Examiner’s report on PhD thesis:  
Analysis of Stability of Steady and Time Periodic Flow in a Pipe 
submitted by Vit Prusa to Charles University, Prague.

The classic hydrodynamic stability problem of flow in a pipe of circular cross-section has been studied. The case of steady flow with no-slip boundary conditions is believed to be stable to all linear disturbances at all Reynolds numbers. In this thesis the effects of relaxing the no-slip condition are investigated through the use of Navier’s slip condition, and the effects of a time-periodic component to the basic flow are also studied. Unlike previous investigations, the present study has included non-axisymmetric disturbances. It is found that there are parameter regimes where a slip boundary condition, and a time-periodic base flow, can each produce modest destabilizing effects. However, the effects are not strong enough to produce instability — instead they reduce the damping rates of some disturbances.

These conclusions were reached through numerical investigations of the linearised disturbance equations, after the parameter regime to be explored had been reduced through the derivation of rigorous bounds on the Reynolds numbers below which the monotonic decay of disturbances is guaranteed. The bounds turn out to be rather weak and do not substantially restrict the part of parameter space where instability could exist. However, in the case of the unsteady basic flow, they do indicate trends that help to focus the numerical search to relatively low frequencies (in the chosen nondimensional variables), and the numerical results confirm that higher frequencies for the basic flow tend to have only a small effect on flow stability. A further contribution of the work is that non-axisymmetric disturbances have been studied for the first time, and it is found that the $n = 1$ mode is less stable than the axisymmetric ($n = 0$) mode at lower Reynolds numbers, but the $n = 0$ mode becomes the least damped at higher Reynolds numbers. In addition, axisymmetric disturbances with non-zero swirl have been studied for the first time, and these are also shown to be less damped than the swirl-free modes considered by previous authors.

The question of whether the time-periodic flow becomes unstable at very large Reynolds numbers has been left open. A recent publication by Blennerhassett & Bassom (2006) claims to have found instability above a certain very high Reynolds number, but the numerical method used in the thesis was unable to definitely confirm or refute this finding, due to slowness of convergence. However, these Reynolds numbers are probably well in excess of those likely to be encountered in experiments. Nonetheless, although the results at the Reynolds numbers where Blennerhassett & Bassom found instability are inconclusive, the candidate hints that instability is unlikely in the time-periodic case.

The thesis is generally well written, the ideas and techniques are carefully explained, however there are quite a number of minor mistakes in the English (and occasionally in notation, e.g. on page 7 velocity switches between $v$ and $u$). Although the question of the existence of linearly unstable modes to time-periodic pipe flow has not been answered, the candidate has made a good number of useful contributions to this problem including bounds for monotonic decay of disturbances in time-periodic flow, study of swirling axisymmetric waves, non-axisymmetric waves and the effects of Navier’s slip boundary
conditions, all of which can make destabilizing contributions in certain parameter regimes. Therefore, I am satisfied that the thesis proves the author's ability for creative scientific work and I recommend that the thesis is suitable for defending.

During the examination, I would like to discuss the following issues:

1. Can a tighter bound for monotonic decay be found for the time periodic flow by using numerical estimates, rather like Joseph & Carmi (1969) did for steady pipe flow? Why is a bound obtained purely from analytic estimates 'more valuable' if it turns out to be weaker? Can't both be considered equally rigorous?

2. Navier's slip condition has a negligible effect on stability at large Reynolds numbers (e.g. see page 51) and the numerical results suggest that if there is instability it will be at large Reynolds numbers. Are there other types of slip boundary condition listed in Rao & Rajagopal (1999) whose effect does not vanish at large Reynolds numbers, and so might cause instability? E.g. for rarified gas?

3. If Navier's slip condition emerges as a model for infinitesimal roughness (page 5), then what determines the weighting parameter $\theta$? Does $\theta \to 1$ as roughness tends to zero in order to recover no-slip for smooth surfaces?

4. In a case where it is shown that over one period of the time-periodic basic-flow disturbances experience a weak net decay, there could, in principle, still be shorter time intervals where strong growth takes place (followed by slightly stronger decay). Such growth could cause amplitudes to become large enough to trigger a nonlinear bypass mechanism producing transition to turbulence. Experiments show transition to turbulence even when there is net decay over a full period, so it would be interesting to see the results of such a study. This is a mechanism that could probably be explored quite easily by making minor adjustments to the codes.

5. The theoretical bounds required expanding the solution using basis functions given by the Stokes operator and the numerical method has used the same basis functions. However, these are not necessarily optimal for numerical purposes. Chebychev polynomials often have good convergence properties in stability calculations at high Reynolds numbers because they are good at resolving thin layers close to the walls, even though they have no physical relation to the flow (unlike those from the Stokes operator), and don't have all the advantages listed on page 54. Nonetheless, given that the current method has convergence difficulties at large Reynolds numbers, it may be worth trying a Chebychev method in these cases to try to confirm or refute Blennerhassett & Bassom's claim of instability at large Reynolds numbers.

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