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Objet : Referee report on Pavel Nejedly doctoral thesis entitled *Structural and agorithmic properties of graph coloring*

The area of this thesis is discrete mathematics, specifically graph theory. Pavel Nejedly tackles some important conjectures and problems on which he makes some significant advances.

The first one regards colourings of the square of a planar graph and their generalisations $L(p,q)$-labellings. An $L(p,q)$-labelling is a colouring of the vertex set with colours $1, 2, \ldots$ such that the difference of colours of adjacent vertices is always p and the difference of colours of vertices at distance 2 is at least q. The $L(p,q)$-span is the smallest number of colours needed. So when $p = q = 1$, it is the chromatic number of the square. The celebrated Conjecture of Wegner states that the chromatic number of such a graph with maximum degree Δ is at most $\lfloor \frac{3}{2}\Delta \rfloor + 1$. More generally, it is conjectured that the $L(p,q)$-span is at most $q\lfloor \frac{3}{2}\Delta \rfloor + 1$ provided Δ is large enough. Wegner's conjecture has been verified for K_4-minor free graphs by Lih, Wang and Zhu. Nejedly with his co-authors extend this results to $L(p,q)$-labelling. They also extend it to list colouring : in this case, each vertex is given a list of $\lfloor \frac{3}{2}\Delta \rfloor + 1$ available colours but the list may vary from vertex to vertex. These results are proof using a structural decomposition theorem of K_4-minor free graphs.

The author also consider planar graphs with large girth. Borodin et al. showed that the chromatic number of the square of a planar graph with girth at least 7 is bounded by $\Delta + 1$ provided that Δ is large enough and that there are planar graphs with girth 6 whose square require at least $\Delta + 2$ colours. Nejedly completes these results by showing that the chromatic number of the square of a planar graph with girth 6 is at most $\Delta + 2$ when Δ is large enough. He also shows that the upper bound $\Delta + 1$ also holds for the $L(2,1)$-span of planar graphs of girth 6. The proofs of these results use the discharging method in a slightly complicated way.

In the second part of thesis, a proof a colouring extension problem on cyclinder graph due to Thomassen is given. The proof is computer assisted.

Finally the author consider the Shortest Cycle Cover Conjecture asserting that every bridgeless graph with m edges has a cycle cover of total length at most $7m/5$. This implies the celebrated Double Cycle Cover Conjecture. Pavel Nejedly and his co-authors showed that every every bridgeless graph with m edges has a cycle cover of total length at most $44m/27$ (improving the previous $5m/3$ of Alon and Tarsi) and at most $34m/21$ when the graph is cubic (improving the previous $44m/27$ of Fan). The proofs use some refinement of classical splitting lemmas that allow to refine the Rainbow Lemma.

Altogether, the thesis contains many interesting results obtained by various techniques. The author demonstrated deep familiarity with the relevant background, as well as technical ability that enable him to obtain new results, and a good taste in selecting problems.

The presentation is also very clear and pedagogical. I only spotted a few minor typos that are not even worth mentioning.

In conclusion, this is a very good thesis, which should certainly be approved as a PhD thesis and I warmly recommend its acceptance for the doctoral degree.

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