CHARLES UNIVERSITY
FACULTY OF SOCIAL SCIENCES
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The BEER Formula: Analysis of Equilibrium Exchange Rate of Czech Koruna

Bachelor's thesis

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Year of defense: 2025

Declaration of Authorship

I declare that I compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain any other academic title.

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During the preparation of this thesis, OpenAI's ChatGPT and GitHub Copilot were used to identify grammatical errors and to improve the R code, respectively. After using the tools, I reviewed and edited the content as necessary and take full responsibility for the content of the publication.

Prague, April 26, 2025

Ondrej Hajek

Abstract

This bachelor's thesis aims to specify models for estimating the equilibrium exchange rate of the Czech koruna against the euro and to examine the properties of its misalignment as derived from these models. In this study, the single-equation Behavioral Equilibrium Exchange Rate approach is employed, utilizing the Error Correction Model. Particular attention is given to the Real Interest Rate Differential fundamental, which, according to the test results, is identified as a statistically significant variable influencing the exchange rate movements. The thesis also investigates the impact of changing the exchange rate deflator on the resulting model estimates and the misalignment. It is found that when the nominal exchange rate is deflated using consumer prices, the estimated misalignment is higher and exhibits greater volatility over most of the sample period than when producer prices are used. These findings contribute to the refinement of the models used to estimate the exchange rate of the Czech koruna, which, in turn, has important implications for the formulation of monetary policy.

JEL Classification	C32, E70, F31, F41, O52
Keywords	Misalignment, ECM, RIRD, BEER
Title	The BEER Formula: Analysis of Equilibrium Exchange Rate of Czech Koruna

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Abstrakt

Cílem této bakalářské práce je zaměřit se na specifikaci modelů pro odhad rovnovážného směnného kurzu české koruny vůči euru a zkoumat vlastnosti vychýlení od rovnováhy, které je odvozené z těchto modelů. V této studii je použit jednorovnicový přístup behaviorálního rovnovážného směnného kurzu, pro jehož výpočet je využit model korekce chyb. Zvláštní pozornost je věnována fundamentu rozdílu reálných úrokových sazeb, který je na základě výsledků testů identifikován jako statisticky významná proměnná ovlivňující pohyby směnného kurzu. Práce se rovněž zabývá dopadem změny zvoleného deflátoru směnného kurzu na výsledné odhady modelů a vychýlení kurzu od své rovnováhy. Bylo zjištěno, že při deflování nominálního směnného kurzu pomocí spotřebitelských cen je odhadované vychýlení vyšší a vykazuje větší volatilitu během většiny pozorovaného období ve srovnání s případem, kdy jsou pro deflaci použity ceny průmyslových výrobců. Tyto poznatky přispívají k upřesnění modelů používaných k odhadu směnného kurzu české koruny, což má následně důležité důsledky pro provádění měnové politiky.

Klasifikace JEL	C32, E70, F31, F41, O52
Klíčová slova	Nesladěnost, ECM, RIRD, BEER
Název práce	Receptura BEER: Analýza Rovnovážného Měnového Kurzu České Koruny

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Acknowledgments

I would like to express my gratitude to prof. Ing. et Ing. Luboš Komárek, M.Sc., MBA, Ph.D. for his professional guidance and for proposing the topic of this thesis. I am also grateful to PhDr. Jaromír Baxa, Ph.D. and Attila Sarkany, M.A. for their helpful responses to my questions and for their valuable comments.

Typeset in FSV LaTeX template with thanks to prof. Zuzana Havrankova and prof. Tomas Havranek of Institute of Economic Studies, Faculty of Social Sciences, Charles University.

Bibliographic Record

Hajek, Ondrej: *The BEER Formula: Analysis of Equilibrium Exchange Rate of Czech Koruna.* Bachelor's thesis. Charles University, Faculty of Social Sciences, Institute of Economic Studies, Prague. 2025, pages 64. Advisor: prof. Ing. et Ing. Luboš Komárek, M.Sc., MBA, Ph.D.

Contents

List of Tables

List of Figures

Acronyms

ADF Augmented Dickey-Fuller

AIC Akaike Information Criterion

ARDL Autoregressive Distributed Lag

BEER Behavioral Equilibrium Exchange Rate

BIC Bayesian Information Criterion

CHEER Capital Enhanced Equilibrium Exchange Rate

CNB Czech National Bank

CPI Consumer Price Index

DFR Deposit Facility Rate

ECB European Central Bank

ECM Error Correction Model

ECT Error Correction Term

EER Equilibrium Exchange Rate

ERM II European Exchange Rate Mechanism II

EU European Union

FDI Foreign Direct Investments

FEER Fundamental Equilibrium Exchange Rate

GFCF Gross Fixed Capital Formation

GDP Gross Domestic Product

HICP Harmonized Index of Consumer Prices

HP Hodrick-Prescott

HQC Hannan-Quinn Criterion

HRF Hamilton Regression Filter

ITMEER Intermediate-term Model-based Equilibrium Exchange Rate

KPSS Kwiatkowski-Phillips-Schmidt-Shin

NAIRU Non-accelerating Inflation Rate of Unemployment

NATREX Natural Real Exchange Rate

NEER Nominal Effective Exchange Rate

NFA Net Foreign Assets

NX Net Export

OLS Ordinary Least Squares

PP Phillips-Perron

PPI Producer Price Index

PPP Purchasing Power Parity

REER Real Effective Exchange Rate

REPO Repurchase Operations

RIRD Real Interest Rate Differential

ULC Unit Labor Cost

VECM Vector Error Correction model

Chapter 1

Introduction

Exchange rates, one of the most widely discussed macroeconomic measures, are monitored not only by corporations, governments and economists, but also by a significant portion of the general public. Exchange rates serve as indicators of a currency's economic strength and significantly influence people's behavior and spending patterns. Within the European Union (EU), the most important exchange rates are those involving the euro, particularly for countries expected to join the common monetary union through the European Exchange Rate Mechanism II (ERM II). Determining an appropriate exchange rate value for a potential transition to the euro is essential, whether to enhance trade efficiency or to mitigate currency risk for businesses. This value should align with the economic conditions of the individual countries undergoing the change, reflecting their underlying economic fundamentals. For this purpose, the economic concept of the equilibrium exchange rate is employed.

The present study focuses on the specification of models for estimating the equilibrium exchange rate of the Czech koruna against the euro, with particular emphasis on the role of the Real Interest Rate Differential (RIRD). The rationale for concentrating on this specific relationship stems from the significant increases in interest rates observed in the post-pandemic era, which have been the largest in over 25 years across most European countries. Therefore, the hypothesis that the interest rate differential has not only an economically, but also a statistically significant effect on the development of the exchange rate will be tested. An additional hypothesis of the thesis posits that, when comparing the misalignment of the observed exchange rate deflated by various price indices, the absolute value of the misalignment will exhibit differences. Several methodologies exist for calculating the equilibrium exchange

rate, such as Fundamental Equilibrium Exchange Rate (FEER), Natural Real Exchange Rate (NATREX) and Behavioral Equilibrium Exchange Rate (BEER), developed by authors Williamson (1994), Stein (1995) and MacDonald (2000), respectively. In this thesis, the BEER approach will be utilized. To ensure accurate estimation, a thorough analysis of the data for stationarity will be conducted using the Dickey & Fuller (1979) and Phillips (1988) tests, as well as cointegration analysis using the Engle & Granger (1987) method. Finally, the development of the misalignment of the exchange rate from its equilibrium over the sample period will be examined, similar to the approach taken by Komárek & Motl (2012).

The structure of this bachelor's thesis is as follows. In Chapter 2, the key economic concepts necessary for understanding the aim of the thesis will be defined, the relevant literature on the topic will be reviewed and various methods for determining the real exchange rate will be introduced. In Chapter 3, the fundamentals and their specific form used in estimating the equilibrium exchange rate will be discussed. Additionally, all necessary tests related to the properties of the data and the applied detrending methods will be described in this chapter. Lastly, the models and their specifications used to calculate the misalignment of the observed exchange rate from its equilibrium level will also be presented. In Chapter 4, the estimates obtained from the calculations will be presented and the corresponding results will be interpreted. Chapter 5 will build upon the previous chapter by discussing the possible causes of the results and exploring how the findings could be further utilized by policymakers and other relevant stakeholders. The thesis will conclude in Chapter 6.

Chapter 2

Literature review

In this chapter, the focus will be on defining fundamental concepts such as the exchange rate and its variation, the real exchange rate equilibrium and its different conceptions across various time horizons, and the basic economic concepts related to the subject of the thesis. Subsequently, the different approaches used for determining the equilibrium exchange rates, along with their various modifications, will be examined.

2.1 Exchange Rate

Exchange rates act as bridges connecting international trade. An exchange rate is the amount of one currency required to purchase a unit of another currency or, conversely, the amount of one currency that is received in exchange for another. This system enables international trade, allowing countries to focus on producing goods in which they have a comparative advantage over others, thus supporting overall economic growth and stability. Accordingly, the exchange rate serves as one of the primary indicators of the economic performance of a country compared to others, particularly its trading partners. This would be entirely true in a scenario in which the exchange rate is determined exclusively by the supply and demand for currencies. Such a situation would only be plausible in the absence of any external influences, which would mean no interventions, immediate market adjustments, and no application of monetary policy, in other words, a purely floating exchange rate. In that case, the exchange rate would always remain in equilibrium. However, in practice, a purely floating exchange rate does not exist and, therefore, it is necessary to account for misalignment from equilibrium.

When examining exchange rates, we must also specify the foreign currency to which we compare our domestic currency. The most commonly used type for comparing economies, and the one employed in this bachelor's thesis, is the bilateral exchange rate. In this case, we express the price of one unit of foreign currency in terms of the domestic currency. In contrast to this relatively straightforward concept, there also exist effective exchange rates. These measure the value of the home currency against a basket of foreign currencies. Effective exchange rates are typically used to obtain an overall measure of a country's external competitiveness, particularly in relation to international trade balances. They are calculated as a weighted sum of individual exchange rates, with weights usually reflecting the significance of each foreign currency in the domestic country's trade. The calculation of Nominal Effective Exchange Rate (NEER) is presented in Equation 2.1.

$$NEER = \sum_{i=1}^{n} (S_i \times w_i) \tag{2.1}$$

where S_i is the bilateral nominal exchange rate between the domestic currency and the currency of trading partner i, w_i is the weight of trading partner i, typically based on trade shares and n is the number of foreign currencies considered.

Effective exchange rates are expressed as an index. As stated by MacDonald (2007), a value above 100 indicates an appreciation of the exchange rate, while a value below 100 signifies a depreciation. This convention is widely used in NEER analysis.

2.1.1 Real Exchange Rate

Most of the time, the focus is on real exchange rates, which are derived by adjusting the nominal exchange rates to account for the relative prices. The real exchange rate Q is calculated by modifying the bilateral nominal exchange rate S, which is defined as the domestic currency price of a unit of foreign currency, where an increase in S represents a nominal depreciation of the domestic currency. As a result, the real exchange rate can be calculated using the following equation:

$$Q = \frac{SP^*}{P}, \tag{2.2}$$

or by using natural logarithms:

$$q = s + p^* - p, \tag{2.3}$$

where lower case letters denote values in natural logarithm, P denotes the price level in the home country and * indicates variables of the foreign country.

For calculating the real exchange rate, various price measures can be employed. Since these measures differ, they yield varying results regarding the influence of economic fundamentals and can even impact their statistical significance. Consequently, using different price indices may result in distinct model specifications and alternative representations of the economic fundamentals. In addition, certain price indicators are more suitable for specific economies, as they emphasize different aspects of economic activity.

Among the most commonly used indices are a country's Consumer Price Index (CPI) or its alternative, the Harmonized Index of Consumer Prices (HICP), which is applied within the EU. Other frequently employed measures include the Producer Price Index (PPI), Unit Labor Cost (ULC), Gross Domestic Product (GDP) deflator, the ratio of tradable to non-tradable prices, and, less commonly, the ratio of export prices to import prices. So far, we have focused on the real bilateral exchange rate, which will be the primary focus of this thesis. However, it is also pertinent to mention the effective version of the real exchange rate. The Real Effective Exchange Rate (REER) is calculated similarly to the NEER, except that in this case, individual real exchange rates are weighted by trade importance, as shown in the following equation:

$$\text{REER} = \sum_{i=1}^{n} w_i \left(\frac{S_i P_i}{P_j} \right) \tag{2.4}$$

where S_i is the domestic currency price of a unit of $i - th$ trading partner foreign currency, j represents the home country and w_i is the weight of trading partner i, typically based on trade shares.

