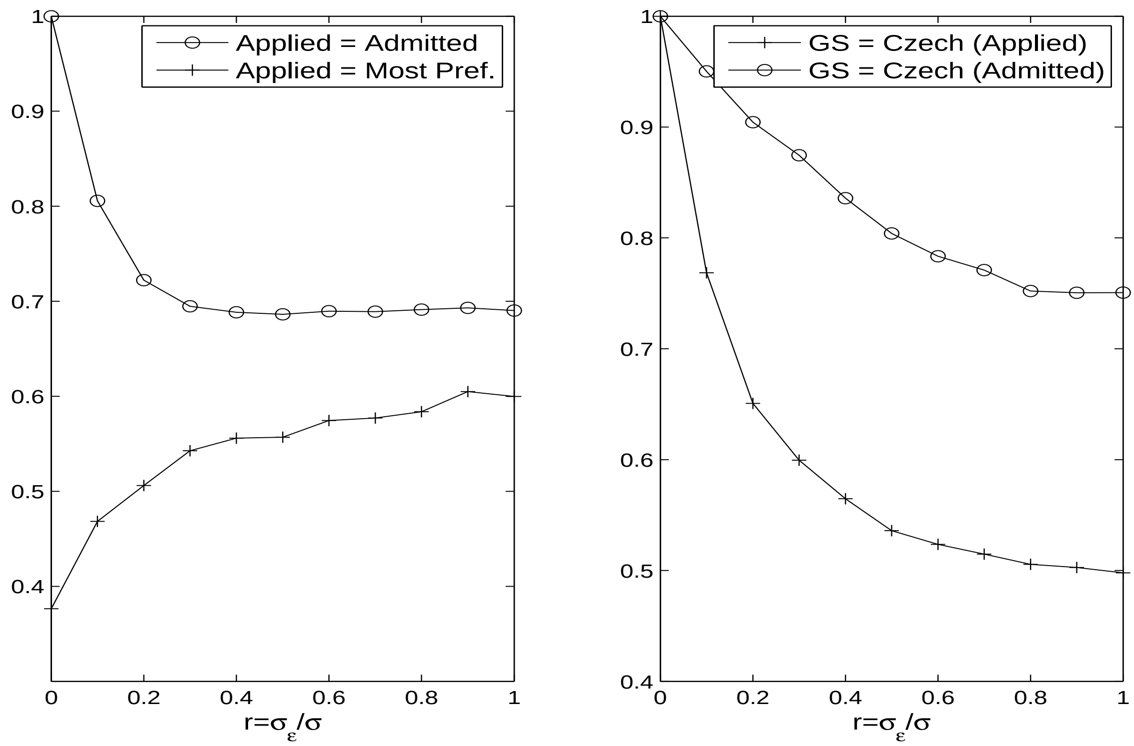
The right panel of figure 1.8 compares application and admission outcomes under the Czech and the GS mechanisms. While under the Czech mechanism, students are forced to strategize, i.e. "lie" about their true preferences, they have no incentive to do so under the GS mechanism – the best strategy here is to truly state the preferences. We also assume that under the GS mechanism, students know their true qualifications (or, equivalently, that there is some sort of nation-wide testing), whereas under the Czech mechanism, students only know their perceived qualifications, which are subject to perception error.

The upper profile in the right panel of figure 1.8 depicts the proportion of applicants admitted to the same school in both mechanisms. This proportion is declining from 100% for $r = 0$ to about 75% for $r = 1$. Therefore, when students perfectly know their qualifications, the Czech mechanism produces the same result as the GS mechanism. This can be understood in the following way: The GS mechanism guarantees each student admission to the best school available for him (eliminates justified envy). When, under the Czech mechanism, students perfectly know their qualifications and the distribution of both qualification and school heterogeneity component in the population, they are able to determine the best available school themselves and apply to this school directly.[62] Of course, the assumption of perfect knowledge of the true qualification and of the two population distributions will not be met in reality.

The bottom profile of figure 1.8 shows the proportion of applicants who, under the Czech mechanism, apply to the same school they would be assigned to by the GS mechanism. In other words, it shows the percentage of students who correctly understand which is their best available school. This ratio also starts at 100% in the case of perfect certainty but declines steeply and for $r = 1$ only every second applicant applies to the school he would be assigned by the GS mechanism. Note that in the Czech mechanism under perfect certainty, the proportion of pupils applying to the school they prefer most

[62] This is in line with Ergin and Sönmez (2005) who show in their Theorem 1, that the set of Nash equilibria in this game corresponds to the set of stable matchings under the given preferences.

Figure 1.8: Variance of qualification perception error and revealed/true school demand



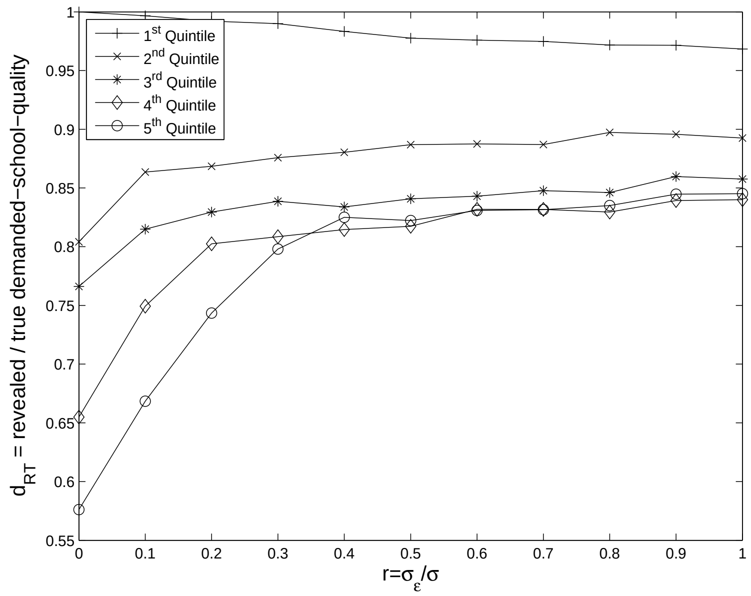
is very small (the left panel of figure 1.8). This is because applicants with qualifications below the threshold given by the capacity of their most preferred school know ex-ante that they would not be admitted. The discrepancy between revealed and true preferences is the highest in the case of perfect certainty. Perfect certainty ($r = 0$) means higher misrepresentation of preferences, but also better elimination of justified envy.

The impact of perception error on preference misrepresentation for applicants with different qualifications is shown in figure 1.9. We plot the demand for school quality, depicted as revealed over true school quality ratio, d_{RT},[63] against r for each quintile of true qualifications.

Except in one singular case, the ratio d_{RT} is different from one in the whole range of r for all qualification quintiles. This confirms that the Czech mechanism always suffers with misrepresentation of preferences. Values of d_{RT} being smaller than one indicate that in the Czech mechanism applicants on average apply to a school of lower quality than they truly prefer. The case of $r = 0$ represents the case of full certainty about one's own qualification. Here, discrepancies between true and revealed demand for quality are caused purely by the

[63] In particular, d_{RT} is defined as a ratio of the average quality of schools applicants apply for and the average quality of schools truly preferred.

Figure 1.9: The effect of qualification noise – revealed over true preferences



strategic behavior of applicants. In this case, the overall misrepresentation of preferences is the highest but varies across qualification groups. Applicants in lower qualification quintiles apply to schools which are on average of lower quality than applicants truly prefer. The highest discrepancy between true and revealed quality demand appears in the lowest qualification quintile. This is because applicants in this quintile perceive lowest probability of admission to their most preferred school given their low qualification determining admission priorities. In the top qualification quintile, all applicants apply to the highest quality school which is also at the top of their preference list.

Uncertainty ($r > 0$) increases the incidence of non-strategic misrepresentations among applicants but the direction of misrepresentation is not uniform across qualification groups. In the top qualification quintile, growing information imperfections decreases the revealed demand for quality. This is because some highly qualified applicants underestimate their true qualification and consider an application to their most preferred school too risky. At lower quintiles, imperfect information has the opposite effect, increasing the demand for quality. This is because some applicants overestimate their qualification and admission chances at the school they prefer more. For $r > 0.5$, the incidence of non-strategic misrepresentation remains almost unchanged.

Figure 1.10 depicts the impact of perception error on revealed demand at the level of individual schools. On the vertical axis, we plot the revealed demand over school capacity ratio. In the case of perfect certainty, revealed demand for the each individual school is equal to its capacity and all applicants are admitted in the 1st round. Growing uncertainty leads to swiftly dropping demand for lowest quality school ($j = 4$) below its capacity. This drop is due to the increasing number of low-qualification applicants with higher positive perception error. Some of these pupils apply to school $j = 3$ as they perceive non-zero probability of admission there and $\Pi_{i3} > \Pi_{im}$.[64] Their opportunity costs of bidding higher are low because they can always secure Π_{im}, productivity gain in lowest quality school ($j = 4$).[65] Dropping demand for school $j = 4$ necessarily causes growing excess revealed demand at other schools. The excess demand is growing with the perception error but at a diminishing rate. Note that true demand for the highest quality school ($j = 1$) is four times higher than its capacity, while revealed demand is higher only by 1.8 times. This reveals a strong discrepancy between observed and true demand from the best school.

Figure 1.11 depicts the impact of perception error on revealed/true demand ratio at the level of individual schools. In the case of perfect certainty, the revealed demand exceeds the true demand almost 10 times at the lowest quality school; the ratio then falls dramatically with growing uncertainty. The picture is more stable for the higher-quality schools, with the ratio fluctuating between 3 and 4 for most values of r in the case of the third-best school, rising from 1 to 1.6 with growing r for the second-best school, and staying far below 1 for all r values for the best school.

Figure 1.12 shows the impact of differing perception error (which, as discussed above, can also be interpreted as risk aversion) on the matching outcomes for individual students. Students have been divided into quintiles by their true qualification and within each quintile a relationship between (i) noise to ability (risk aversion) and (ii) the utility obtained from the school where the student was admitted has been examined. We find that differing levels of perception error (risk aversion) among students have no systematic effect on the utility from matching outcomes. In some cases, the aggressive strategy pays off and the students with higher perceived ability / lower risk aversion is admitted to the school of his choice, whereas in other cases, he is rejected and ends up in the lowest-quality school.

[64]Note that π_{im} is the expected productivity obtained from the least quality school.

[65]We assume that when faced with two strategies yielding the same expected productivity a student will choose the strategy with higher probability of being admitted. Thus, when a student has zero probability of being admitted to a school different from the fourth school he will apply to the fourth school directly even though he would end up there anyway.

Figure 1.10: The effect of perception error on excess demand

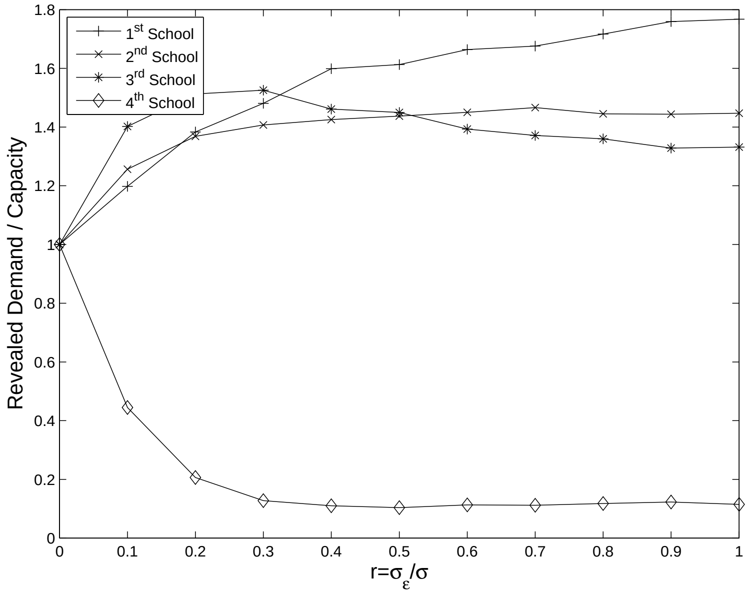


Figure 1.11: The effect of perception error on revealed/true demand ratio

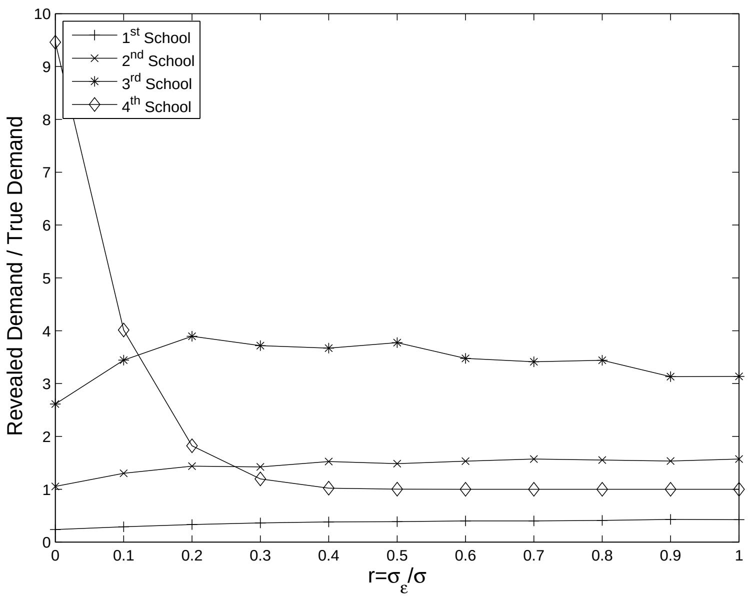
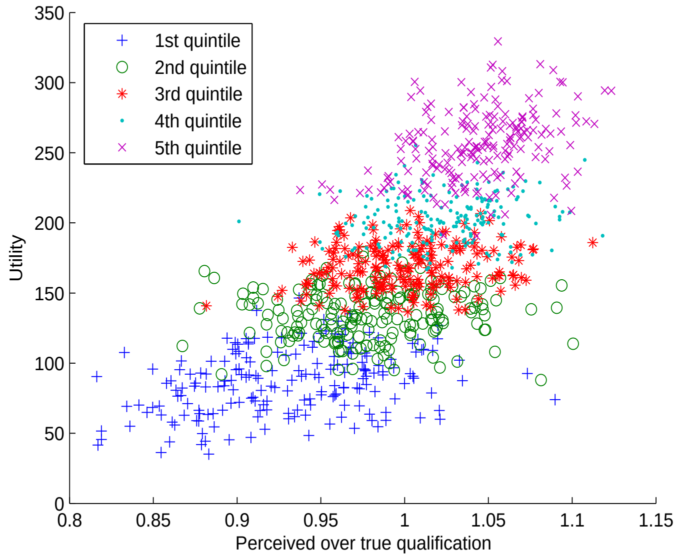


Figure 1.12: Effect of perception error on matching outcomes for individual students



1.4.4 Efficiency

In this section we investigate and compare the aggregate efficiency of matching outcomes under the Czech and the GS mechanism. As pointed out in section 1.2, the GS mechanism is strategy-proof,[66] it eliminates justified envy but is not Pareto-efficient. The Czech mechanism is not strategy-proof and therefore is not Pareto-efficient either. The notion of Pareto-efficiency defined in section 1.2 is not a useful concept for our purposes.[67] We need to assume a cardinal utility – or, in our particular case, productivity – allowing to sum up across all the students and compare the aggregate measure.[68] The measure we use is

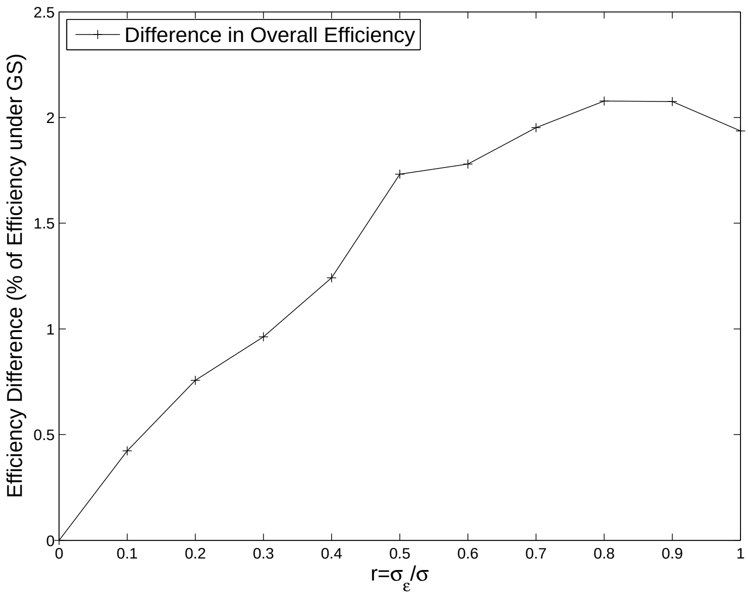
$$\Pi = \sum_{i=1}^{n} \Pi_i. \qquad (1.7)$$

[66]See Roth (1982).

[67]It does not enable us to compare two mechanisms, which both either are or are not Pareto efficient.

[68]The productivity Π can also be understood as a student's future earnings potential, a measure of welfare, and the efficiency comparisons can then also be interpreted as welfare comparisons. On the other hand, the link to future earnings might in practice be more complicated and non-linear. If, for example, the labor market has the nature of a tournament, then first place yields a much higher prize than the second one, although the difference in productivity may be small. Making this assumption would also change the strategic behavior of the students, making them more willing to take risky bets. A proper analysis of welfare impacts of the mechanisms should also take into account the disutility due to the justified envy of students who ended up in a bad school, although, under a different strategy, they could have been admitted to a better school.

Figure 1.13: Impact of perception error on efficiency

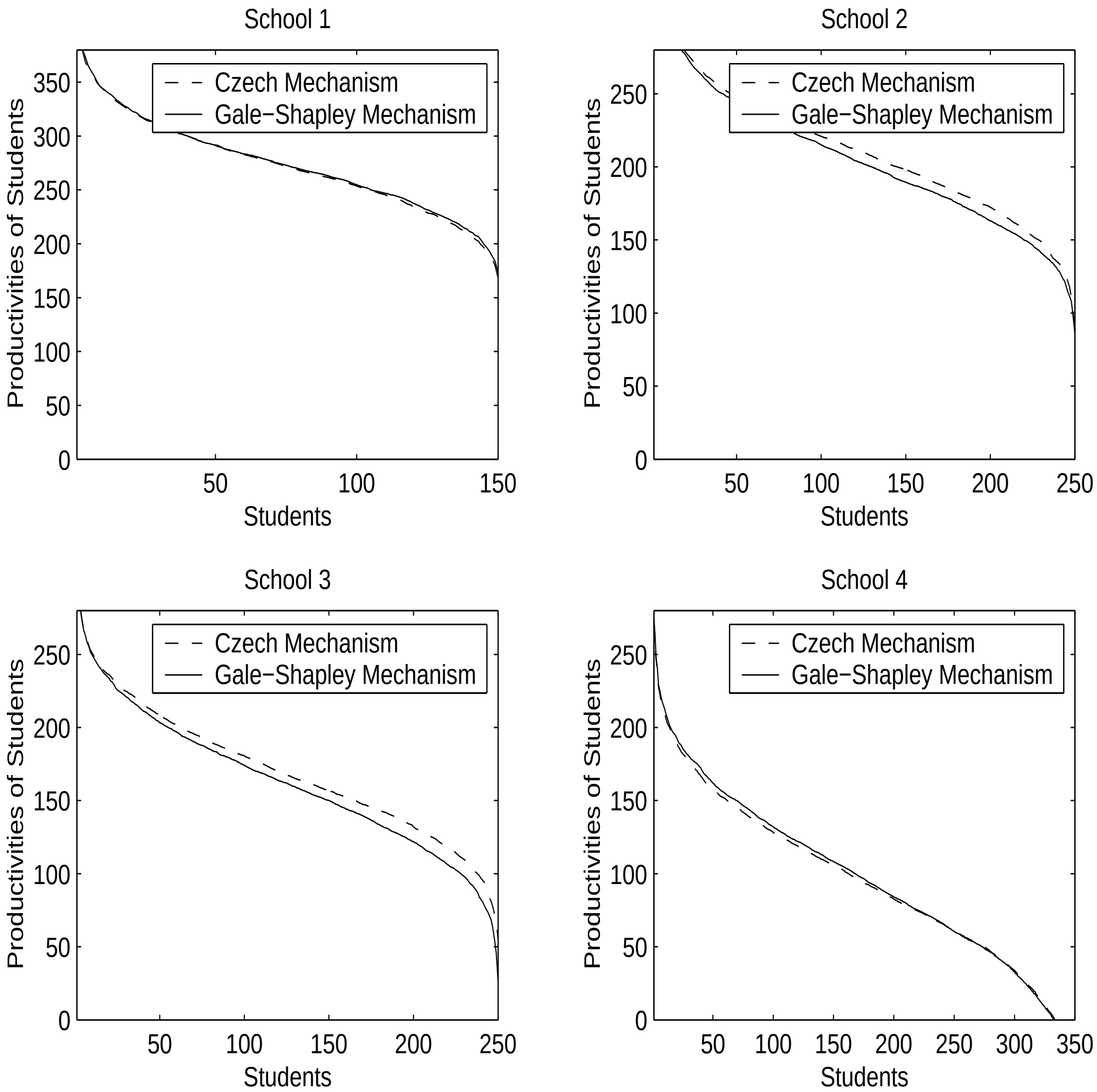


representing the arithmetic sum of individual ex-post realized productivities 1.6. Note that formula 1.7 contains actual realizations of productivities, Π_i and not ex-ante expectations π_i. Also note that 1.7 contains individual specific school heterogeneity components, which means that the productivities are not determined only as a product of a student's ability and the school quality, but also reflect the heterogeneous preferences of students for different types of schools.

Figure 1.13 shows the impact of perception error variance on the aggregate efficiency. On the vertical axis, we plot the percentage difference of (1.7) in the Czech and the GS mechanism. In the case of perfect certainty, both mechanisms exhibit the same aggregate productivity, which is in line with the above observation that in this case all the students apply to and are admitted to the schools they would be assigned to under the GS mechanism. Increasing perception error makes the Czech mechanism slightly more efficient than the GS one. Although the percentage difference in aggregate efficiency between the two mechanisms is minor, it points to an important feature.

The difference in efficiencies is driven by two effects having opposite impacts on the efficiency of the Czech mechanism. The first effect (*matching effect*) makes the GS mechanism more efficient because it leads to a higher degree of positive matching between true qualifications of pupils and school qualities. Better positive matching is due to the absence of strategic behavior in the GS mechanism. The second effect is due to the

Figure 1.14: Productivities of students by schools



heterogeneous preference component, ω_i which determines productivities (and therefore school preferences of individual applicants) but does not determine admission since this depends on qualification only. To explain, consider two applicants a and b, both preferring school s the most,[69] having identical perception error, somewhat different qualifications (pupil a is only slightly less qualified than pupil b), and substantially different school preference components such that

$$A_a - A_b < 0, \omega_{a,s} - \omega_{b,s} \gg 0, \text{and } \epsilon_a = \epsilon_b, \tag{1.8}$$

This implies that for sufficiently high difference in ω, student a would obtain notably higher productivity at school j than student b because

$$\begin{aligned} \Pi_{as} - \Pi_{bs} &= [Q_s A_a + \omega_{as}] - [Q_s A_b + \omega_{bs}] = \\ &= Q_s[A_a - A_b] + [\omega_{as} - \omega_{bs}] \approx [\omega_{as} - \omega_{bs}] \gg 0. \end{aligned} \tag{1.9}$$

The relationship follows due to $A_a - A_b$ being very close to zero. However, the GS mechanism would more likely enroll pupil b in school s than pupil a due to $A_b - A_a > 0$ irrespective of $\omega' s$. In the Czech mechanism, where pupils apply strategically, pupil a is more likely to apply to s than pupil b and is more likely to be admitted to school s. Therefore, higher productivity of pupil a in school s will more likely be realized in the Czech mechanism. This effect (*cardinality effect*) makes the Czech mechanism more efficient. Note that school preference heterogeneity is not revealed by tests, yet it is known by the applicant ex-ante. The Czech mechanism does not account for the school preference heterogeneity directly, but it does so indirectly through the applicant's strategic behavior.

Although the *matching* and *cardinality* effects have an opposing impact on aggregate efficiency and compensate each other, the latter one dominates in our plausible parameterizations of r, which makes the Czech mechanism perform slightly better according to the aggregate productivity measure.[70]

The distribution of the difference in individual productivities, broken down by individual schools, is depicted in figure 1.14. Students within each school are ordered by their

[69] I.e. $s = \arg\max_j(\Pi_{aj}) = \arg\max_j(\Pi_{bj})$.

[70] A potential modification to be explored in further research is to assume risk-averse agents and compare their ex-ante utilities obtained in the two mechanisms. Our hypothesis is that, for reasonable parameterizations, the GS mechanism would perform better.

productivity. In school $j = 1$, the productivity distribution induced by the GS mechanism is higher than that induced by the current mechanism, but the differences are small. The situation is the opposite in schools $j = 2$ and $j = 3$ and the differences are substantially larger. In school $j = 4$, the GS mechanism again leads to higher productivity but the differences are again small. In terms of the above discussed effect, the matching effect dominates in schools $j = 1$ and $j = 4$ while the cardinality effect dominates in schools $j = 2$ and $j = 3$. Overall, the cardinality effect has a higher absolute impact than the matching effect.

