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Review for Dr. Vojtěch Patkóš habilitation thesis

To whom it may concern,

The habilitation thesis *'Precision calculations of light atomic spectra'* submitted by Dr Vojtěch Patkóš to the Faculty of Mathematics and Physics, Charles University in Prague is about the development and application of relativistic quantum electrodynamics (QED) to small and light atomic systems. For these types of systems, a non-relativistic reference state provides a good starting point, and perturbative corrections can be constructed using the theoretical framework called non-relativistic QED (nrQED). Dr Patkóš, in collaboration with Professors Krzysztof Pachucki and Vladimir Yerokhin, has developed higher-order corrections, up to α^6 and α^7 in terms of the fine-structure constant (in natural units), as well as recoil corrections due to the motion of the nuclei. The formal calculations were used in numerical applications and allowed direct comparison with experimental measurements. The results serve for continued testing of QED (and its practical adaptations), probing nuclear structure effects, and refining fundamental physical constants. The calculations and results represent the current frontier in the field.

The 45-page thesis includes five chapters. The thesis is well written and provides an easily accessible overview of the twelve enclosed scientific publications (as Appendix B), which form the basis of this habilitation work. Appendix A lists the publications of the Author. After an introduction and motivation of the work (Chapter 1), Chapter 2 reviews the theoretical framework and the main idea of the nrQED approach used during the work. Chapters 3 and 4 are about specific applications to hydrogen-like and helium-like atoms, respectively. The difficulties related to cancelling infinities of singular and divergent nrQED terms are briefly mentioned in the thesis and elaborated in detail in the author's scientific publications. Chapter 5 summarizes the work in seven thesis points and ends with a brief account of further plans.

The 91 bibliographic items give an appropriate overview of the literature. However, additional citations for (other variants and other applications of) the nrQED scheme (in particular, V. Korobov, but also A. Yelkhovsky and J-Ph. Karr) would be appropriate *(vide infra).*

I have collected the further small comments and corrections for consideration:

- Ref. 36: 'A' is missing in Phys. Rev. A.

- A general comment for the references: if possible, it is better to avoid *et al.* in the authors' names. It is nice to see all the names of the authors who contributed to each paper.
- $\mathrm{i} = \sqrt{-1}$ imaginary unit should be everywhere upright (not italic).
- The 'the' article is missing in several instances in the text.
- p. 6, 2nd paragraph:
  - 'For helium, the accuracy of' should be 'For triplet helium, the accuracy of'.
  - '$2S - 2P$' should be '$2\,^3S - 2\,^3P$', *i.e.,* the indication of the triplet spin state, should be added.
  - The citations [5], [6] (and similarly [9], [10]) should be separately included after the theoretical and the experimental uncertainties, respectively (to help the reader and for clarity).
- p. 7, above Eq. (1.2): 'For the hydrogen atom, ' should be 'For the hydrogen atom with a fixed nucleus, '
- p. 7, Eq. (1.3): It could be noted that the latest CODATA recommendation is slightly different for the inverse fine structure constant. I assume the authors used the (1.3) value in (all/most of?) their earlier work related to this dissertation.
- p. 7, the paragraph below Eq. (1.4): 'partialy' should be 'partially'.
- p. 8, Concerning nrQED, citations to Vladimir Korobov's work (and A. Yelkhovsky, and J-Ph. Karr) are missing. Also, a recent review by Adkins could be cited. On the other hand, the citation of Ref. 21 (also in papers by the co-authors) is not very useful; conceptually OK, but a bit misleading since many more fundamental (and numerical) nrQED papers are not cited at all.
- p. 12, 2nd paragraph: 'si' should be 'is'.
- p. 17, above Eq. (2.29): Is this the most appropriate citation for the Bethe logarithm?
- p. 19, Eq. (3.3): A comment on whether this expression is valid for any spin or just spin-1/2 and spin-0 would be appropriate. If it is also for spin-0, a short comment instructing the reader why it is the case and what happens with spin-1, spin-3/2, etc.
- p. 29, the paragraph below Eq. (4.20): Please check the sentence 'Only the recoil…'.

As a reviewer, I am expected to make an explicit statement about the originality of the work using the Turnitin system. I have studied both the dissertation and the Turnitin report as they were provided to me by the Charles University habilitation office. Based on these documents, I do not see any sign of plagiarism.

Regarding my overall assessment, the scientific content is highly original and important for the development of the field, both from a theoretical and, as far as I can tell, from an experimental point of view. The habilitation work has been reported in high-quality publications by the Author and his co-workers that are well-known and closely followed by the scientific community. Overall, this is excellent work and outperforms the standard requirements for a habilitation thesis. For these reasons, I recommend further progressing the habilitation procedure.

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