2.2 Equilibrium Exchange Rate

For our research it is essential to define the Equilibrium Exchange Rate (EER). In essence, the EER is a theoretical concept that seeks to identify the true underlying exchange rate, one that purely reflects the fundamental conditions of the economy, thus eliminating the effects of economic bubbles, irrational behavior of economic agents, and similar distortions. This means that we

are seeking an exchange rate at which all its economic fundamentals are in equilibrium. The general concept of EER and the evolution of its interpretation over the past decades have been extensively studied by economists, such as Milgate (1991). However, there is still no consensus on the precise definition of the equilibrium exchange rate or the appropriate time horizon for its estimation. This issue will be examined in a later section of the thesis. In theory, the current exchange rate should fluctuate around the equilibrium rate, and the misalignment between the equilibrium and the current exchange rate should be stationary. This means that the misalignment has an expected value of zero and its variance remains constant over time.

Understanding exchange rate misalignment is essential for monetary policymakers, as there is substantial evidence provided by Shatz & Tarr (2002) that an overvalued currency leads to a decline in economic growth of a country, primarily through reduced export competitiveness. In contrast, currency undervaluation does not appear to have such a negative impact on the economy. Conversely, it can stimulate economic growth by making exports relatively more affordable. However, imports become more expensive and, therefore, the overall economic impact depends on the structure of the economy. Some countries, such as China, Turkey and even the Czech Republic between 2013 and 2017, have systematically devalued their currencies to support domestic exporters.

2.2.1 Equilibrium Across Different Time Horizons

We have already discussed that the concept of equilibrium exchange rate is understood differently by various scholars. Now, we aim to emphasize how the chosen time horizon influences the relevant interpretation of equilibrium. According to Driver & Westaway (2005), the time scale under consideration affects the key questions of interest and, consequently, the significance of a given equilibrium. In addition, it affects the selection of fundamental variables in our modeling approach. Similarly to Pavlikova (2019), the interpretation of the equilibrium exchange rate will now be divided into three subsections: short-run, medium-run and long-run.

Short-run equilibrium

The concept of the short-run equilibrium exchange rate, as outlined by Clark & MacDonald (1998), refers to the exchange rate that would emerge when its fundamental determinants remain at their current levels, excluding the influence

of unpredictable factors such as financial market bubbles or external shocks. Equation 2.5 presents the mathematical representation of the concept.

$$e_t^{ST} = \beta' Z_t + \theta' T_t \tag{2.5}$$

where e_t^{ST} represents the short-run equilibrium exchange rate at time t, Z_t is a vector of economic fundamentals used to estimate equilibrium, T_t consists of short-term transitory factors, including current and lagged variables as well as dynamic effects from the fundamentals Z. Finally β and θ are vectors of estimated coefficients.

This concept aligns with Williamson (1983) definition of the current equilibrium exchange rate, which he describes as the exchange rate that would arise if financial markets were fully informed and acted rationally. This form of equilibrium is particularly valuable for examining temporary market distortions and detecting the potential presence of speculative bubbles or economic shocks. When the economy is in a state of disequilibrium, short-term exchange rate adjustments, whether through appreciation or depreciation, are essential to allow markets to absorb the impact and restore stability. To estimate the short-term exchange rate, models such as Capital Enhanced Equilibrium Exchange Rate (CHEER), Intermediate-term Model-based Equilibrium Exchange Rate (ITMEER), or BEER can be employed. In this thesis, we will focus on the BEER model, as it is particularly suited to analyze short-term equilibrium, which is the only type of exchange rate equilibrium that can be influenced by central banks through their monetary policy.

Medium-run equilibrium

The medium-term EER is contingent on both the internal and external economic balance. Firstly, to achieve internal balance, the economy must operate at its normal capacity, where supply and demand are in equilibrium, resulting in a zero output gap. Additionally, unemployment should be at the Non-accelerating Inflation Rate of Unemployment (NAIRU). Secondary, the economy must achieve an external balance, which implies that other countries must also achieve their own internal equilibrium. It is important to note that an external balance does not necessarily require a zero current account balance. Instead, it should be at a sustainable level, meaning it is consistent with eventual convergence toward stock-flow equilibrium. This process of convergence leads to domestic interest rates aligning with global levels.

In the medium run, short-term discrepancies are expected to be corrected through exchange rate adjustments and, to some extent, market mechanisms, resulting in real rigidities. Unlike short-term equilibrium, which is based on the current values of economic fundamentals, the medium-term equilibrium relies on their trend values. As a consequence, if the assumption of monetary policy superneutrality holds, monetary policy becomes ineffective in influencing the medium-term exchange rate. Based on these considerations, the medium-run equilibrium exchange rate can be defined as follows:

$$\hat{e}_t = \beta' \hat{Z}_t \tag{2.6}$$

where $\hat{Z}_t$ denotes a vector of economic fundamentals at their trend values, which converge towards long-term equilibrium, and β' represents the corresponding vector of coefficients.

Long-run equilibrium

A long-term equilibrium is achieved when both the domestic and global economies maintain stable stock variables such as wealth, capital and foreign debt, and flow variables such as trade and investment over time, ensuring long-term sustainability. According to Milgate (1998), long-run equilibrium can be understood as a state in which the economy does not exhibit at endogenous tendency to change. The long-run equilibrium exchange rate is then defined as:

$$\bar{e}_t = \beta' \bar{Z}_t \tag{2.7}$$

where $\bar{Z}_t$ is a vector of economic fundamentals at their long-run values and again β' represents the corresponding vector of coefficients.

2.3 Models of Equilibrium Exchange Rate

In this section, we present the four most commonly used methods to estimate the real equilibrium exchange rate. As mentioned in the previous section, each model is suited to a different time horizon and has its own strengths and weaknesses. The models discussed include Purchasing Power Parity (PPP), the Fundamental Equilibrium Exchange Rate (FEER), the Natural Real Exchange Rate (NATREX), and finally, the Behavioral Equilibrium Exchange Rate (BEER), which will be employed in our calculations.

2.3.1 PPP

The Purchasing Power Parity (PPP) approach, one of the oldest methods of determining the equilibrium exchange rate, was originally formulated by Cassel (1924) and was previously referred to as "buying or paying power". PPP is typically categorized into absolute and relative forms. The absolute PPP is defined as the ratio of price levels between the compared countries, while the relative PPP is calculated as the actual exchange rate in a given period multiplied by the ratio of price indices of the respective countries during the same period. According to Officer (1976), the actual exchange rate used in the relative PPP approach should be taken from a stable period, excluding times of crisis.

$$PPP_j^{abs} = \frac{PL_j^B}{PL_j^A} \quad \text{and} \quad PPP_j^{rel} = \frac{P_j^B}{P_j^A} \cdot R_j \tag{2.8}$$

where PL_j^i represents the price level in country i in period j, P_j^i is the price index in country i in period j, PPP_j^{abs} denotes the absolute PPP in period j, PPP_j^{rel} refers to the relative PPP in period j and R_j is the actual exchange rate in period j.

From the formulas alone, the limitations of the PPP approach as a measure of EER become immediately apparent. It neglects to account for key real determinants of the exchange rate, such as relative productivity, interest rate differentials, net foreign asset positions, and capital flows. As a result, PPP proves to be an inadequate method for estimating the EER in developing countries, where these fundamental factors evolve over time, thereby shifting the equilibrium exchange rate, an effect that PPP does not capture. Due to these shortcomings, PPP is primarily used for very long-term estimates of the equilibrium exchange rate in already developed economies.

To address some of these limitations, MacDonald (2000) proposes a less restrictive form of PPP, which assumes that the real exchange rate is mean reverting, defined by the following equation:

$$q_j = \rho q_{j-1} + \alpha + \epsilon_j, \tag{2.9}$$

where q_j is the exchange rate in period j and ρ is expected to be between zero and plus infinity ($0 < \rho < \infty$).

It is thought that Cassel (1924) assumed the half-life of a shock to PPP to be approximately one year. However, more recent studies, such as those of Bayoumi & MacDonald (1999) and Frankel & Rose (1996), generally suggest a

half-life of around four years. As mentioned above, this has led to PPP being rarely used in contemporary research to estimate EERs. Nevertheless, it remains a popular simplification among the general public, often employed in the form of indices such as the Big Mac Index.

2.3.2 FEER

Another widely used approach is the Fundamental Equilibrium Exchange Rate (FEER) methodology, developed by Williamson (1983), extended and tested for robustness by Wren-Lewis (1998). This medium-run approach is based on macroeconomic equilibrium at both the internal and external levels, as discussed above. The sustainable external balance varies between countries and the determination of its appropriate level is generally addressed by Williamson (1994). Unlike the PPP approach, it does not depend on changes in inflation differential. Rather, maintaining the exchange rate at a constant FEER requires the nominal exchange rate to change at a rate equal to the inflation differential. However, there are three well-established reasons why the FEER should be viewed as a trend path for the exchange rate rather than a constant. First, differences in productivity growth rates, as explained by Balassa (1964), contribute to this trend. Second, the accumulation of net foreign liabilities leads to a depreciation of the FEER. Third, if the product of the income elasticity of import demand and the domestic growth rate exceeds the product of the income elasticity of export demand and the foreign growth rate, there will be a long-term trend toward a deterioration of the current account balance.

For defining the FEER we need to start with a core of macroeconomic approach by equating the current account (CA) to the negative of capital account (KA):

$$CA \equiv -KA \tag{2.10}$$

Furthermore, we express the current account in terms of its determinants. The current account is typically modeled as a function of domestic and foreign aggregate output or demand, denoted as Y_d and Y_f, respectively, along with the real effective exchange rate, q. Substituting these variables into the previous equation, we obtain the following.

$$CA = b_0 + b_1 q + b_2 \bar{y}_d + b_3 \bar{y}_f = -\overline{KA} \tag{2.11}$$

where $b_1 < 0$, $b_2 < 0$, and $b_3 > 0$, and $\overline{KA}$ is the equilibrium capital account over the medium term.

The FEER, or the real effective exchange rate (q), is derived from the current account model in equation (2.11) and represents the exchange rate that maintains macroeconomic balance. It ensures that the current account is consistent with the sustainable capital account, given that its determinants reflect full employment conditions. Solving equation (2.11) for q determines the FEER:

$$FEER = \frac{-\overline{KA} - b_0 - b_2\bar{y}_d - b_3\bar{y}_f}{b_1} \tag{2.12}$$

For calculating FEER of the Czech koruna, Komárek & Motl (2012) have used a slightly different approach. The basic version of the FEER model contains a block of foreign trade equations and other complementary equations, or identities, ensuring simultaneous achievement of the internal and external equilibrium of the economy. The Export and Import equations are as follows:

Export:

$$\ln X = f(\ln MEMU; \ln RER; \ln DPROD; \ln FDI^X), \tag{2.13}$$

Import:

$$\ln M = f(\ln DD; \ln RER; \ln X), \tag{2.14}$$

where X is real export, M is real import, DD is real total domestic demand, $MEMU$ is real imports of euro area goods and services, RER is the real exchange rate, $DPROD$ is the labour productivity share of the Czech Republic relative to the EMU, and FDI is foreign direct investment.

Exports and imports equations represent long-run parts of cointegration vectors estimated by Vector Error Correction model (VECM). The full FEER model is then constructed by augmenting the behavioral equations of real exports and imports with the identities of net exports, real GDP, and the identities that define the internal and external equilibrium.

Identities:

$$NX_t = X_t - M_t \tag{2.15}$$

$$NX_t^N = P_t^X NX_t + M_t \left(1 - \frac{P_t^M}{P_t^X}\right) P_t^X \tag{2.16}$$

$$Y_t = DD_t + NX_t \tag{2.17}$$

$$Y_t^{GAP} = Y_t - Y_t^{EQ} \tag{2.18}$$

$$Y_t^N = Y_t P_t^Y \tag{2.19}$$

$$CA_t^{EB} = CA_t^{EQ} Y_t^N \tag{2.20}$$

$$CA_t^{GAP} = NX_t^N - CA_t^{EB} \tag{2.21}$$

where NX is real export, NX^{N} is nominal export, P^{X} are export prices, P^{M} are import prices, P^{Y} is deflator of the GDP. Y is real GDP, Y^{N} is nominal GDP, Y^{EQ} is the potential GDP and finally Y^{GAP} is the difference between the potential and real GDP. CA^{EQ} is the long-term current account balance, CA^{EB} is the sustainable amount of the long-term current account relative to the GDP and CA^{GAP} is the difference between the long-term and actual current account balance. The model is then solved as an optimization problem, minimizing the size of the squares of the variables Y^{GAP} and CA^{GAP} ,when bounded by the individual equations of the model. Since two variables are subject to optimization, two instruments are required to facilitate the adjustment mechanism, namely the real exchange rate and domestic demand.