1.5 Discussion of findings and conclusions

In the previous section, we thoroughly reviewed properties of the current pupil-school matching mechanism in terms of quantitative results using our computational model and we contrasted outcomes to those under the GS mechanism. In this section, we put identified properties and quantitative findings into the broader perspective of their implications, including policy ones.

One of the key features of a matching mechanism is whether it is strategy-proof. Strategy proofness allows parents to rank schools in order of their true preferences, simplifies the instructions on how to make a school-choice, does not penalize pupils for poorly informed and/or poorly strategizing parents, provides school management better information about the positive or negative demand effects of their managerial decisions, and supplies school system administrators with better information about demand for individual schools and the impact of policy changes. Also, policy discussions that have developed regarding the pupil-school matching mechanisms show that one advantage of strategy-proof mechanisms is that they level the playing field between the strategically sophisticated and well-informed and those who may be unsophisticated or poorly informed.

As we stated at the beginning, it follows from the properties of the Czech mechanism that it is not strategy-proof. However, the level of preference misrepresentation is not known and it is not clear how significant an issue this is in practice. One of our important findings, therefore, is that the current mechanism indeed induces widespread misrepresentation of school preferences among applicants. Depending on the chosen parameters, misrepresentation of preferences is the optimal strategy for 40% – 60% of rationally behaving students.[71]

[71] Misrepresentation of a similar kind had been identified by Chen and Sönmez (2004) (experimentally)

The large incidence of strategic misrepresentation of preferences has a number of implications for the functioning of the schooling system as a whole. The number of applications submitted is the only non-anecdotal aggregate information on demand which can be compared to the supply of schools and school type capacities. For the sake of our following argument it is important to recall that the misrepresentation of preferences we have identified creates bias in the demand for individual schools toward their capacities. In particular, according to demand revealed through the number of actual applications, high-quality schools are seemingly less demanded than they actually are. Similarly, low-quality schools face a demand higher than they would face would pupils not apply strategically. In the presence of pupils' strategic behavior, observable demand quantities do not allow us to distinguish between large and small demand-supply discrepancies and a lack of information on actual demand-supply gaps can harm the schooling system in various ways.

Within Europe, schools are commonly financed by formula-based schemes with quotas imposed on the number of pupils enrolled by individual schools and school types. Hidden demand-supply discrepancies make identification of poor-performing schools or outdated school types, and their restructuring or closures, difficult. Consequently, desirable adjustments of the schooling system could be slowed down or suspended. In this respect, Abdulkadiroglu and Sönmez (2003), Ergin and Sönmez (2005) and Abdulkadiroglu et al. (2006) note that for a long time, the Boston mechanism had been defended by its proponents arguing that a high percentage of applicants were granted their first choice school. This argument is false and misleading in not recognizing that many first choice schools are not those most preferred.

The Czech mechanism resembles the Boston one in the sense that proponents of the scheme stress the high proportion of matches realized in the 1st round. Proponents of the current scheme do not recognize the large scope of preference misrepresentation. In our opinion, the current scheme could be an important factor slowing down the restructuring of the outdated 3-track structure of the upper secondary schooling level in the Czech Republic. The apparent equilibrium underpinned arguments for insider lobby groups defending the status quo. By not recognizing the hidden discrepancies, the general public and policy makers in particular did not pursue the necessary initiative to reform the upper-secondary level of schooling.

Biases in revealed demand can also adversely affect the decision-making of school man-

and Abdulkadiroglu et al. (2006) (empirically) in the case of Boston mechanism.

agement. The nature of know-how building in schooling is - as in many other complex productive processes - an experience of an evolutionary nature. In particular, educational know-how is formed through a kind of adaptive learning based on a long sequence of partial trials and errors. In this process, revealed demand for a school is important feedback, enabling management to evaluate the (in)effectiveness of its previous decisions, including the internal organization of the school, curriculum content, teachers' incentive schemes, etc. In the presence of school preferences biased toward school capacities, a management introducing proper or improper components of educational technology cannot receive positive or negative feedback through revealed demand. The limited sensitivity of revealed demand to changes in true demand limits the accountability of schools. Overall, misrepresentation of school preferences of the kind we identify in the current mechanism can limit desirable school competition[72] and foster stagnation of a schooling system. This drawback can have a serious adverse impact on the overall quality of the education, especially in decentralized schooling systems which rely on a high degree of school autonomy and which stress school-choice principles.

As shown quantitatively in Section 1.4, matching of pupils to schools has equity and efficiency implications. By efficiency, we do not mean here the efficiency of schools' internal operations but the efficiency consequence of a particular matching between heterogeneous qualifications of pupils and qualities of the school as stipulated by equation (1.3). There, productivity of a school graduate is a product of the pupil's initial study qualification and of school quality. Our quantitative results indicate only small aggregate efficiency differentials between the Czech and the GS mechanisms, the former doing slightly better. Since the differences are marginal and due to the limitations of the chosen measure (sum of individual productivities), we do not see this result as making a strong case in favor of the Czech mechanism. However, exploring different measures of efficiency and conducting analysis on empirical data can be a possible area for a future research.

Equity, on the other hand, seems to be a bigger concern. In our model, pupils act rationally: they make no mistakes other than those implied by the imperfect perception of their own qualifications. This may not necessarily be so in reality. Recall that in the Czech mechanism, applicants deal with the probability of admission to each individual school, while this knowledge is not needed in the GS mechanism. In the Czech system, some applicants may count on wrong probabilities and follow a sub-optimal strategy. In this case, the slight productive superiority of the Czech mechanism can disappear.

[72]See Hoxby (2000) for an argument about the effect of competition on school productivity.

Abdulkadiroglu et al. (2006) present empirical evidence from the Boston scheme that a non-negligible proportion of applicants employs incorrect strategies, and it is likely that erroneously strategizing applicants exist in the Czech mechanism too.

Sub-optimal strategy choices adversely affect future admission to college and labor market career. This is likely to happen among well-qualified pupils with weak parental background due to lack of parental support in the acquisition of information and school-choice strategizing. In the GS mechanism, pupils have to reveal ordinal preferences of schools only. In the Czech mechanism, the task is more complicated. One needs to deal with cardinal preferences and estimate admission probabilities at all accessible schools.

Sub-optimal strategizing is less costly for students from wealthier families since they can opt out and choose a private school. This is usually not an option for well qualified pupils from poor families who cannot afford to pay tuition. Those well-qualified pupils end up in a low-quality school. In this respect, the Czech mechanism is most harmful to smart pupils from socially and financially poor families.

There are two more important features of the Czech mechanism worth mentioning here. Strong incentives to apply strategically may become cumbersome and can cause trauma and represent non-negligible psychic costs incurred by applicants and their parents. Anecdotal evidence on this phenomena appears in popular media every year during high-school admission period.

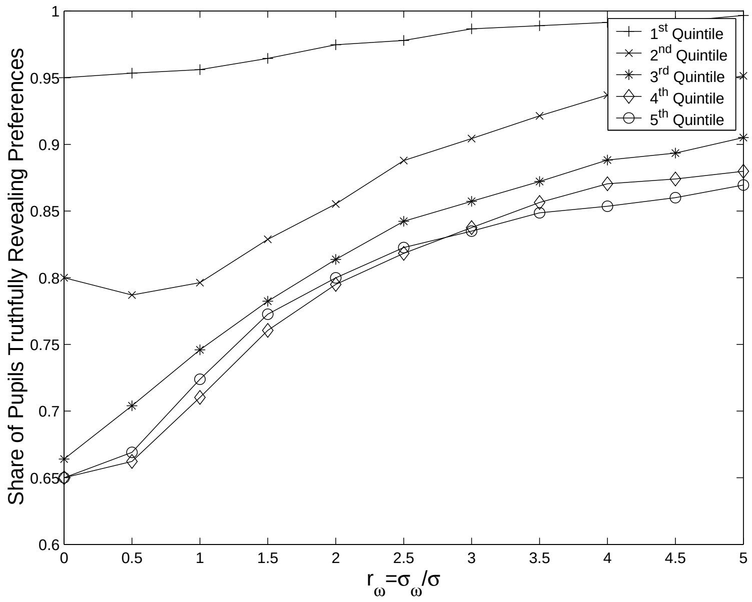
Another phenomena is justified envy. This formally defined property of a matching mechanism has its counterpart in real life. In particular, parents and their children whose admission outcome does not fulfill a pairwise stability property can feel they are treated unfairly. This may lead them to initiate a legal dispute. Our quantitative results show that in the current mechanism, for reasonable parameter values, between 10-20% of pupil-school matches does not fulfill the pairwise stability condition (these are matches different from those under the GS mechanism). This means that 10-20% of students prefer another school to the school they are matched to and their qualification is higher than the qualification of at least one student admitted to their preferred school.

A natural alternative to the decentralized Czech mechanism is the GS mechanism with the school priorities determined centrally, based on transparent criteria. It is then a political decision what exactly these criteria should be, whether only students' study aptitude, as determined by a central test, or a wider set of criteria with stronger emphasis on equity objectives. That the GS mechanism is not just a theoretical concept can be seen by the fact that it has been used for years to match medical interns to hospitals in the US. More recently, it has also been successfully applied in the Boston area to replace the well-known Boston mechanism, which suffered from similar flaws as the Czech one.

Any changes to the current system will, however, be very difficult to achieve. This is obvious from the ongoing debate about standardizing final examinations in secondary schools. Opponents of nation-wide testing warn that it would create a strong push for a unified curriculum and would limit diversity. Some also argue that nation-wide testing would create grounds for the introduction of output based-funding, arguing that rewarding output skills instead of skills-added would adversely affect the financing of those schools that enroll low-skill pupils from a weak social background. These concerns will need to be properly addressed before any change is implemented.

1.A Appendix – effect of school preference heterogeneity component

Figure 1.15: The effect of heterogeneity component – revealed over true preferences



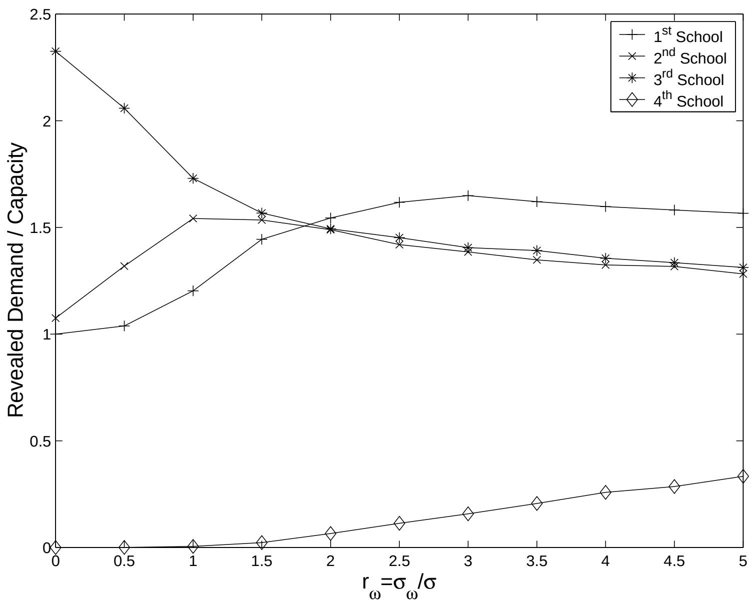
In this section we investigate the role played by school preference heterogeneity, captured by ω_{ij}. As described above, we consider $\omega_{ij} \sim iid\ N(0, \sigma_\omega)$. In the initial set-up, $\sigma_\omega = 50$. We now vary σ_ω in the range of 0 - 100 and study the effect of the changes in this parameter on students' behavior.

Figure 1.15 shows the impact of variance in ω_{ij}, measured by ratio $r_\omega = \sigma_\omega/\sigma$, on the misrepresentation of school preferences. On the vertical axis we plot the ratio of pupils, sorted into quantiles by perceived qualifications, who do not misrepresent school preferences. For all values of σ_ω, the preference misrepresentation is lower in the case of students with higher perceived ability. However, increasing σ_ω diminishes the incidence of preference misrepresentation in all quantiles of pupils. The most sensitive impact appears in the middle range of r_ω.

Misrepresentations of preferences translate into differentials between true and revealed demand at the level of individual schools. These differentials are depicted in figure 1.16. In the absence of school preference heterogeneity, $r_\omega = 0$, students' true school preferences are exclusively determined by the product of perceived qualification and school quality, and the ordinal school preference of all pupils is identical.[73] In this border case, nobody applies to the lowest-quality school, $j = 4$, because admission to this school is always a fall-back option. Revealed demand for high quality schools, $j = 1, 2$, is almost equal

[73]Note that in this case, school quality is the only variable of choice affecting the productivity of an individual pupil. Since all pupils choose from the same pool of schools, all pupils prefer the most the school of the highest quality.

Figure 1.16: The effect of heterogeneity component – revealed demand over capacity

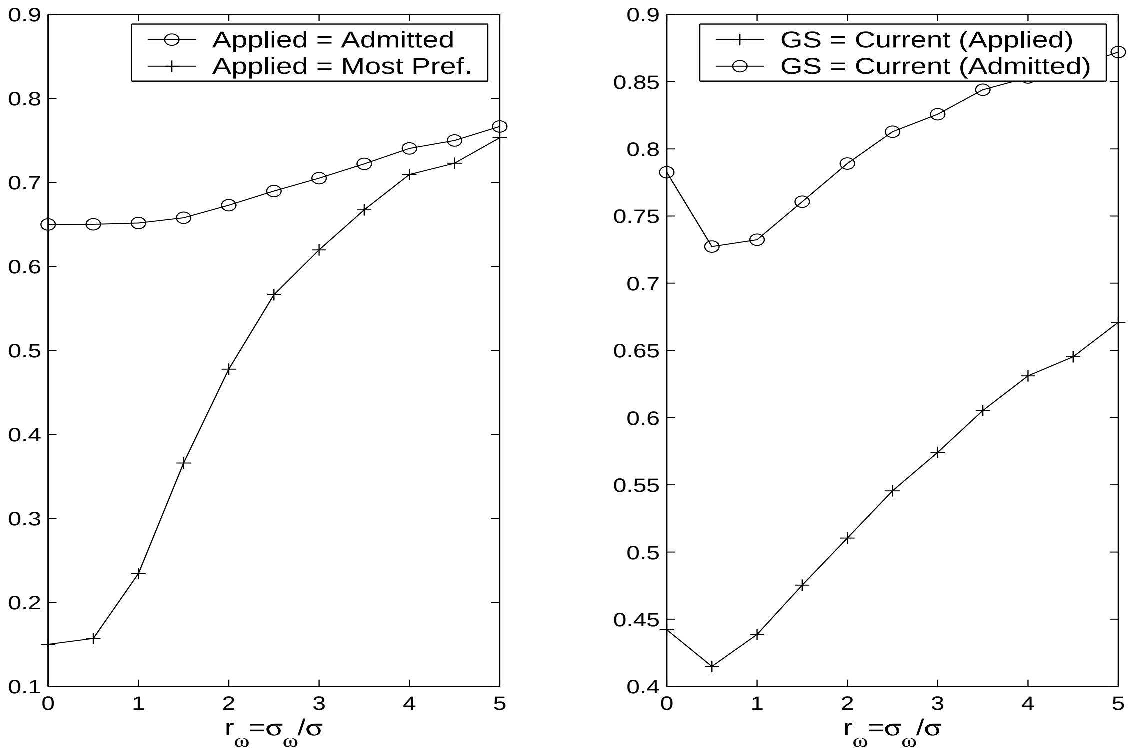


to a school's capacities. Recall that school 1 is truly demanded by all pupils and the difference between true and revealed demand is extraordinary. High excess demand for school $j = 3$ is due to the low qualification pupils who bear no risk applying to school 3 instead of school 4.

With increasing variance of school preference heterogeneity, the role of school quality in the formation of school preferences diminishes and the heterogeneity between revealed and true demands across schools declines. Note that for high values of r_ω the lowest-quality school 4 becomes the most preferred by some pupils. This could be due to its proximity or to particular taste or local skill match, effects being captured by ω.

The left panel in figure 1.17 shows the percentage of students admitted to the school to which they apply and the percentage applying to their most preferred school. The share of students who are admitted to the same school where they apply only slightly increases with the increasing heterogeneity component. While this ratio is heavily dependent on the qualification noise, as shown in section 1.4.3, the size of the heterogeneity component does not have much effect. The ratio of students applying to their most preferred schools, on the other hand, rises substantially with the increasing heterogeneity component size. The main reason is that the underlying preferences become more heterogenous and some students will most prefer some other school than school 1. If this is the case, then applying to such a school will more often be the best strategy than applying to school 1 was under the zero heterogeneity component. Based on this finding, we may say that the greater the

Figure 1.17: The effect of heterogeneity component – Czech vs. GS mechanism



heterogeneity of the underlying preferences, the smaller the preference misrepresentation and the greater the percentage of students, who are admitted to their most preferred school.

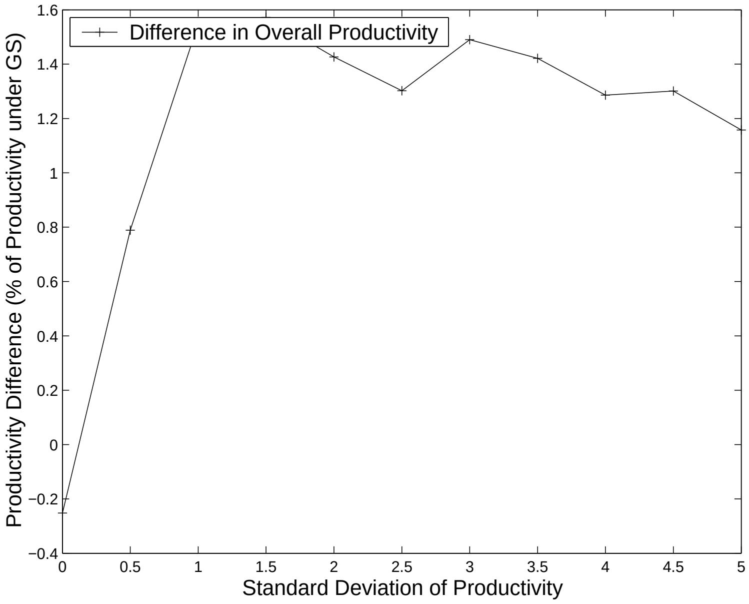
Figure 1.17 shows the impact of school preference heterogeneity on dissimilarity of outcomes under the current and the GS mechanism. On its vertical axis we depict the same ratios as figure 1.10, but on the horizontal axis we plot ratio r_ω. The left panel shows that the proportion of pupils who apply to the same school in both mechanisms (note that in the GS mechanism, all pupils apply to their truly most preferred schools) is growing fast with growing school preference heterogeneity, from very low proportions to 70%. This is in line with findings and interpretations in the previous section which stresses that the ranking of school qualities is unique but school preference components are pupil specific. As preferences become more heterogenous, students, especially those with lower perceived qualification, have higher chances of admission at their top choice school. This corresponds to a growing, although slowly, proportion of pupils admitted in the 1st round, as depicted by the upper profile on the left panel.

The right panel of figure 1.17 compares matching outcomes under both mechanisms. The upper profile shows that the proportion of pupils matched identically in both mechanisms declines slightly from almost 80% for relatively low values of r_ω, and increases afterwards to almost 90% for $r_\omega = 1$. Shifted down, but of very similar shape is the locus depicting the share of pupils in the current mechanism applying to the school they would be admitted to in the GS mechanism. Without school preference heterogeneity, only about 45% of students would apply to the school they would be admitted to by the GS mechanism. The proportion is growing with the growing role of school preference heterogeneity and reaches above 65% for $r_1 = 100$. This indicates that the incidence of instability in the current mechanism resulting from the growing role of school preference heterogeneity diminished. In other words, the proportion of pupils who prefer a school other than their actual match (in the current mechanism) school and whose qualification exceeds the qualification of at least one pupil admitted to the former school is decreasing with school preference heterogeneity.

The impact of school preference heterogeneity on the aggregate productivity is shown in figure 1.18. As in figure 1.13, we present the percentage difference from the productivity achieved in the GS mechanism. The current mechanism outperforms the GS mechanism for the whole range of values r_ω, except very small ones. In the absence of school preference heterogeneity, when all the students have the same ordinal preferences, the efficiency difference is given solely by the imperfect matching effect[74] in the current mechanism. Increasing variance of school preference heterogeneity makes the cardinality effect dominate the negative matching effect and the current mechanism outperforms the GS mechanism in terms of aggregate efficiency. The difference in efficiency is very small in the examined interval of r_ω.

[74]Matching of some high-quality students, who were rejected by top schools, to low quality schools.

Figure 1.18: The effect of heterogeneity component – productivity difference



Chapter 2

What determines school demand: empirical evidence from the Czech Republic

Abstract

Pupil-school matching mechanisms play a critical role in the schooling system. They affect the behavior of students and – through the information they convey – also the behavior of the schools and the authorities responsible for education policy. In this paper, I empirically study a type of decentralized, ability-based matching mechanism, variants of which are used in several European countries. The variant studied herein is the one used to match 9th grade students to upper-secondary schools in the Czech Republic. Using district-level data on demand for public gymnasia, I find significant evidence that students do not apply to their most preferred schools, opting rather for a less-preferred, but safer option. Furthermore, using data on individual student school choices, I also find that students with weak socio-economic backgrounds misrepresent their preferences more often than the other students.

Keywords: pupil-school matching, matching mechanisms

JEL classification codes: C12, D60, I20

2.1 Introduction

The spread of school choice programs in the US in the last decade has attracted the attention of economists to the mechanisms matching students to schools.[1] Once school choice is enabled, then – except in the improbable case that each school could admit all the applicants – there must be some mechanism that determines which students will be matched to which schools. The particular design of such a mechanism is important because it affects both the outcomes of the matching given the revealed preferences of the applicants and, as a result, the application choices themselves. Ill-designed mechanisms can substantially diminish the benefits of school choice.

In this paper I empirically study the mechanism used to match 9th grade students to upper-secondary schools in the Czech Republic.[2] As other European countries use similar mechanisms, my findings are likely to have broader relevance. The Czech mechanism is also similar to the Boston mechanism – in fact, they are both examples of a priority matching mechanism – so the results presented here can also be related to studies of the Boston mechanism. In the type of mechanisms discussed herein, school choice does not actually mean that a student can choose the school where he will study but that he indicates his preferences; a mechanism is then used to match him to a particular school. In the Czech mechanism, a student chooses the school where he will take an entrance examination and the school will then decide whether to admit him or not. If not admitted, the student can try other schools but, after being rejected by his first-choice school, he may find that all the seats at the other good schools are already taken. Therefore, it may be an optimal strategy for a student to indicate as his first-choice school a school different from that he actually prefers most, if the admission chances at his most-preferred school are low.

My first hypothesis assumes that students actually do behave in this way, i.e. that in the current mechanism they misrepresent their preferences for schools. I study this hypothesis on the case of demand for public gymnasia, the type of school most suitable for those intending to continue to university and which faces highest excess demand. The fact that gymnasia are most over-demanded and that they represent a kind of elite school in each district makes them a good object for analysis. Indeed, if the preference

[1] See Abdulkadiroglu and Sönmez (2003), Chen and Sönmez (2004), Ergin and Sönmez (2005), Abdulkadiroglu et al. (2005), Abdulkadiroglu, Pathak, and Roth (2005), Abdulkadiroglu et al. (2006).

[2] In the Czech Republic, children typically attend grades 1 – 9, corresponding roughly to primary and lower-secondary level in the US and other countries, in one school. The transition to upper-secondary level means transition to other school, which is why a matching mechanism is needed at this stage.

misrepresentation hypothesis is correct then it would concern mainly public gymnasia. My main finding with respect to this hypothesis is that revealed demand for gymnasia is significantly affected by the capacity of gymnasia, while the true demand should not be (indicating preference misrepresentation). Increasing the capacity by 1 seat increases the revealed demand on average by 1.1 – 1.2 seats. This finding holds in various specifications, in which I control for potential endogeneity of capacity.

My second hypothesis states that students with weaker socio-economic backgrounds misrepresent their preferences more often than students from well-off families. In particular, this means that for a given ability, the lower the social status of a student, the lower the probability that he will apply to an over-demanded school, even if there is no tuition. This stifles social mobility by limiting access to a high-quality education for socially disadvantaged students. In support of my hypothesis, I find that an increase in the per capita income of the family by one standard deviation increases the probability of applying to a gymnasium by 8.3%. When at least one of the parents has a university education then the probability of applying to a gymnasium increases by 10% – 15%, everything else held constant.

Given that I find indications of preference misrepresentation by students the question is whether it is a reason for concern. Is it not the small capacity of the most demanded schools that causes the real problem? Should the relevant authorities, rather than changing the mechanism, increase the capacity of the most demanded schools sufficiently such that all those interested could be admitted?

The message that my results convey is that, although adjusting the capacity to demand is desirable, the matching mechanism will always be important.

