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Opponent's Report on the Habilitation Thesis
by
Šárka Hudecová (Prague)
on
"Structural tests for time series"

The habilitation thesis under review is an extended collection of five original research papers, which have already been published in high-level mathematical journals. One paper is a single-authored work and the others are joint papers together with well-known coauthors in the field. Altogether they cover a considerably broad spectrum of topics and methods in mathematical and applied statistics, in particular in the statistical analysis of various classes of time series. Focus is on the discussion of goodness-of-fit tests and change-point procedures in various stochastic frameworks. Goal is to extend and further generalize procedures, which are capable of detecting and handling structural breaks in statistical data of different types. The latter are relevant in a variety of applied fields, e.g., in engineering, finance, insurance, economics, medicine, biology, hydrology, climatology, or ecology, to give just an incomplete list. The thesis essentially consists of three main chapters (Chapters 2-4) according to the different types of statistical framework under investigation.

First, in an introductory Chapter 1, the motivation and the structure of the thesis are described in more detail. The various models under consideration are introduced such as, e.g., binary time series models (Ch. 1.1.1), count time series like INGARCH models and models based on thinning operators (Ch.1.1.2), or general multivariate time series based on copulas (Ch. 1.2). Relevant mathematical tools for the goodness-of-fit testing and change-point detection in the latter models are briefly discussed in Chapters 1.3 and 1.4.

Chapter 2 deals with change-point problems in binary time series. It is based on two original, single-authored publications from 2013. The time series under consideration forms a binary autoregressive time series (cf. (2.1) and (2.2)) with a logit link function and with *atmost one abrupt change in the underlying intercept parameter* β_0 (cf. (2.4)). In order to test for a possible change in the mean, a CUSUM type score statistic is suggested (cf. (2.6)-(2.10)), which is sensitive to any parameter change leading to a change in the unconditional success probabilities of the binary time series. The main result derives an extreme value limiting distribution of the test statistic (Theorem 2.1), which allows for a comparison of the test statistic with the asymptotic critical value. Moreover, if the null hypothesis of ``no change´´ is rejected, an estimator $\widehat{m}$ of the unknown change-point m can be proposed. The above result can be generalized in various ways, for example by allowing for exogenous covariates (cf. Ch. 2.3). Then, under suitable regularity assumptions, the asymptotics of Theorem 2.1 can be retained (cf. Hudecová (2013a,b)).

Chapter 3, based on the results from Hudecová et al. (2015) and Hudecová et al. (2017), deals with goodness-of-fit (GOF) tests and sequential change-point procedures for count data. Since the underlying distributions are discrete, an appropriate statistical tool here is provided by the probability generating function and its empirical counterpart. The models under investigation are specific INAR and INARCH time series of counts, which are briefly revisited in Chapters 3.2.1 and 3.2.2 together with some important identities in Chapter 3.2.3.

In Chapter 3.3, GOF tests for INAR and INARCH data are suggested, which are based on weighted integral type test statistics comparing two different estimators of the underlying marginal distribution (cf. (3.10)). For the INAR model, it is shown in Theorem 3.1 that, under suitable assumptions, the test statistic has an asymptotic null distribution, which is the distribution of an integral of a Gaussian process with some specified covariance structure, i.e., it is equal to an infinite weighted sum of independent χ_1^2 random variables. Since there is no explicit form of this limiting distribution, this test can only be used, if there exists a consistent estimator for the unknown covariance structure. Therefore, in order to carry out the test and to compute critical values, a parametric bootstrap procedure is additionally suggested (see Algorithm 1 on p. 28). Moreover, Theorem 3.2 proves consistency of the test under fixed alternatives. Similar results are obtained for a linear Poisson INARCH(1) model, that is, an asymptotic null distribution is derived (cf. Theorem 3.3) and a corresponding parametric bootstrap procedure is suggested (cf. Algorithm 2 on p. 30).

In Chapter 3.4, change-point statistics are constructed for the sequential detection of structural breaks in INAR and Poisson INARCH time series data. For the INAR model, the proposed test statistic (cf. (3.21)-(3.22)) has an asymptotic null distribution, which is again the distribution of a zero-mean Gaussian process with an unknown covariance structure (cf. Theorem 3.4) and thus can only be used to obtain critical values, if this covariance structure can be estimated consistently. Therefore a resampling procedure based on historical data is proposed, which provides an asymptotically correct approximation of the null distribution and thus gives asymptotic critical values for the test (cf. Theorem 3.5 and Algorithm 3 on p. 36). Moreover, consistency of the test is proved under a large class of fixed alternatives (cf. Theorem 3.6). Corresponding results are obtained for the INARCH model, that is, an asymptotic null distribution of the suggested test statistic (cf. Theorem 3.7), a suitable resampling procedure (cf. Algorithm 4 on p. 37), and consistency of the test under a large class of fixed alternatives (cf. Theorem 3.8).