2.3.3 NATREX

Another approach to modeling the real equilibrium exchange rate is the NATREX model, developed by Stein (1995). NATREX represents the equilibrium real exchange rate that balances the balance of payments in the absence of cyclical factors, speculative capital flows and fluctuations in international reserves. A similar concept was introduced by Nurkse (1945), who proposed his own standard for an equilibrium exchange rate during the IMF's period of loosely defining 'fundamental equilibrium' under the Bretton Woods system.

As noted earlier, NATREX is a medium- to long-term approach, since short-term fluctuations are primarily driven by speculative capital flows rather than movements in real fundamentals, which remain relatively stable over short periods. The medium-term market clearing equilibrium in the NATREX framework can be expressed through the national income accounts equation:

$$I - S + CA = 0 \tag{2.22}$$

where I represents the desired national investment, S denotes the desired national saving, and CA is the desired current account, all measured under conditions of capacity output and fulfilled inflation expectations. The real exchange rate then adjusts through appreciation or depreciation to balance excess or insufficient demand, ensuring that equilibrium is maintained.

The NATREX model forecasts the extent to which changes in fundamentals will impact the equilibrium real exchange rate, without assessing the desirability of the disturbance or its consequences. This defines NATREX as a purely positive approach, in contrast to FEER model by Williamson (1994) , which incorporates a normative aspect by assuming that capital flows should correspond to the difference between investment and savings that are not distorted by public policy.

In the first category, which addresses asset markets, three distinct cases can be identified. According to studies by Crouhy-Veyrac & Saint Marc (1998), the first group consists of small countries with high capital mobility, such as Germany and France. These countries can freely borrow at the world real interest rate r', meaning that the market adjustment falls entirely on the real exchange rate R to align the current account with the investment-savings balance $I - S$.

The second group, examined by Stein (1998), includes large economies such as the United States, which are influential enough to affect the world interest rate and also exhibit high capital mobility. In this case, an increase in the desired borrowing, $I - S$, leads to a rise in the world interest rate and to a medium-run appreciation of the NATREX. However, since higher interest rates negatively impact $I - S$, the extent of NATREX appreciation is ultimately lower. The third group consists of small economies with imperfect capital mobility, such as Australia. As studied by Lim & Stein (1998), these countries face borrowing constraints due to a finite foreign demand for their liabilities. Consequently, an increase in the desired borrowing raises the domestic real interest rates, leading to a lower degree of NATREX appreciation, similar to the previous case. The second categorization of the NATREX models is based on the treatment of the goods market, which considers three types of goods: nontradables, exportables, and importables. In the first version, applicable to large open economies, the country's deflator converges to the price of exportable goods, making the real exchange rate equivalent to the terms of trade. In

the second version, used for countries small in the tradable goods market, endogenous changes in the real exchange rate are proportional to changes in the relative price of nontradables.

2.3.4 BEER

The most widely researched and commonly used approach to estimate the equilibrium exchange rate is BEER. This methodology employs econometric techniques to establish a behavioral relationship between the real exchange rate and the relevant economic variables. It is represented by a single reduced form equation in the following form:

$$q_t = \beta_1' \mathbf{Z}_{1t} + \beta_2' \mathbf{Z}_{2t} + \tau' T_t + \varepsilon_t \tag{2.23}$$

where Z_1 and Z_2 represent vectors of economic fundamentals that affect the real exchange rate over the long and medium term, respectively. In addition, the Z_2 factors may be related to the business cycle. T is the vector of transitory fundamentals that affect the exchange rate in the short term and ε_t is the random disturbance term.

Based on the values of the fundamentals incorporated into the model, different types of misalignment are obtained. This topic will be examined in more detail in the next subsection. However, rather than focusing on the specific form of the fundamentals, researchers primarily emphasize the correct selection of individual fundamentals. There are numerous possible approaches and combinations of fundamentals for estimating the equilibrium exchange rate by the BEER methodology, which vary according to country-specific characteristics. Given the large number of potential fundamentals, careful selection is necessary to avoid model overspecification, which could lead to misleading results. Several studies focus on summarizing all possible fundamentals used in the BEER model, such as Adler & Grisse (2017). Similarly, Table 2.1 provides an overview of various BEER model specifications employed by different authors to estimate the equilibrium exchange rate of the Czech koruna.

Table 2.1: Fundamentals of the BEER Model for the Czech koruna

Fundament	Babetskii & Égert (2005)	Malecký & Komárek (2007)	Pošta (2010)	Motl & Komárek (2012)	Jančovič (2017)	Novotný (2022)
FDI		−				
PROD	−	−	−	−	−	−
GS/GOVDEBT		−	−		−	−
INTV					+	
NFA	−	−	−	−	+	−
NX				−		
OIL			+		+	
OPEN		−				
RIRD			+		−	
SPT					+	
GFCF				−		
TOT		−				

Table 2.1 summarizes various specifications of the BEER model for the Czech koruna. The first column lists the acronyms for the fundamentals. FDI refers to Foreign Direct Investment, $PROD$ represents the productivity differential between the Czech Republic and the Eurozone, GS and $GOVDEBT$ stand for government spending and government debt, respectively. $INTV$ refers to interventions by Czech National Bank (CNB), Net Foreign Assets (NFA) denotes net foreign assets, NX represents net exports, OIL proxies oil prices, $OPEN$ measures the openness of the economy, RIRD is the real interest rate differential, SPT refers to spot trades made by the CNB, $GFCF$ represents real investments and TOT stands for terms of trade. A detailed discussion of most of these fundamentals is provided in Chapter 3.

The other columns show individual models by different authors. The plus sign indicates real exchange rate depreciation, while the minus sign indicates real exchange rate appreciation, based on the respective study results.

2.4 Misalignment

The primary objective of estimating the real equilibrium exchange rate using the aforementioned models is to assess misalignment, i.e. deviations of the actual exchange rate from its estimated equilibrium value. Within our behavioral approach, we differentiate between two types of misalignment. The first is the current, or speculative, misalignment, defined as the difference between the actual real exchange rate and the estimated equilibrium rate based on actual fundamentals. This captures short-term deviations of the domestic currency from its equilibrium value.

The second is the total misalignment, including both current and cyclical misalignment, defined as the gap between the actual and estimated equilibrium real exchange rate, where the latter is derived using a cyclical filter, such as the Hodrick-Prescott or Hamilton filter, to capture sustainable fundamental values. Misalignment is a critical concern for all economic agents interested in exchange rate developments, particularly for central banks, which incorporate misalignment of the exchange rate alongside other indicators when formulating monetary policy.

Chapter 3

Data and Methodology

In this chapter, we will provide a detailed explanation of the data used in our real equilibrium exchange rate estimation model. In addition, we will discuss the most appropriate methods for filtering our data using various statistical filters. Furthermore, we will present all the necessary tests conducted to ensure the robustness of our EERs calculations. Moreover, we will examine the optimal data frequency for our analysis and provide a comprehensive explanation of the methodologies employed to estimate the BEER.

3.1 Data

3.1.1 Data frequency

Data frequency is a crucial specification of the BEER model, as it directly affects the significance of fundamental variables and consequently their inclusion in the model. Selecting an appropriate frequency is essential to ensure the reliability of estimates. Certain variables, such as the real interest rate differential, may be significant when using quarterly data but not when using annual data, as highlighted by Giordano (2021).

There are three possible data frequencies for modeling the BEER model: monthly, quarterly, and annual. Monthly data presents challenges due to limited availability, as only a few time series are collected on a monthly basis. Even when available, these time series often begin around 2007 or later. Quarterly data face similar availability issues, but all quarterly time series are available at least from 1999 onward, which is sufficient for our research, as the euro was introduced at the beginning of 1999. Annual time series typically extend back to 1980, providing the longest time horizon. Giordano (2021) demonstrates that

quarterly data exhibit strong predictive power, making them a suitable choice for the modeling of exchange rates. Additionally, quarterly data are the preferred choice in most studies, particularly in research conducted by Komárek & Motl (2012) at the CNB. Based on these considerations, we also adopt a quarterly data frequency for our BEER model estimation.

3.1.2 Fundamentals

We now describe the sources and time coverage of the fundamental variables used in our model. It is crucial to select only variables that have a strong theoretical foundation in economic theory, as including an excessive number of independent variables may lead to overfitting. Ca´ Zorzi & Rubaszek (2023) recommend using approximately three to five independent variables to achieve optimal results. Our model extends the framework employed by the CNB, as specified by Komárek & Motl (2012). The first fundamental variable is $PROD$, representing the productivity differential. It is calculated as the Czech Republic real GDP, deflated by the GDP deflator, per person employed, divided by the equivalent measure for the eurozone. This ratio expresses the Czech productivity as a percentage of the productivity of the eurozone. Data are sourced from Eurostat. The second fundamental variable is NFA, defined as the nominal net foreign assets relative to nominal GDP. The data for NFA are obtained from the CNB database, while GDP figures are sourced from Eurostat. Another independent variable in our BEER model is Gross Fixed Capital Formation (GFCF), which represents the real investment as a percentage of GDP. Given that investment is expected to influence the exchange rate with lag, we include GFCF with a delay of one period in the static model. Both time series are retrieved from the CNB database. The next fundamental variable is NX, which represents net exports. It is defined as the difference between total exports and total imports, both adjusted by their respective deflators. All relevant data, including export and import deflators, are obtained from the CNB database.

The final independent variable is the Real Interest Rate Differential (RIRD), between the Czech Republic and the Eurozone, which is considered in three different versions depending on the maturity of the interest rate:

- $RIRD_S$ (short-term): The differential between the Czech Repurchase Operations (REPO) rate and the Eurozone Deposit Facility Rate (DFR), adjusted for inflation using the HICP from each region. Data on short-

term interest rates are sourced from the ARAD and European Central Bank (ECB) databases, while HICP data come from Eurostat.

- $RIRD_M$ (medium-term): Similar to $RIRD_S$ but calculated using the 6-month $PRIBOR$ and 6-month $EURIBOR$ rates, also adjusted for inflation using HICP. The interest rate data are retrieved from the $ARAD$ and ECB databases.

- $RIRD_L$ (long-term): Defined as the difference between the 10-year Czech government bond yield and the 10-year Euro area benchmark bond yield, again adjusted for inflation using HICP. The bond yield data are obtained from the $FRED$ database.

The dependent variable in our model is the real exchange rate, calculated from the nominal exchange rate obtained from the CNB database. It is deflated using two alternative measures: first, the PPI and second, the HICP. Both deflators are sourced from Eurostat. All fundamental variables are expressed in constant 2015 prices to ensure comparability.

3.1.3 Data Filtering Methods

Most econometric models, such as static model or Autoregressive Distributed Lag (ARDL) model in levels, require the stationarity of time series. However, a significant portion of macroeconomic time series exhibit a trend and are integrated at higher orders. Consequently, detrending or filtering is commonly applied to separate the trend and cyclical components of these series. One of the most widely used detrending techniques is the Hodrick-Prescott (HP) filter, introduced by Hodrick & Prescott (1997). This method decomposes a time series into a smooth trend and a cyclical component by minimizing the following equation.

$$\min_{\tau} \left(\sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right]^2 \right) \tag{3.1}$$

In this technique, it is necessary to select the smoothing parameter λ. This parameter controls the trade-off between the smoothness of the trend and the fit to the original series. A higher λ places greater weight on the second term of the equation, resulting in a smoother trend. In the extreme case, if λ approaches infinity, the estimated trend becomes linear. Conversely, a lower λ produces a trend that more closely follows the original series. Setting λ to zero would

eliminate the second term entirely, causing the estimated trend to be identical to the original series and the cyclical component to be zero.

For quarterly data, as used in this thesis, the commonly recommended value for λ is 1600, with higher values suggested for higher frequency data. Some researchers, such as Franke *et al.* (2023), have explored the "optimal" value of λ by applying the HP filter to data sets with known trend and cyclical components. Their findings suggest that for quarterly data, the appropriate λ should be 8 to 12 times higher than the conventional value of 1600.