One reason is that because capacity adjustments cannot occur instantaneously, there will always be some over- and under-demanded schools. In these circumstances, the current mechanism, in combination with the entrance examinations as a way to determine students' priorities, leads to stressful situations which some students may prefer to avoid and hence apply to less-demanded schools where they are sure of being admitted. On the other hand, some of those who take the risk and apply to the highly-demanded schools will necessarily be rejected and may end up in poor-quality schools. These are the sources of instability and unfairness in the mechanism which will be present whenever there is some mismatch between supply and demand.

The mechanism is also important because of the information it provides. In a market in which there is no price mechanism to convey information, like the upper-secondary education market described here, it is crucial to ascertain what kind of information is provided by the mechanism and used to reconcile demand and supply. It would be desirable if the mechanism provided undistorted information about the demand for individual schools. Without this, it is difficult to tell which schools are truly over-demanded because some apparently over-demanded schools may simply benefit from the fear of students to apply to their truly most preferred schools. Having the correct information would provide better incentives for school principals and teachers to make the school more desirable for students. Such information would also make a stronger case for extending the capacity at the over-demanded schools and decreasing it at the less demanded ones. The possibility of these capacity adjustments would also provide incentives for improving school quality.[3]

The paper builds on a growing body of literature on the design of school choice mechanisms which itself emanates from the literature on matching in two-sided markets. I do not discuss in much detail the basic concepts used in this literature, such as stability or strategy-proofness. The interested reader is referred to, for example, Dubins and Freedman (1981), Roth and Sotomayor (1990), Roth (2002) or Roth (2008). The same holds for the description of individual matching mechanisms which can be found, for example, in Abdulkadiroglu and Sönmez (2003).

The first to consider school choice from a mechanism design perspective were Abdulkadiroglu and Sönmez (2003). They analyze various mechanisms at work in practice and compare them with two theoretically superior mechanisms, the Gale-Shapley (GS) mechanism and the Top Trading Cycles (TTC) mechanism. Of the practically used mechanisms, special attention is devoted to the Boston mechanism, which is quite similar to the one analyzed in this paper. Both the GS and TTC mechanisms are strategy-proof.[4] The GS mechanism is stable (produces stable matchings) but need not be Pareto efficient,[5] while the TTC mechanism is always Pareto efficient but may produce unstable matchings.[6] The Boston mechanism, which is a form of a priority-matching mechanism, is neither strategy-proof nor is it likely to produce stable and Pareto-efficient matchings.

The Boston, GS and TTC mechanisms have been studied experimentally by Chen

[3]This effect is analogous to an increase in competition among schools. Hoxby (2000) presents some evidence using US data that stronger competition among schools leads to higher school productivity.

[4]Strategy-proofness means that the dominant strategy for students is to submit their true preferences.

[5]A matching is stable if there is no student-school pair such that the student prefers the school to his match and has higher priority at the school than some of the actually admitted students.

[6]Pais and Pintér (2008) compare the TTC, GS and the Boston mechanisms in an experimental setting; both TTC and GS mechanisms outperform the Boston mechanism, with TTC delivering slightly better results than GS. Kesten (2004) suggests two new mechanisms that to some degree solve the conflict between stability and Pareto efficiency.

and Sönmez (2004). The experimental results suggest a high rate of preference misrepresentation under the Boston mechanism which leads to significantly lower efficiency in the laboratory environment compared to the two competing mechanisms. Ergin and Sönmez (2005) study theoretically the equilibria of the preference revelation game induced by the Boston mechanism. Their key result is that under the assumption of perfect information the set of Nash equilibria of this game is equal to the set of stable matchings under the true preferences. Both the theoretical and experimental evidence suggests that from the studied mechanisms both the GS mechanism and the TTC mechanism perform better than the Boston mechanism (and similar priority-matching mechanisms used in practice). In addition, Abdulkadiroglu et al. (2006) present some empirical evidence of preference misrepresentation under the Boston mechanism, confirming the theoretical and experimental results. They show, however, that not all the students (their parents) act strategically and that those who state their true preferences without regard for the capacity limitations of their favorite schools are likely to be the losers in the school choice game. In 2005, this evidence persuaded the administration in Boston to replace the existing mechanism with the GS mechanism.

The rest of the paper is organized as follows. Section 2.2 describes in more detail the admission procedure I analyze. Section 2.3 presents a model of the determination of true and revealed preferences and of true and revealed demand. Section 2.4 describes the data I use. Section 2.5 analyzes my first hypothesis (that there is significant preference misrepresentation) using the district-level revealed demand for all public gymnasia in the Czech Republic. Section 2.6 deals with my second hypothesis (that the preference misrepresentation concerns mainly students from lower social backgrounds) using individual-level data from the PISA project. Section 2.7 discusses the policy implications of my findings and concludes the analysis.

2.2 Description of the admission procedure

I study the procedure used to assign 9th grade students to upper-secondary schools in the Czech Republic. I focus on this transition since this is when most students change school and must go through the admission procedure.[7] The procedure is very simple

[7]Apart from that, many gymnasia also offer 8-year programs, to which students can transfer from the 6th grade. The number of seats offered is quite limited, so most gymnasia students join after 9th grade.

to understand.[8] Each school is required to hold an admission proceedings. This does not mean that all schools have to hold entrance examinations; they may choose students by their elementary school grades or they may simply accept all the applicants if their capacity allows. Virtually all of the good-quality schools, however, administer an entrance examination which takes place at a single common date for all schools. A student chooses in advance a school at which he takes the entrance examination and, based on the results, the school decides which applicants will be admitted. This admission decision is final and cannot be later repealed by the school.

Students not admitted in the first round have another chance in the second round. Most of the good-quality schools, however, fill most of the available seats in the first round so the chances of admission in the second round are small. With quite a high probability, a student not admitted in the first round will end up in a low-quality school. The student could have avoided this consequence by applying in the first round to a less-demanded school where the admission chances are higher. This behavior is referred to as preference misrepresentation and I present some evidence suggesting that it is a common phenomenon among students.

The studied mechanism is similar to the well-known Boston mechanism. In each round, the admission decisions are final and by not applying to a school in a particular round, a student loses his priority he would otherwise have at that school in that round. The determination of priorities in the studied mechanism and in the Boston mechanism, however, is different. In the Boston mechanism, the priorities of all students at all the schools are in fact known in advance (or they can be established with certainty). They are determined by factors such as proximity to the school or having siblings attending the school. The only uncertainty concerns which students will actually apply to which schools. In the Czech mechanism, the priorities themselves are not known in advance. They are determined in a single step with the admission-rejection decision which means that the uncertainty is even higher than in the Boston mechanism.

My data cover four consecutive years from 2002 – 2005. Effective from 2005, two changes were made to the admission procedure. Until then, the rules of the procedure only applied to public schools. The admission procedure to private schools was in fact unregulated and alongside an application to a public school a student could apply to as many private schools as he wished. By applying to a private school the student did

[8] This does not imply that it is easy to determine what should be one's optimal strategy, which, on the other hand, is highly complicated.

not lose his priority at a public school and vice versa. After the change, the admission procedure rules now apply to all schools, both public and private. A student can now choose a single school for the first round, either public or private.

The second change concerns the choice of school (schools) for each round, or rather the timing thereof. Under the previous rules, a student chose two schools, one for the first round and the other for the second round. When choosing the school for the second round, the student did not know how many seats (if any) there would be left at this school after the first round. Under the new rules, the student chooses a single school for the first round. Only when not admitted does he choose schools for the second round in which he may choose more than one school.

While the first change made the situation of students relatively worse by restricting their choice, the second change probably made them better off by letting them choose the second-round schools only after the first round is over and with the knowledge of which schools have free seats and how many. The second change is usually interpreted as reducing choice but, given that schools, for some reason, do not respond by limiting the number of seats for the second round, this is not correct.[9]

Neither of the changes, however, essentially changes the nature of the mechanism. Preventing students from submitting parallel applications to private schools is not likely to change their behavior. If rejected by their preferred public school, students interested in getting a seat in a private school are likely to get it, since the private schools are typically flexible and do not face high excess demand. The possibility to select a school for the second round only after the first round is finished is really a formalization of an informal status quo. The second round is already less formal and centralized than the first round. Students who have to participate in the second round (or rather their parents) usually research which schools still have open seats and how many; then they may decide to change the ex-ante choice, which is typically feasible.

An appealing feature of the admission procedure (both before and, even more so, after the modification) is that its rules are clear and easy to understand. This, however, is a fallacy because although the rules are easy to understand, it is very difficult to determine what one's optimal strategy should be. The problem is that the game induced by the procedure does not have an equilibrium in undominated strategies, which means that one's optimal strategy depends on the strategies of the others. The strategy choice is

[9]The mechanism was further amended in 2009, when students were allowed to submit 3 applications in the first round.

further complicated because the optimal strategy also depends on a student's ability (relative to others), which students do not know with certainty.

Administratively, this procedure is quite convenient for the schools. The strict limitation to one application in the first round guarantees that no student will have more than one offer. In addition, the schooling law stipulates that students admitted in one round cannot participate in further rounds, so the schools are in fact sure that their offers will be accepted. This means that to fill a certain number of places, schools need not make more offers, and they have almost perfect control over the number of students who will enroll at the school.

2.3 Model

This section describes the theoretical motivation for the subsequent empirical analysis. It captures the basic features of the demand for public gymnasia in the first round of the admission procedure but the framework is more general. I start with an assumption that if the population in individual districts had the same characteristics, the true (underlying) demand (the share of 9th grade students that would like to go to a gymnasium) would be the same. I further assume that the true demand will be higher in districts with a higher share of a college-educated population. More educated people tend to value education more and, therefore, given ability and other characteristics, a student is more likely to prefer a gymnasium (and a college afterwards) to other educational paths if his parents are themselves college-educated. The true demand should also be higher, everything else held constant, in districts with a college because the costs of going to college are lower, more students should be interested in a college education, and, therefore, more students should prefer gymnasia to other schools.

If students state their preferences truthfully, then the revealed demand (the share of students that actually apply to gymnasia) is the same as the true demand. If they do not, as I assume, then it should be higher if true demand is higher but it should also be affected by additional factors that do not affect the true demand. These factors fall into two categories – the risk of rejection and the consequences of rejection. For a given true demand, the risk of rejection decreases with increasing capacity of gymnasia and, therefore, the higher the capacity the higher the revealed demand should be.[10]

[10] This assumes the capacity is exogenous and does not adjust to the true demand. I will discuss this issue in more detail in section 2.5.

The consequences of rejection depend on the quality of school to which the rejected student is eventually admitted. Chances of being admitted to a relatively good public school in the second round are small because these schools will have all or almost all of their seats filled and, therefore, by applying to a school with a high risk of rejection, a student risks ending up in some low-quality school. Presence of a comparable private school could provide a good hedging option, at least for students who can afford to pay tuition. These schools typically fill their seats by the students not admitted to the over-subscribed public school and, hence, they usually have free seats after the first round of the admission procedure.

Among the available private schools I focus on the role of the private gymnasia. A private gymnasium represents a competition to the public gymnasia but the public ones are still preferred by most of the students, not least because there is no tuition. The "competition" effect decreases the true demand and through it also the revealed demand while the "hedging" effect leaves the true demand unchanged and increases the revealed demand. If the "hedging" effect prevails I should see more "aggressive" bidding for the public gymnasia, i.e. higher revealed demand, in districts with a private gymnasium.

2.3.1 The true demand for gymnasia

I start the formal analysis by defining a function of a student's utility derived from attending a certain school. Student i in a district j has utility

$$U_{ij} = U(S, A_{ij}, E_{ij}, C_j, \xi_{ij}) \tag{2.1}$$

The utility depends on the type of school a student attends (S), his ability (A_{ij}), the education of his parents (E_{ij}), whether there is a college in the district j (C_j) and some other unobserved and random characteristics (ξ_{ij}). Students with high ability will on average receive highest utility in gymnasia, while students with low ability will on average receive highest utility in less demanding schools. Everything else held constant, students with more educated parents are more likely to value education more and will receive higher utility from more demanding schools – gymnasia. Availability of a college in a district means lower costs of college education and higher utility from a gymnasium (the most suitable secondary school for those attempting to continue in college).

For simplicity, I assume students are risk-neutral. With risk-aversion, the incentive for preference misrepresentation would be even stronger.

Denote the utility from a public gymnasium of student i in district j as U_{ij}^G and that from any other school $S \neq G$ as U_{ij}^S. The probability that student i in district j will prefer a gymnasium to other schools can be written as

$$P_{ij} = P(U_{ij}^G \geq max_{\forall S \neq G}\{U_{ij}^S\}) = P(A_{ij}, E_{ij}, C_j) \tag{2.2}$$

The probability $P(\ldots)$ is increasing in all the arguments because U_{ij}^G is increasing in all the arguments and, at the same time, the utilities from the other schools are either decreasing or increasing at a lower rate. Formally, the exact form of this function is given by the distribution of ξ_{ij} but the above general formulation is sufficient for the purposes of this paper.

Denote the number of students in the ninth grade (the grade in which students apply to upper secondary schools) in district j as N_j and the share of students preferring gymnasium as D_j. Then, I can write

$$D_j \simeq \frac{\sum_i^{N_j} P_{ij}}{N_j} \tag{2.3}$$

Making a simplifying assumption that the distribution of ability of students is the same across districts ($\bar{A}_j = \bar{A}$), I can write the true demand for gymnasia as a function of the average values of the remaining variables[11]:

$$D_j = D(\bar{E}_j, C_j, \bar{\xi}_j) \tag{2.4}$$

where $\bar{E}_j$ is the share of university-educated population and $\bar{\xi}_j$ represents the average of the idiosyncratic characteristics of the individuals in district j. The function $D(\ldots)$ is increasing in both $\bar{E}_j$ and C_j because the probability $P(\ldots)$ is increasing in E_{ij} and C_j.

2.3.2 The revealed demand for gymnasia

Consider the expected utility from *applying to* a gymnasium (as opposed to *studying at* a gymnasium). As I argued in section 2.2, under the analyzed admission mechanism, a student who would like to study at a gymnasium may still prefer to apply to another, less attractive school if the probability of *being admitted* to a gymnasium is low. This

[11] Although in reality the average utility will not be the same across districts, this does not change anything about the effects of the other variables and does not affect the validity of the analysis.

probability depends on various factors but for simplicity assume that it only depends on the student's ability, true demand for gymnasia, and the capacity of gymnasia[12]:

$$\pi_{ij} = \pi(A_{ij}, D_j, K_j), \tag{2.5}$$

where K_j denotes the capacity of gymnasia in district j. The expected utility a student obtains from applying to a gymnasium, u_{ij}^G, is

$$u_{ij}^G = \pi_{ij} U_{ij}^G + (1 - \pi_{ij}) U_{ij}^{Al}, \tag{2.6}$$

where U_{ij}^{Al} denotes the utility from the alternative the student ends up with if rejected by the gymnasium. Defining the expected utilities u_{ij}^S from applying to the other schools in a similar way, the probability that a student applies to a gymnasium is

$$p_{ij} = P(u_{ij}^G \geq max_{\forall S \neq G}\{u_{ij}^S\}) = p(\pi_{ij}, U_{ij}^G, U_{ij}^{Al}). \tag{2.7}$$

As one would assume, the probability $p(\ldots)$ depends on the probability that he is admitted, the utility he gets if admitted, and the utility he gets if not admitted. The share of students in a district who will apply to gymnasia, d_j, can be written as

$$d_j \simeq \frac{\sum_i^{N_j} p_{ij}}{N_j}. \tag{2.8}$$

I can again write d_j as a function of the factors affecting p_{ij}, making the same assumption as above (distribution of ability is the same across districts):

$$d_j = d(D_j, K_j, \bar{U}_{ij}^{Al}) = d(D(\bar{E}_j, C_j, \bar{\xi}_j), K_j, \bar{U}_j^{Al}). \tag{2.9}$$

The revealed demand d_j depends on the true demand D_j, the capacity of the gymnasia, and the quality of the alternative school for the rejected students. Through the true demand, the revealed demand depends on the share of college-educated population and the presence of a college in the district. The function $d(\ldots)$ is increasing in D_j because the higher the true demand, the more students should be willing to take the risk of applying

[12] One could argue that the probability of admission should rather depend on the *revealed* than on the *true* demand. This is certainly the case, but it would result in a circular definition, since π_{ij} will be used to define the revealed demand. I am using here the actual demand as an exogenous variable affecting the probability of admission indirectly, through the revealed demand.

to a gymnasium. It is increasing in K_j because the higher the capacity the higher the probability of being accepted and, thus, the higher revealed demand. It is also increasing in $\bar{U}_j^{Al}$, the utility from the alternative school, because when a student is rejected, his loss is lower the higher $\bar{U}_j^{Al}$ is and he is, therefore, more likely to apply.

In any strategy-proof mechanism, unlike the Czech mechanism, the revealed demand is the same as the true demand. The revealed demand does not at all depend on school capacity, since students are not "punished" for not being accepted to their most preferred school.

2.4 Data

I use two data sets to analyze my two hypotheses. To analyze the first hypothesis about preference misrepresentation in general, I use a data set of school-level information on upper-secondary schools containing information on school type, location, number of admitted students, number of applicants and other school-specific variables. Because students in one district represent a natural demand for schools in that district I pool together the information on schools in each district and create a district-level data set. To analyze the hypothesis that preference misrepresentation concerns mainly students with weak socio-economic backgrounds I use an individual-level data set from the PISA project containing information on students' ability (test results), social status, school attended, and choice of secondary school.

2.4.1 The school-level data set

The school-level data set was obtained from the Office for Educational Information (Ústav pro informace ve vzdělávání), an institution collecting and publishing various information on the educational sector, and it covers all upper-secondary schools in the Czech Republic (the data are not a sample but represent the whole population of schools). The data form a panel, covering four consecutive admission procedures to the upper-secondary schools, namely school years 2002/2003 to 2005/2006. There is one observation for each school in each year. Apart from information on the admission procedure, there are many other school characteristics in the data set, most of which I do not use in this paper.

In my analysis, I focus on admission to 4-year public gymnasia.[13] I pool together

[13] Apart from the 4-year gymnasia, there are also 8-year gymnasia, to which some students transfer

data on all these gymnasia (namely their capacity and number of applicants) within one district, forming a panel data set where one observation corresponds to a district in a given year. This means that for each year I have 75 observations, or 300 observations in total. I pool the data at the district level because within a district, the demand for individual gymnasia may depend on district- and school-specific factors I do not observe. In particular, the district student population forms a natural demand for the district's schools, while for individual schools within a district, such a natural demand pool is not possible to identify.[14]

The dependent variable I use is the relative first-round revealed demand for gymnasia in each district, calculated as the share of all 9th grade students who in the first round of the admission procedure applied to gymnasia on the total 9th grade student population. I call this variable revealed demand. I only focus on the first-round demand because the chances of admission to a gymnasium in the second-round are very small and students really interested in a gymnasium usually apply in the first round.[15] Because of the small admission chances, second round school choices need not be so carefully contemplated and they may be driven by some unobserved factors.

The first explanatory variable is the capacity of public gymnasia (I further refer to it as capacity), measured as the share of the first-round-admitted students on the total population of 9th grade students.[16] In addition I use further variables identified as relevant in my model: the presence of a private gymnasium and the proxies for true demand – the share of college educated population and the presence of a university in a district. In the case of the share of college educated population, I use data from 2001 for all the years. This should not be problematic because the potential changes are negligible (the natural rate of change in this variable is low and the mobility between districts is low as well). I consider a district to be a university district if a public university is available in

between their 5th and 6th grade. In the first 4 grades they offer similar curriculum as the elementary schools. Their students then gradually continue to the standard 4-year gymnasium program without having to take the admission exams. Most of the students studying at gymnasia, however, join after 9th grade for the 4-year program.

[14]In the metropolitan districts (Prague, Brno, Plzen, Ostrava), the natural demand may be even broader and may include students from the neighboring districts. To check that this does not significantly distort my results, I have tried specifications excluding the metropolitan and the neighboring districts from the data set. The results are not significantly different from those presented here and are available upon request.

[15]The median percentage of those admitted in the first round is 91% in 2002, 92.5% in 2003, and 93.3% in 2004 (in 2005, I only have data on the whole admission procedure).

[16]The values would be very similar if I used the number of students admitted in both rounds but for consistency I focus again on the first round only.

it [17].

Table 2.1 in the Appendix summarizes the demand and capacity in individual years. The first panel of the table shows the values for the first round, i.e. those I use further in the analysis, and the second panel, for comparison, shows the values for the whole admission procedure. The figures in the table are weighted averages and standard deviations (in parentheses) where weights are the number of 9th grade students in each district. In the years 2002-2004 the average number of first-round applicants stays almost constant, between 15.5% and 16.2% of 9th grade students; the number of first-round admitted rises slightly, from 10.7% to 11.4% of 9th grade students. As a result, the (revealed) excess demand in the first round falls from 46.5% to 36.3%. Similarly, the share of applicants to gymnasia in the whole admission procedure (1st and 2nd round) in 2002-2004 is in fact constant, at a level around 24%. The share of admitted students in the overall procedure increases slightly. The overall excess demand falls slightly but less than the first-round excess demand.

For the year 2005 I only have information on the total number of applicants and admitted students. The share of applicants is 17.6%, the share of admitted students is 13.1%, and the overall excess demand is 32.4%. Because of the change in the admission procedure the 2005 data are similar in nature to the first-round data in the previous years. The main reason is that in 2005, unlike the previous years, only students not admitted in the first round submitted applications in the second round, so the total number of applications in the second round is significantly lower. Hence, it seems appropriate to treat the 2005 data in the same way as the data on the first rounds in the previous years and to use them, in some specifications, together. However the evidence I present is robust, even without the specifications using the 2005 data.

2.4.2 The individual-level data set (PISA)

To verify my second hypothesis – that preference misrepresentation is more prevalent among students with weaker socio-economic backgrounds – I use an individual-level data set from the PISA project which assesses the abilities of 15-year-old pupils in OECD countries and several other countries. The data were gathered in 2003, i.e. before the change in the admission mechanism described above. For the Czech Republic, the data set includes both 9th grade students (last grade at the lower-secondary level) and 10th

[17] There are 12 such districts in the Czech Republic.

grade students (first grade at the upper-secondary level) aged 15. Since I am interested in the school choice behavior of 9th grade students, I only use the corresponding part of the data set, which leaves me with 5047 observations. The following variables from the data set are of interest:

- School identifier – a randomly generated number so that I know which students come from the same school;
- Test scores in mathematics, reading and problem-solving tests that were part of the project;
- Grades in mathematics, Czech language, foreign language, physics, chemistry, biology, and geography taken from the last school report students were given;
- Income of a student's family and number of family members (I am able to calculate per capita family income);
- Parents' education;
- The choice of school for the first round of the admission procedure.

Optimally, I would want to use this individual-level data set to analyze preference misrepresentation in the school choice instead of the district-level data. Unfortunately, it is not possible to match the data from the PISA data set with my data on gymnasia capacity in individual districts since the PISA data set does not include district codes and the school codes have been replaced by random indicators. I do not know in which district a student participating in the PISA project lives and hence I am unable to find out either the capacity of the gymnasia in this district or the other explanatory variables.

Table 2.2 in the Appendix shows the choices of secondary school type for the first round of the admission procedure. Most students (51%) apply to vocational schools, approximately 31% apply to apprentice schools (of which 19% to specializations without a graduation exam), and approximately 18% to gymnasia (this is similar to the values of 16-17% from the school-level data set).

2.5 Empirical analysis of revealed demand for gymnasia

2.5.1 Derivation of estimable model

In this section I confirm my first hypothesis regarding preference misrepresentation in the demand for gymnasia. The structural relationship I am interested in is:

$$d = \beta_0 + \beta_1 D + \beta_2 K + \beta_3 G_{Priv} + \epsilon, \tag{2.10}$$

where d is the observed relative demand for public gymnasia (the share of 9th grade students who applied), D is the relative true demand (the share of students who applied), K is the relative capacity of the public gymnasia (the share of those admitted), and G_{Priv} is the dummy variable for the presence of a private gymnasium. ϵ is the error term which has zero mean and is uncorrelated with the regressors. If students stated their preferences truthfully, then hypothesis H_0 specified as $\beta_1 = 1, \beta_2 = 0, \beta_3 < 0$[18] should hold. If, on the other hand, students misrepresent their preferences, $H_1 : \beta_1 < 1, \beta_2 > 0, \beta_3 > 0$ should hold. The problem is that to estimate equation 2.10 I would need to observe the true demand D which, of course, I do not.

To avoid the bias due to the omission of the true demand from the regression I use as proxies the variables that, according to my model in section 2.3.1, should determine the true demand, namely the share of college-educated population (E) and the presence of a university in the district (C). The relationship for the true demand is then

$$D = \delta_0 + \delta_1 E + \delta_2 C + v. \tag{2.11}$$

I assume that v has zero mean and is uncorrelated with E, C, K and G_{Priv}. Substituting into the structural relationship, I obtain the estimable model

$$d = \beta_0 + \beta_1\delta_0 + \beta_1\delta_1 E + \beta_1\delta_2 C + \beta_2 K + \beta_3 G_{Priv} + \beta_1 v + \epsilon \tag{2.12}$$

The error term is now $\beta_1 v + \epsilon$ which, according to my assumptions, has mean zero and is uncorrelated with the regressors; therefore, running OLS on this model should provide consistent parameter estimates. The inclusion of E and C, which can be viewed as the

[18] $\beta_3 < 0$ due to the competition effect of private gymnasia.

proxies for the true (unobserved) demand, should solve the omitted variable problem.

