Referee report on the dissertation thesis "Essays on Mathematical Methods for Economics"

by František Brázdik

Disertation thesis by František Brázdik consists of three independent chapters. The first two chapters are devoted to Data Envelopment Analysis and the third chapter is focused on DSGE models. While the second and third chapters can be characterized mostly as nice applications of known models and methods, the first chapter has a theoretical character and attempts to establish new modifications of some known models.

The starting point for the **first chapter** analysis is the paper by Huang and Li (2001), where the stochastic version of the additive DEA model is developed. Based on this result, oriented versions of the model are formulated (called the almost 100% chance constraint model) in the thesis together with the α chance constrained versions of additive and oriented models. The possible usage of these models is illustrated by evaluation of the efficiency of Indonesian rice farms. Derivation of the new models is made rather mechanically, by analogy with the deterministic case and, to my taste, it lacks more precise mathematical rationale. Moreover, this chapter contains a number of minor inaccuracies and shortcomings in the formulation of results. Some of them, together with my comments, are formulated as follows:

1. Property 1 (page 9) represents only conical convexity, which corresponds to the constant returns to scale. For the case of variable returns to scale it is needed to assume convexity, i.e. to add $\mathbf{1}^T \lambda = 1$ to the left hand side of the implication.

2. From the derivation on page 12 it follows that the condition on the line 10 is sufficient for the DMU_j to be α -stochastically efficient (and not necessary as it is claimed in the thesis). The fact that this condition is also necessary follows from Theorem 1 of Huang-Li (2001). The presentation of this part of thesis should be corrected and the two side implication between the α -stochastically efficiency and the non-positive optimal value of the objective in problem (1.2) should be formulated (this was done in the paper by Huang-Li). The same relations should be formulated and proved for the new models of the thesis, especially for the problem (1.15).

3. Theorem 1 of the thesis formulates one side implication mentioned in the comment 2. Theorem 2 is, however, a pure reformulation of Theorem 1. Instead of it the reverse implication should be provided.

4. If the linear programs of DEA models are solved by simplex method, the non

Archimedean ϵ , or the two stage methods must be used to recognize if the the slacks are zero in any optimal solution. This, however, is not needed when an interior point method (IPM) is used for solving the linear program. In fact, the IPMs provide solutions with maximal number of positive components and hence, if the obtained IPM-solution has zero slacks, then also the slacks of any other solution vanish. The author uses an IPM for solving the linear programs in chapters 1 and 2. However, he also derives the ϵ versions of oriented stochastic models. I am not sure whether the author only does not fully appreciate the advantages of IPM, or there are some deeper hidden reasons for introducing the ϵ multiple of slacks into the objective function in the stochastic case. In the latter case, the reasons should be provided.

5. The claim in the sentence: "From this comparison, I deduce that on average the considered Indonesian rice farms were operating at lower efficiency levels than rice farms in Bangladesh." on p. 29 is wrong, because of the relative character of efficiency scores obtained by DEA. It has no sense to compare the efficiency scores between two different data sets. The only thing we can deduce is that "on average" the Bangladesh farms are "closer" to their best peers than the Indonesian's are to their ones.

The **second chapter** is essentially a case study, where using DEA (output oriented CCR and BCC model) and econometric techniques (Tobit model), the efficiency of rice farms in Indonesian is analyzed. In this chapter author demonstrates an overview of the numerous papers and knowledge of a wide range of instruments used in this field. The author widely comments the obtained results and puts them in the context with other works in the field.

I have two major and two minor comments to this part of the work. The major comments relate to the model orientation choice as well as to the choice of particular inputs and outputs. Taking into account these comments would require repeated (with slight variations) all the calculations and subsequent analysis. Because of this as well as of a slightly polemical nature of the comments I think that taking them into account is not necessary in this thesis. Moreover, I guess that this would cause only small change in the results.

1. It is not clear to me, why the input oriented model is used in this analysis. Although the choice of the input orientation is discussed on p. 50, the author's reasons are not convincing enough. The situation, when the given level of inputs (land, labor and subsidized seed, urea and phosphate) should be turned into the highest return (in order to achieve self-sufficiency in the production) clearly points to the use of output-oriented model. The oriented models are preferably used in the world journal analysis, perhaps because of their simplicity and nice geometric interpretation. However, I believe that in this case the usage of the SBM model by Tone (the properties of this slack-based nonoriented model are described for example in the book by Cooper, Seiford, Tone (2000)) would be much more appropriate.

2. It seems to me that there are some discrepancies in the choice of particular inputs and outputs in the analysis. On p.7 it is written that the input Labor includes harvest and thresh. On the other hand it is mentioned on p. 48 that the cost for the harvest is sometimes paid separately, which means that this part of costs is not included into the input Labor. This indicates that the one-output model is not correct since for some DMU the harvest cost (as labor) is incorporated into input Labor and for the other is not. Regarding the two-outputs model a question arises if the choice of Harvest Cost as the output is correct. It seems to me that Harvest Cost should be considered as input and Gross Yield and Net Yield should be chosen as two outputs.

3. Instead of "all values of slacks are zero" it should be "all values of slacks are zero for any optimal solution of (2.1)" in Theorem 4 on p. 51.

4. There are some missing indexes, stars, etc. in the text and formulas on pages 54 and 55.

The **third chapter** deals with macroeconomic modeling of the transition period from the floating to a fixed exchange rate using a relatively small DSGE model. Author's contribution lies in extending of the model by Justinian and Preston (2004) to simulate the transition period, since the change of floating exchange rate on fixed one, until the completion of this change. Model parameters were estimated for the Czech Republic. Results are evaluated using several approaches including modern macroeconomic methods.

Overall, I conclude that the work meets the requirements for the dissertation thesis and after taking into account and incorporating my comments, the work can be submitted to the defense.

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