A more recent study by Hamilton (2017) introduces an alternative approach to detrending time series data, known as the Hamilton Regression Filter (HRF), which is based on a fundamentally different principle. Instead of smoothing the time series, Hamilton argues that when predicting the future value of a variable based on currently available information, any misalignment between the predicted and actual observed values must be attributed solely to the cyclical component of the series. Consequently, by subtracting the predicted values from the observed values, the cyclical component of the series can be extracted. In his paper, Hamilton proposes that for quarterly data, the trend value should be estimated by regressing the value of the variable eight periods ahead on its current value and its three most recent lags. The following equations illustrate the methodology. The first equation presents the calculation of the predicted value of the variable and the estimation of its trend, while the second equation determines the cyclical component of the time series.

$$y_{t+h} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + v_{t+h}, \tag{3.2}$$

The cyclical components (residuals) are calculated as follows:

$$\hat{v}_{t+h} = y_{t+h} - \hat{\beta}_0 - \hat{\beta}_1 y_t - \hat{\beta}_2 y_{t-1} - \hat{\beta}_3 y_{t-2} - \hat{\beta}_4 y_{t-3} \tag{3.3}$$

Hamilton claims that his method addresses several limitations of the HP filter, such as the need to arbitrarily select the smoothing parameter λ, the potential introduction of artificial relationships within the data, and the risk of incorrectly filtering data points at the beginning and end of the sample. However, in contrast, Hodrick (2020) and Franke *et al.* (2025) demonstrate in their studies that the HP filter outperforms the HRF in most modeled scenarios. We estimate models using both detrending approaches. Due to the limited sample size, we will apply the classical HP-filter approach with a lambda value of 1600 for detrending, as the alternative method with a higher lambda is more

likely to impact the results due to higher changes in the boundary values of our limited sample. It is also important to note that both filtering techniques have the potential to remove not only deterministic trends but also stochastic trends from the data. This implies that applying these filters may lead to the loss of long-term relationships within the dataset.

3.2 Methodology

3.2.1 Tests of Stationarity

In this thesis, we perform primarily two types of unit root tests to assess the stationarity of the time series of our fundamentals. First, we apply the Augmented Dickey-Fuller (ADF) test, developed by Dickey & Fuller (1979). Second, we use the Phillips-Perron (PP) test, introduced by Phillips (1988). The PP test accounts for heteroskedasticity and autocorrelation without the need for lagged differences, but may be less powerful than the ADF test when autocorrelation is well specified. Thus, we conduct both tests to ensure robustness of the results. In both tests, the null hypothesis posits that the series has a unit root, implying nonstationarity, while the alternative hypothesis indicates that the series is stationary, free from a unit root. The outcomes of both tests are presented below.

Table 3.1: Stationarity Tests Results Unadjusted Data

Variable	ADF Statistic	ADF P-Value	PP Statistic	PP P-Value
$\ln RER_{PPI}$	-2.266178	0.46629915	-9.174889	0.5842546
$\ln RER_{HICP}$	-2.527566	0.35791799	-9.178603	0.5840388
$PROD$	-2.686416	0.29205319	-12.904015	0.3675992
NFA	-1.049551	0.92668054	-1.072806	0.9850785
$GFCF$	-1.635555	0.72777843	-5.213376	0.8144112
NX	-2.258464	0.46949785	-16.407078	0.1640777
$RIRD_S$	-4.201338	0.01000000	-153.814072	0.0100000
$RIRD_M$	-4.005785	0.01160821	-156.000750	0.0100000
$RIRD_L$	-4.248556	0.01000000	-154.898869	0.0100000

From the Table 3.1, we observe that in both tests, most variables are nonstationary, except for the three variants of real interest rate differentials, which are stationary in both tests. Next, it is necessary to determine the order of integration for the nonstationary variables, meaning the number of times the time series must be differenced to achieve stationarity. This step is crucial, as certain models, such as the ARDL and its variant, the Error Correction

Model (ECM), require the series to be at most I (1), which means integrated of order 1. To verify whether the series are of order I(1), they must be differenced, and the unit root test should be performed again. In this second test, all variables should exhibit p-values lower than the chosen significance level, confirming their stationarity.

Table 3.2: Stationarity Tests Results after Differentiation

Variable	**ADF Statistic**	**ADF P-Value**	**PP Statistic**	**PP P-Value**
$\ln RER_{PPI}$	-5.532477	0.01000000	-77.20721	0.01
$\ln RER_{HICP}$	-4.712247	0.01000000	-105.48533	0.01
$PROD$	-4.338457	0.01000000	-82.10269	0.01
NFA	-3.432313	0.05290343	-148.11649	0.01
$GFCF$	-4.108174	0.01000000	-125.58009	0.01
NX	-4.786239	0.01000000	-109.36218	0.01
$RIRD_S$	-4.104423	0.01000000	-152.16866	0.01
$RIRD_M$	-3.886986	0.01737483	-154.62352	0.01
$RIRD_L$	-4.216735	0.01000000	-154.42007	0.01

The results presented in Table 3.2 indicate that most of the variables are now stationary at least at the 1 percent significance level, with the exception of NFA in the ADF test, which is nearly stationary at the 5 percent significance level. In contrast, the results of the PP test show that all variables are now stationary.

To further assess the stationarity of the NFA after differencing, we employ the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test originally developed by Kwiatkowski *et al.* (1992). It is important to note that, unlike the ADF and PP tests, which assume nonstationarity under the null hypothesis, the KPSS test takes stationarity as the null hypothesis. Based on data visualization and the results of the KPSS test, we conclude that NFA, after differencing, is stationary. This suggests that all three real interest rate differential variables can be considered stationary, whereas the remaining time series are integrated of the first order, confirming the need for first differencing to achieve stationarity.

We also apply the same tests to the detrended data, obtained by using the statistical filters outlined in the subsection Data Filtering Methods. It has been demonstrated that the time series, after isolating the cyclical component, becomes stationary without the need for differentiation. This is beneficial for estimation using models such as the static model or the ARDL model in levels, provided that there is no cointegration within the data. The results of the ADF

and PP tests on the detrended data can be found in Table A.1 and Table A.2 in appendix of the thesis.

3.2.2 Optimal Number of Lags

Before proceeding with a more detailed analysis of the impact of fundamental variables on the EER, such as estimating the number of cointegration relationships among them, it is essential to determine the optimal number of lags that we need to use. This will be accomplished using the Akaike Information Criterion (AIC), Hannan-Quinn Criterion (HQC), and Bayesian Information Criterion (BIC). Although all of these criteria serve to identify the most appropriate number of lags, they differ in their approach. The AIC prioritizes predictive precision by imposing a relatively low penalty for the inclusion of additional parameters. The HQC strikes a balance between predictive accuracy and model simplicity, introducing a penalty for complexity that grows logarithmically with the size of the dataset. Lastly, the BIC applies even a stronger penalty for additional parameters, making it a more suitable choice for smaller data samples.

For the present analysis and based on the number of variables and their quarterly frequency, we will adopt the number of lags recommended by the HQC. This will involve selecting four lags for all variables and model specifications in order to maintain consistency throughout the calculations. The decision to use four lags is also justified from the perspective of adequately covering the entire year, thus accounting for seasonal regularities. Given that some of the series are integrated of order one, it is necessary to differentiate them to achieve stationarity. The results of all combinations and different information criteria are presented in Table 3.3.

Table 3.3: Optimal Lag Selection Based on AIC, HQC and BIC

Combination with	**AIC Lags**	**HQC Lags**	**BIC Lags**
$lnRER_{PPI}$ & $RIRD_S$	5	2	1
$lnRER_{HICP}$ & $RIRD_S$	4	4	1
$lnRER_{PPI}$ & $RIRD_M$	5	2	1
$lnRER_{HICP}$ & $RIRD_M$	4	4	1
$lnRER_{PPI}$ & $RIRD_L$	5	4	1
$lnRER_{HICP}$ & $RIRD_L$	4	4	1

3.2.3 Cointegration

In this subsection, we focus on examining the data to identify potential cointegration relationships among the time series. Cointegration refers to the situation where nonstationary time series, which may display different short-term movements, are linked by a long-term relationship, implying that they influence each other over time and share a common or inverse underlying trend. Determining whether the data are cointegrated is crucial for selecting an appropriate model to estimate the relationship between the equilibrium exchange rate and its fundamental determinants. Since we are dealing with multiple time series, we employ the Johansen cointegration test with decomposition of eigenvalues, as introduced by Johansen (1988). The null hypothesis of the Johansen test posits that there is no cointegration, whereas the alternative hypothesis suggests the presence of one or more cointegration relationships. The test can also be used to determine the number of cointegration relationships, up to $k - 1$, where k represents the number of potentially cointegrated time series. Additionally, the Johansen test requires the specification of the optimal number of lags, which was determined in the previous subsection Optimal Number of Lags. To ensure the robustness of our findings, we also tested the residuals of the static model estimated by Ordinary Least Squares (OLS) on both unadjusted and seasonally adjusted data. If the variables are cointegrated, the residuals should be stationary. For both approaches, we exclude the real interest rate differential variables as they are stationary and therefore cannot be cointegrated by definition.

After testing the residuals, we find that they are stationary regardless of the data used. However, a stronger significance of stationarity is observed in the residuals when seasonally adjusted data are employed. The Johansen cointegration test is conducted for various combinations of the time series, typically revealing three to four cointegration relationships out of a possible four. The results of these tests are provided in Table A.3 and Table A.4 in the appendix. Based on these findings, we conclude that the variables are cointegrated, indicating the existence of a long-term relationship among the fundamental determinants.

3.2.4 Static Regression Model

The first model used to estimate the equilibrium exchange rate is the one frequently used by the Czech National Bank (CNB). Initially, the data will be detrended using statistical filters and the cyclical component of the series will be extracted. Since all time series are now stationary, a linear regression model in levels can be applied. Most variables will not include lagged values. However, the variable GFCF_t will be replaced by its lagged value, GFCF_{t-1}, based on the assumption that real investment has a lagged effect on the real equilibrium exchange rate. The following equation is then estimated using the OLS methodology.

$$\begin{aligned} \ln RER_t = \beta_0 + \beta_1 DPROD_t + \beta_2 NFA_t + \beta_3 GFCF_{t-1} \\ + \beta_4 NX_t + \beta_5 RIRD_t + \varepsilon_t \end{aligned} \tag{3.4}$$

3.2.5 Autoregressive Distributed Lag Model

For the unfiltered data, where some series are stationary while others are integrated on order one, a more advanced model is required to accommodate this mixture of different integration orders. The ARDL model, first introduced by Pesaran & Shin (1999), meets these criteria. As the name suggests, the model consists of an autoregressive component, incorporating lagged values of the dependent variable. In addition, the lagged values of the independent variables are included, as shown in Equation 3.5.

$$y_t = \alpha + \sum_{i=1}^{p} \beta_i y_{t-i} + \sum_{j=0}^{q} \gamma_j X_{t-j} + \epsilon_t \tag{3.5}$$

where p and q represent the optimal number of lags for the dependent and independent variables, respectively. In our case, both p and q are equal to four, as determined in subsection Optimal Number of Lags. Additionally, a necessary condition for the ARDL model is that none of the time series is integrated on the order of two or higher. This condition was verified in the Tests of Stationarity subsection. The Equation 3.5, along with the parameters of the independent variables and their lags, is then estimated using the OLS methodology. Particular attention should be given to the number of independent variables, as an increase in the number of lags leads to a rapid decline in the degrees of freedom.

3.2.6 Autoregressive Distributed Lag Bound testing

The ARDL model can be further extended to an ECM if the data are found to be cointegrated. The presence of cointegration is tested using the ARDL Bounds Testing approach, which was first formulated by Pesaran *et al.* (2001). This method tests the null hypothesis of no cointegration against the alternative hypothesis that the time series are cointegrated. If there is sufficient evidence to reject the null hypothesis, ECM should be preferred over the ARDL model. Our ARDL bound testing results were unable to reject the null hypothesis in the case of the real exchange rate deflated by the PPI. However, based on our findings in subsection Cointegration, economic theory and the results of the ARDL bound test for the real exchange rate deflated by the HICP, we still proceed with transforming the ARDL model into an ECM in both cases.