There is, however, still some risk that the omitted variable problem is not fully solved by the inclusion of the two proxies. If capacity adjusts to true demand, an unobserved shock to true demand would also increase capacity, showing up as a causal relationship from capacity to revealed demand even though there is none. In particular, if the error term v in equation 2.11 is correlated with K, then running OLS on equation 2.12 produces an inconsistent estimate of β_2. This would also be the case if there were another omitted variable that correlated with the true demand. Then I could still not rule out the possibility that the positive coefficient at K does not reflect the difference between true and revealed demand, but that there is a reverse causality, meaning that higher values of K are caused by higher true demand. In addition, the proxies for true demand may also be problematic. Both the share of university-educated population and the presence of a university can actually affect the capacity of public and private gymnasia through political pressure and market demand.

Because of these issues, I use two different ways to control for the potential endogeneity and to verify my results. First, I use an instrumental variable regression in which I instrument for the share of students admitted to gymnasia (K) by the share of students admitted in the year 1991.[19] Since the beginning of the 1990s, due to subsequent shocks, the demand for gymnasia has evolved differently in individual districts while the capacity only partly reacted to this development. The capacity of gymnasia at the beginning of the 1990s should, thus, be determined independently of the demand in the period 2002-2005 and should be usable as an instrument for the capacity in this period. I do not instrument for the private gymnasium dummy because I lack a good instrument. Because I am not completely able to control for the possible endogeneity of the private gymnasium variable and because my hypothesis can also be tested using only the capacity variable, I also estimate models in which I exclude the districts with private gymnasia. The results of fixed effects regression can be viewed as further evidence in support of my hypothesis.

Second, I take advantage of different rates of change in a district 15-year-old population (roughly corresponding to 9th grade students) between 1991 and the period of my analysis, relative to the change in capacity of gymnasia. In all the districts, the size of the 15-year-old population fell between 1991 and 2002 – 2005, but the rate of change

[19]The instrument contains the total number of admitted students, while the instrumented variable contains the number of admitted students in the first round. This should be unproblematic since more than 90% of the students are, on average, admitted in the first round.

differed. Also, the capacity of the gymnasia changed at different rates. For each district, I thus define a Distortion Coefficient in the following way:

$$DistortionCoefficient = \frac{POP15_{200X}/POP15_{1991}}{CAPG_{200X}/CAPG_{1991}}. \quad (2.13)$$

The nominator is the ratio of the size of the 15-year-old population in 2002 – 2005 to the 15-year-old population in 1991 (for each year in 2002 – 2005 one value). The denominator is the ratio of district gymnasium capacity in 2002 – 2005 to capacity in 1991. I then examine specifications of the model in which the capacity of public gymnasia is replaced by the Distortion Coefficient. The higher the value of the Distortion Coefficient, the lower the revealed demand should be because the competition for the available gymnasium capacity got tougher and, thus, the probability of rejection increased.

2.5.2 Estimation results

Table 2.3 in the Appendix presents the results of the OLS and IV regressions on pooled data from all districts and all years.[20] Because the admission procedure changed in 2005 and in this year I only have data on the overall numbers of applicants and those admitted, I estimate models both for the period 2002-2004 and for the period 2002-2005.[21] The table reports four specifications: OLS on data from 2002-2004, OLS on 2002-2005, IV estimation on 2002-2004, and IV on 2002-2005.

In accordance with my hypothesis, all the coefficients are positive and are statistically significant at a 1% level. The coefficients of gymnasium capacity fall within the range of 1.135 and 1.176, which means that increasing the capacity by 1 seat increases the revealed demand by more than one applicant. An interesting question is whether the relationship between capacity and revealed demand stays the same in the whole data range. The revealed demand should be a concave function of capacity: in districts with low capacity the relative excess is bound to be higher than in districts with high capacity, because the marginal applicant in a low-capacity district will derive higher utility from a gymnasium than a marginal applicant in the high-capacity district and is thus willing to accept a higher risk of rejection. If the districts in the upper capacity range had high

[20] I have also run regressions on individual year data. As the results were very similar, I only report the results obtained using the pooled data.

[21] As I argued above, the data on the total number of applicants and those admitted in 2005 correspond in nature closely to the data on the first rounds in the previous years. Therefore, in the analysis covering the whole period I treat them the same as data on the first round and stack them together with the data from the previous years.

enough capacity, this effect should be observed in the data. To check this hypothesis, I also run a regression where, besides capacity, I include capacity squared as a regressor. Should the hypothesis hold, the coefficient of this regressor should be significant and have negative sign. However, in both the 2002 – 2004 and in 2002 – 2005 specifications, the coefficient is highly insignificant (p-values 0.516 and 0.712, respectively) and has positive sign.[22] Therefore, it seems that all the districts in the present data set lie in a range, where the capacity is still low enough to induce the relative excess demand to decrease.

The coefficient at the private gymnasium dummy falls within 0.008 and 0.012, which means that having a private gymnasium in a district increases the revealed demand for public gymnasia by between 0.8 and 1.2 percentage points. Given that the revealed demand is on average around 15% of the student population, the increase is on average between 5% and 8%. According to my hypothesis, this suggests that the presence of an acceptable alternative increases the revealed demand for public gymnasia because it decreases the opportunity cost of rejection. The positive coefficient, however, can also be due to reverse causality – that the private gymnasia were established in the districts with the highest true demand for gymnasium education. Although I am not able to rule this out, one bit of supporting evidence is that the positive coefficient was replicated in the fixed effects estimation (see below).[23]

Consistent with my hypothesis, the coefficients of variables representing true demand are also positive and significant. If the share of university-educated population is higher by one percentage point, the revealed demand for public gymnasia is higher by between 0.178 and 0.218 percentage points. In addition, the presence of a university in the district increases the revealed demand by between 1.3 and 1.6 percentage points.

As a check, I repeat the procedures reported in table 2.3 on data which exclude all the districts with private gymnasia, which means I focus solely on the effect of the capacity of public gymnasia. If private gymnasium capacity were endogenous, this could bias other results, so I am interested in whether the results can be replicated on districts without private gymnasia. Table 2.4 in the Appendix shows the results of the modified estimations. The coefficients of the capacity variable, those of largest interest, are very close to those in the basic specification with all the districts included; this supports my

[22] Results are available upon request.

[23] Yet another explanation for a positive relationship between the presence of a private gymnasium and the demand for public gymnasia may be improved quality of the public gymnasia. The private gymnasium represents competition for the public one and may force it to offer a better curriculum, hire better teachers, etc. For a discussion of school competition, see Hoxby (2000).

preference misrepresentation hypothesis.

I perform the Hausman specification test to compare the results of the OLS and IV estimation (with the gymnasium capacity in 1991 used as an instrument for capacity in 2002 – 2005). Neither in the specifications including all districts nor in those including only districts without private gymnasia can I reject the hypothesis that the coefficients are equal.[24] Provided the gymnasium capacity in 1991 is a good instrument,[25] then endogeneity does not appear to be a serious issue in this context.

The results of the OLS regression using the Distortion Coefficient instead of public gymnasia capacity are reported in table 2.5 in the Appendix. In support of my hypothesis, the coefficient at the Distortion Coefficient is negative and significant (in both the 2002 – 2004 and 2002 – 2005 specifications). This means that in districts with unfavorable development of student population vs. gymnasium capacity, the revealed demand is lower than in districts with more favorable development. If students revealed their preferences truthfully, no such effect could be observed.

The results of the panel data analysis are reported in table 2.6. The first two columns show the results of, respectively, fixed and random effects procedures for the period 2002-2004. The last two columns show the results of fixed and random effect procedures for the period 2002-2005. Note that variables that are fixed within each district across the period fall out of the regression in the fixed effects treatment. Overall, the panel data analysis confirms my results obtained by running the OLS and IV regressions on the pooled data. The coefficients of the capacity variable are positive and significant in all the specifications. Similarly to those obtained by standard OLS and IV regressions, they are significantly larger than one, confirming that the demand is not satiated even in districts at the high end of the capacity range.[26]

For both the 2002-2004 and 2002-2005 periods, I perform the Hausman specification test to compare the consistent estimates from the fixed effects procedure with the efficient (and under the null hypothesis also consistent) estimates from the random effects procedure. The results of these tests suggest that I can use the random effects procedure. In

[24]When all the districts are included then for the 2002-2004 period I have chi2(4)=0.92 and P-value = 0.92 and for 2002-2005 period I have chi2(4)=0.43 and P-value = 0.98. When only districts without private gymnasia are included then for the 2002-2004 period I have chi2(2)=3.09 and P-value = 0.38 and for 2002-2005 period I have chi2(4)=1.87 and P-value = 0.60.

[25]In the first stage of the IV regression, the coefficient of gymnasium capacity in 1991 is significant at 1% level and has a value of around 0.7 in both the 2002 – 2004 and 2002 – 2005 specifications.

[26]With the exception of the coefficient in the fixed effects specification for 2002-2005, which is significant at 10% level, all the other coefficients are significant at 5% level.

neither of the periods can I reject the null that the two sets of estimates are the same.[27]

In table 2.7 in the Appendix, I report the effects of an analysis exploring the effects of the size of the 9th-grade-student population on the revealed demand. While in the previous specifications I normalized both supply and demand by the cohort size, here I conduct a fixed effect regression of the relative demand (demand / cohort size) on the cohort size, controlling for capacity. My hypothesis states that, keeping the capacity constant, larger cohort size means that a smaller percentage of the students will apply. I conduct the estimation both on data including all the districts and on data including only districts without a private gymnasium. As expected, the effect of the population size is negative and significant. An increase in cohort size by 100 decreases the revealed demand (the number of students applying to gymnasia) by between 10.2 students (the specification with all the districts) and 13.5 students (the specification with districts without private gymnasia). This reinforces the results of the IV regression and of the OLS regression with gymnasium capacity replaced by the Distortion Coefficient, indicating that the observed relationship between capacity and revealed demand is due to preference misrepresentation and not due to adjustments of capacity to true demand.

The panel data analysis points in the same direction as the analysis using pooled data. The results allow me to conclude that there is convincing empirical evidence in support of my hypothesis of preference misrepresentation. In all the specifications I consider, I find that revealed demand is affected by factors that should not affect the true demand, which leads me to conclude that the true and revealed demand differ. These factors affect the revealed demand in the assumed direction and the coefficients are statistically significant.

2.6 Individual application behavior by social status

In this section I examine my second hypothesis, namely that preference misrepresentation mainly concerns students with weaker socio-economic (SE) backgrounds. Under this assumption the current admission procedure may limit social mobility by limiting access to a high-quality education for these students. I show that by holding ability constant, a student from a SE family or whose parents have lower education is less likely to apply to a gymnasium.[28]

[27] Chi2(2) = 2.4 and P-value = 0.301 for the period 2002-2004; Chi2(3) = 3.29 and P-value = .349 for the period 2002-2005

[28] Finding this effect may, however, as well reflect different preference ordering of individual schools by the SE students and it does not allow me to conclude with certainty that SE students misrepresent their

There may be several reasons the SE students are less likely to apply to public gymnasia. First, although they may still prefer it to other schools, SE students may actually value a gymnasium less than wealthier students do. SE students may be less sure about continuing to university, in which case a good vocational school may be a more acceptable alternative for them. Second, for a student from a well-off family a rejection by a public gymnasium need not be so serious, since the family may afford to pay tuition at a private gymnasium willing to accept the student. This option may not be available for a student from a SE family who would thus prefer a less attractive school with higher admission probability. Third, a lower family social status may lead to lower self-confidence, which in turn may discourage even a high-ability student from applying to a gymnasium. The current system sometimes gives rise to cronyism and corruption, the perception of which alone may discourage students from SE families to apply.

In terms of my model of section 2.3.2, these factors can be interpreted as variables entering the expected utility derived from applying to a gymnasium u_{ij}^G and the probability of applying to a gymnasium p_{ij}^G. If a SE student values gymnasium less than a well-off student does, it means that his true utility from studying at a gymnasium is lower, which also lowers the expected utility from applying and, thus, the probability of applying. Lower self-confidence decreases the (perception of) the student's probability of being admitted π_{ij}. The unavailability of the private gymnasium option means lower alternative utility U_{ij}^{Al}.

I use the probit estimation to determine the effect of a student's ability (measured by grades in school and the test scores in the PISA project) and his socioeconomic characteristics (family income and parental education) on the probability of applying to a gymnasium.

Overall, I estimate 8 specifications. As the dependent variable in all the specifications I use the binary variable whether or not a student applied to a gymnasium. In specification (1), I use as explanatory variables the following:

- Average grades from the last school report. School grades represent the main information on study aptitude for the student and his parents. Although the scale is the same in all the schools, the criteria differ across schools, which means that the information may be distorted. Students were divided into 4 categories by average

preferences more often than do wealthier ones, especially given my hypothesis that students with more educated and richer parents will value education more. Although I am not able to rule out the different-preferences effect completely, I present some evidence suggestive of the preference-misrepresentation effect.

grades in all the study subjects: average between 1 and 1.5 (1 is the best mark in the Czech Republic), 1.5 – 2, 2 – 2.5, and > 2.5. The first category was omitted.

- Average test score from the three PISA tests (mathematics, reading and problem-solving)

- The per capita family income; the PISA data set reports family income which I divide by the number of family members, counting both adults and children.

In specification (2) I use math grades and math test scores instead of the average grades and average test scores from all the study subjects. Specifications (3) and (4) are, respectively, the same as specifications (1) and (2), with the difference that dummies for parental education have been added. Parents were divided into three groups by the highest attained educational level: parents with apprentice school or elementary school,[29] parents with upper-secondary school finished by graduation, and parents with a university diploma. Specifications (5) – (8) are like specification (1) – (4) but with the school dummies included. In all the specifications, average test scores and income were normalized by subtracting the mean and dividing by the standard deviation in order to obtain marginal effects in terms of standard-deviation change.

Results of the probit estimation – the marginal effects on the probability of applying to a gymnasium – are reported in table 2.8 in the appendix. Results of specification (1) show that the effect of income is positive and significant. An increase in the per capita income of the family by one standard deviation increases the probability of applying to a gymnasium by 8.3%.

Having average grades between 1.5 and 2 decreases the probability of applying to a gymnasium by 17.6% compared to the omitted group with average grades between 1 and 1.5; average grades between 2 and 2.5 decreases the probability by 24.1%, and an average higher than 2.5 by 22.2%. A change in the average score in the three tests by one standard deviation (when evaluated at the mean test score) changes the probability by 10.2% (see row 6 in table 2.8). Both the school grades and the test score results are significant in determining whether the student applies to a gymnasium.

Using math grades and math test scores I obtain similar results (see specification 2 in table 2.8). The omitted category is a grade of 1 from math. Having a grade of 2 decreases the probability of applying by 13.9% and having a grade of 3 or worse by

[29]In the Czech Republic, elementary school also includes the lower-secondary level.

30.4%. Holding the grade constant, a change in math test score by one standard deviation changes the probability by 8.4%. The effect of income in this specification is also positive and significant although slightly smaller in magnitude: one standard deviation in per capita family income changes the probability of application by 7.1%.

When I include parental education as an explanatory variable (specifications 3 and 4), the effect of income decreases in magnitude but remains positive and significant. The educational dummies are significant by themselves and are quite large in magnitude. When at least one of the parents has a university education then the probability of applying to a gymnasium increases by 10% – 15%, everything else held constant. In this case, the fact whether parents have graduated from an apprentice school (or less) or a secondary school does not make much difference.

In specifications 5 – 8 I control for school dummies.[30] First, I observe that the effects of average grades and, although less so, math grades are larger. For example, a student with average grades between 2 and 2.5 is now 37.7% less likely to apply to a gymnasium than a student with average grades between 1 and 1.5, whereas without controlling for school dummies, the difference was only 24.1%. The reason is that, given average grades, students from better schools are more likely to apply to a gymnasium and the true (within-school) sensitivity to grades is thus larger than the sensitivity obtained when all the students are pooled together.

Family income also has a larger effect when controlling for school dummies.[31] A change by one standard deviation in income changes the probability of applying to gymnasium by 8.3% in specification (1) and by 5% in specification (3); the values are 14.5% and 11.1% in specifications (5) and (7), respectively. It is possible to explain this difference consistently with my hypothesis in that the observed effect of income is due to higher preference misrepresentation among the students from a weaker social background and not due to different preference ordering of these students, although this need not be the only possible explanation.

The argument is similar as for grades. First note that controlling for individual schools also means controlling for districts. Students from a district with higher gymnasium capacity (relative to true demand) will be more likely to apply to a gymnasium, given

[30] Because of the low number of observations, the school dummies are not jointly significant in specifications 5 (chi2(64) = 50.09, Prob > chi2 = 0.8983) and 7 (chi2(64) = 49.26, Prob > chi2 = 0.9128). They are, however, significant in specifications 6 (chi2(97) = 168.21, Prob > chi2 = 0.0000) and 8 (chi2(97) = 167.26, Prob > chi2 = 0.0000).

[31] This holds only in the specification using average grades and average scores; when using math grades and math scores, the coefficients remain almost the same.

family income, than students from a district with lower gymnasium capacity. In a relationship between income and probability of applying, controlling for districts means that each district will have a different intercept and the coefficient in the true (within district) relationship between income and probability will be higher.

A similar argument could be made for parental education which also has higher coefficients in specifications (5) and (7) than in specifications (1) and (3).[32] While having parents who completed secondary school decreases the probability of applying by 15.5% without controlling for school dummies, it decreases the probability by 20.7% when I control for school dummies.

Table 2.9 in the Appendix shows the fitted probabilities from a probit estimation that a student with given characteristics will apply to a gymnasium. In this table, students are divided into groups by parental education and, respectively, math grades, math test scores, average grades, and average test scores. This table shows that for students in the same ability group, parental education plays a crucial role. For example, when I look at math grades, a student with grade of 1 and at least one of the parents having a university education, has a probability of applying to a gymnasium of 0.675; parents with a secondary school education decreases the probability to 0.446; and apprentice or elementary school decreases it further to 0.340.

2.7 Discussion and conclusion

This paper presents an empirical evidence for two hypotheses, namely that the current admission procedure encourages students to misrepresent their preferences for schools, and that this preference misrepresentation more often concerns students from weaker socioeconomic backgrounds. In this final section I offer, on the one hand, some ideas about what kind of development in students' and schools' strategies might be expected if the procedure stays in its current form and, on the other hand, some suggestions for reform of the procedure.

Let me begin with recent trends in the behavior of schools and students. There seems to be a parallel between their behavior and the behavior, described in Roth (1991), of agents in other two-sided markets which use a non-centralized or an unstable matching mechanism. The parallel consists in the tendency toward unraveling, i.e the process of

[32] The difference is again significant only in specifications with average grades; with math grades, the coefficients are practically the same.

moving back the dates when matches are effectively agreed upon. The context in Roth (1991) is one of new physicians and surgeons being matched to hospitals; unraveling means that the date when matches are agreed upon occurs long before the completion of the study and moves still further to the back. The official matching mechanism is then used only to formalize the previously agreed matches.

A similar tendency has slowly emerged in the studied context. Indeed, some schools devised a way to sidestep the limitation to a single application. A school holds a "rehearsal" entrance examination before the deadline for submitting applications, formally to allow students to field-test their abilities. In reality, there is a common understanding between the school and the students that this is a real exam and the school uses it to select applicants. The school then also holds some form of entrance examination on the nation-wide standardized date, but for those who succeeded at the "rehearsal" this is just a formality. The advantage for those rejected is that they did not waste their official application on the single shot they have in the official admission procedure. As the "rehearsal" exam enables a school to choose the best students ahead of time, one can expect that the other schools will react similarly. More and more schools will likely use this strategy by administering "preliminary" examinations at ever earlier dates.

Unraveling is a consequence of the instability of the official mechanism. By agreeing on matches earlier, students and schools may achieve a matching with better properties than the one produced by the official mechanism but this matching is not likely to be optimal. For example, if the match is agreed too far ahead, neither schools nor students may know with certainty what their preferences will be at the time the match is to be consummated. The problem could be solved by introducing a stable mechanism. This is supported by the empirical evidence in Roth (1991) who shows that in markets that introduced a stable mechanism, unraveling was curtailed and high voluntary participation in the mechanism was achieved. One such mechanism is the Gale-Shapley (GS) matching mechanism, which the Boston school administration recently decided to use.

There are two concerns regarding the implementation of the GS mechanism. Administratively, the system could be more demanding than the current system, mainly because schools would have to be able to rank more students than they do now. This could be solved in one of two ways. Either a student would have to take an entrance exam at all the schools by which he would like to be considered, which is in fact the same as when applying to university,[33] or a common test, something like the SAT in the US, would be

[33] The similarity concerns only the possibility of the choice of multiple schools; Czech universities do

taken by all students and the schools would produce the rankings based on the results of this test. Students would also have to rank all the schools at which they would like to be considered, but since most students have some preference ordering for the schools, this should not be a problem. The additional administrative costs need not be high and given the importance of the secondary-school choice they are certainly worth taking.

The other concern is more fundamental and deals with the possibility of separation of the best students in the best schools. Under the current system it typically happens that some less talented students (who mustered the courage and were lucky) end up in a high-quality schools while some highly talented students end up in a worse school. In the GS mechanism, the variance of students' abilities within a single school would most likely decrease and the highly talented students would be more likely to cluster together. The mixing of students with different talents is viewed by some as advantageous because of the potentially positive peer effects. Although some mixing may be a legitimate aim, it could be achieved in a way which is more transparent, less unfair, and which does not induce preference manipulation. Quite simply, the over-demanded schools would be required to assign a certain share of their seats by lottery, while the rest would be assigned using the centralized matching mechanism.

not use any centralized mechanism.

2.A Appendix – tables and regression results

Table 2.1: Demand for and capacity of gymnasia (school data)

	First Round			Total		
Year	Applicants	Admitted	Excess	Applicants	Admitted	Excess
2002	0.157	0.107	1.465	0.235	0.120	1.953
	(0.061)	(0.038)	(0.245)	(0.097)	(0.044)	(0.338)
2003	0.162	0.116	1.378	0.240	0.129	1.849
	(0.067)	(0.040)	(0.225)	(0.098)	(0.046)	(0.305)
2004	0.155	0.114	1.363	0.233	0.126	1.835
	(0.059)	(0.036)	(0.230)	(0.095)	(0.041)	(0.342)
2005	NA	NA	NA	0.168	0.127	1.301
	NA	NA	NA	(0.073)	(0.045)	(0.201)

Source: Office for Educational Information (UIV). "Applicants" and "Admitted" are the shares of applying and admitted students on the population of 9th-grade students. "Excess" is the ratio of applying to admitted students. The values in parenthesis are standard deviations. Observations are weighted by the number of gymnasia in each district.

Table 2.2: Secondary school applications, 1st round (individual data)

Secondary School	Number of Applicants	Per cent
No Application	14	0
Apprentice School w/o Graduation	972	19
Apprentice School with Graduation	585	12
Vocational School	2592	51
Gymnasium	884	18
Total	5047	100

Source: PISA. The sample includes all the 9th grade students in the PISA sample with the exception of students of 8-year gymnasia

Table 2.3: Demand for public gymnasia (OLS and IV)

	OLS-02-04	OLS-02-05	IV-02-04	IV-02-05
	(1)	(2)	(3)	(4)
Capacity	1.205***	1.159***	1.213***	1.204***
	(.040)	(.035)	(.090)	(.082)
Private gym. in district	.012***	.008***	.012***	.008**
	(.004)	(.003)	(.004)	(.003)
Share Uni. Educ.	.215***	.299***	.211***	.282***
	(.045)	(.041)	(.053)	(.049)
University Distr.	.014***	.012***	.014***	.011***
	(.004)	(.004)	(.004)	(.004)
Const.	-.012**	-.017***	-.012	-.019***
	(.005)	(.004)	(.007)	(.007)
Obs.	225	300	216	288

*** significant at 1% level; ** significant at 5% level; * significant at 10% level. The dependent variable is the share of first-round applicants to gymnasia on the population of 9th grade students. The explanatory variables are the share of students (on the population of 9th grade students) admitted to gymnasia in the first round of the admission procedure (Capacity); the share of university educated population in a given district (Share Uni. Educ.); dummy for a university being present in the district (University Distr.); and dummy for a private gymnasium present in the district. Observations are weighted by the number of 9th grade students in respective districts. In the IV specifications, the Capacity variable is instrumented by the capacity from 1991.

Table 2.4: Demand for public gymnasia (OLS and IV, districts without private gymnasium)

	OLS-02-04	OLS-02-05	IV-02-04	IV-02-05
	(1)	(2)	(3)	(4)
Capacity	1.222***	1.167***	1.077***	1.075***
	(.037)	(.033)	(.101)	(.088)
Share Uni. Educ.	.436***	.479***	.579***	.572***
	(.122)	(.112)	(.172)	(.154)
University Distr.	-.004	-.002	-.004	-.002
	(.004)	(.004)	(.005)	(.004)
Const.	-.033***	-.033***	-.031***	-.032***
	(.011)	(.010)	(.012)	(.010)
Obs.	181	239	172	227

*** significant at 1% level; ** significant at 5% level; * significant at 10% level. The dependent variable and the independent variables (with the exception of the dummy for private gymnasia which falls out) are as in table 2.3 and so are the weights and the instruments.