Chapter 4, based on Neumeyer et al. (2019) and Omelka et al. (2020), deals with the estimation and GOF tests in multivariate time series modelled via copulas. In Chapter 4.1, the copula time series models under consideration, which allow for the presence of exogenous covariates, are introduced and some basic facts concerning parametric, semiparametric or fully nonparametric estimation approaches are collected. More details of the model estimation are provided in Chapter 4.2, making use of estimated residuals $\hat{\varepsilon}_t$ and local polynomial estimators $\hat{m}_j$ and $\hat{\sigma}_j$ of the conditional mean and volatility functions m_j and σ_j, respectively (see (4.5) and (4.6)). Concerning the estimation of the underlying copula function C, a nonparametric approach is discussed in Chapter 4.2.1 as well as a semiparametric approach in Chapter 4.2.2, if the copula function belongs to a parametric family involving an unknown finite-dimensional parameter θ. Estimation of θ, making use of an inversion of Kendall's tau, as well as an M-estimation method are suggested.

Chapter 4.3 deals with asymptotics for the suggested copula estimators. In the nonparametric approach, Theorem 4.1 provides uniform consistency (in probability) of the copula estimator $\tilde{C}_n$ from (4.8) towards the ``oracle'' estimator $C_n^{(or)}$ as introduced in (4.13). This, together with the weak convergence of $\sqrt{n}\,(C_n^{(or)} - C)$, immediately results in the weak convergence of $\sqrt{n}\,(\tilde{C}_n - C)$ to a centered Gaussian process G_C (see Corollary 4.1). In the semiparametric case, asymptotic normality of the Kendall's tau estimator $\hat{\theta}_n^{(ik)}$ (see p. 43) is also immediate from Theorem 4.1 (cf. Corollary 4.2). Concerning the asymptotics of an M-estimator $\hat{\theta}_n$ of θ, asymptotic normality can also be proved under suitable assumptions (see Theorem 4.2). In fact, the latter asymptotic coincides with that of the consistent root $\hat{\theta}_n^{(or)}$ of the estimating equations (4.14) and it can be shown that $\hat{\theta}_n$ and $\hat{\theta}_n^{(or)}$ have the same asymptotic representation. The assumptions of Theorem 4.2, however, exclude some useful models, which is further discussed in Chapter 4.5.

For practical relevance, Chapter 4.4 deals with model verification and goodness-of-fit testing for the procedures discussed in Chapter 4.2. Based on the asymptotics of Chapter 4.3, the use of a Cramér-von Mises type test statistic S_n is proposed (cf. (4.18)) and a parametric bootstrap procedure is suggested to obtain critical values (see Algorithm 5 on p. 48). Since the assumptions of Theorem 4.2 are somewhat restrictive, the final Chapter 4.5, based on Omelka et al. (2020), discusses a simpler regression model and appropriate regularity conditions, which ensure validity of the stated asymptotics for a larger class of copula families. The considered setup is introduced in Chapter 4.1, whereas the relevant regularity conditions are given in (C.1) on p. 50 and (C.2)–(C.8) in Chapter 4.5.4. The main statements are given in Theorems 4.3 and 4.4, showing that, when fitting the copula C, one can (under the stated assumptions) ignore the fact that the true innovations were replaced by the estimated residuals. A brief discussion of practical implications is sketched in Chapter 4.3.

In conclusion, this habilitation work under review shows a broad and sound competence of the candidate in her field. She proves to be familiar with deep and sophisticated methods in probability theory, mathematical statistics and their applications as well as to be able to provide significant contributions to the state of the art with new ideas and suggestions. The habilitation thesis is very well written and organized, although there are highly involved techniques behind. Altogether it is an outstanding piece of scientific work, which certainly proves the candidate's ability for doing independent and successful research work in her field and for dealing with relevant applications. Moreover, the total list of publications, containing 25 items since 2009, mostly in high-level journals, definitely shows that Šárka Hudecová is an active researcher and well-integrated in her field. So, I can only strongly recommend to accept this thesis as the author's cumulative habilitation work in probability and mathematical statistics.

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(Prof. i.R. Dr. Josef G. Steinebach)