3.2.7 Error Correction Model

As previously mentioned, the ECM model is employed when cointegration is present in the data. In other words, the time series exhibit long-run dependency, making the ECM model more appropriate to estimate the equilibrium value of the real exchange rate. The model was first introduced by Sargan (1964) and was further enhanced by Engle & Granger (1987), who formally linked ECM with cointegration analysis. The general form of the ECM model is as follows:

$$\Delta y_t = \alpha + \sum_{i=1}^{p-1} \lambda_i \Delta y_{t-i} + \sum_{j=0}^{q-1} \delta_j \Delta X_{t-j} + \gamma(y_{t-1} - \theta X_{t-1}) + \varepsilon_t \tag{3.6}$$

where $(y_{t-1} - \theta X_{t-1})$ represents the Error Correction Term (ECT). ECT captures the deviation from the long-run equilibrium at time $t-1$. Its parameter γ represents the speed of adjustment in the model and is expected to be negative and statistically significant when the series are cointegrated. It should be emphasized that both the dependent and independent variables are now expressed in first differences, which must be considered when interpreting the effects and deriving the real equilibrium exchange rate from the model's fitted values. The focus remains on the variables in levels, as they reflect the long-term relationship and allow for comparison with the outcomes of the static model, which, on the other hand, captures only short-term relationships.

3.2.8 Models Residuals testing

To assess the performance of our model, we examine various properties of the residuals to determine whether any systematic information remains unexplained. As a first step, the residuals are tested for stationarity using the ADF test, as discussed in the previous section, but this time applying stricter critical values. Any potential nonstationarity in the residuals, more likely observed in the simpler static model, could indicate the omission of information.

Next, we apply the Jarque-Bera test for normality. The authors Jarque & Bera (1980) propose the test with the null hypothesis assuming normality. If the test statistic is lower than the critical value, it indicates that the residuals do not follow a normal distribution, which could lead to invalid results in statistical tests, such as the t-test, that assume normality. Finally, we will also test the residuals for autocorrelation. If the residuals are autocorrelated, it would violate one of the fundamental assumptions of OLS, potentially making the regression results invalid. To test for autocorrelation in the residuals of both models, we will use the Ljung-Box test, named after its authors Ljung & Box (1978). For the results to be valid, we must fail to reject the null hypothesis of no autocorrelation in the residuals.

Chapter 4

Results

In this chapter, we examine the properties of our time series data. We discuss potential issues that may arise on the basis of the analysis and their impact on the estimates. Subsequently, we comment on the results of both models and interpret the effects of the independent variables on the dependent variable. Finally, we calculate the misalignment of the actual real exchange rate from its equilibrium and analyze the differences in misalignment based on the choice of the deflator for the dependent variable.

4.1 Data Structure and Descriptive Analysis

Prior to estimating the equilibrium exchange rate using the models outlined in the previous Chapter 3, it is essential to carry out a comprehensive examination of the data. This step enables a deeper understanding of the underlying relationships and facilitates a more accurate interpretation of the results. Initially, all time series were plotted to gain a basic visual insight. From the plots in Figure A.1, it became evident that some series are probably nonstationary a hypothesis that was subsequently confirmed through the results of the ADF and PP tests for stationarity. To further investigate these trends, statistical filters were applied to decompose the series into their cyclical and trend components.

Furthermore, a possible structural break was identified in the series of net foreign assets. Prior to 2012, NFA experienced a gradual decline, followed by a brief period of stability with some volatility. Thereafter, NFA began to increase. To formally test for the presence of a structural break in the NFA series, we used the Chow test, introduced by Chow (1960). This test divides the sample into two subperiods at the potential break point, estimates separate models for each

subsample, and compares the estimated coefficients. The null hypothesis of the Chow test posits that the coefficients in both subsamples are equal. Rejection of this hypothesis indicates the presence of a structural break in the data. The results of our Chow test strongly reject the null hypothesis, confirming a structural break in the NFA series. This finding may contribute to differing results compared to those of Komárek & Motl (2012), as their study was carried out just before the identified structural break occurred.

Additional insights derived from the graphical analysis includes a significant decline in net exports relative to GDP from the second half of 2021 to the end of 2022. Further investigation revealed that this period was characterized by a sharp decrease in goods exports, whereas imports remained stable or even slightly increased. It is plausible that this decline in exports was an effect of the COVID-19 pandemic. Aside from these observations, the remaining time series generally behave in accordance with economic theory, the existing literature and prior expectations. As noted in the previous Chapter 3, we also find that the unfiltered data exhibit cointegration, particularly after the application of seasonal adjustment procedures. Graphical representations of both unadjusted and detrended data are provided in Figure A.1, Figure A.2 and Figure A.3 in the appendix.

4.2 Model Estimations and Interpretations

4.2.1 Static Model results

We estimate the static model using both variations of the detrended datasets, those filtered with the HP filter and those using the HRF method. As the time series become stationary after filtering, we can apply the OLS estimation in levels. Furthermore, GFCF is included with a one period delay, similarly to Komárek & Motl (2012), and for the reasons discussed above. For each filtered dataset, we estimate the model with different combinations of the dependent variable, $lnRER_{HICP}$ or $lnRER_{PPI}$. Additionally, we consider three variations of the RIRD independent variable, as mentioned in Chapter 3. This results in a total of six model specifications for each filtering method.

Across all specifications, the productivity ratio exhibits a statistically significant negative effect on the real exchange rate, which aligns with theoretical expectations. Similarly, the variable for net foreign assets is significant in all models. However, we observed an unexpected positive effect on the real ex-

change rate. That could be the impact of the structural break in NFA series, which will be further examined in Chapter 5. Furthermore, the independent variable GFCF_{t-1} appears to be statistically insignificant in all model specifications. This insignificance may be due to an incorrect selection of the lag, which will be further investigated in the ECM model, where the effects of different lag structures can be assessed more thoroughly.

The remaining results show considerable distinctions. Most model specifications suggest a negative effect of the Net Export (NX) variable, which is consistent with economic theory. However, NX is not in all cases statistically significant. The most noticeable differences appear in the behavior of the RIRDs variables. When using data detrended by the HP filter, the model produces consistent results. In all specifications, the RIRD variables are positively related to the real exchange rate and statistically significant. In contrast, when using data detrended by the Hamilton regression filter, the effects have the opposite sign and only the RIRD variables explaining the real exchange rate deflated by HICP are statistically significant. This may result from a potential spurious regression caused by both the real exchange rate $lnRER_{HICP}$ and the RIRD variables incorporating the HICP index in their calculations. The negative sign appears to be a direct consequence of applying the Hamilton regression filter. This assumption is supported by an analysis of the correlations between the real exchange rate and the RIRD variables in unadjusted data, HP-filtered data and HRF-filtered data. The results show that the Hamilton filter changes the correlation sign. This finding provides further motivation to avoid filtering techniques, because of their impact on the data and the relationships between variables remain uncertain. For this reason, we adopt a more advanced ECM for further analysis, as this approach does not require data filtering. Results of the static regressions models are available from Table A.5 to Table A.8 in the appendix.

4.2.2 Error Correction Model results

The second model used to evaluate the REER is the ECM applied to unadjusted data. As mentioned in subsection Optimal Number of Lags, we selected four lags for each variable, allowing a more comprehensive understanding of the impact of independent variables in different time lags. The model was again estimated for all six combinations of variables. For the sake of simplicity, we present the results solely for the model that employs $RIRD_M$. When interpret-

ing the results, it is essential to recognize that ECM use variables in differences. Consequently, the correct interpretation is that a one-unit change in an independent variable affects the change in the dependent variable. This effect is denoted in the tables by the symbol d(), while L(variable, X) represents the X-th lag of the variable specified in parentheses. All outputs are rounded to two decimal places.

Table 4.1: Results of ECM with $lnRER_{HICP}$

Variable	Estimate	Std. Error	t-statistic	p-value
Intercept	0.45	0.15	3.07	0.00
d(L($lnRER_{HICP}$, 1))	-0.02	0.12	-0.14	0.89
d(L($lnRER_{HICP}$, 2))	-0.06	0.13	-0.50	0.62
d(L($lnRER_{HICP}$, 3))	-0.26	0.11	-2.37	0.02
d($PROD$)	-0.94	0.14	-6.54	0.00
d(L($PROD$, 1))	-0.21	0.18	-1.14	0.26
d(L($PROD$, 2))	-0.29	0.17	-1.67	0.10
d(L($PROD$, 3))	-0.30	0.17	-1.71	0.09
d(NFA)	0.05	0.02	2.46	0.02
d(L(NFA, 1))	0.04	0.02	1.74	0.09
d(L(NFA, 2))	-0.01	0.02	-0.42	0.68
d(L(NFA, 3))	0.01	0.02	0.26	0.79
d($GFCF$)	-0.07	0.41	-0.17	0.86
d(L($GFCF$, 1))	0.69	0.39	1.75	0.08
d(L($GFCF$, 2))	0.32	0.40	0.79	0.43
d(L($GFCF$, 3))	-0.76	0.42	-1.81	0.07
d(NX)	-0.12	0.14	-0.84	0.40
d(L(NX, 1))	-0.16	0.15	-1.07	0.29
d(L(NX, 2))	0.05	0.15	0.34	0.74
d(L(NX, 3))	0.11	0.14	0.78	0.44
d($RIRD_M$)	0.81	0.21	3.97	0.00
d(L($RIRD_M$, 1))	-0.84	0.37	-2.26	0.03
d(L($RIRD_M$, 2))	-0.71	0.31	-2.29	0.03
d(L($RIRD_M$, 3))	-0.31	0.21	-1.49	0.14
ECT	-0.13	0.04	-3.07	0.00

Table 4.2: Results of ECM with $lnRER_{PPI}$

Variable	Estimate	Std. Error	t-statistic	p-value
Intercept	1.22	0.26	4.67	0.00
d(L($lnRER_{PPI}$, 1))	0.09	0.10	0.85	0.40
d(L($lnRER_{PPI}$, 2))	0.16	0.10	1.57	0.12
d(L($lnRER_{PPI}$, 3))	0.06	0.09	0.61	0.54
d($PROD$)	-0.51	0.09	-5.77	0.00
d(L($PROD$, 1))	-0.05	0.09	-0.51	0.61
d(L($PROD$, 2))	0.04	0.09	0.43	0.67
d(L($PROD$, 3))	0.02	0.09	0.20	0.84
d(NFA)	0.03	0.01	2.00	0.05
d(L(NFA, 1))	0.00	0.01	0.07	0.94
d(L(NFA, 2))	-0.02	0.01	-1.48	0.14
d(L(NFA, 3))	0.00	0.01	0.02	0.99
d($GFCF$)	-0.27	0.29	-0.94	0.35
d(L($GFCF$, 1))	0.56	0.27	2.04	0.04
d(L($GFCF$, 2))	0.82	0.29	2.83	0.01
d(L($GFCF$, 3))	0.19	0.30	0.64	0.52
d(NX)	-0.09	0.09	-0.90	0.37
d(L(NX, 1))	-0.26	0.11	-2.50	0.01
d(L(NX, 2))	-0.16	0.11	-1.45	0.15
d(L(NX, 3))	0.06	0.11	0.58	0.56
d($RIRD_M$)	0.21	0.13	1.56	0.12
d(L($RIRD_M$, 1))	-0.36	0.19	-1.89	0.06
d(L($RIRD_M$, 2))	-0.77	0.19	-4.10	0.00
d(L($RIRD_M$, 3))	-0.58	0.14	-4.11	0.00
ECT	-0.32	0.07	-4.67	0.00

The first three explanatory variables represents the autoregressive component of the model. The results already differ depending on the chosen deflation method. The effect on $lnRER_{PPI}$ is positive and statistically insignificant, whereas for $lnRER_{HICP}$, the effect is negative and, in some cases, statistically significant, particularly at the third delay. This negative estimate suggests that, in the short run, lagged values may counteract a previous increase or decrease in the real exchange rate.

The variable $PROD$ exhibits a consistent effect across all models in the current period, with a statistically significant negative impact on the real exchange rate. In the case of $lnRER_{PPI}$, the lagged coefficients are largely statistically insignificant. In contrast, for $lnRER_{HICP}$, all the lagged values are consistently negative and statistically significant, indicating a persistent effect of $PROD$ on the real exchange rate.

All ECMs exhibit similar results to the static model regarding the NFA variable in the current period, as it has a positive and statistically significant effect on the real exchange rate. The second lag appears to have a negative effect, which would align with economic theory, but these impacts are largely statistically insignificant.