Table 2.5: Demand for public gymnasia (OLS using distortion coefficient)

	OLS-02-04-DistC	OLS-02-05-DistC
	(1)	(2)
Distortion Coefficient	-.078***	-.081***
	(.006)	(.006)
Private gym. in district	.012*	.008
	(.006)	(.005)
Share Uni. Educ.	.553***	.633***
	(.074)	(.066)
University Distr.	.021***	.022***
	(.007)	(.006)
Const.	.165***	.159***
	(.010)	(.009)
Obs.	216	288

*** significant at 1% level; ** significant at 5% level; * significant at 10% level. The Distortion Coefficient is defined in the following way: $(POP15_{200X}/POP15_{1991})/(CAPG_{200X}/CAPG_{1991})$, where $POP15_{year}$ stands for the size of 15-year-old population in the respective year and $CAPG_{year}$ stands for the capacity of public gymnasia in the respective year. The higher the value of the coefficient, the tougher the competition for public gymnasia became since 1991 and the higher the distortion to the true demand.

Table 2.6: Demand for public gymnasia (fixed and random effects)

	Fixed-02-04	Random-02-04	Fixed-02-05	Random-02-05
	(1)	(2)	(3)	(4)
Capacity	1.122***	1.176***	1.060***	1.125***
	(.062)	(.001)	(.052)	(.001)
Private gym. in district	-.004	.012***	.004	.008***
	(.021)	(.0001)	(.021)	(.0001)
Share Uni. Educ.		.228***		.313***
		(.002)		(.001)
University Distr.		.014***		.012***
		(.0001)		(.0001)
Const.	.072***	-.010***	.075***	-.015***
	(.024)	(.0002)	(.022)	(.0002)
Obs.	225	225	300	300

*** significant at 1% level; ** significant at 5% level; * significant at 10% level. The dependent variable and the independent variables are as in table 2.3 and so are the weights. Results of the Hausman specification test for fixed versus random effects procedure: Chi2(2) = 0.49 and P-value = 0.781 for the period 2002-2004; Chi2(2) = 1.7 and P-value = 0.427 for the period 2002-2005.

Table 2.7: Changes in population size (fixed effects)

	OLS-All	OLS-No-Private
	(1)	(2)
Capacity	.630***	.946***
	(.050)	(.063)
Cohort Size	-.102***	-.135***
	(.024)	(.032)
Const.	.275	.197**
	(.258)	(.078)
Obs.	225	181

*** significant at 1% level; ** significant at 5% level; * significant at 10% level. The dependent variable is the share of first-round applicants to gymnasia on the population of 9th grade students. The independent variables are the absolute capacity and the size of the student population.

Table 2.8: Individual application behavior – marginal effects from probit estimation

	Specification							
Explanatory Var.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Avg. Mark 1.5-2	-0.176		-0.171		-0.188		-0.186	
	(0.032)		(0.032)		(0.050)		(0.050)	
Avg. Mark 2-2.5	-0.241		-0.237		-0.377		-0.368	
	(0.029)		(0.029)		(0.044)		(0.045)	
Avg. Mark ≥2.5	-0.222		-0.214		-0.312		-0.293	
	(0.026)		(0.026)		(0.036)		(0.037)	
Math Mark 2		-0.139		-0.133		-0.149		-0.143
		(0.018)		(0.018)		(0.019)		(0.019)
Math Mark ≥3		-0.304		-0.293		-0.325		-0.316
		(0.017)		(0.017)		(0.019)		(0.019)
Avg. Score	0.102		0.099		0.099		0.105	
	(0.022)		(0.022)		(0.038)		(0.037)	
Math Score		0.084		0.075		0.075		0.067
		(0.010)		(0.010)		(0.011)		(0.011)
Income	0.083	0.071	0.050	0.050	0.145	0.065	0.111	0.047
	(0.018)	(0.008)	(0.020)	(0.008)	(0.035)	(0.009)	(0.035)	(0.009)
P. Educ. Low			-0.120	-0.127			-0.166	-0.118
			(0.039)	(0.019)			(0.060)	(0.020)
P. Educ. Med.			-0.155	-0.102			-0.207	-0.110
			(0.041)	(0.019)			(0.064)	(0.020)
Sch. Dummies	NO	NO	NO	NO	YES	YES	YES	YES
Observations	544	2427	544	2427	399	2328	399	2328

Grades are taken from the last school report given to a student. The average mark is the average from the following subjects: Mathematics, Czech Language, Foreign Language, Physics, Chemistry, Biology and Geography. Students are divided into 4 categories by average mark, the omitted category is the one with best average mark (1-1.5). Test scores are taken from the PISA tests. The average score is the average of math, reading and problem solving test scores. Both math and average test scores are normalized by subtracting mean and dividing by standard deviation. Income is the per capita family income, normalized in the same way as test scores. Parents were divided into three categories by their maximum educational level: university (high), secondary school with graduation (medium), and apprentice or elementary school (low). If the education differed between the two parents, the education level of the more educated parent was taken as relevant. The "high" category was omitted.

Table 2.9: Probability of applying to gymnasium

	Parental Education			Parental Education		
Math Mark	Low	Medium	High	Low	Medium	High
1	0.340	0.446	0.675	0.311	0.445	0.670
2	0.137	0.227	0.433	0.121	0.213	0.428
3	0.020	0.040	0.111	0.019	0.039	0.107
School Dummies	Yes	Yes	Yes	No	No	No

	Parental Education			Parental Education		
Math Test Score	Low	Medium	High	Low	Medium	High
> 1	0.300	0.397	0.640	0.269	0.389	0.629
$0 - 1$	0.139	0.221	0.404	0.129	0.208	0.405
$-1 - 0$	0.054	0.111	0.236	0.053	0.109	0.232
< -1	0.030	0.046	0.082	0.019	0.041	0.079
School Dummies	Yes	Yes	Yes	No	No	No

	Parental Education			Parental Education		
Average Mark	Low	Medium	High	Low	Medium	High
1-1.5	0.474	0.512	0.801	0.374	0.435	0.746
1.5-2	0.196	0.208	0.532	0.108	0.152	0.434
2-2.5	0.046	0.035	0.109	0.018	0.022	0.116
>2.5	0.007	0.029	0.023	0.006	0.011	0.053
School Dummies	Yes	Yes	Yes	No	No	No

	Parental Education			Parental Education		
Avg. Test Score	Low	Medium	High	Low	Medium	High
> 1	0.618	0.516	0.724	0.437	0.441	0.703
$0 - 1$	0.186	0.231	0.578	0.119	0.199	0.477
$-1 - 0$	0.098	0.118	0.241	0.058	0.084	0.201
< -1	0.009	0.050	0.155	0.003	0.010	0.091
School Dummies	Yes	Yes	Yes	No	No	No

The figures in the tables are fitted values from the probit regression. The parental education categories are the same as in table 2.8. The math test score and average test score categories divide the students into groups by average deviation distance from mean. For example, category >1 includes students with test score by more than 1 standard deviation better than average.

Chapter 3

Bankruptcy Regimes and Gambling on Resurrection

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Abstract

We develop a model of a debt contracting problem under bankruptcy regimes differing by a degree of softness. In the model, the degree of softness is associated with the extent to which the absolute priority rule can be violated. We show that when the degree of softness can be set individually for each project, then the debtor's tendency to excessive risk-taking can be eliminated and the first best solution can be attained. When it is given exogenously by a bankruptcy law, then a completely tough law results in a lower distortion from the first best than a soft law with a moderate degree of softness.

Keywords: corporate bankruptcy, debt contracts, ex ante efficiency
JEL classification codes: G33, K12, K39

3.1 Introduction

A question often debated among bankruptcy scholars is whether firm value in bankruptcy should be divided in accordance with the absolute priority rule (APR). In particular, the issue is whether it is optimal that shareholders receive some payoff only after all the creditors have been paid in full. In the bankruptcy literature,[1] bankruptcy laws are usually divided into tough and soft, depending on how a firm's management is treated. But because providing the management with a favorable treatment is usually associated with more APR violations, we use the categories "tough" and "soft" to denote whether the law enables APR violations (soft law) or prevents them (tough law).

In this paper, we analyze the debt contracting problem in the presence of *gambling on resurrection*, i.e. debtor's excessive risk taking in the situation of privately observed high probability of financial distress, under different bankruptcy regimes.[2] We first show that if the degree of softness can be determined endogenously, i.e. agreed upon between the debtor and the creditor, the first-best solution can always be attained. In reality, however, this is not possible due to the multiplicity of creditors with different seniority levels, for each of which the optimal degree of softness would be different. We then examine the situations, in which, as in practice, the degree of softness is given exogenously by the bankruptcy law. A sufficiently soft law can eliminate the debtor's moral hazard problem and leads to optimal investment level, though at the cost of higher interest rates. Under a law that is insufficiently soft, however, this moral hazard problem gets even worse than under a completely tough law. We also show that the gambling on resurrection argument for soft law is further weakened if a possibility for creditors to verify the firm's situation is introduced.

In the literature, one may find arguments both in favor of soft law and tough law.[3] In line with the claim of Hart (2000) that there is no "one size fits all" solution in bankruptcy

[1] For an up-to-date survey of the economic literature on both personal and corporate bankruptcy see White (2007). In our paper, we deal with corporate bankruptcy only.

[2] The debtor's gambling on resurrection is necessarily accompanied with misreporting (not confessing the observed situation to creditors). Infamous examples of such "cooking of books" prior to the failure include the cases of Enron's top management prior to the company's bankruptcy, or Mr. Bernard Madoff within his Ponzi scheme of about $ 50 bn. On the sovereign level, one may argue about the Greek government's alleged misreporting on its fiscal situation. A clear example from economies that have transformed from plan to market within the last two decades is the behavior of the top management and some shareholders of Investicni a Postovni Banka few years prior to the third largest Czech bank's failure in 2000 which costed Czech tax payers about $ 8 bn.

[3] For a summary of some *pros* and *cons* of soft and tough bankruptcy laws see Knot and Vychodil (2005).

legislation, one may say that each of these *pros* and *cons* is of different relevance and strength in different countries.[4]

One of the ex ante efficiency arguments for soft law has been the *gambling on resurrection* hypothesis, which states that under APR, debtors tend towards excessive risk-taking and delaying bankruptcy filing once they privately observe that they are on the verge of bankruptcy. Violation of APR is believed to suppress this type of moral hazard problem by giving shareholders a positive payoff even though the creditors are not paid in full. In this paper, we show that this is not generally valid and should thus be viewed with caution.

The most discussed example of a law enabling APR violations has been Chapter 11 of the U.S. Bankruptcy Code of 1978. Although *de iure* APR is supposed to hold in Chapter 11, vast empirical evidence has been collected to support the hypothesis that *de facto* APR is violated in bankruptcy cases under Chapter 11.[5] After a bankruptcy filing, the automatic stay prevents creditors from further debt collection efforts, the management has an exclusive position to present a plan of reorganization, and the consent of a class of creditors with the plan can be replaced, under the cram down procedure, by a court decision. These are just examples of rules that enable the management to enforce APR violations on the creditors. Certainly, the U.S. case is just one of many and we can observe very different bankruptcy laws around the world with different degrees of APR violation.

There are several papers similar to our paper. In the model of Bebchuk (2001) the APR violations increase the distortions of management's decision-making in favor of risky projects (they worsen the gambling on resurrection problem). In our model, too, we observe this effect for certain parameter values. Unlike Bebchuk, however, we find that under different parameter values a soft law can actually eliminate the gambling on resurrection problem and lead to optimal investment level. The law, however, needs to be "soft enough" to have this effect.[6]

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[4]Some authors explicitly studied various country-related specific factors that should be taken into account when designing an optimal bankruptcy law. For instance, Baird and Rasmussen (2002) and Baird and Rasmussen (2003) stress the importance of capital structure and the functioning of asset markets, Berkovitch and Israel (1998) emphasize information structure, while Lambert-Mogiliansky, Sonin, and Zhuravskaya (2003) and Biais and Recasens (2002) study the effects of corruption among judges.

[5]See, e.g., Franks and Torous (1989), Eberhart, Moore, and Roenfeldt (1990), LoPucki and Whitford (1990), Weiss (1990), Bebchuk and Picker (1993), Franks and Torous (1994), Betker (1995), Longhofer and Carlstrom (1995), Weiss and Wruck (1998), Carapeto (2000).

[6]The difference between our model and Bebchuk's comes from the fact that Bebchuk assumes the project characteristics are given *ex ante* and are private information of the firm. In Bebchuk's model,

Another related paper is Bester (1994). In Bester's model, the low state automatically implies default, while in the high state the debtor can either repay or default strategically. Thus the high-type debtor might pretend to be a low-type and the creditor cannot distinguish between financial and strategic default. In our model, instead, we focus on the situation of the low-type pretending to be the high-type and of the creditor's lowered ability to distinguish between success-driven continuation and gambling on resurrection.

Finally, in the model of Povel (1999), the debtor also receives a private signal on the project's type, unobservable by creditors, and decides either to file for bankruptcy or continue running the firm. Nonetheless, in Povel's model the debtor, in addition, chooses her effort level between the initial financing period and receiving the signal. The main idea of that model lies in the trade-off between incentives to invest effort and incentives to reveal private information about the project's type. Soft bankruptcy law worsens the former while improving the latter. In our model, we assume away the effort choice and show that even the pure effect of the law's softness on incentives to reveal true information is twofold. Under some circumstances, softening bankruptcy law strengthens the debtor's motives for gambling on resurrection and misreporting.

The model we present in this paper is, we believe, both realistic and tractable. In general, it draws the connection between financial contracting and bankruptcy law. More specifically, it allows – among other things – for inspecting the links between bankruptcy law design, credit rationing, company's misreporting, cost of monitoring, profitability of projects, and size of firms. An important part of the paper are simulations showing, for each of the bankruptcy regimes, the sensitivity of the individual variables to parameter changes.

The paper is structured as follows. The following section describes the setup of the model and defines contracts and strategies. Section 3.3 analyzes the benchmark situation when the degree of softness of bankruptcy law is specified endogenously in the contract. In the fourth section we treat the degree of softness as an exogenous parameter and analyze the effects of different values of this parameter on investment level, interest rate and strategy choice. Section 3.5 introduces the possibility for the creditor to verify, with a certain cost, the debtor's report. The sixth section considers what happens when we allow the parties to renegotiate the contract in period 1. Section 3.7 concludes.

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once the project is started, there are no more decisions concerning its characteristics. On the other hand, we assume that the project's characteristics are common knowledge at the time when the project is financed. Only after that, the debtor privately learns information about how the project's chances to succeed changed and may choose a risky or a safe strategy.

3.2 The model

3.2.1 Setup

We study a relationship between a debtor, who owns a firm,[7] and a bank. We assume the debtor is both the firm's owner and its manager and has limited liability. The debtor has an opportunity to undertake a profitable project and needs financing from the bank in order to do so. Bank credit is the only source of financing for the debtor.[8] The initial investment in the project is determined by both parties. During the life of the project the debtor receives private information about the probability of the project's success. The information may be either good or bad. The project's characteristics are such that it is optimal to continue if the information is good (the probability of success is high) and to quit if the information is bad (the probability of success is low). The incentives of the debtor, however, may be to continue the project even if the information is bad.

The project, if successful, can bring $\beta(K)$ where K is a non-negative initial investment. We assume a particular form of $\beta(K)$, namely $\beta(K) = B \ln(K+1)$ where $B > 0$. Note that $\beta(0) = 0$, $\beta'(K) > 0$, and $\beta''(K) < 0$. The decreasing returns imply that there is an optimal level of investment, one at which the marginal benefit equals the marginal cost. An important property of different bankruptcy regimes that we examine is whether they induce the optimal investment level. The whole investment K is financed by debt which means that the debt level equals the size of investment. In exchange for the provided financing, the bank is promised to obtain $(1+r)K$ at the end of the game, unless the firm goes bankrupt. We assume the risk-free interest rate is zero. The credit market is competitive which means that, in equilibrium, the bank's expected profit will be zero and the debtor of the firm will capture all the surplus from the relationship which also means that the debtor's expected profit is a perfect measure of the social gain from the project.

The relationship extends over three periods. In period 0 a credit contract is signed and investment is realized. The contract specifies the principal K, the interest rate r, and the strategy to be followed in period 1. In period 1 the debtor receives private information about the state of the world, either truthfully or untruthfully reveals it to the creditor, and decides on a further strategy – either continue running the project (strategy S_C) or quit the project (strategy S_Q). In period 2 outcomes are realized and returns divided

[7] Under a firm we understand primarily a collection of assets used in a particular business.

[8] This assumption is common in existing models on ex ante effects of a bankruptcy law and does not limit the validity of the model's implications.

according to the contract and, in the case of bankruptcy, according to the bankruptcy law.

There may be two states of the world in period 1, the good state (H) and the bad state (L), with probabilities p and $(1-p)$, respectively, where $0 < p < 1$. If the debtor decides to quit the project (strategy S_Q), a recovery value γK, where $0 < \gamma < 1$, is obtained with certainty, no matter whether the state of the world is H or L. If the debtor decides to continue the project (strategy S_C) in state H, the project continues and yields the good outcome, $B \ln(K+1)$, with certainty. However, in state L, strategy S_C results in the good outcome, $B \ln(K+1)$, only with probability π, and in the bad outcome, 0, with probability $1-\pi$, where $0 < \pi < 1$. For a project that had been financed in period 0 we must have $B \ln(K+1) > \gamma K$, otherwise the project would always be liquidated in period 1 and would have never been financed in period 0. Therefore, if the debtor observes that the state of the world is H, she continues the project for sure. The only decision node regarding the choice between strategy S_C and strategy S_Q is thus in state L.

The firm's value before the start of the project is $V > 0$. This can be thought of as the value of the assets the firm possesses and that may serve as collateral.

Throughout the paper, besides providing analytic derivations of optimal contracts under different legal and institutional setups, we illustrate these contracts by simulations on a numerical example with parameters given as $p = 0.6$, $\pi = 0.2$, $\gamma = 0.65$, and $V = 1$, unless stated otherwise. Graphical representations of these simulations are given in Appendix 3.A.2.

3.2.2 Contracts and strategies

Both the debtor and the bank are risk-neutral agents who maximize their expected profits. A strategy $S_i \in \{S_C, S_Q\}$ is the debtor's decision whether to continue (S_C) or quit (S_Q) the project in state L. A contract is a triple $\{K, r, S_i\} \in \Re^2_+ \times \{S_C, S_Q\}$. The bank lends the firm K in period 0 and the debtor promises on behalf of the firm to repay $(1+r)K$ in period 2. The debtor also commits to follow strategy S_i in period 1 if state L occurs. Denote the debtor's and the bank's expected profit in period t as $F_t(K, r, S_i)$ and $G_t(K, r, S_i)$, respectively, where $t = 0, 1$.

A contract $\{K, r, S_i\}$ is *incentive compatible* if in period 1 – when the debtor decides whether to quit or continue – $F_1(K, r, S_i) \geq F_1(K, r, S_j), i \neq j$. A contract is *feasible* if it is incentive compatible and $G_0(K, r, S_i) \geq 0$. The debtor's maximization problem has, thus, the following form:

$$\max_{(K,r,S_i)\in\Re^2_+\times\{S_C,S_Q\}} F_0(K,r,S_i) \tag{3.1}$$

s.t.

$$F_1(K,r,S_i) \geq F_1(K,r,S_j), \qquad i \neq j, \tag{3.2}$$

$$G_0(K,r,S_i) \geq 0. \tag{3.3}$$

In period 1 the debtor privately learns the state of the world, reports it to the creditor, and chooses between strategies S_C and S_Q. The creditor cannot observe the state of the world, but only observes the choice of strategy.[9] If the period 1 state of the world is H, there is no moral hazard as continuation (S_C) is the optimal strategy for both the debtor and the creditor. Thus, if state H occurs, the project continues smoothly to period 2. If the period 1 state is L and the contract requires the debtor to follow S_Q, then, for certain levels of K and r, the debtor has an incentive to misreport (i.e., report state H) and to follow S_C.

This is where our model differs from the previous literature, which usually defines a good state as a realization of high cash flows which the debtor can divert instead of paying to the lender.[10] There the principal-agent problem is particularly salient in the good state. Our model explores the case – more common in reality, we believe – in which the moral hazard problem occurs in the bad state, in which the debtor is tempted to continue and gamble on resurrection, while the optimal solution is to quit.

A crucial parameter of our model, in addition to those defined above, is the degree of softness of bankruptcy law, α, satisfying $0 \leq \alpha \leq 1$. This parameter determines the fraction of the residual value of a bankrupt firm that is captured by the debtor. Thus, $\alpha = 0$ means completely tough law and the higher the value of α the softer is the law. Bankruptcy laws do not specify α explicitly and it is, therefore, not possible to find its value directly in the laws themselves. α is rather a consequence, sometimes unintended, of the way a law is written and applied by the courts. An important factor affecting α is whether the law enables for reorganization and under what circumstances. Whereas in liquidation, most bankruptcy laws ensure (at least on the paper) full adherence to

[9]Formally, the assumption that the report will be made by the debtor seems redundant, but it will become utilized later in the treatment with verification. In fact, here we assume that the debtor can report untruthfully without any risk of detection because the cost of verification is infinitely high.

[10]See, e.g., Bester (1994), Berglof and von Thadden (1994), Bolton and Scharfstein (1996), Hart and Moore (1998), and Berglof, Roland, and von Thadden (2003).

the absolute priority rule (APR), deviations from APR are typically possible in reorganization, which means that the equity holders can retain part of the value even though the creditors are not repaid in full. Another factor affecting α is the way the control of the bankruptcy process is split between the debtor, the creditors and the court. When the debtor retains some control, she may be able to capture some value even though she should receive nothing according to strict interpretation of the law.

Even though α cannot be found directly in the bankruptcy law, there is some empirical evidence concerning its value. A number of studies have estimated the size of deviations from the APR under the U.S. Chapter 11 (reorganization chapter), typically measured – following Franks and Torous (1989) – as the amount paid to equity divided by the amount distributed to creditors under the reorganization plan. This measure, denoted as ΔAPR, slightly differs from the degree of softness α, which represents the share of the amount paid to equity on the total amount paid to equity and creditors. Thus, to translate ΔAPR to α, one needs to use $\alpha = \frac{1}{1+\frac{1}{\Delta}}$. Using this transformation it can be stated that, for example, Eberhart, Moore, and Roenfeldt (1990) found an average degree of softness in their sample of 7.0%, ranging from 0% to 26.5%, while Betker (1995) saw it in another sample at 2.8%.[11]

In the following two sections, we solve the model for two different setups: one in which α is determined endogenously within the contract (section 3.3) and another in which α is given exogenously by the law (section 3.4).[12]

As we will show below, the setup with α specified endogenously weakly dominates, in terms of social welfare, the setup with α specified exogenously in the bankruptcy law. The reason is that the law will necessarily fix α at some constant level, which will not be optimal for most of the projects in the economy. Within the boundaries of this paper, it would, therefore, be better to leave it on the parties to specify the degree of softness themselves, setting it at a level optimal for the project in question and avoiding the application of the law altogether.[13] The law, however, exists also for other reasons than specifying the ratio, in which the value is divided between the debtor and the creditor(s).

[11]In terms of ΔAPR, the estimate of an average deviation from the APR by Eberhart, Moore, and Roenfeldt (1990) was 7.5%, ranging from 0% to 36%, while that of Betker (1995) was 2.9%. A summary of similar empirical observations can be found in White (2007).

[12]For a discussion of the possibility of voluntary contracting for the violation of APR in the case of bankruptcy, see Povel (1999) and Schwartz (1998).

[13]Specifying the exactly optimal α in a contract is, however, not possible either. This is, among others, due to complex and constantly changing debt structure (multiple creditors with different seniorities), changing values of underlying parameters and impossibility to measure some of the underlying parameters.

Its main function is to provide a collective framework for the resolution of a firm's debts in a situation when its assets are less than its liabilities and, as such, it also determines the rules for the division of the firm value, i.e. specifying the degree of softness.[14]

3.3 Endogenous choice of the degree of softness

Assume, first, that the contract in period 1 specifies α in addition to the principal (K), the interest rate (r), and the strategy (S_i). When α is endogenous, then α, K and r will always be set at such levels that the debtor prefers the socially optimal strategy S_Q in state L because this maximizes his expected profit. The debtor's maximization problem then is

$$\max_{K\geq 0,\, r\geq 0,\, 0\leq\alpha\leq 1} \quad p[V + B\ln(K+1) - (1+r)K] + \\ +(1-p)\max\{V+\gamma K-(1+r)K;\alpha(V+\gamma K)\} \tag{3.4}$$

s.t.

$$\text{ICC:} \quad \max\{V+\gamma K-(1+r)K;\alpha(V+\gamma K)\}\geq \\ \geq \pi[V+B\ln(K+1)-(1+r)K]+(1-\pi)\alpha V, \tag{3.5}$$

$$\text{PC:} \quad p(1+r)K+(1-p)\min\{(1+r)K;(1-\alpha)(V+\gamma K)\}-K\geq 0. \tag{3.6}$$

When designing the contract, the debtor maximizes her expected payoff formulated in the maximand. With probability p, state H will occur in period 1 and the project will continue till period 2. Then the debtor retains the value of the firm's assets (independent of the project) and the project's upside payoff and is able to repay the whole debt.[15] With probability $(1-p)$, state L occurs and the debtor quits the project. Depending on what yields her higher payoff, she chooses between *out-of-bankruptcy liquidation*, which gets her full value of the firm minus the value of the debt, and *filing for a bankruptcy*

[14] The main justification for the law's existence is muting creditors' incentives to race to be the first to collect. As White (2007) puts it: *"When creditors realize that a debtor firm might be insolvent, they have an incentive to race against each other to be first to collect. This is because, as in a bank run, the earliest creditors to collect will be paid in full, but later creditors will receive nothing. The race to be first is inefficient, since the first creditor to collect may seize assets that the firm needs for its operations and, as a result, may force the firm to shut down. Early shutdown wastes resources because the piecemeal value of the firm's assets may be less than their value if the assets are kept together and the firm sold as a going concern."*

[15] Note that if the occurrence of state H had not implied full repayment in period 2, the creditor would not have been willing to lend in period 0.

reorganization, which frees her from the full debt repayment and gets her fraction α of the total remaining firm's value.

