Review by the opponent of the bachelor thesis

Thesis title: Fully symmetric operations
Thesis author: Jan Kubica

Summary of the thesis content

The author tries to understand which finite algebras have symmetric operations of all arities. Surprisingly, it is not even known whether the problem of checking this property is decidable and the author makes first steps in this direction. This work is based on the ideas of Zarathustra Brady who first showed that it is sufficient to check the existence of symmetric operations of arity up to the size of the domain if the domain has at most 4 elements.

Overall evaluation of the thesis

Thesis topic. Symmetric operations are very important not only for understanding of finite algebras but also to classify the complexity of the Constraint Satisfaction Problem (CSP) over a fixed constraint language. For instance, only constraint languages admitting symmetric operations of all arities can be solved by Basic Linear Programming Relaxation (BLP), which is one of the basic algorithms for the CSP. Thus, a resolution of the problem suggested by the author would give an algorithm to check whether the constraint language corresponding to the algebra is solvable by BLP.

Author's contribution. The main contribution of the author is the combinatorial lemma showing that we can define 6 linear orders on subsets satisfying the following property. For any partition of the whole set into three subsets A, B, and C these 6 linear orders restricted to $\{A, B, C\}$ give all 6 possible linear orders on $\{A, B, C\}$. Using this lemma the author shows that a symmetric operation of any arity (on a 3-element domain) can be obtained as a composition of binary and ternary symmetric operations and term operations corresponding to all linear orders, which we always have by the result of Brady.

Mathematical level. Mathematical level of the paper is high, definitions are precise, and the proofs seem to be correct. Although, I have a few recommendations to improve the text:

- It is good when statements are short. What you write in the statement of Lemma 1 after "Moreover..." seems to me an observation or a trivial corollary, and I would definitely exclude this from the statement. Also, I don't see why you introduce a subset X as for $X = 2^{[n]}$ the claim would be exactly the same.
- When you define the order $\prec_i$ after Lemma 2 you assume that $<_i$ is defined on subsets of of any size but earlier you define it only on subsets of size n.
- Being symmetric is a property of an operation (mapping) and I find it strange that you define everything for terms.
- In several proofs the author consider many cases to cover all possible sizes of sets and all possible orders. I strongly believe that some cases can be considered simultaneously, which would make the proof shorter.

Work with sources. The citations seem to be correct.

Conclusion

I consider the thesis to be very good, the author understood a very difficult concept of LO-system of operations and provided his own proof of nontrivial statements. That is why, I recommend it to be accepted as a bachelor's thesis.

The classification will be communicated by the opponent to the chair of the examination committee.

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