Another variable under consideration is gross fixed capital formation. This variable yields the most inconsistent results across the model specifications. In most results, a statistically significant impact is observed in the second or third lag, which is reasonable, as investments support the domestic economy with a delay and their effects are not immediately visible upon implementation. The unexpected is the difference in the sign of the effect. The impact on $lnRER_{PPI}$ appears to be positive, whereas for $lnRER_{HICP}$, it is negative. It can be argued that the impact of real investments on various types of real exchange rates is highly dependent on the economic sector in which the investments are made and the resulting price adjusted.

The next variable in the results is net exports. Its effect appears to be consistent across all model specifications. The first two estimates are typically negative, as theoretical expectations would suggest. The subsequent positive effect observed in the lagged terms could be attributed to the strong interdependence between the exchange rate and a country's net exports. Specifically, if the real exchange rate initially fall, meaning the currency appreciates, this would negatively affect net exports by reducing the competitiveness of domestic goods in foreign markets. Consequently, net exports would decline as the real exchange rate appreciates.

The final independent variable included in our initial model is the real interest rate differential in its various forms. Through all its time variants, RIRD exhibits a similar pattern. The coefficients for the current period are consistently positive and statistically significant, while the lagged effects are generally negative and, in most cases, also statistically significant. The initial positive effect may be explained by short-term uncertainty among investors when interest rates rise, leading to reduced demand for the currency and a temporary depreciation. However, in the following quarter, investors may return to the currency with higher interest rates, attracted by the prospect of greater returns, ultimately causing the currency to appreciate. The initial depreciation could also be interpreted in line with uncovered interest parity, this explanation would be inconsistent with the appreciation observed in later periods.

On the final line of our results, we present the estimated effect of the Error

Correction Term. In all models, the effect of the ECT has the expected sign and is statistically significant, suggesting that the real exchange rate returns to its long-term equilibrium. The speed of adjustment, as indicated by the ECT coefficient, varies depending on the choice of the dependent variable. Specifically, it appears that the speed of adjustment for the real exchange rate deflated by the PPI is faster than that for the rate deflated by the HICP. Based on economic theory and the values of the adjusted R-squared, the model appears to be valid. Considering the remaining degrees of freedom, there is no immediate concern, however, it is still important to remain cautious about the potential risk of overfitting.

4.2.3 Results of Residuals testing

After conducting tests on our static model with detrended data, we observe similar results regardless of the combination of variables.

Figure 4.1: Histogram HP-filter (HICP)

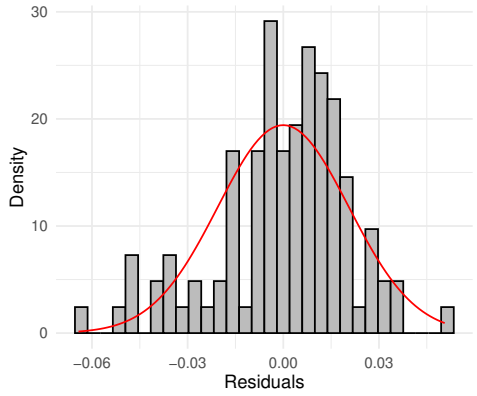


Figure 4.2: Autocorrelation HP-filter (HICP)

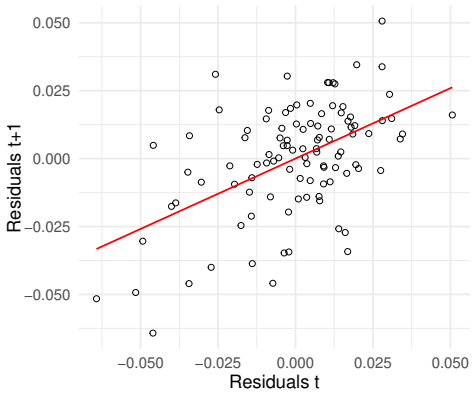


Figure 4.3: Histogram HP-filter (PPI)

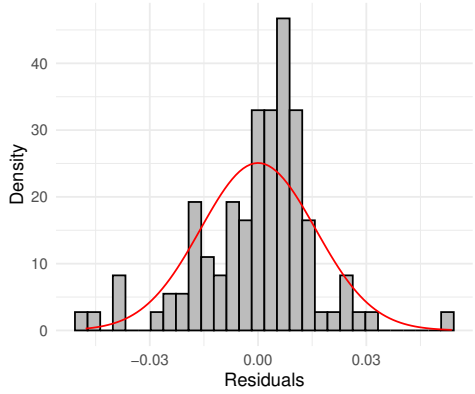
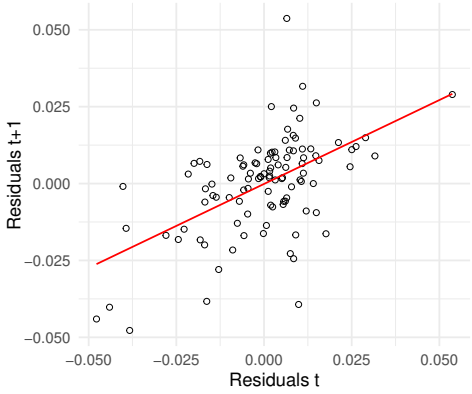


Figure 4.4: Autocorrelation HP-filter (PPI)



As illustrated in the Figure 4.1 to Figure 4.4, the residuals exhibit a normal distribution. However, they also display visible positive autocorrelation and nonstationarity. This indicates certain limitations of the model and suggests that some information remains embedded within the residuals.

When applying the same tests to the residuals derived from the more advanced ECM, we obtain better results for both variations of the dependent variable. In case of ECM the ADF test confirms that the residuals are stationary. Furthermore, the results from Figure 4.6 and Figure 4.8, along with the Ljung-Box test outcomes, indicate that the residuals are not autocorrelated. The results of the normality test vary depending on the deflation method employed. Specifically, the residuals of the model using $lnRER_{PPI}$ conform to a normal distribution according to the test. However, the normality test for residuals from the model using $lnRER_{HICP}$ rejects the null hypothesis at the 5 percent significance level. This could be attributed to an insufficiently large sample size. At the 1 percent significance level, both residuals can be considered normally distributed. Therefore, the test results suggest that the ECM is a more appropriate model, as most likely no systematic information remains in the residuals.

Figure 4.5: Histogram ECM (HICP)

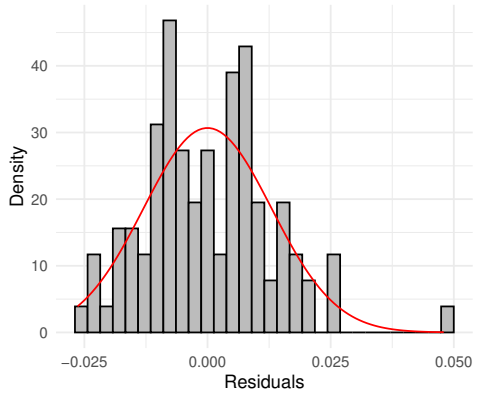


Figure 4.6: Autocorrelation ECM (HICP)

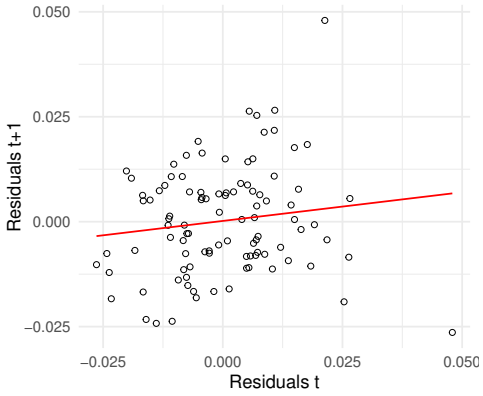


Figure 4.7: Histogram ECM (PPI)

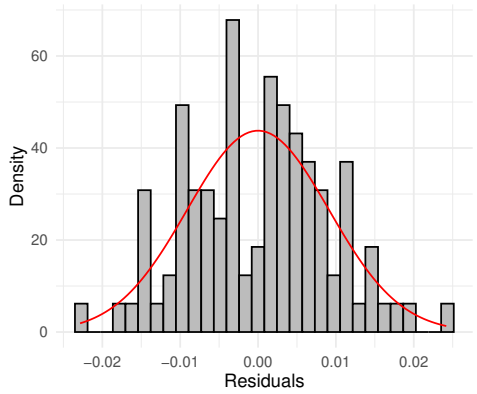
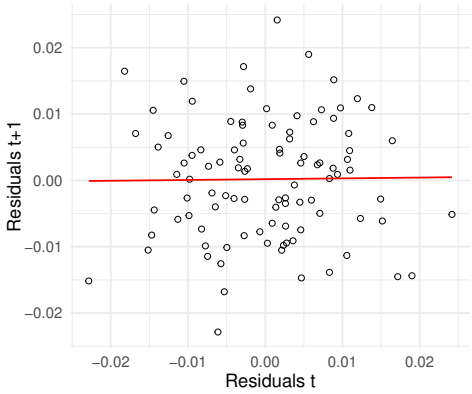


Figure 4.8: Autocorrelation ECM (PPI)



4.3 Assessing Real Exchange Rate Misalignment

In this section, we focus on interpreting the results of the real exchange rate misalignment from its equilibrium. We examine the degree of misalignment for both models utilized in this thesis. The analysis is performed for the real exchange rates deflated by both the HICP and PPI indices to capture potential differences in their variation and absolute values. For clarity and consistency, we again restrict our analysis to models that incorporate RIRD in its medium-term horizon form.

Misalignment is defined as the percentage deviation of the observed real exchange rate from its estimated Equilibrium Exchange Rate (EER) value, which is derived from the models, first using the static model and subsequently the

ECM. The misalignment is computed according to the following equation:

$$Misalignment = \frac{\text{Observed } RER - \text{Estimated } EER}{\text{Estimated } EER} \times 100 \tag{4.1}$$

4.3.1 Static Model Misalignment

First, we evaluate the misalignment of the real exchange rate from its equilibrium level as estimated by the static model. Misalignments are computed for both HRF detrended and HP filtered data. Positive values in Figure 4.9 and Figure 4.10 indicate an undervalued currency, while negative values reflect overvaluation.

Figure 4.9: Misalignment using HP-filter

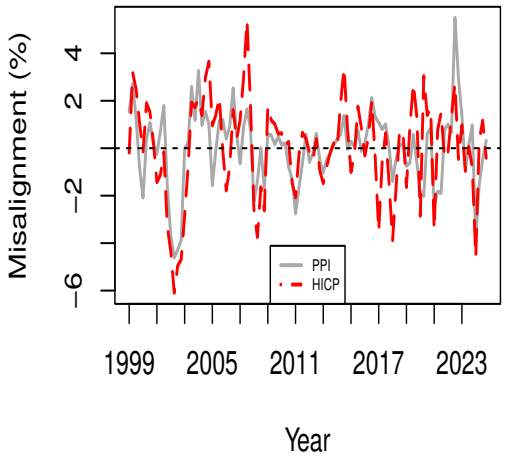
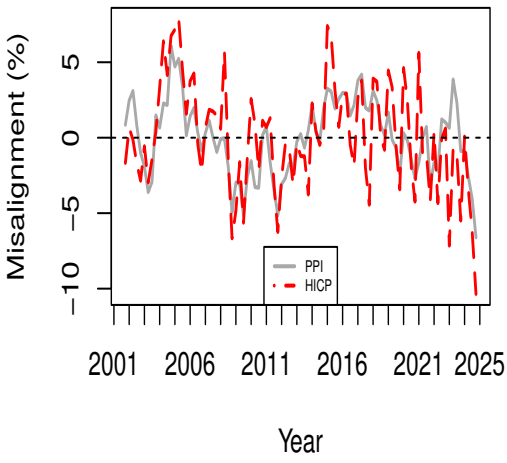


Figure 4.10: Misalignment using HRF



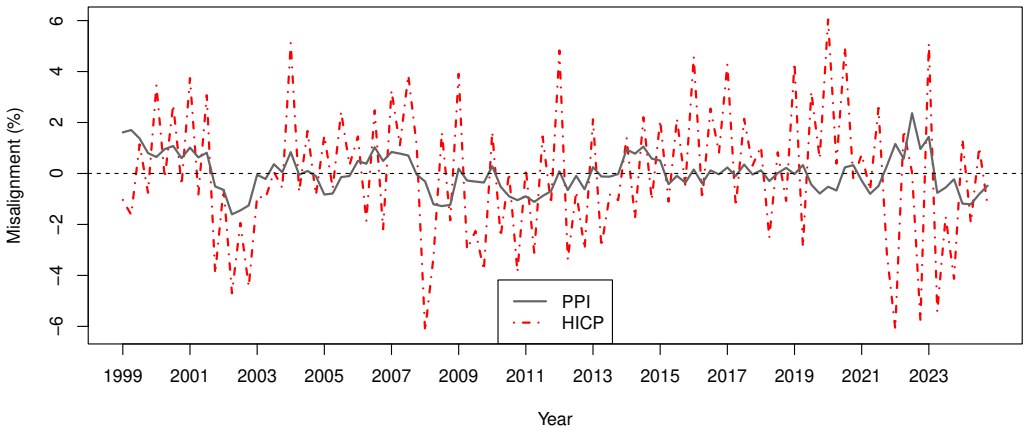
From Figure 4.9 and Figure 4.10, it is evident that regardless of the detrending method used, the periods of currency overvaluation and undervaluation largely coincide. However, the plots differ slightly in magnitude of deviation and behavior at the end of the sample. The HP filter suggests that the Czech koruna, after being overvalued for most of 2024, returns to equilibrium at the end of the year. In contrast, the HRF based model indicates that the koruna remains overvalued. The latter interpretation originally seems more plausible, since the HP filter is known to distort trends at the beginning and end of the sample. However, the results must first be compared with the misalignment from the ECM. Overall, it appears that the Czech koruna tends to be overvalued relative to its equilibrium value, particularly after major economic crises, such as the bursting of the dot-com bubble, the Great Recession, and the post

COVID-19 crisis. The results also indicate that the Czech koruna was persistently undervalued during the CNB intervention period from 2013 to 2017.