Obviously, the debtor's payoff after quitting the project, both from out-of-bankruptcy liquidation and from bankruptcy reorganization, must be higher than her expected profit from continuation. The incentive compatibility constraint (3.5) assures that the debtor will never gamble on resurrection in state L. The gamble would have got him the upside payoff with probability π and fraction α of the firm's assets with probability $(1-\pi)$. Since α is endogenous, the contract sets it at such a level that incentivizes the debtor to choose the first best strategy, S_Q, in state L. Such α can always be found as the creditor is willing to accept higher α when compensated by a higher interest rate. From the creditor's participation constraint (3.6) we can see that in the extreme case of $\alpha = 1$, the interest rate would reach $r = \frac{1-p}{p}$.

Let us first consider what the first best solution looks like. The maximization problem in this case is

$$\max_{K \geq 0} \; V + pB\ln(K+1) + (1-p)\gamma K - K \tag{3.7}$$

and the first best level of K is

$$K^{FB} = \frac{pB}{1-(1-p)\gamma} - 1. \tag{3.8}$$

K^{FB} is the level of K that generates the highest surplus. Because the debtor has all the bargaining power and captures all the ex ante surplus, she would like to set $K = K^{FB}$. This will, therefore, be the optimal level of K with α and r adjusted to satisfy the ICC (3.5) and PC (3.6). Because the debtor's maximization problem does not lead to a unique solution for α and r – higher α implies higher r, but the optimal level of K and the ex ante expected profits of the debtor and the creditor remain the same – we assume that they are both set to the minimum level still satisfying the constraints. If $\alpha = 0$ and $r = 0$ satisfy the ICC for $K = K^{FB}$, then these are the optimal values. Whether this is possible depends on the model parameters. In particular, consider parameter B, which can be thought of as the project's upside or profitability, and denote the maximum value for which $\alpha = 0$ and $r = 0$ is compatible with the ICC as B_1. With these assumptions and notation the solution to the debtor's maximization problem can be shown to take on the values stated in the following proposition.

Proposition 1. The solution to the debtor's maximization problem with endogenous determination of α is

$$\begin{aligned}
K^{en} &= \frac{pB}{1-(1-p)\gamma} - 1 \quad = K^{FB}, \\
r^{en} &= \begin{cases} 0 & \text{if} \quad B \leq B_1, \\ \frac{1-p}{p}\left[1-(1-\alpha^*)\frac{V+\gamma K^{FB}}{K^{FB}}\right] & \text{otherwise,} \end{cases} \\
\alpha^* &= \begin{cases} 0 & \text{if} \quad B \leq B_1, \\ \frac{V+pB\ln(K^{FB}+1)-[1-(1-p)\gamma]K^{FB}}{V+\frac{p+(1-p)\pi}{\pi}\gamma K^{FB}} & \text{otherwise,} \end{cases} \\
S &= S_Q.
\end{aligned} \tag{3.9}$$

The main point of the previous proposition is that when the degree of softness can be determined freely in the contract, the first best solution (level of investment and the optimal strategy choice) can always be attained. When the project is not too profitable ($B < B_1$), a completely tough law ($\alpha = 0$) will produce the first best solution. The debtor is never tempted to continue the project in the bad state and the creditor is always repaid in full, which means he is willing to accept $r = 0$. When, on the other hand, the project's profitability exceeds a certain threshold ($B \geq B_1$), the debtor needs to be incentivized to liquidate the project in the bad state by receiving a fraction of the firm's residual value. The creditor is not always repaid in full and needs to receive positive r in order to satisfy his participation constraint. The threshold B_1 depends positively on the firm's value V and the degree to which the project assets can be re-deployed elsewhere (γ). It depends negatively on the probability of the good state (p) and on the probability of project success in the bad state (π). The negative dependence on p results from the dependence of K^{FB} on p: higher p leads to higher K^{FB}, which increases the value of the project and makes it more tempting for the debtor to continue in the bad state.

In Appendix 3.A.2 we demonstrate the dependence of K, α and r on the model's parameters. The optimal investment K is linearly increasing in the project upside parameter B. α and r are discontinuous functions of B. They both equal zero as long as the ICC (3.5) can be satisfied for $K = K^{FB}$, $r = 0$, $\alpha = 0$; they both jump up discontinuously when this is no longer possible.

3.4 Exogenously given degree of softness

The framework developed in the previous section allows us to analyze bankruptcy laws with various degrees of softness. In this section, we assume that α is given exogenously. We begin with the tough law case where $\alpha = 0$ and continue with the analysis of the soft law case where $\alpha > 0$.

3.4.1 Tough bankruptcy law

Under the tough law regime, α is exogenously set to 0, which means that the debtor receives nothing whenever the creditor is not paid in full. Only the choice of K and r in period 0 can be used to affect the debtor's decision on which strategy to choose in period 1. For the extensive form representation of the game under tough bankruptcy law see Figure 3.1 in Appendix 3.A.1.

There are two possible situations. First, the level of K is such that the debtor prefers S_Q in state L. In this case the debt is riskless and $r = 0$. The reason is that for such a contract to be feasible the debtor must obtain some payoff after quitting the project which, given $\alpha = 0$, also means that the bank will be repaid in full. Given that the debtor can credibly commit to strategy S_Q, the maximization problem becomes

$$\max_{K \geq 0} \{V + pB\ln(K+1) + (1-p)\gamma K - K\} \tag{3.10}$$

s.t.

$$\text{ICC:} \quad V + \gamma K - K \geq \pi[V + B\ln(K+1) - K]. \tag{3.11}$$

Second, the level of K is such that the debtor prefers S_C in state L, which means that full repayment is no longer guaranteed (the project fails with probability $1-\pi$) and, hence, r will be positive to compensate the bank for the risk. Because of the credit market competitiveness and the risk-neutrality assumption, the interest rate will only ensure that the bank will just break even in expected terms and its expected profit will be zero. Given that the debtor prefers strategy S_C in state L, the maximization problem becomes

$$\max_{K \geq 0, r \geq 0} \{[p + (1-p)\pi][V + B\ln(K+1) - (1+r)K]\} \tag{3.12}$$

s.t.

$$\text{PC:} \quad [p + (1-p)\pi](1+r)K + (1-p)(1-\pi)V - K \geq 0. \tag{3.13}$$

In the former maximization problem, the participation constraint would be redundant as the creditor gets repaid for sure and is willing to lend at a riskless interest rate. On the other hand, in the latter problem, the incentive compatibility constraint is not needed because the contract involving S_C becomes optimal only when the distortion associated with satisfying the ICC for S_Q is so large that it becomes too costly to deter the debtor from the choice of the risky strategy S_C. Such a contract will automatically involve $r > 0$.[16]

We denote the contracts that solve the two problems as $\{K_Q^T, 0, 0, S_Q\}$ and $\{K_C^T, r_C^T, 0, S_C\}$, respectively. The superscript T stands for tough law and the subscripts Q and C for, respectively, quitting and continuation. The debtor decides which of the two types of contract to offer to the creditor. Thus she compares her ex ante payoff from the contract $\{K_Q^T, 0, 0, S_Q\}$ with that from the contract $\{K_C^T, r_C^T, 0, S_C\}$.

Quitting the Project in the Bad State

When the optimal contract is the one involving quitting in state L, then the problem given by (3.10) and (3.11) can be solved in the following way:

- As long as the ICC (3.11) is not binding for $K = K^{FB}$, the first best can be implemented: $\{K, r, \alpha, S_i\} = \{K^{FB}, 0, 0, S_Q\}$.

- When the ICC (3.11) is binding for $K = K^{FB}$ (K^{FB}, is too large for S_Q to be incentive compatible), we need to decrease K below its first best level to K_Q^T given by

$$(\gamma + \pi - 1)K_Q^T - \pi B \ln(K_Q^T + 1) + (1-\pi)V = 0. \tag{3.14}$$

  Although the solution cannot be obtained in the closed form, it can be shown that

$$\frac{\partial K_Q^T}{\partial V} > 0; \qquad \frac{\partial K_Q^T(\pi)}{\partial \pi} < 0; \qquad \frac{\partial K_Q^T(B)}{\partial B} < 0; \qquad \frac{\partial K_Q^T}{\partial p} = 0. \tag{3.15}$$

[16] This can be shown as follows. Suppose the debtor can repay the bank in full even after the project fails and there is only V left. The debtor thus remains in the residual claimant position in all the situations that may occur which rules out the gambling on resurrection type of moral hazard. Absent this type of moral hazard, the debtor would always choose the socially optimal strategy, which is S_Q.

Thus, under the tough law, given strategy choice S_Q,

$$K_Q^T = \begin{cases} \frac{pB}{1-(1-p)\gamma} - 1 & = K^{FB} \quad \text{if (3.11) is not binding,} \\ K_Q^T \text{ given by (3.14)} & < K^{FB} \quad \text{otherwise.} \end{cases} \tag{3.16}$$

Continuing the Project in the Bad State

When the optimal contract involves project continuation in state L, we can derive the following solutions to the problem given by (3.12) and (3.13):

$$K_C^T = [p + (1-p)\pi]B - 1 \quad = K_C^{FB} < K^{FB}, \tag{3.17}$$

$$r_C^T = \frac{1 - [p + (1-p)\pi]}{p + (1-p)\pi} \left(1 - \frac{V}{[p + (1-p)\pi]B - 1}\right) > 0 \tag{3.18}$$

where K_C^{FB} is the first best K *given that the project is always continued in period 1.* In this case, the investment level is smaller than the first best level and there is always a positive interest rate.

Optimal Contract under Tough Law

Ex ante the debtor decides which of the two types of contract to offer to the creditor. Thus she compares her ex ante payoff from the contract $\{K_Q^T, 0, 0, S_Q\}$ with that from the contract $\{K_C^T, r_C^T, 0, S_C\}$. The debtor will prefer the latter contract to the former, iff

$$\begin{aligned} &[p + (1-p)\pi][V + B\ln(K_C^T + 1) - (1 + r_C^T)K_C^T] > \\ &\quad > V + pB\ln(K_Q^T + 1) + (1-p)\gamma K_Q^T - K_Q^T. \end{aligned} \tag{3.19}$$

If the debtor could always commit to S_Q in the contract, she would prefer this strategy ex ante and set the investment level to $K = K^{FB}$ which would maximize her ex ante expected payoff. However, for parameter values such that ICC (3.5) is violated for $K = K^{FB}$, the debtor would not honor the commitment. Thus, in order to make the commitment to S_Q incentive compatible, we need to have $K_Q^T < K^{FB}$, which leads to a debtor's profit smaller than the first best social gain. If this dead-weight loss becomes large enough, it is no longer optimal to decrease K any further. At this point, giving the debtor incentives to choose S_Q becomes more costly than accepting the choice of S_C. This means that S_C becomes the optimal strategy with investment level $K = K_C^T$ and interest rate $r = r_C^T$.

We now examine how K^T, r^T and the strategy choice evolve when we change the upside of the project, B, holding the other parameters – p, π, γ, V – constant. To facilitate the analysis, we use a special notation for two particular values of B. As defined already in section 3.3, B_1 denotes the level of B at which ICC (3.5) becomes binding for $K = K^{FB}$, i.e., the level at which the first best is no longer achievable under tough law. Now we denote B_2 the level of B at which the costs of inducing the debtor to choose S_Q equal the costs of choosing S_C over S_Q, i.e., for $B > B_2$ the optimal contract assumes continuation of the project in state L. It can be shown that $B_1 \leq B_2$. Referring to these two thresholds, we can describe the dependence of K^T on B.[17]

Proposition 2. Under the tough law,

$$K^T = \begin{cases} \frac{pB}{1-(1-p)\gamma} - 1 & = K_Q^{FB} = K^{FB} & \text{if} \quad B \leq B_1, \\ K_Q^T \text{ given by (3.14)} & < K^{FB} & \text{if} \quad B_1 < B \leq B_2, \\ [p + (1-p)\pi]B - 1 & = K_C^{FB} < K^{FB} & \text{otherwise,} \end{cases}$$

$$r^T = \begin{cases} 0 & \text{if} \quad B \leq B_2, \\ \frac{1-[p+(1-p)\pi]}{p+(1-p)\pi} \left(1 - \frac{V}{[p+(1-p)\pi]B-1}\right) & \text{otherwise,} \end{cases}$$

$$S_i^T = \begin{cases} S_Q & \text{if} \quad B \leq B_2, \\ S_C & \text{otherwise.} \end{cases}$$

Proposition 2 divides the projects under tough law environment into three categories. First, for projects whose upside relative to the firm's value is rather small ($B < B_1$), the first best amount of K can be lent by the creditor, since the debtor can credibly commit to liquidate the project in the bad state and repay the creditor in full. In this case, the debtor is able to cover the whole loss from the firm's residual value V and, therefore, acts in a socially optimal way, liquidating the project in the bad state. Second, for projects with medium upside relative to the firm's value ($B_1 \leq B < B_2$), if lent the first-best amount, the debtor would prefer to continue the project, because the vision of the project succeeding, however unlikely, is attractive enough for him to gamble on resurrection. K has to be decreased to make the quitting strategy more attractive (see Figure 3.7 in Appendix 3.A.2), which means the debtor can only obtain debt financing below the efficient scale. Third, for projects with upside too high relative to the firm's value ($B_2 \leq B$), it is not efficient to deter the debtor from continuing by reducing the

[17]Note again that we are interested only in those situations when $K_Q^{FB} > 0$ and the socially optimal strategy is S_Q.

invested amount. The best available solution is to accept ex ante that the project will continue regardless of the state of the world and reflect the risk of less-than-full repayment at a higher interest rate.

For projects in the second and third category, the problem with gambling on resurrection in the bad state arises due to the fact that the firm's value V is assumed to be fixed. The problem could be eliminated if the firm's value could be increased sufficiently, relative to the size of the project. This could be done by increasing equity financing. Therefore, in countries with functioning capital markets, the ex ante inefficiencies of the tough law are less of an issue because when facing such a project the debtor could raise new capital to bring the resulting leverage ratio to an acceptable level.[18]

The dependence of K on B is illustrated in Figure 3.8 in Appendix 3.A.2. Figure 3.9 then illustrates the debtor's expected profit in period 0 as a function of B. As we assume the credit market to be perfectly competitive, the debtor's expected profit represents the whole social surplus generated by the project.

3.4.2 Soft bankruptcy law

Having analyzed the moral hazard problem in a regime of tough bankruptcy law in section 3.4.1, we now move to a regime of soft bankruptcy law characterized by $\alpha > 0$. This regime enables the debtor of a bankrupt firm to always keep a fraction of the firm's value, even if the creditors are not paid in full. In other words, the soft law enables violation of APR.

An often-cited example of a soft bankruptcy law is the U.S. Bankruptcy Code, especially its reorganization chapter, Chapter 11. There is substantial evidence that the APR is often violated in Chapter 11 cases. Longhofer and Carlstrom (1995), for example, survey the existing empirical literature on APR violations and find, based on that literature, that in a sample of large corporations with publicly traded securities, APR violations occur in 75% of reorganizations.

We model the soft law by assuming exogenously given $\alpha > 0$ (the extensive form representation of this game is shown in Figure 3.2 in Appendix 3.A.1). As before, we are particularly interested in the effects on the debtor's strategy choice in state L and, implicitly, on the level of investment K and interest rate r.

[18] A testable hypothesis would be that in countries with tough bankruptcy laws, we should observe lower leverage ratios and also lower real interest rates.

The difference from the case of $\alpha = 0$ analyzed in section 3.4.1 is that if the debtor gambles on resurrection in state L and this gamble fails, she still keeps fraction $\alpha > 0$ of the firm's remaining value. Thus her expected payoff from continuation is larger by $(1-\pi)\alpha V$, which is added to the right hand side of ICC (3.11). To induce her to choose S_Q in state L for $K = K^{FB}$ and $r = 0$ the following modified ICC must hold:

$$V + \gamma K^{FB} - K^{FB} \geq \pi[V + B\ln(K^{FB}+1) - K^{FB}] + (1-\pi)\alpha V. \tag{3.20}$$

In this case, by increasing the payoff from continuation in the bad state, the soft law makes the gambling on resurrection more attractive.

For $K = K^{FB}$ and $r = 0$ to be the solution, the value kept by the debtor after out-of-bankruptcy liquidation and full repayment must be larger than what she could obtain from filing for bankruptcy, i.e., the following must hold

$$V + \gamma K^{FB} - K^{FB} \geq \alpha(V + \gamma K^{FB}). \tag{3.21}$$

If the condition of having both (3.20) and (3.21) hold is violated, the solution with $K = K^{FB}$ and $r = 0$ cannot be achieved. It may still be possible to achieve a solution with $K = K^{FB}$ and $r > 0$, provided α is high enough to induce the debtor to choose S_Q, i.e., provided that

$$\alpha(V + \gamma K^{FB}) \geq \pi[V + B\ln(K^{FB}+1) - (1+r^S)K^{FB}] + (1-\pi)\alpha V, \tag{3.22}$$

where r^S is given by solving the bank's participation constraint (holding with equality) as

$$r^S = \frac{1-p}{p}\left[1 - (1-\alpha)\frac{V + \gamma K^{FB}}{K^{FB}}\right]. \tag{3.23}$$

In this case, as in the case with endogenous α, the debtor obtains a large enough share of the pie to induce him to act in the socially optimal way and to liquidate the project in the bad state. The bank, although not repaid in full in state L, is willing to finance the project at the socially efficient level because it is compensated by a higher payoff in state H.

When the project upside given by B is high and the law's degree of softness given by α is low (both (3.20) and (3.22) are violated), the first best cannot be achieved for the given α. The situation is then similar to the one under tough law. The optimal contract

$\{K^S, r^S, S_i\}$ will be determined as the solution to one of the following maximization problems:

1. **Quitting in state** L. In the optimal contract, $r = 0$ and the investment K is such that the debtor prefers S_Q in state L so that the creditor gets repaid in full. The maximization problem then becomes

$$\max_{K} \quad p[V + B\ln(K+1)] + (1-p)(V + \gamma K) - K \tag{3.24}$$

s.t.

$$V + \gamma K - K \geq \pi[V + B\ln(K+1) - K] + (1-\pi)\alpha V. \tag{3.25}$$

The optimal K is obtained by solving (3.25) held with equality, i.e., K^S is given by

$$(\gamma + \pi - 1)K^S - \pi B \ln(K^S + 1) + (1-\alpha)(1-\pi)V = 0. \tag{3.26}$$

2. **Continuation in state** L. K and r are such that in the bad state the debtor prefers to continue the project. The maximization problem becomes:

$$\max_{K,r} \quad [p + (1-p)\pi][V + B\ln(K+1) - (1+r)K] + (1-p)(1-\pi)\alpha V \tag{3.27}$$

s.t.

$$[p + (1-p)\pi](1+r)K + (1-p)(1-\pi)(1-\alpha)V - K \geq 0. \tag{3.28}$$

Here, the optimal investment is $K = [p+(1-p)\pi]B - 1 = K_C^{FB} < K^{FB}$. The optimal interest rate is positive and is obtained by substituting K_C^{FB} in the participation constraint (3.28) holding with equality.

When deciding which of the two possible contracts described above is the best, the debtor compares the expected payoffs from each, i.e., the values of the objective function at the optimal solution, and chooses the one with the highest payoff.

We now summarize the above derivations in the following proposition.

Proposition 3. Under the soft law with exogenously given α, the optimal levels of K and r are determined as follows.

- If (3.20) and (3.21) hold, $K^S = K^{FB}, r^S = 0, S_i^S = S_Q$.
- If (3.20) and (3.22) hold but (3.21) does not hold, $K^S = K^{FB}$, r^S is given by (3.23), and $S_i^S = S_Q$.
- If neither (3.20) nor (3.22) hold, then the first best is not attainable and $K < K^{FB}$. The debtor will decide between a contract involving quitting in the bad state (case 1 above) and a contract involving continuation in the bad state (case 2 above), depending on which of the contracts yields her higher expected profit in period 0.

The key conclusion of the soft law analysis is that if the law is not soft enough (if the constraint (3.22) is violated), it further worsens the gambling on resurrection problem observed under tough law by making the continuation strategy in the bad state more attractive for the debtor. The consequence is higher inefficiency given by the higher difference between the feasible (K^S) and the optimal (K^{FB}) investment level. If, on the other hand, the law is soft enough (if the constraint (3.22) holds), then the debtor behaves in the socially optimal way, liquidating the project in the bad state. The practical question is when the law is soft enough. For the parameters here, this is the case for $\alpha = 0.5$, meaning that the debtor would have to retain 50% of the firm's value in bankruptcy. Compared to the empirically observed values of the degree of softness under Chapter 11 – which range from 0% to 26.5%, with an average below 10% (as discussed in section 3.2.2) – this seems to be unrealistically high. If such a high level cannot be achieved in practice, then tougher law will produce better results.

3.5 Possibility of verification under tough law

In this section, we explore the possibility of creditors' verification of the firm's information about the state of the world in period 1. We focus on the tough law setup ($\alpha = 0$) and introduce a new parameter to the analysis – the cost of verification, c. Verification offers another instrument, besides those explored in previous sections, to solve the gambling on resurrection problem and, possibly, to improve efficiency. We will show under what conditions it will be used and what benefits it brings. For the extensive form representation of the game, see Figure 3.3 in Appendix 3.A.1.

As discussed before, the debtor would like to commit to the socially efficient strategy, S_Q, ex ante because this would enable him to obtain financing in the amount K^{FB} and would maximize the debtor's expected payoff. However, since the state of the world is the debtor's private information, such a commitment would not be credible if *ex post*, in state L, the debtor would prefer S_C. Previously we assumed that the only ways to solve this problem were to reduce K below the optimal level or to use APR violation to incentivize the debtor to choose the safe strategy. In this section we instead introduce the possibility that the bank is able to verify, at a certain cost, the debtor's report of the state of the world and, thus, make sure that the debtor liquidates the project in the bad state.

If the debtor reports state H, the bank can decide to verify this information, which costs it c. We assume a perfect monitoring technology is used: If the bank decides to verify, it will learn the true state with certainty. If it finds the state is H, nothing happens and the project continues to period 2. If it uncovers misreporting, i.e., if it finds that the state is L, it will take control of the business and obtain the lesser of full payoff $(1+r)K$ and the firm's entire value $V+\gamma K$. We denote q the probability that the bank will decide to verify the state of the world reported by the debtor. We also assume that following the discovery of misreporting, the debtor obtains nothing even if the bank is paid in full. This reflects the fact that the bank is in control and it will not exert any effort to obtain value in excess of $(1+r)K$.

Note that without this "punishment" assumption, partial verification ($q < 1$) would never be sufficient to induce the debtor to choose S_Q in situations in which she, without verification, would prefer S_C. This is simply because she could never do worse by lying than by truth-telling. Full verification ($q = 1$) would always be necessary in this case. Note also that the cost of verification, although paid by the bank, will eventually be borne by the debtor. This is because we assume a competitive credit market, which means that the bank's expected profit is zero under all circumstances and the whole surplus goes to the debtor. The verification cost cuts into this surplus.

Depending on the parameter values, the optimal solution for $\{K, r, S_i\}$ and q can take four different forms:

1. **No Verification, First Best**, $\{K^{FB}, 0, S_Q\}$ and $q = 0$. After borrowing $K = K^{FB}$ at $r = 0$, the debtor chooses S_Q in state L even without verification. Verification is unnecessary and will not be used. Note that this is the case of $B \leq B_1$.

2. **No Verification, First Best not Attainable**, $\{K < K^{FB}, 0, S_Q\}$ and $q = 0$. At $K = K^{FB}$ the debtor would choose S_C in state L but lowering K below the first

best level costs her less than the expected cost of verification she would have to pay the bank in the form of interest rate.