4.3.2 Error Correction Model Misalignment

Next, the misalignment of the real exchange rate with the equilibrium is examined based on the ECM. The patterns observed in Figure 4.11 are in line with the interpretation framework outlined above.

Figure 4.11: Misalignment of ECM



Based on the ECM misalignment results, as depicted in Figure 4.11, it appears that the overall patterns of the high misalignment periods remain largely consistent with the results of the static model. However, the results of the misalignment in the model based on the PPI appear to be more stable over the entire sample period compared to the static model. The same cannot be said for the HICP misalignment, which exhibits values similarly high to those of the static model. It almost seems that the volatility is even greater in this case, particularly in the latter part of the sample. This is probably caused by the drastic change in the NX variable, as discussed in Section 4.1. In all models, it appears that those incorporating the HICP deflator are more sensitive to changes in the variables than those using the PPI. Finally, it is evident from all estimated models that the misalignment is significantly higher when the HICP is used to deflate the exchange rate. These findings are consistent with our initial hypothesis.

Chapter 5

Discussion

The first topic for discussion emerged during the analysis of the input data. As mentioned in Chapter 4, we observe a structural break in the NFA time series, which may explain why, in most models, the coefficient measuring the impact of NFA on real exchange rate is relatively small and often statistically insignificant. Notably, in our data, the sign of NFA changes from positive to negative during the later lags. According to economic theory, an increase in NFA could initially depreciates the currency due to lower demand for domestic currency, but eventually leads to appreciation as a result of higher productivity. This may explain the change in the sign of NFA in our results. Furthermore, it is crucial to highlight that the static model used by the CNB was developed prior to the occurrence of the structural break in NFA. For these reasons, the model, along with its underlying assumptions and fundamentals, should be updated to improve the estimation of the equilibrium exchange rate.

As demonstrated throughout our research, the NFA variable could potentially be replaced by the Real Interest Rate Differential (RIRD). The test results indicate that, regardless of the time horizon of the RIRD, the variable remains statistically significant. For central bank use, we recommend selecting the $RIRD_M$ based on medium-term interest rates, calculated by using $PRIBOR$ and $EURIBOR$. A longer $RIRD_L$ setting, based on government bonds, is not optimal for the focus of CNB. Additionally, a short-term $RIRD_S$ is not preferred, as the REPO rate and the DFR from the ECB do not change frequently, making the variable more influenced by inflationary movements. Given that our hypothesis regarding the significant statistical and economic impact of the RIRD on the real exchange rate holds, we suggest including the RIRD variable in the BEER calculation. Further investigation should be conducted on the GFCF

variable and the effects of its lag values. GFCF in our model could potentially be replaced by Foreign Direct Investments (FDI), as is done in models employed by other authors, also shown in Table 2.1. Given the strong relationship between these fundamentals, FDI may produce more statistically significant estimates while preserving the underlying relationship. The behavior of the other independent variables was as expected and in line with economic theory.

The thesis focused on employing the most appropriate model that would minimize the information left in the residuals. Based on the residuals testing, estimation and misalignment results, we recommend using more advanced ECM method that account for the long-term relationships between the variables. In specifying the ECM, we focused on estimations in levels to support our second hypothesis regarding both the short-term and long-term relationship. In addition, Figure A.8 in the appendix shows that the same applies to the ECM when estimated as a whole, confirming the robustness of the results. Overall, one should avoid relying on simple static models, even after filtering the data. This is primarily because it remains uncertain what exactly filtering does to the data, and secondly, after applying these methods, the residuals still appear to contain substantial information. Somewhat unexpectedly, the static model using data detrended by the HP filter produced misalignment estimates more closely aligned with those of the ECM, suggesting that HP filtering outperforms the HRF approach. Several factors may account for this result, however, the most plausible explanation lies in the limitations of the HRF, which tends to shift crises to later periods due to the methodology by which it is constructed. To address this issue, the data should ideally be adjusted for shocks prior to applying HRF detrending.

The results presented in Chapter 4 confirm the second hypothesis, namely that the degree of misalignment varies depending on the deflator used in the calculation of the real exchange rate. The findings suggest that the degree of misalignment tends to be higher when the real exchange rate is deflated using the HICP as opposed to the PPI. The Czech Republic is a small open economy, and PPI prices are typically tied to inputs used in production, often raw materials. This may explain why HICP, which is less connected to tradable goods, is also less directly affected by exchange rate movements. In contrast, the PPI reflects the price dynamics of internationally traded goods more strongly and therefore responds more clearly to changes in the exchange rate. Additionally, the models used in this thesis are based on the approach applied by the CNB, which uses PPI as the deflator. This may also contribute to the fact that mis-

alignment appears lower when using PPI, since the model structure is more aligned with that methodology.

Furthermore, we observe that misalignment is considerably more volatile when the real exchange rate is deflated using the HICP. A plausible explanation is the broader scope of consumer prices, which encompass a wide range of goods and services purchased by households and tend to respond more sensitively to domestic economic fluctuations. In contrast, the PPI tends to exhibit greater price stability, as it is primarily shaped by input costs and often governed by forward contracts, which limit short-term variability. Moreover, PPI prices are generally less affected by seasonal patterns. These structural differences likely contribute to the observed divergence in misalignment volatility.

Chapter 6

Conclusion

To conclude, the objective of this thesis was to develop an optimal model for estimating the equilibrium exchange rate using a specific set of fundamental variables, with particular emphasis placed on the Real Interest Rate Differential (RIRD), in order to provide valuable insight into its effect on the real exchange rate. Thereafter, the misalignment of the observed exchange rate from its equilibrium level was examined, with attention given to the differences in misalignment arising from the choice of the exchange rate deflation method.

Initially, an analysis of the input data was performed, in which the properties of the dataset were discussed, such as the structural break identified in the Net Foreign Assets (NFA) variable, the stationarity of the series and, consequently, the cointegration relationships among the time series selected to estimate the equilibrium exchange rate. To allow the use of the model in its level form and to ensure stationarity, detrending was performed using both the Hodrick-Prescott (HP) filter and the Hamilton Regression Filter (HRF). The resulting filtered data were subsequently employed in the estimation of the equilibrium exchange rate using a static regression model and in the following estimation of exchange rate misalignment.

To enhance the results, the Autoregressive Distributed Lag (ARDL) model, based on the positive outcomes of the cointegration tests, was implemented in its Error Correction Model (ECM) form. This approach allowed for the accounting of long-term relationships within the data without the need for filtering. The results obtained from both the ECM and the static model indicated that RIRD is not only economically, but also statistically significant in the estimation of the real exchange rate and therefore should be taken into consideration in the Behavioral Equilibrium Exchange Rate (BEER) modeling framework. Fur-

thermore, the more advanced ECM was found to be superior to the static model, based on the properties of the residuals and the extent to which its estimates aligned with economic theory.

Lastly, the misalignment of the exchange rate from its equilibrium was calculated from both models and its evolution over the sample period was analyzed. In addition, the reasons for the higher level of misalignment associated with the Harmonized Index of Consumer Prices (HICP) based approach, as anticipated in the hypothesis, were discussed. It was also found that the misalignment calculated using HICP is more volatile across the entire sample period in comparison to the Producer Price Index (PPI) based measure.

The estimated coefficients and the resulting misalignments were found to be consistent across all estimated models and various combinations of independent variables. ECM appeared to be an appropriate framework to evaluate the real exchange rate misalignment within the BEER approach. However, the application of alternative methods, such as the Fundamental Equilibrium Exchange Rate (FEER) or Natural Real Exchange Rate (NATREX), would be beneficial in assessing the robustness of the results. Specifically, the NATREX model could offer valuable insight as a potential avenue for future research, given that it has not yet been specified for the Czech economy.