3. **Probabilistic Verification**, $\{K < K^{FB}, r = \frac{pcq}{K}, S_Q\}$ and $q > 0$. Without verification the debtor would choose S_C and a probabilistic verification $(0 < q < 1)$ is sufficient to induce her to choose S_Q. This is the case when full repayment is possible after quitting the project, i.e., the debtor still receives a certain payoff following the choice of S_Q. $K < K^{FB}$ because the marginal cost of increasing K is higher than in the social planner's problem by $c\frac{\partial q}{\partial K}$. The bank has to be compensated for the expected verification cost pqc, thus $r = pqc/K > 0$. If the debtor reports H, the bank verifies with probability q which is set endogenously in such a way that the debtor never lies. He reports state H only when it really occurs, which happens with probability p. Thus, the ex ante probability that the bank will need to bear the verification cost c is pq.

4. **Full Verification**, $\{K = K^{FB}, r = \frac{pc+(1-p)[(1-\gamma)K-V]}{pK} > 0, S_Q\}$ and $q = 1$. If full repayment is impossible after the choice of S_Q in state L, the debtor would choose S_C for any $q < 1$. In order to induce the debtor to choose S_Q we therefore need to have $q = 1$. At this level of q, the marginal cost of increasing K is the same as in the social planner's problem (since $\frac{\partial q}{\partial K} = 0$) and we will have $K = K^{FB}$. The interest rate will again compensate the bank for the verification cost and also for the risk of less than full repayment if state L occurs.

5. **No Verification, Continuation**, $\{K_C^{FB} < K^{FB}, r = \frac{1-[p+(1-p)\pi]}{p+(1-p)\pi}\left(1 - \frac{V}{K_C^{FB}}\right), S_C\}$ and $q = 0$. The debtor may always offer a contract involving the choice of S_C in state L if she compensates the bank for the risk of less than full repayment in the case of project failure and keeps her participation constraint satisfied.

From these alternatives, the debtor will propose a contract that yields her the highest expected payoff and is feasible. Cases 1, 2 and 5 are the same as under tough law without verification. In what follows, we analyze problems 3 and 4 in more detail. Before that, however, we make some comments common to both of them.

First, we assume that the creditor can credibly commit to verifying with the probability $q^*(K, r)$ ex ante.[19] Otherwise, the creditor would have an inconsistency problem:

[19] This is a realistic assumption in the sense that the banking business is based on reputation and, thus, the bank's commitment is actually enforced by the other business it has. Committing to verification and then not doing it would have a reputational cost for the bank.

he would like to commit to verifying with probability q^* but – once this commitment is made and the debtor adapts his behavior in the desired way – to renounce this commitment and save the cost c. We would then have mixed-strategy equilibria which would complicate the analysis and lead us away from the point of our interest.

Second, unlike in the situation without verification, the expected payment of the firm to the bank is $K + pcq^*(K,r) > K$. If there is verification, the interest serves to compensate the bank for the actual verification cost that it incurs, not (or not only) for the risk of less than full repayment. Compared with the social planner solution, the verification cost is therefore a source of inefficiency.

Finally, note that the creditor will not want to increase q above $q^*(K,r)$ because this increases his cost without any increase in return – for $q = q^*(K,r)$ the debtor will choose S_Q anyway. Note also the discontinuity in the returns to increasing the verification probability q – for some $q < q^*(K,r)$ the bank will only ensure the choice of S_Q if it actually verifies, while for $q = q^*(K,r)$ the debtor will always prefer S_Q. Because of this discontinuity, the bank will either verify with probability $q = q^*(K,r)$ or not verify at all. Any intermediate level of q cannot be optimal.

3.5.1 Probabilistic verification

Consider first the problem when full repayment is possible after the project is quit, which means that for the optimal K and r we have $V+\gamma K \geq (1+r)K$. In this case, probabilistic verification is sufficient to induce the debtor to quit in state L. In other words, q may always be set to such a level that the debtor prefers the sure payoff from quitting to the lottery induced by continuation.

The firm's maximization problem is then

$$\max_{K,r} \quad \{V + pB\ln(K+1) + (1-p)\gamma K - (1+r)K\} \tag{3.29}$$

s.t.

$$V + \gamma K - (1+r)K \geq (1-q)\pi[V + B\ln(K+1) - (1+r)K], \tag{3.30}$$

$$rK \geq pcq. \tag{3.31}$$

Equation (3.30) is the incentive compatibility constraint which ensures that the debtor will prefer S_Q. It is the analogue of (3.11) in the case without verification, the difference being that the expected payoff from continuation is multiplied by the probability $(1 - q)$ that misreporting will go through; with probability q, the bank will discover the misreporting, will seize control, and the debtor will receive nothing. Equation (3.31) is the participation constraint. We can express the optimal q from (3.30) held with equality as $q^*(K,r) = 1 - \frac{V+\gamma K-(1+r)K}{\pi[V+B\ln(K+1)-(1+r)K]}$, substitute it into (3.31) and solve the modified maximization problem. This yields the following first-order conditions:

$$(K) \quad p\frac{B}{K+1} + (1-p)\gamma - (1+r) + \lambda[r - pcq_K^*(K,r)] = 0, \tag{3.32}$$

$$(r) \quad - K + \lambda[K - pcq_r^*(K,r)] = 0. \tag{3.33}$$

Using (3.33) to express λ and substituting back to (3.32) yields

$$p\frac{B}{K+1} + (1-p)\gamma - (1+r) + \frac{K[r - pcq_K^*(K,r)]}{K - pcq_r^*(K,r)} = 0. \tag{3.34}$$

From equation (3.34) and from the participation constraint (3.31) holding with equality we can obtain the optimal levels of K and r for the situation in which the creditor verifies the firm's report in period 1 with probability $q \in (0,1)$, i.e., $K_p^V(q)$ and $r_p^V(q)$. Here, the superscript V denotes "verification", while the subscript p denotes that the verification is "partial".

Denote F_C the firm's payoff if the project ends successfully and F_Q the firm's payoff if the project is quit in period 1. Differentiating $q^*(K,r)$ with respect to K and r, we obtain

$$q_K^* = \frac{[(1+r)-\gamma]F_C - [(1+r) - \frac{B}{K+1}]F_Q}{\pi(F_C)^2} \geq 0, \tag{3.35}$$

$$q_r^* = \frac{K(F_C - F_Q)}{\pi(F_C)^2} \geq 0. \tag{3.36}$$

Because $q_r^* \geq 0$ and from (3.33) $\lambda = \frac{K}{K - pcq_r^*}$, we have $\lambda \geq 1$. The shadow cost associated with the participation constraint (in which q is endogenously determined to satisfy the ICC) is in general higher than one. This means that, in this regime, increasing the amount borrowed, K, by one dollar increases the expected costs (here the value of the debt) by more than one dollar because the verification probability q needs to be increased as well. This formally shows what we have already mentioned before, namely the fact that with probabilistic verification we will have $K < K^{FB}$.

The optimal probability of verification, $q^*(K, r)$, does not depend on c, but c affects whether verification will or will not be used. If the creditor verifies with probability $q^*(K, r)$, the debtor will always choose S_Q and the bank will always be repaid in full. The gain from verification for the bank is $(1-p)(1-\pi)[(1+r)K - V]$ and the cost is $pcq^*(K, r)$. The bank will, therefore, want to verify the firm's report if

$$c \leq \frac{(1-p)(1-\pi)}{p\, q^*(K_p^V, r_p^V)}[(1+r_p^V)K_p^V - V]. \tag{3.37}$$

3.5.2 Full verification

Consider now the case when after quitting the project in state L full repayment is impossible. Because S_C offers her a positive payoff with at least some probability, the debtor would never choose S_Q for $q < 1$ and, therefore, we need to have $q = 1$. In this case the firm's maximization problem can be written as

$$\max_{K,r} \quad p[V + B\ln(K+1) - (1+r)K] \tag{3.38}$$

s.t.

$$p[(1+r)K - c] + (1-p)(V + \gamma K) - K \geq 0. \tag{3.39}$$

Because the bank always verifies the debtor's report ($q = 1$) the expected verification cost is pc. The first-order conditions are:

$$(K) \qquad \frac{pB}{K+1} - (1+r)p + \lambda[p(1+r) + (1-p)\gamma - 1] = 0, \tag{3.40}$$

$$(r) \qquad -pK + \lambda pK = 0. \tag{3.41}$$

From the FOC for r we have $\lambda = 1$. The marginal cost of borrowing an additional dollar is just one dollar. Using this in the FOC for K, we can obtain the solution for K,

$$K_f^V = \frac{pB}{1-(1-p)\gamma} - 1 = K^{FB}. \tag{3.42}$$

The intuition for this result is that in this regime, the verification probability is fixed, $q = 1$, and therefore the verification cost that the bank needs to be compensated for incurring is fixed as well at $rK = pc$. Therefore, if the cost does not change with K, the optimal K will be the one maximizing the overall surplus, which is K^{FB}. Substituting K^{FB} into the participation constraint (3.39) holding with equality we obtain the following solution for r:

$$r_f^V = \frac{c}{K^{FB}} + \frac{1-p}{p}\left(1 - \gamma - \frac{V}{K^{FB}}\right). \tag{3.43}$$

Because the gain from verification for the bank is $(1-p)[V+\gamma K - \pi(1+r)K - (1-\pi)V]$ and the cost of full verification is pc, the bank will want to verify the firm's report if

$$c \leq \frac{1-p}{p}\left\{\pi V + \left[\gamma - \pi(1+r_f^V)\right] K^{FB}\right\}. \tag{3.44}$$

3.5.3 Optimal contract under the possibility of verification

In the previous sections, we have shown the optimal contracts under tough and soft bankruptcy laws, assuming the creditor cannot verify the state of the world in period 1. In other words, we have implicitly assumed the cost of verification to be prohibitively high. Under these circumstances, the feasible investment (K) was at social optimum until a certain level of the project upside (B), but fell short of the efficient level for higher upsides.

Once the cost of verification falls below a critical threshold, it starts to pay off for the creditor to verify the state of the world and the under-investment problem starts to diminish. For zero verification cost, the socially optimal investment level becomes feasible for all values of the project upside.

Figures 3.13 through 3.16 in Appendix 3.A.2 illustrate these findings by means of simulations.

3.6 Allowing for renegotiation

So far, we have assumed away the possibility of renegotiation. Now, although we believe and argue below that this is not an unreasonable assumption, we will consider how the situation changes when renegotiation is allowed.[20] The first argument for not including renegotiation in the basic setup is that the bank may want to build a reputation of not being willing to renegotiate in order to prevent strategic defaults by other debtors. In our model, the bank does not need such a reputation because the debtor has nothing to gain from defaulting in the good state (we assume all the firm's value consists of verifiable

[20] For a general analysis of debt-renegotiation in bankruptcy see, e.g., Janda (2002).

assets, so the bank could turn to court to enforce the payment). However, in reality, strategic default may be an issue for the bank and it may have an incentive to develop such a reputation. The second reason is that under renegotiation, the bank effectively forgives part of the debtor's non-contingent payment specified in the contract. Although this may actually be more profitable for the bank than doing nothing and allowing the debtor to continue the project, relevant laws may treat such debt forgiveness by bank officers as illegal.

We describe the effects of renegotiation for the case of tough law and then only mention the differences under soft law. The only node in the game where renegotiation can take place is state L in period 1. In addition, considering renegotiation only makes sense in the suboptimal case when the first best cannot be reached, i.e., for $B > B_1$. In this case, the debtor has an incentive to continue the project although the action maximizing the firm's value is to quit the project. Therefore, there is room for a mutually advantageous renegotiation of the initial contract.

The bargaining situation is shown in Figure 3.17 in Appendix 3.A.3. The x-axis denotes the debtor's payoff, the y-axis the bank's payoff. The maximum payoff for both is $V + \gamma K$ and the line connecting these payoffs on the x- and y-axis is the Pareto frontier, with the slope -1. In the status quo point without renegotiation, the debtor's expected payoff is $\pi[V + B\ln(K+1) - (1+r)K]$ and the bank's expected payoff is $\pi(1+r)K + (1-\pi)V$. These payoffs also determine the threat points of the debtor and the bank denoted by P_d and P_b, respectively. The bargaining takes place between these two points on the Pareto frontier.

In state L, the debtor can contact the bank, reveal that state L occurred, and offer to quit the project if she receives a certain payoff. The maximum payoff the debtor can obtain depends on the bargaining powers of the debtor and the bank. We analyze two cases – first, when the debtor has all the bargaining power, and second, when the bank has all the bargaining power.

3.6.1 Different allocations of bargaining power

Suppose first that the debtor possesses all the bargaining power within the renegotiation process, i.e., that the debtor is able to hold the bank down to its threat point P_b where its payoff is $\pi(1+r)K + (1-\pi)V$. The debtor's payoff from renegotiation in state L is, therefore, $V + \gamma K - \pi(1+r)K - (1-\pi)V$. The bank's and debtor's payoff in the good state are the same as without renegotiation, i.e., $V + B\ln(K+1) - (1+r)K$ for the debtor and $(1+r)K$ for the bank. The debtor's maximization problem in period 0 can thus be written as

$$\max_{K,r} \quad [p + (1-p)\pi][V - (1+r)K] + pB\ln(K+1) + (1-p)\gamma K \tag{3.45}$$

s.t.

$$\text{PC:} \quad [p + (1-p)\pi](1+r)K + (1-p)(1-\pi)V - K \geq 0. \tag{3.46}$$

If, alternatively, the bank has all the bargaining power, the debtor is held down to her threat point and her payoff from the renegotiation in state L is therefore the same as from continuation, i.e., $\pi[V + B\ln(K+1) - (1+r)K]$. The bank captures the rest of the firm's value after quitting the project, which is equal to $V + \gamma K - \pi[V + B\ln(K+1) - (1+r)K] < (1+r)K$.[21] In state H the payoffs are again the same as without renegotiation and the debtor's maximization problem can be written as

$$\max_{K,r} \quad [p + (1-p)\pi][V + B\ln(K+1) - (1+r)K] \tag{3.47}$$

s.t.

$$\text{PC:} \quad p(1+r)K + (1-p)\{V + \gamma K - \pi[V + B\ln(K+1) - (1+r)K]\} - K \geq 0. \tag{3.48}$$

[21] The inequality can be explained as follows. As mentioned above, renegotiation will only take place in the suboptimal case where the debtor would prefer to continue the project at $K = K^{FB}$, while the optimal strategy is to quit the project. This means that the debtor's expected payoff from continuation is higher than from quitting and paying the bank in full. Therefore, if after quitting the debtor receives as much as she expects to gain from continuation, the bank cannot be repaid in full.

3.6.2 Optimal contract under renegotiation

The solutions to the two alternative maximization problems are

$$K_{Rd}^T = K_{Rb}^T = \frac{pB}{1-(1-p)\gamma} - 1 = K^{FB}, \tag{3.49}$$

$$r_{Rd}^T = \frac{1-[p+(1-p)\pi]}{p+(1-p)\pi}\left(1 - \frac{V}{\frac{p}{1-(1-p)\gamma}B - 1}\right), \tag{3.50}$$

$$r_{Rb}^T = \frac{1-[p+(1-p)\pi]}{p+(1-p)\pi}\left[1 - \frac{V}{\frac{p}{1-(1-p)\gamma}B - 1}\right. \tag{3.51}$$

$$\left. - \frac{\gamma - \pi B \ln\left(\frac{p}{1-(1-p)\gamma}B\right)}{(1-\pi)\left(\frac{p}{1-(1-p)\gamma}B - 1\right)}\right] < r_{Rd}^T, \tag{3.52}$$

where subscripts Rd and Rb denote the treatment with renegotiation when all the bargaining power resides with the debtor (Rd) or the bank (Rb), respectively; T indicates the tough law regime ($\alpha = 0$).

We see that when renegotiation is possible, then irrespective of whether the debtor or the bank is in the position of making the take-it-or-leave-it offer, the first best can be attained. The optimal investment level in both cases is $K = K^{FB}$ and the debtor follows strategy S_Q in state L. The distribution of the bargaining power only affects, in a predictable way, the interest rate. The intuition behind having $r_{Rd}^T > r_{Rb}^T$ is the following. Because the bank is supposed to just break even in period 0, then a higher payoff from renegotiation in state L enables it to decrease the payoff in state H, which means to decrease the interest rate.

Under the soft law, renegotiation would also occur only in state L and only if the debtor would, without renegotiation, prefer to continue the project. The situation would be similar as under tough law; only the status quo payoffs and therefore the threat points of the parties would shift. Renegotiation would again enable the parties to attain the first best. The interest rate would be higher than under tough law because the debtor's threat point is higher and the bank's threat point is lower, which increases the debtor's and decreases the bank's payoff from renegotiation. This holds irrespective of who has more bargaining power.

3.7 Conclusion

Soft bankruptcy laws are believed to mitigate the gambling on resurrection problem by not fully wiping out the existing shareholders in the case of financial distress. Our model confirms this stylized fact; our main conclusion is, however, that the degree of softness needs to be sufficiently high to achieve this result. If the law is not soft enough, the situation reverses to “the tougher, the better”.

We see two practical issues with the soft law. First, although a sufficiently soft law outperforms a completely tough one, it can be very difficult to find the optimal degree of softness for a given economy – there is no one-size-fits-all solution both in terms of different projects and different creditor types. Hence it might seem reasonable for the policy maker to fully preserve APR, rather than trying to find the optimal degree of APR violation. Moreover, could the optimal degree be found, it might still be impossible to reach it in practice. Even the best-known example of a clearly soft law, Chapter 11, is empirically documented as being substantially tougher than the optimal degree of softness found in our paper.

This paper deals with the situation in which the debtor can stop a project at a time when substantial value can still be recovered. The practical benefits of the APR violation may occur once the bankruptcy procedure has been started, by motivating the debtor to cooperate. How these benefits compare with the drawbacks identified in our paper is a question which deserves further research.

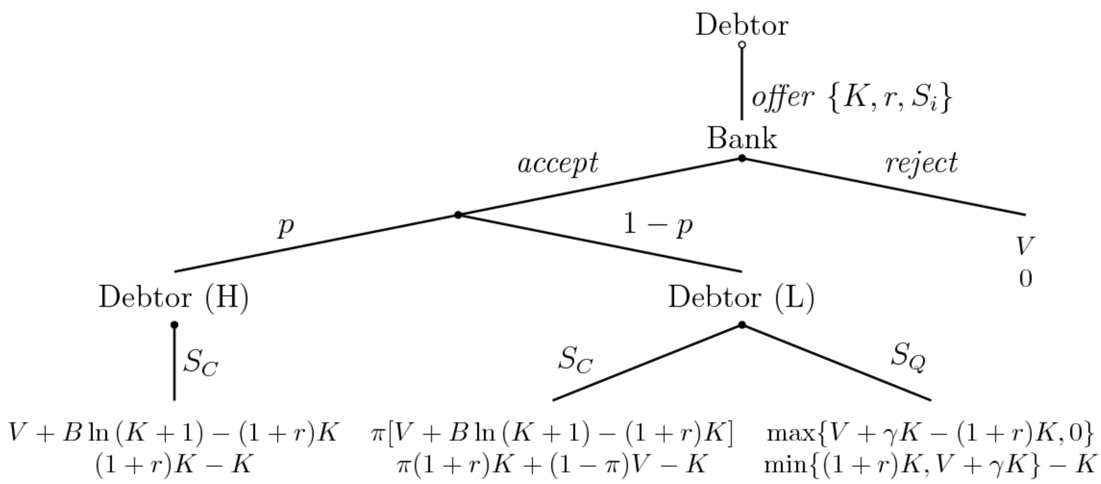
3.A Appendix – extensive form representations and graphical simulations

3.A.1 Extensive form representations

Tough law

Figure 3.1 provides the extensive form representation of the tough law regime as analyzed in section 3.4.1. If state H occurs, the debtor continues for sure in order to get the upside payoff $V + B \ln(K+1) - (1+r)K$ (because quitting would yield him $V + \gamma K - (1+r)K$, which is lower) and the creditor is repaid in full. However, if state L occurs, the debtor can either misreport and choose a risky continuation to get the upside with probability π or safely quit. If she quits, then either she can repay full $(1+r)K$ to the bank and keep $V + \gamma K - (1+r)K$ or the residual value is insufficient for full repayment so that the debtor gets nothing and the bank gets back less than what was specified in the contract. If the debtor misreports in state L and follows S_C, the creditor gets full repayment with probability π and a partial repayment V with probability $(1-\pi)$. The creditor's payoff from rejecting the offered contract in the beginning, 0, represents the outside option from which the participation constraint is derived.

Figure 3.1: Extensive Game under Tough Law

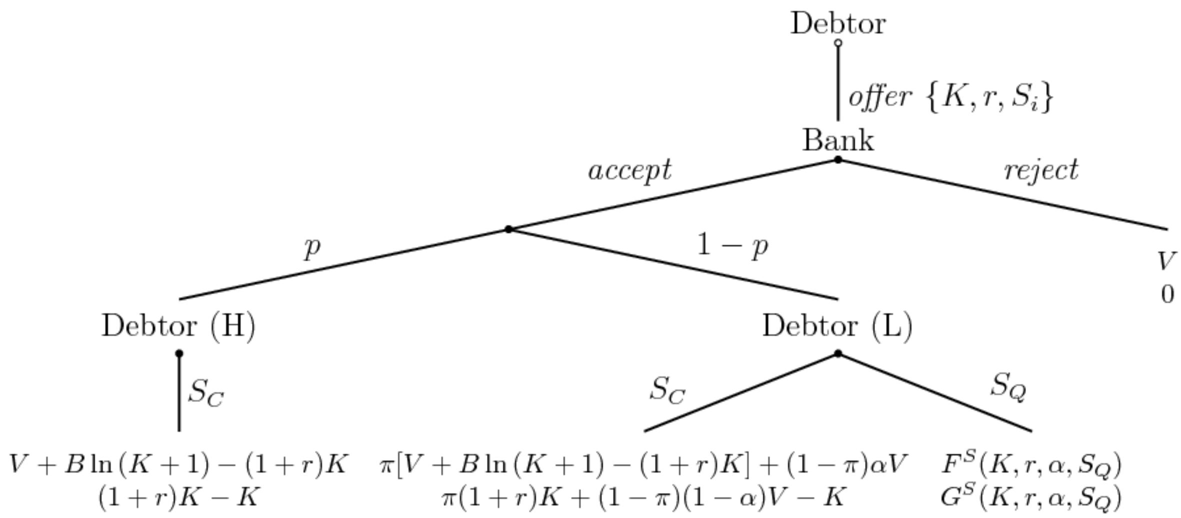


When analyzing the tough law regime, we assumed $V + \gamma K \geq (1+r)K$ so that quitting of the project in state L does not lead to bankruptcy. Had this assumption been violated, there would be no way to induce truth-telling and choice of S_Q in state L. However, as we show in the following paragraph, that assumption did not limit our analysis in any way – whenever our solution in Proposition (2) implies $K^T = K^{FB}$, the assumption that $V + \gamma K \geq (1+r)K$ always holds.

Soft law

The extensive form representation of the soft law regime with exogenous α analyzed in section 3.4.2 is shown in Figure 3.2. The game with endogenous determination of α as analyzed in section 3.4.2 would look the same with α being added as the fourth parameter of the contract offered by the debtor to the creditor in period 0.

Figure 3.2: Extensive Game under Soft Law



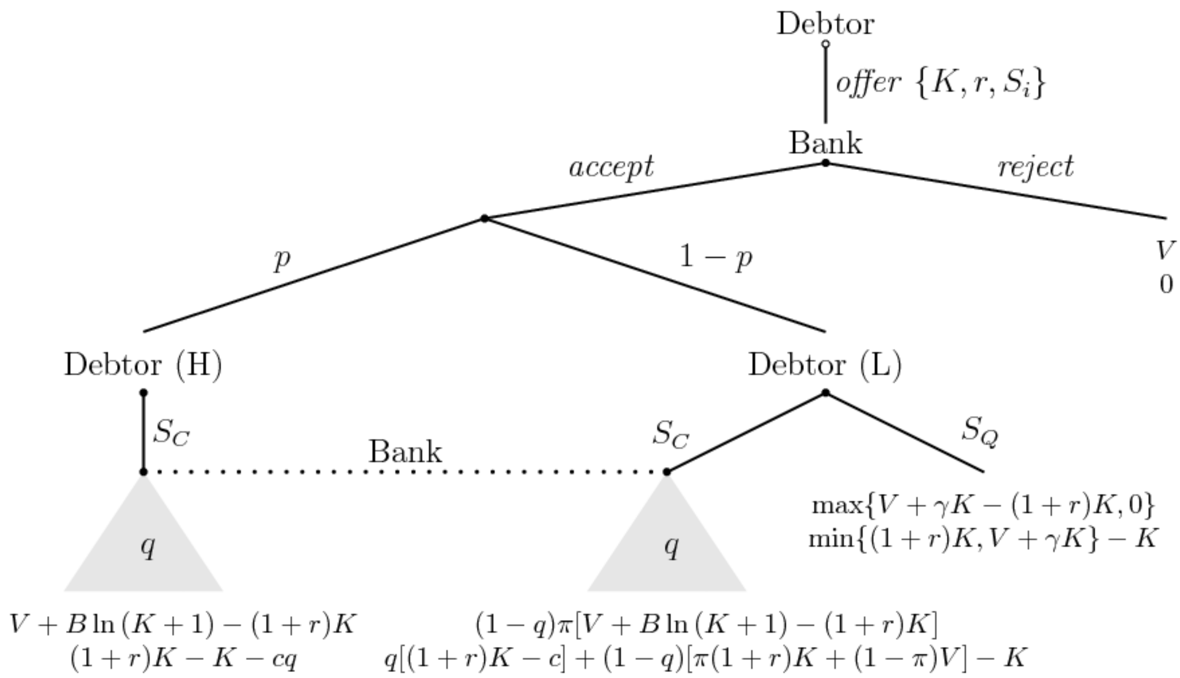
$$\text{where} \quad \begin{bmatrix} F^S(K,r,\alpha,S_Q) \\ G^S(K,r,\alpha,S_Q) \end{bmatrix} = \begin{bmatrix} \max\{V+\gamma K-(1+r)K; \alpha(V+\gamma K)\} \\ \min\{(1+r)K; (1-\alpha)(V+\gamma K)\} - K \end{bmatrix}$$

When state H occurs, the firm remains solvent and the payoffs are the same as under tough law. What changes are payoffs from both strategies after state L is observed by the debtor. The debtor's payoff from continuation is increased at the expense of the creditor by $(1-\pi)\alpha V$. The debtor's payoff from quitting becomes either $V+\gamma K-(1+r)K$ with full repayment $(1+r)K$ to the creditor or $\alpha(V+\gamma K)$ with partial repayment $(1-\alpha)(V+\gamma K)-K$ to the creditor.