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Appendix A

Appendix

Figure A.1: Raw Data Visualization

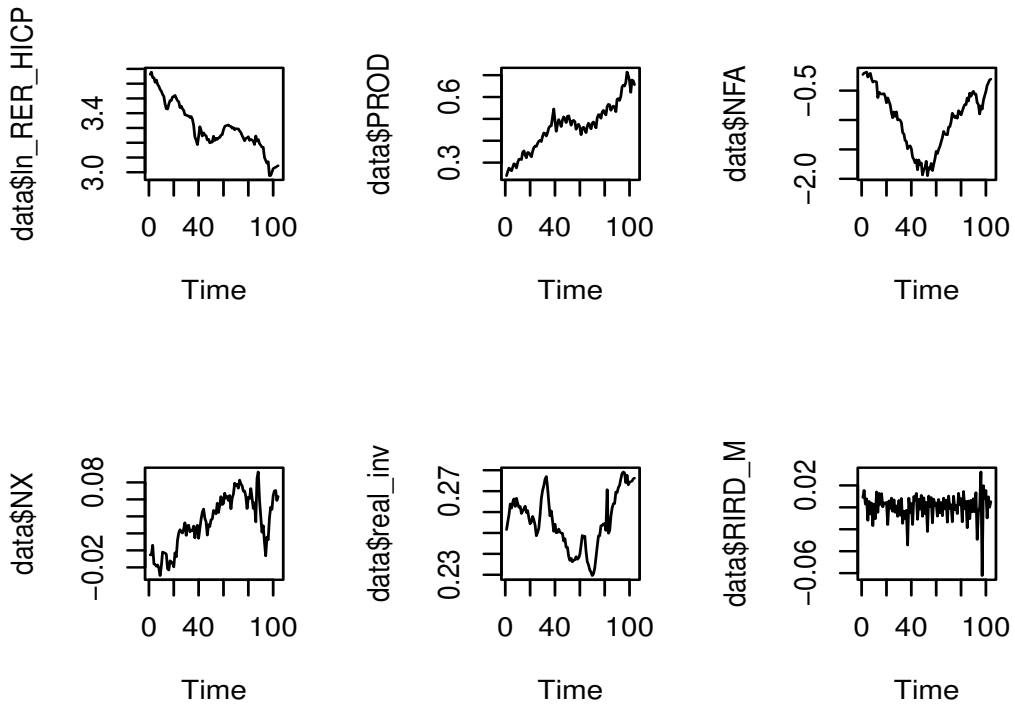


Figure A.2: HP-filtered Data Visualization

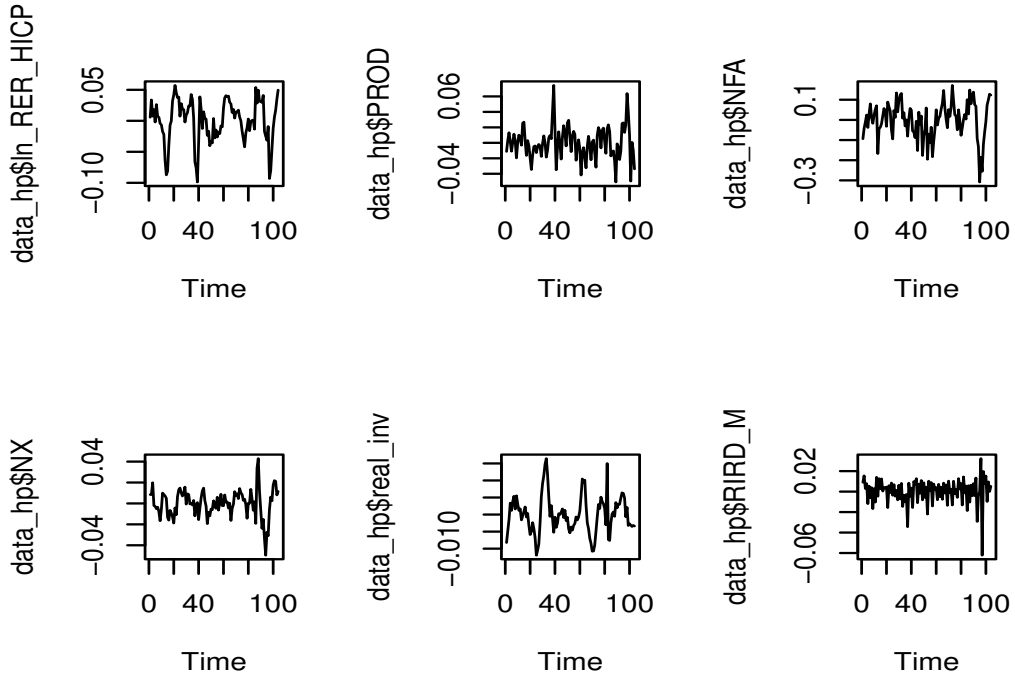


Figure A.3: HRF Data Visualization

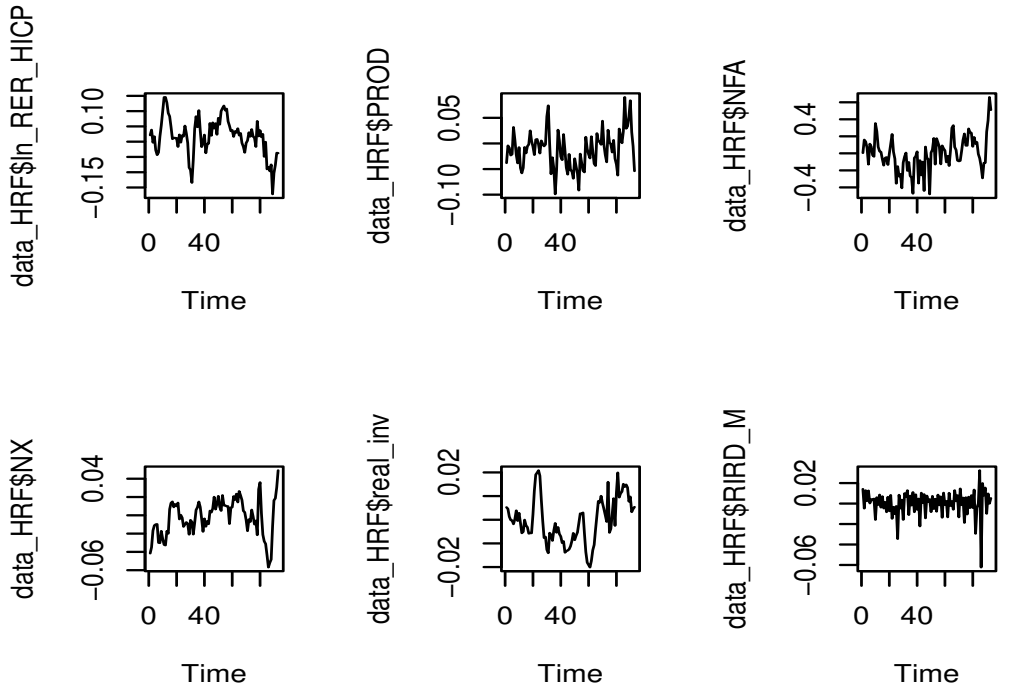


Table A.1: Stationarity Tests Results after applying HP-filter

Variable	ADF Statistic	ADF P-Value	PP Statistic	PP P-Value
$\ln RER_{PPI}$	-5.987127	0.01000000	-36.44032	0.01
$\ln RER_{HICP}$	-4.837441	0.01000000	-30.76671	0.01
$PROD$	-5.909252	0.01000000	-58.04924	0.01
NFA	-4.202154	0.01000000	-50.39102	0.01
$GFCF$	-4.992579	0.01000000	-34.57811	0.01
NX	-4.382588	0.01000000	-41.99436	0.01
$RIRD_S$	-4.201338	0.01000000	-153.81407	0.01
$RIRD_M$	-4.005785	0.01160821	-156.00075	0.01
$RIRD_L$	-4.248556	0.01000000	-154.89887	0.01

Table A.2: Stationarity Tests Results after applying HRF

Variable	ADF Statistic	ADF P-Value	PP Statistic	PP P-Value
$\ln RER_{PPI}$	-3.534308	0.04326374	-18.81244	0.07751828
$\ln RER_{HICP}$	-4.400549	0.01000000	-23.22678	0.02658851
$PROD$	-5.183111	0.01000000	-41.10376	0.01000000
NFA	-4.008791	0.01231397	-37.98368	0.01000000
$GFCF$	-3.951728	0.01501924	-17.56524	0.09720265
NX	-3.639570	0.03397154	-23.37801	0.02526289
$RIRD_S$	-3.976270	0.01385570	-132.02184	0.01000000
$RIRD_M$	-3.733165	0.02570931	-133.82964	0.01000000
$RIRD_L$	-3.982895	0.01354166	-133.27224	0.01000000

Table A.3: Johansen Cointegration Test Results (HICP)

Hypothesis	Test Statistic	10% Critical Value	5% Critical Value	1% Critical Value
$r \leq 4$	2.07	10.49	12.25	16.26
$r \leq 3$	11.90	16.85	18.96	23.65
$r \leq 2$	27.18	23.11	25.54	30.34
$r \leq 1$	31.57	29.12	31.46	36.65
$r = 0$	46.93	34.75	37.52	42.36

Table A.4: Johansen Cointegration Test Results (PPI)

Hypothesis	Test Statistic	10% Critical Value	5% Critical Value	1% Critical Value
$r \leq 4$	6.41	10.49	12.25	16.26
$r \leq 3$	23.26	16.85	18.96	23.65
$r \leq 2$	27.92	23.11	25.54	30.34
$r \leq 1$	33.10	29.12	31.46	36.65
$r = 0$	52.63	34.75	37.52	42.36

Table A.5: Static Model results HRF (HICP)

Variable	Estimate	Std.Error	t-statistic	p-value
Intercept	-0.00	0.00	-0.51	0.61
PROD	-1.45	0.13	-11.43	0.00
NFA	0.06	0.02	2.67	0.01
$GFCF_{t-1}$	-0.70	0.44	-1.58	0.12
NX	-0.38	0.22	-1.73	0.09
$RIRD_M$	-1.93	0.33	-5.80	0.00

Table A.6: Static Model results HRF (PPI)

Variable	Estimate	Std.Error	t-statistic	p-value
Intercept	-0.00	0.00	-0.47	0.64
PROD	-0.51	0.09	-5.66	0.00
NFA	0.03	0.01	1.75	0.08
$GFCF_{t-1}$	0.20	0.31	0.63	0.53
NX	-0.72	0.16	-4.61	0.00
$RIRD_M$	-0.23	0.23	-0.98	0.33

Table A.7: Static Model results HP-filter (HICP)

Variable	Estimate	Std.error	t-statistic	p-value
Intercept	-0.00	0.00	-0.04	0.97
PROD	-1.13	0.11	-10.03	0.00
NFA	0.11	0.03	4.23	0.00
$GFCF_{t-1}$	0.09	0.38	0.23	0.82
NX	-0.16	0.17	-0.93	0.36
$RIRD_M$	0.79	0.19	4.27	0.00

Table A.8: Static Model results HP-filter (PPI)

Variable	Estimate	Std.error	t-statistic	p-value
Intercept	-0.00	0.00	-0.02	0.98
PROD	-0.53	0.09	-6.08	0.00
NFA	-0.00	0.02	-0.17	0.86
$GFCF_{t-1}$	0.38	0.30	1.28	0.20
NX	-0.45	0.13	-3.35	0.00
$RIRD_M$	0.31	0.14	2.13	0.04

Figure A.4: Histogram HRF (HICP)

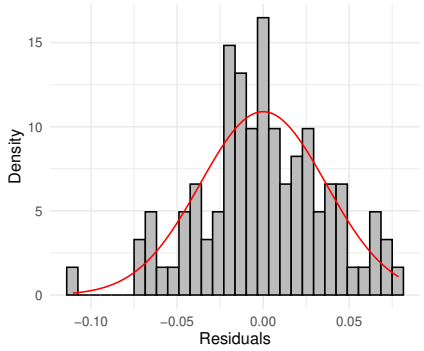


Figure A.5: Autocorrelation HRF (HICP)

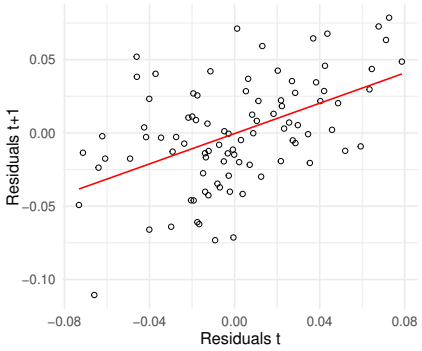


Figure A.6: Histogram HRF (PPI)

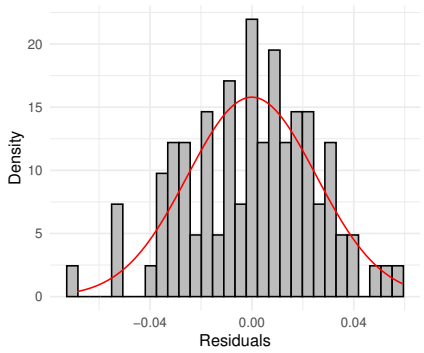


Figure A.7: Autocorrelation HRF (PPI)

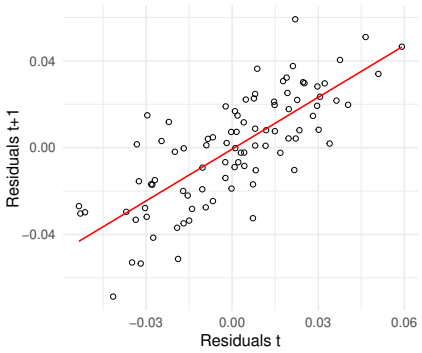
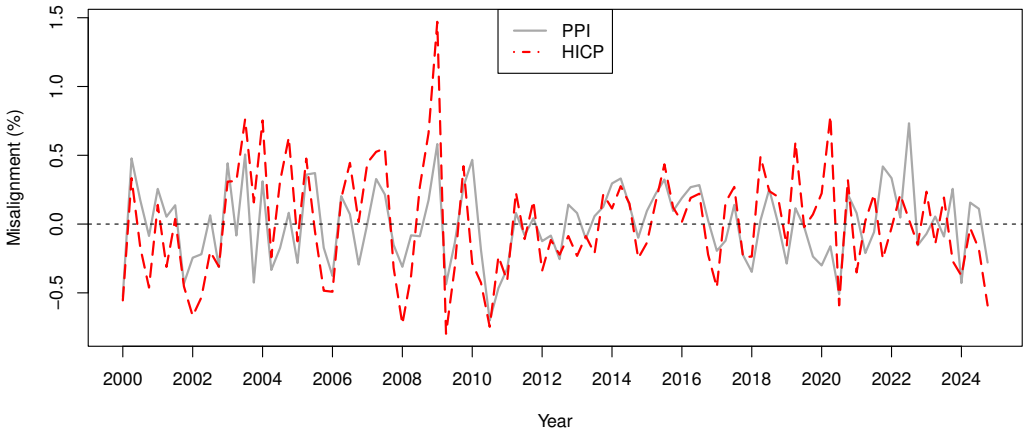


Figure A.8: Misalignment of the Full ECM Model



Appendix B

Data and Source Codes

The data and R source codes used to generate the results presented in this thesis are available upon request to those interested.