Tough law with verification

Finally, Figure 3.3 depicts the game under the tough law regime with verification. In addition to the situation depicted in Figure 3.1, the bank has a chance to verify the state of the world if the debtor claims to be in state H and continues. Thus the verification cost cq enters the bank's payoffs. The debtor's payoff from misreporting is decreased by fraction q which represents the probability of being caught lying.

Figure 3.3: Extensive Game under Tough Law with Verification

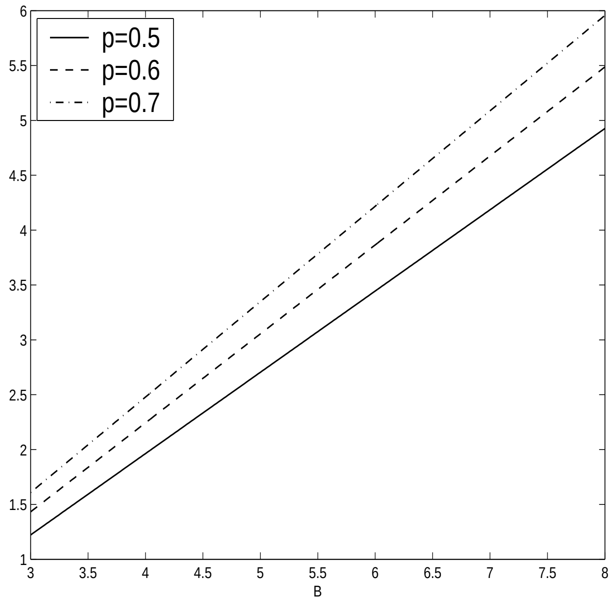


3.A.2 Graphical simulations

Endogenous degree of softness

We consider the following parameter values: $V = 1$, $\gamma = 0.65$, and $\pi = 0.2$. We further consider three different values of p, namely 0.5, 0.6, and 0.7. Given these parameter values, we consider the dependence of K, α and r on the project upside B. Figure 3.4 depicts the dependence of K on B. In the whole range, the dependence of the investment level K on the project upside B is positive and K is higher for higher probability of high state p.

Figure 3.4: Optimal investment level under endogenous degree of softness



More interesting are the functions for α and r, which are depicted in Figure 3.5 and Figure 3.6. They both equal zero as long as the ICC (3.5) can be satisfied for $K = K^{FB}$, $r = 0$, $\alpha = 0$; they both jump up discontinuously when this is no longer possible. The discontinuous jump in α is necessary to satisfy the ICC. For $B \leq B_1$ the amount that remains to the debtor after quitting and paying back in full is sufficient to induce her to quit because it is larger than the expected payoff from continuation. For $B > B_1$ this no longer holds and the debtor must receive more than after full repayment to be induced to quit, which is achieved by setting α to such a level that $\alpha(V + \gamma K)$ is larger than the expected return from continuation. When α jumps up then r must jump up as well to satisfy the creditor's PC.

Figure 3.5: Degree of softness consistent with the optimal strategy

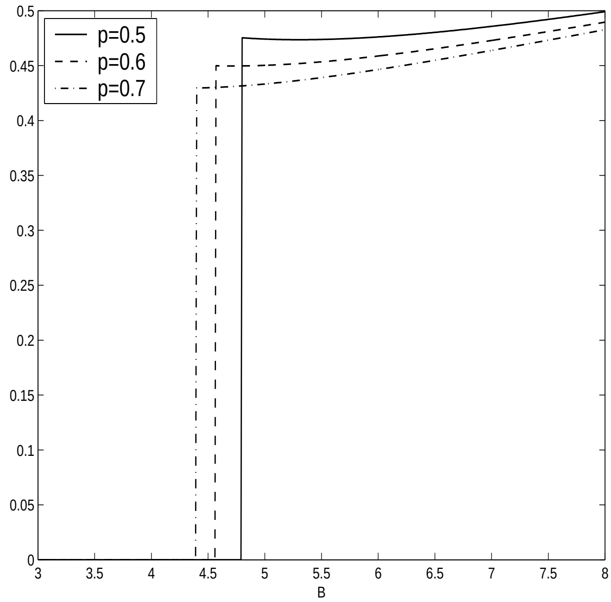
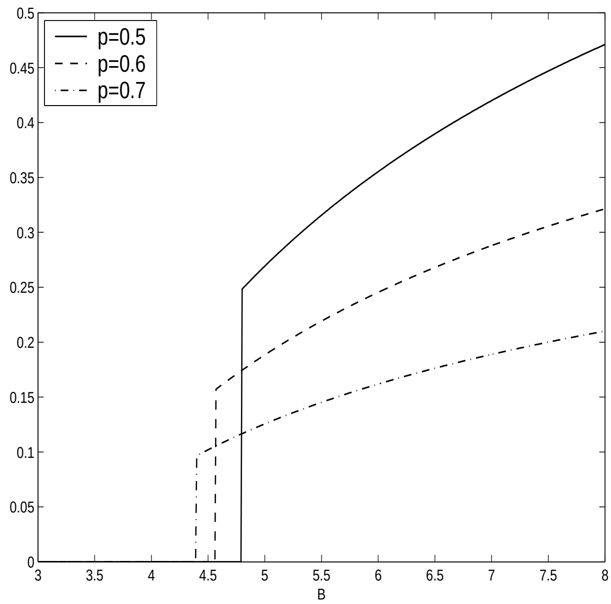


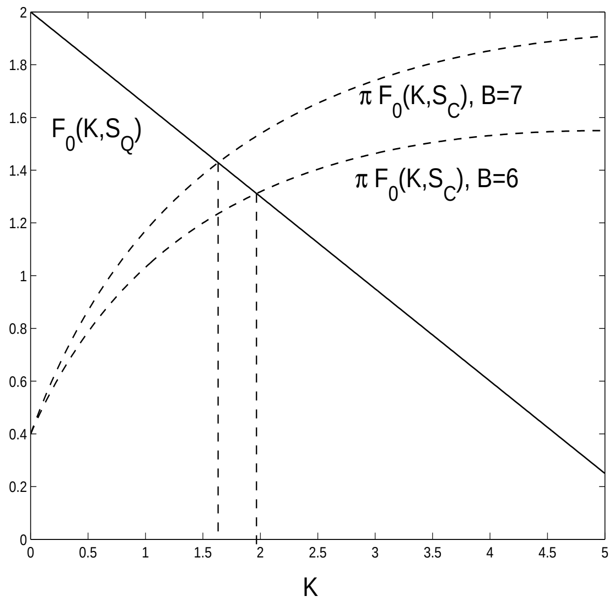
Figure 3.6: Interest rate under endogenous degree of softness and optimal strategy



Exogenous degree of softness, tough law

For a given K higher B increases the payoff from continuation while leaving the payoff from quitting unchanged (Figure 3.7). Thus, the higher B is the lower level of K compatible with the strategy to quit.

Figure 3.7: Debtor's payoff from continuation and from quitting



The dependence of K on B is illustrated in Figure 3.8. Assuming $p = 0.6$, $\pi = 0.2$, $\gamma = 0.65$, and $V = 1$, the firm is able to finance the project on the efficient level $K = K^{FB}$ for $B \leq B_1 = 4.56$. At $B = B_1$, $K^T = K^{FB} = 2.70$. For values of $B > B_1$, K^T is decreasing in B to satisfy the incentive compatibility constraint (3.11). It reaches a minimum of 2.17 at $B_2 = 5.52$. At this point the inefficiency from further decreasing K exceeds that from choosing strategy S_C and, thus, S_C becomes the optimal strategy for the debtor. K jumps up discontinuously to 3.77. At this point, the interest rate also becomes positive, in particular, at $B = B_2$, $r = 0.11$. At the level of B_2, the expected profits of the debtor from both S_C and S_Q are the same and equal to 4.21.

Figure 3.9 then illustrates the debtor's expected profit in period 0 as a function of B. As we assume the credit market to be perfectly competitive, the debtor's expected profit represents the whole social surplus generated by the project. For $B \leq B_1$, the profit is the same as the first best social gain and the debtor follows S_Q. For $B_1 < B \leq B_2$, the profit falls short of the first best social gain but the debtor still follows S_Q. For $B > B_2$, the debtor prefers S_C and the profit falls short of the first best, but with B increasing the gap attenuates.

Figure 3.8: Investment level under tough law

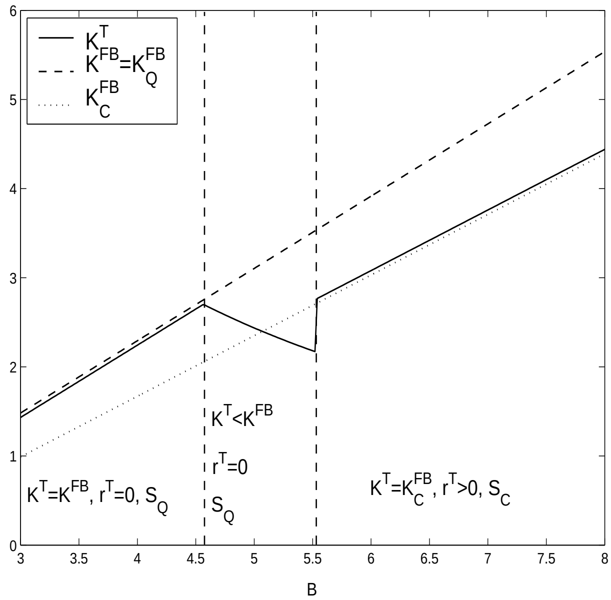
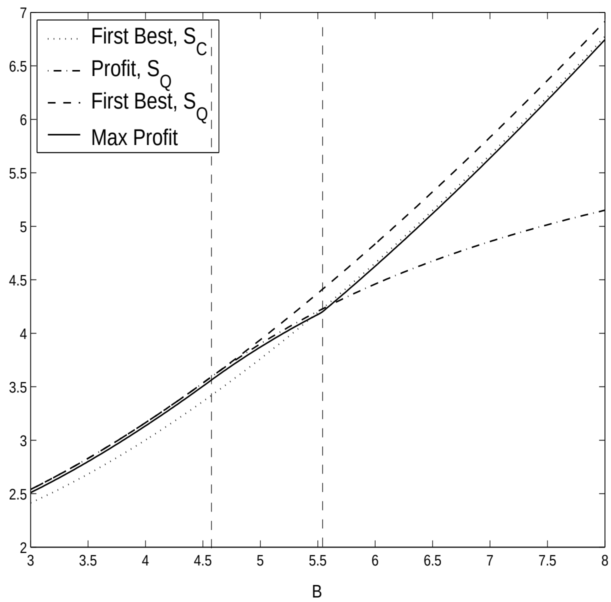


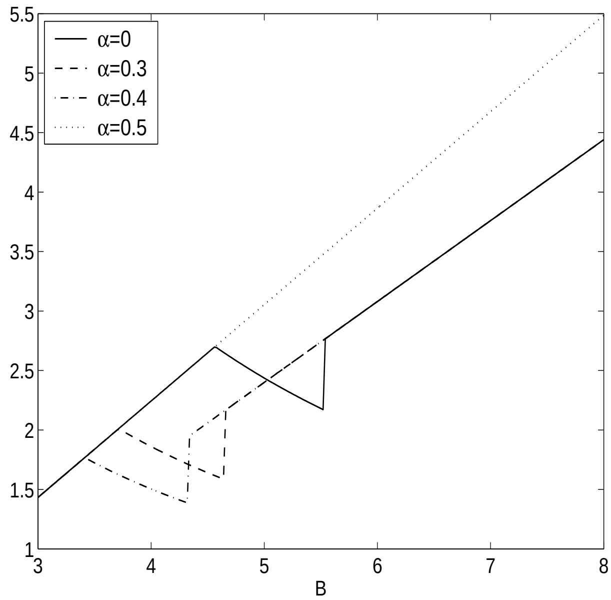
Figure 3.9: Debtor's profit under tough law



Exogenous degree of softness, soft law

Simulations of the dependence of optimal K, r, and the debtor's profit on B and α are provided in Figures 3.10 through 3.12. Figure 3.10 shows that if α is high enough, the first best can be achieved. This is the case for $\alpha = 0.5$ which is sufficient to induce the debtor to choose S_Q for all levels of B. If α is not high enough to make the debtor choose S_Q, K essentially follows the same pattern as under tough law. For sufficiently low $B's$, K is identical with the first best and the debtor chooses S_Q. For $B's$ above a certain level (B_1), choosing S_Q is made credible only by decreasing K below K^{FB}. When ensuring S_Q by further decreasing K becomes too costly, S_C becomes the strategy to be chosen in state L and optimal K is adjusted accordingly, i.e., it jumps upward to K_C^{FB}, the optimal level *given the choice of* S_C *in state* L. The difference is, as already mentioned above, that S_C is now more attractive due to the APR violation after the project failure (the payoff from S_C rises by the term $(1-\pi)\alpha V$), so the constraint making S_Q incentive compatible starts to bind for lower upsides, and K needs to drop below the efficient level at a lower B.

Figure 3.10: Investment level under soft law



Figures 3.11 and 3.12 then show the optimal interest rate and corresponding debtor's expected payoff, respectively.

Figure 3.11: Interest rate under soft law

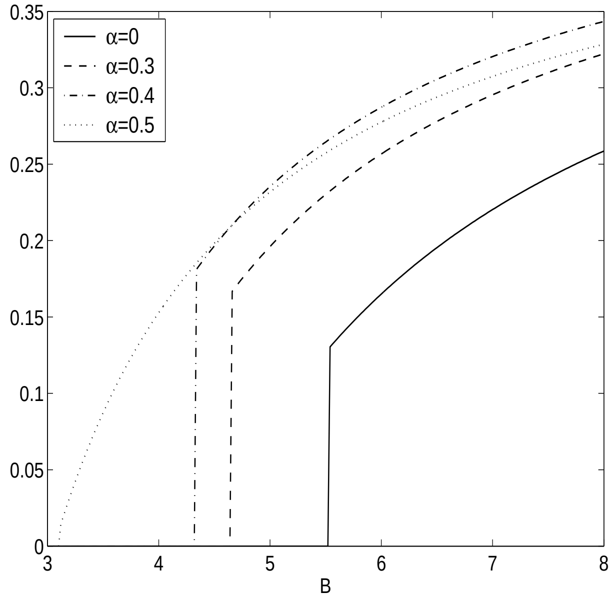
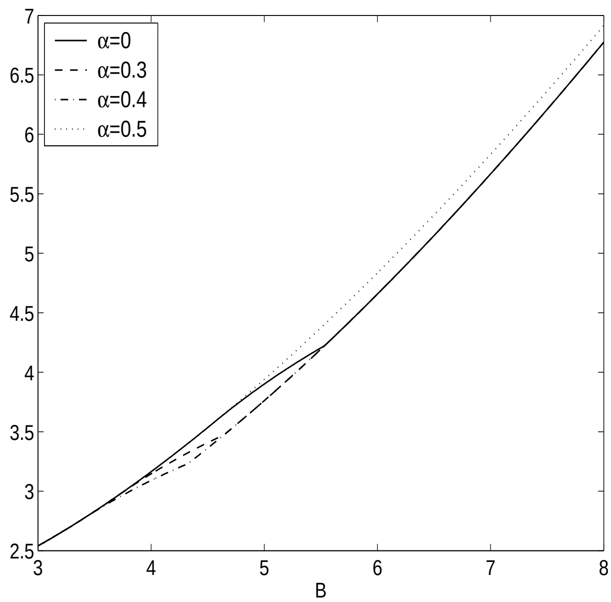


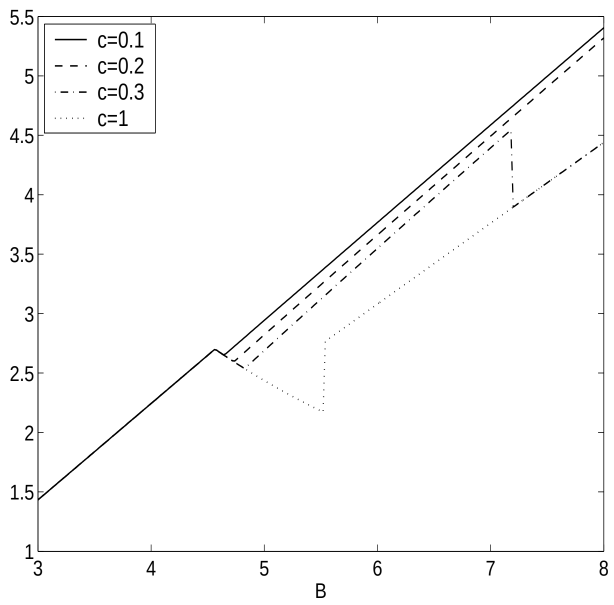
Figure 3.12: Debtor's profit under soft law



Tough law, verification

When verification is possible, then for $B \leq B_1$, the solution is the same as without verification and is identical with the first best. This represents the first part of the line in Figure 3.13 common to all levels of c. Above this level of B, quitting the project for $K = K^{FB}$ is not incentive compatible. For $B's$ only slightly above B_1 it is less costly to induce the debtor to quit by decreasing K below K^{FB} than to use verification. The lower the verification cost c, the lower the level of B at which verification starts to be used. From this point on, K is increasing in B although lower than K^{FB} as long as $q < 1$. For $c = 0.2$ and $c = 0.3$ the payoff from S_C eventually exceeds that from S_Q at a certain level of B. From this point on, the debtor will offer a contract assuming continuation in state L and K will be adjusted accordingly – it will be equal to K_C^{FB}, the optimal level *given the project is always continued.*

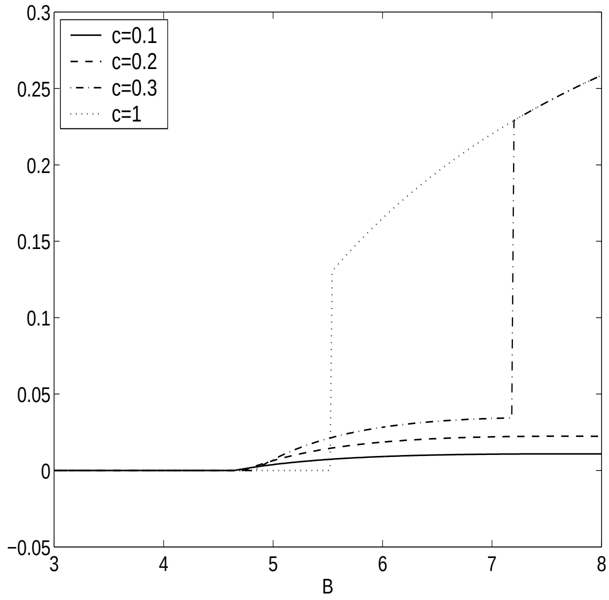
Figure 3.13: Investment level with verification possibility



The interest rate, depicted in Figure 3.14, becomes positive at the level of B at which verification starts to be used. As mentioned above, it compensates the bank for the verification cost and, therefore, is lower for lower c. When S_C becomes the debtor's optimal strategy, the role of the interest rate changes – it compensates the bank for the risk of less than full repayment, as under the case without verification. At this point, the interest rate jumps upward.

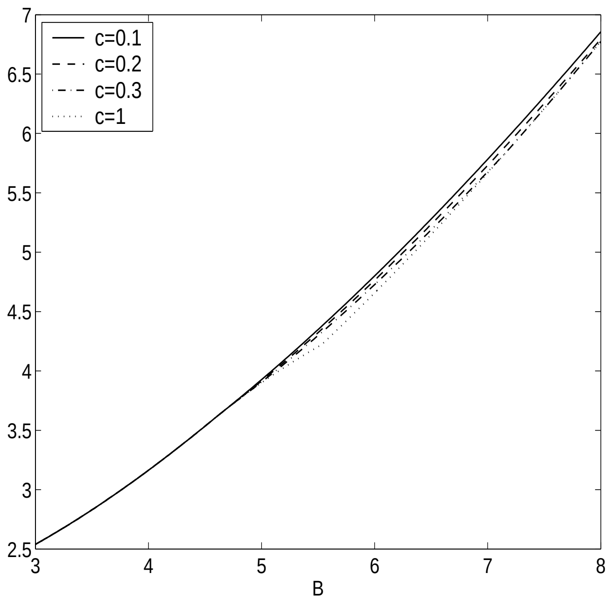
The debtor's payoff, depicted in Figure 3.15, is the same for all levels of c as long as verification is not used. From this point on, the lowest c, naturally, is associated with the

Figure 3.14: Interest rate with verification possibility



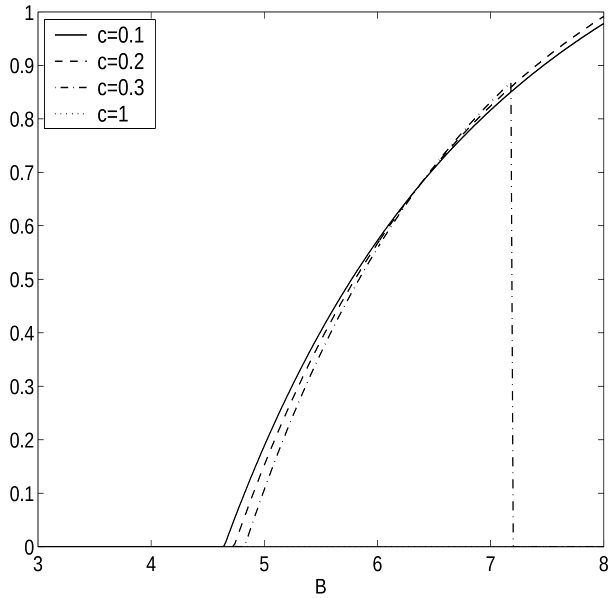
highest expected payoff. When S_C becomes the optimal strategy, c does not affect the expected payoff any longer and, therefore, from this point on, it is the same for $c = 0.2$ and $c = 0.3$.

Figure 3.15: Debtor's profit with verification possibility



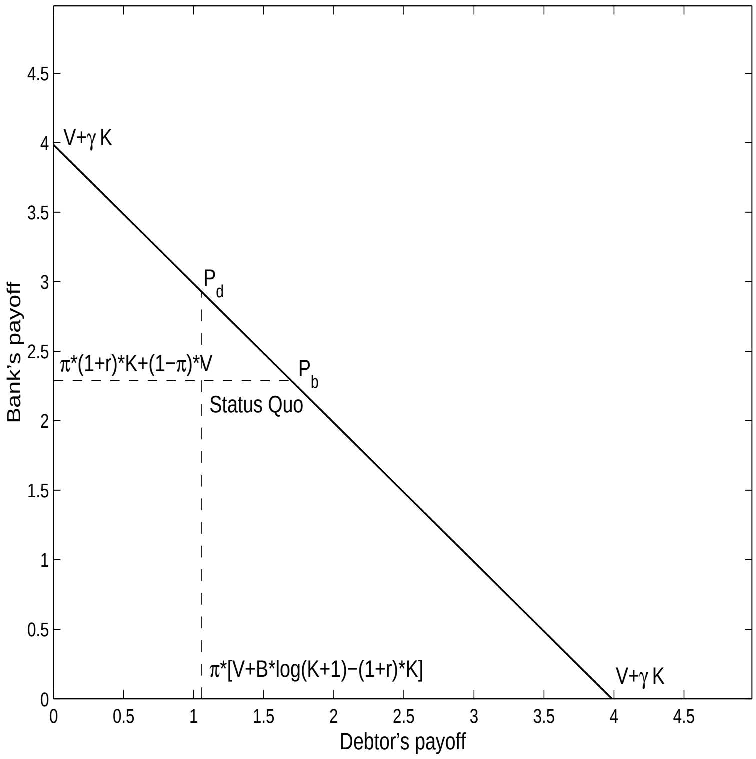
As Figure 3.16 shows, higher c means that verification starts to be used ($q > 0$) at higher B and that S_C becomes the optimal strategy at lower B. The range of $B's$ for which verification is used attenuates.

Figure 3.16: Probability of verification



3.A.3 Renegotiation

Figure 3.17: Renegotiation



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